



Simplified approach to generate controlled-NOT gates with single trapped ions for arbitrary Lamb–Dicke parameters

Miao Zhang, H.Y. Jia, L.F. Wei *

Laboratory of Quantum Opt-electronic Information, Southwest Jiaotong University, Chengdu 610031, China

ARTICLE INFO

Article history:

Received 30 November 2008

Received in revised form 16 January 2009

Accepted 16 January 2009

PACS:

03.67.Lx

42.60.By

37.10.Ty

Keywords:

Trapped ions

Controlled-NOT gates

Lamb–Dicke parameters

ABSTRACT

For certain *specific* (or “*magic*”) Lamb–Dicke (LD) parameters, [Phys. Rev. A 55 (1997) R2489] showed that a two-qubit quantum operation, between the external and internal degrees of freedom of a single trapped ion, could be implemented by applying a single carrier laser pulse. Here, we further show that, such a two-qubit operation (which is equivalent to the standard CNOT gate, only apart from certain phase factors) could also be significantly well realized for *arbitrarily* selected LD parameters. Instead of the so-called “ π -pulses” used in the previous demonstrations, the durations of the pulses applied in the present proposal are required to be accurately set within the decoherence times of the ion. We also propose a simple approach by using only one off-resonant (e.g., blue-sideband) laser pulse to eliminate the unwanted phase factors existed in the above two-qubit operations for generating the standard CNOT gates.

© 2009 Elsevier B.V. All rights reserved.

It has been shown that a quantum computer can be built using a series of one-qubit operations and two-qubit controlled-NOT gates, because any computation can be decomposed into a sequence of these basic logic operations [1]. Therefore, precondition work is to effectively implement these fundamental logic gates [2]. Since the first idea, proposed by Cirac and Zoller [3] in 1995 for implementing quantum computation with trapped cold ions, much attention has been paid to implement the fundamental quantum logic gates in the systems of trapped cold ions [4–7]. Actually, a single CNOT logic operation between the external and internal states of a single trapped had been experimentally demonstrated with $^9\text{Be}^+$ ion in 1995 [4]. Later, the CNOT gate between two individual trapped ions (i.e., $^{40}\text{Ca}^+$ ions) had also been experimentally implemented in 2003 [6]. Recently, quantum manipulations on eight trapped ions had already been realized [7].

However, most of these demonstrated experiments are operated within the usual LD limit (wherein the spatial extent of the complete ion motion is required to be much smaller than the effective wavelength of the applied laser wave), i.e., the so-called LD parameters should be sufficiently small (see, e.g., [4]). In principle, quantum motion of a single trapped ion beyond the above LD limit is also possible [8]. Furthermore, it has been shown that utilizing

the laser–ion interaction outside the LD regime might be helpful to reduce the noise in the trap and improve the cooling rate of the ion [9]. Indeed, several approaches have been proposed to coherently operate trapped ions beyond the LD limit for implementing the desirable quantum logic gates [10,11].

More interestingly, beyond the LD limit Ref. [12] had shown that two-qubit quantum gates could be implemented by using only one laser pulse. This is significantly different from the previous scheme demonstrated within LD regime, wherein three steps laser pulses are usually required. Note that the approach in Ref. [12] only work well for certain *specific* LD parameters, although accurately setting the desirable LD parameter is possible in experiments [13]. In this Brief Report, given the LD parameter is arbitrarily set we show that the two-qubit operations proposed in Ref. [12] could still be implemented sufficiently well. Furthermore, by adding only one off-resonant (e.g., blue-sideband) pulse, we propose a simple way to eliminate the unwanted phase factors existed in the above two-qubit operation for generating the standard CNOT gates. This means that, for arbitrary LD parameters the exact single-ion CNOT gate could be sufficiently well implemented by using two laser pulses. Besides the requirement of an auxiliary atomic level, the present proposal for implementing the standard CNOT gate is really simpler than many previous ones, including that proposed recently in Ref. [14]. Here, the so-called standard CNOT gate between the external and internal degrees of freedom of the ion reads [4]

* Corresponding author. Tel.: +86 28 87600789.

E-mail address: lfwei@swjtu.cn (L.F. Wei).

$$\hat{C}_N = |0\rangle\langle g| \langle 0| \langle g| + |0\rangle\langle e| \langle 0| \langle e| + |1\rangle\langle g| \langle 1| \langle e| + |1\rangle\langle e| \langle 1| \langle g| \quad (1)$$

with $|g\rangle$ and $|e\rangle$ being two selected internal atomic levels and $|0\rangle$ and $|1\rangle$ the two lowest motional Fock states of the ion's external vibration.

We consider that a single ion is trapped in a coaxial resonator RF (radio frequency)-ion trap [15,16], and assume that only the quantized vibrational motion along the principal x axis is important for the cooled ion. Following Monroe et al. [12] the ion is driven by a classical traveling-wave laser field (with frequency ω_L and initial phase θ_L). In the rotating framework (rotating with the angular frequency ω_L), the system can be described by the following Hamiltonian [12,17]

$$\hat{H} = \hbar\nu\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \frac{\hbar\delta}{2}\hat{\sigma}_z + \frac{\hbar\Omega}{2}\{\hat{\sigma}_+ \exp[i\eta(\hat{a} + \hat{a}^\dagger) - i\theta_L] + H.c.\}. \quad (2)$$

Here, \hat{a}^\dagger and \hat{a} are the bosonic creation and annihilation operators of the external vibrational quanta (with frequency ν) of the ion. The Pauli operators $\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|$ and $\hat{\sigma}_+ = |e\rangle\langle g|$ are defined by the internal ground state $|g\rangle$ and excited state $|e\rangle$ of the ion, respectively. The Rabi frequency Ω describes the coupling strength between the ion and the applied laser beam. Also, $\delta = \omega_0 - \omega_L$ with ω_0 being the eigenfrequency of the target qubit generated by the two selected atomic levels $|g\rangle$ and $|e\rangle$. Finally, η is the LD parameter describing the coupling strength between the atomic levels and external vibrational quanta of the ion.

Suppose that the ion is driven by applying the laser to the k th blue-sideband vibration, i.e., the frequency of the applied laser beam is chosen as $\omega_L = \omega_0 + k\nu$ with k being a positive integer. Without performing the LD approximation and under the usual rotating-wave approximation, we have the following simplified Hamiltonian [8,10]

$$\hat{H}_I = \frac{\hbar\Omega}{2}e^{-\eta^2/2}\left[(i\eta)^k e^{-i\theta_L}\hat{\sigma}_+ \sum_{j=0}^{\infty} \frac{(i\eta)^{2j}}{j!(j+k)!} (\hat{a}^\dagger)^{j+k}\hat{a}^j + H.c.\right] \quad (3)$$

in the interaction picture defined by the unitary operator $\hat{U}_0 = \exp\{-it[\nu(\hat{a}^\dagger\hat{a} + 1/2) + \delta\hat{\sigma}_z/2]\}$. The dynamics defined by this Hamiltonian is exactly solvable [10], and its corresponding dynamical evolutions are:

$$\begin{cases} |m\rangle|g\rangle \rightarrow \cos(\Omega_{m,k}t)|m\rangle|g\rangle + i^{k-1}e^{-i\theta_L}\sin(\Omega_{m,k}t)|m+k\rangle|e\rangle, \\ |m\rangle|e\rangle \rightarrow |m\rangle|e\rangle, \quad m < k, \\ |m\rangle|e\rangle \rightarrow \cos(\Omega_{m-k,k}t)|m\rangle|e\rangle - (-i)^{k-1}e^{i\theta_L}\sin(\Omega_{m-k,k}t)|m-k\rangle|g\rangle, \quad m \geq k \end{cases} \quad (4)$$

with $|m\rangle$ being the number state of the external vibration of the ion, and

$$\Omega_{m,k} = \frac{\Omega\eta^k}{2}\sqrt{\frac{(m+k)!}{m!}}e^{-\eta^2/2}\sum_{j=0}^m \frac{(i\eta)^{2j}m!}{j!(m-j)!(j+k)!} \quad (5)$$

being the effective Rabi frequencies. If $k = 0$, i.e., a resonant laser pulse (the “carrier” pulse) of frequency $\omega_L = \omega_0$ is applied to drive the trapped ion, then the above exact dynamical evolutions take the time evolution operator

$$\hat{C}(t_1, \theta_1) = \begin{pmatrix} C_{11} & C_{12}e^{-i(\theta_1+\frac{\pi}{2})} & 0 & 0 \\ C_{21}e^{i(\theta_1-\frac{\pi}{2})} & C_{22} & 0 & 0 \\ 0 & 0 & C_{33} & C_{34}e^{-i(\theta_1+\frac{\pi}{2})} \\ 0 & 0 & C_{43}e^{i(\theta_1-\frac{\pi}{2})} & C_{44} \end{pmatrix} \quad (6)$$

with

$$\begin{aligned} C_{11} &= \cos(\Omega_{0,0}t_1), & C_{12} &= \sin(\Omega_{0,0}t_1), \\ C_{21} &= C_{12}, & C_{22} &= C_{11}, \\ C_{33} &= \cos(\Omega_{1,0}t_1), & C_{34} &= \sin(\Omega_{1,0}t_1), \\ C_{43} &= C_{34}, & C_{44} &= C_{33} \end{aligned} \quad (7)$$

in the subspace $\Gamma = \{|0\rangle|g\rangle, |0\rangle|e\rangle, |1\rangle|g\rangle, |1\rangle|e\rangle\}$. Above, θ_1 and t_1 are the initial phase and duration of the applied carrier laser pulse, respectively. They should be set up properly for realizing the expected quantum logic operation within this subspace.

Obviously, if

$$C_{11} = C_{34} = 1, \quad (8)$$

then a two-qubit quantum operation [12]

$$\hat{C}_1(\theta_1) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & e^{-i(\theta_1+\frac{\pi}{2})} \\ 0 & 0 & e^{i(\theta_1-\frac{\pi}{2})} & 0 \end{pmatrix} \quad (9)$$

could be implemented. This operation is equivalent to the standard CNOT gate (1) between the external and internal states of the ion, apart from the phase factors $\exp[-i(\theta_1 + \pi/2)]$ and $\exp[i(\theta_1 - \pi/2)]$.

The above condition (8) could be satisfied by properly setting the relevant experimental parameters: t_1 and η , as

$$t_1 = \frac{2n\pi}{\Omega_{0,0}}, \quad \eta^2 = 1 - \frac{m-\frac{3}{4}}{n}, \quad n, m = 1, 2, 3 \dots \quad (10)$$

with n and m being arbitrary positive integers. Note that in the scheme of Ref. [12], a slightly different condition (from Eq. (8)): $|C_{11}| = |C_{34}| = 1$ is required. Under such a condition the uncertain phase factors depend not only on the initial phase θ_1 but also on the LD parameters. This may complicate the progress to eliminate the unwanted phase factors for practically realizing the standard CNOT gate. So, we consider the relatively simple condition of Eq. (8) in this paper.

Theoretically, condition (10) is always satisfied for arbitrary-selected LD parameters by properly selecting the values of the integers $n, m = 1, 2, 3, \dots$. As a consequence, the two-qubit operation (9) could be, in principle, implemented for arbitrary LD parameters by properly setting the durations of the applied carrier laser pulse. However, because the practical existence of decoherence, as we discussed in [14], the duration of the present pulse should be shorter than the decoherence times of both the atomic and motional states of the ion [18,19]. This means that the integers n could not take arbitrary large values to let Eq. (10) be exactly satisfied. Experimentally, the lifetime of the atomic excited states $|e\rangle$ reaches 1 s [15,16] and the coherence superposition of $|0\rangle$ and $|1\rangle$ can be maintained up to 1 ms [19]. Based on these data we can always find, via numerical method, sufficiently well approximated solutions to Eq. (8) for implementing the quantum operation (9) with sufficiently high fidelities.

In Table 1 we present some numerical results, for setting proper experimental parameters Ωt_1 , to implement quantum operation (9) robustly for the arbitrarily selected LD parameters (not limited within the LD regime requiring $\eta \ll 1$) from 0.18 to 0.98. It is seen that, the probability amplitudes $C_{11} = C_{22}$ and $C_{34} = C_{43}$ are desirably large, most of them could reach to 0.99. While, unwanted probability amplitudes $C_{12} = C_{21}$ and $C_{33} = C_{44}$ are really significantly small; all of them are less than 0.32. This implies that the lowest fidelity for implementing the quantum operation (9) is larger than 90%. Obviously, the durations t_1 presented in the table can be less than the decoherence time 1 ms by properly setting the carrier Rabi frequency Ω . For example, it is about 0.018–0.37 ms for $\Omega/2\pi = 150$ kHz. However, for the typical experimental parame-

Table 1

Numerical results for implementing quantum operation (9) for arbitrary selected LD parameters; from 0.18 to 0.98 (step by 0.02). The duration of the applied carrier pulse is given by Ωt_1 .

η	$t_1 \Omega$	$C_{11} = C_{22}$	$C_{12} = C_{21}$	$C_{33} = C_{44}$	$C_{34} = C_{43}$
0.18	267.75	0.97520	-0.22135	-0.21954	0.97560
0.20	243.65	0.99948	0.03218	0.03193	0.99949
0.22	179.76	0.97284	-0.23146	-0.23053	0.97306
0.24	168.13	1.00000	0.00000	0.00000	1.00000
0.26	129.49	0.97165	-0.23640	-0.24006	0.97076
0.28	130.92	0.99376	0.11153	0.11041	0.99389
0.30	104.95	0.99505	-0.09935	-0.09778	0.99521
0.32	92.35	0.99374	-0.11172	-0.10790	0.99416
0.34	79.49	0.98271	-0.18517	-0.18852	0.98207
0.36	80.64	0.99586	0.09088	0.09407	0.99557
0.38	67.33	0.99541	-0.09572	-0.09377	0.99559
0.40	68.44	0.98505	0.17225	0.16794	0.98580
0.42	54.57	0.98859	-0.15063	-0.15383	0.98810
0.44	55.56	0.99646	0.08411	0.08530	0.99636
0.46	111.30	0.97957	-0.20111	-0.19843	0.98012
0.48	41.83	0.97796	-0.20881	-0.20607	0.97854
0.50	42.72	1.00000	0.00000	0.00000	1.00000
0.52	43.67	0.97497	0.22234	0.21915	0.97570
0.54	87.23	1.00000	0.00000	0.00000	1.00000
0.56	132.93	0.96361	0.26731	0.26592	0.96400
0.58	118.42	0.97544	-0.22027	-0.22025	0.97544
0.60	29.81	0.99320	-0.11640	-0.11358	0.99353
0.62	30.64	0.99720	0.07473	0.07201	0.99740
0.64	31.52	0.96241	0.27158	0.26906	0.96312
0.66	62.42	0.99952	-0.03096	-0.03027	0.99954
0.68	95.26	0.99509	0.09898	0.09985	0.99500
0.70	353.53	0.99224	0.12437	0.12458	0.99221
0.72	178.61	0.97983	-0.19981	-0.20082	0.97963
0.74	82.49	0.99876	-0.04974	-0.05286	0.99860
0.76	50.12	0.99715	-0.07548	-0.07608	0.99710
0.78	186.67	0.96719	-0.25406	-0.25835	0.96605
0.80	155.88	0.99888	0.04737	0.04576	0.99895
0.82	175.54	0.99258	-0.12162	-0.12311	0.99239
0.84	17.27	0.97693	-0.21356	-0.21395	0.97685
0.86	18.04	0.99867	-0.05149	-0.05191	0.99865
0.88	18.88	0.99205	0.12582	0.12453	0.99221
0.90	19.79	0.95031	0.31129	0.31157	0.95022
0.92	153.85	0.99324	0.11610	0.11507	0.99336
0.94	39.40	0.99517	0.09817	0.09647	0.99534
0.96	60.04	0.99627	0.08632	0.08611	0.99629
0.98	122.12	0.99709	0.07617	0.07482	0.99720

ter: $\Omega/2\pi = 500$ kHz [20] these durations could be further shortened to $t_1 = 5.5 \times 10^{-3}$ –0.11 ms.

Certainly, the above approximated solutions could be further improved by either relaxing the limit from the decoherence time or increasing Rabi frequency Ω (via increasing the powers of the applied laser beams) to shorten the operational time. Thus, the two-qubit gate (9) could always be realized for arbitrarily LD parameters. In principle, designing the applied laser pulse with so short duration is not a great difficulty for the current experimental technology, e.g., the femto-second (10^{-15} s) laser technique. Also, our numerical calculations show that the influence of the practically-existing fluctuations of the applied durations is really weak. For example, for the Rabi frequency $\Omega/2\pi \approx 500$ kHz, the fluctuation $\delta t \approx 0.1 \mu\text{s}$ of the duration lowers the desirable probability amplitudes, i.e., C_{11} and C_{34} presented in Table 1, just about 5%. Thus, even consider the imprecision of the durations, the amplitude of the desirable elements, C_{11} and C_{34} , are still sufficiently large, e.g., up to about 0.95. Therefore, the approach proposed here to implement the desirable quantum operation (9) for arbitrary LD parameters should be experimentally feasible.

Finally, we consider how to generate the standard CNOT gate (1) with a single trapped ion from the quantum operation (9) produced above. This could be achieved by just eliminating the unwanted phase factors in (9) via introducing another off-resonant

laser pulse. Indeed, a first blue-sideband pulse (of frequency $\omega_L = \omega_{ea} + \nu$ and initial phase θ_2) induces the following evolution:

$$|1\rangle|e\rangle \rightarrow \cos(\Omega_{0,1}t_2)|1\rangle|e\rangle - e^{i\theta_2} \sin(\Omega_{0,1}t_2)|0\rangle|a\rangle, \quad (11)$$

but does not evolve the states $|0\rangle|g\rangle$, $|1\rangle|g\rangle$ and $|0\rangle|e\rangle$. Above, $|a\rangle$ is an auxiliary atomic level [4], and ω_{ea} being the transition frequency between it and the excited state $|e\rangle$. Obviously, a “ π -pulse” defined by $\Omega_{0,1}t_2 = \pi$ generates a so-called controlled-Z logic operation

$$\hat{C}_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (12)$$

For the LD parameters from 0.18 to 0.98, and $\Omega/2\pi \approx 500$ kHz, the durations for this implementation are numerically estimated as 3.3×10^{-3} – 1.2×10^{-2} ms. Therefore, the standard CNOT gate (1) with a single trapped ion could be really implemented by only two sequential operations demonstrated above, i.e., $\hat{C}_N = \hat{C}_1(\pi/2)\hat{C}_2$.

In summary, we have rechecked the scheme of Ref. [12] for implementing a two-qubit quantum operation with a single trapped ion by using only a single carrier “ π -pulse” laser beam. We found that, if the limit of definite decoherence time is not considered, then such an approach works really for arbitrarily selected LD parameters, not limits to the so-called “magic” values. Our numerical results indicated that, if the durations of the applied carrier pulses are properly set (rather than that in the so-called “ π -pulse”), then the above two-qubit quantum operation could still be implemented within the definite decoherence time for arbitrarily selected LD parameters. Also, we have discussed the influence from the possible fluctuations of the durations on the implementations of the quantum operation, and shown that such a influence is really weak. In addition, by using a single blue-sideband laser pulse we have shown that the unwanted phase factors induced by the above carrier driving could be eliminated. Therefore, a standard CNOT gate with a single trapped ion could be practically implemented by using only two laser pulses; one carrier pulse pluses one off-resonant one. Finally, we hope that the numerical results, presented in Table 1, might be useful for the future experiment.

Acknowledgement

This work is partly supported by the NSFC Grant No. 10874142.

References

- [1] T. Sleator, H. Weinfurter, Phys. Rev. Lett. 74 (1995) 4087; D.P. DiVincenzo, Phys. Rev. A 51 (1995) 1015.
- [2] A. Barenco, D. Deutsch, A. Ekert, R. Jozsa, Phys. Rev. Lett. 74 (1995) 4083; D. Leibfried, B. DeMarco, V. Meyer, D. Lucas, M. Barrett, J. Britton, W.M. Itano, B. Jelenković, C. Langer, T. Rosenband, D.J. Wineland, Nature 422 (2003) 412; K. Kim, C.F. Roos, L. Aolita, H. Häfner, V. Nebendahl, R. Blatt, Phys. Rev. A 77 (2008) 050303; L.F. Wei, J.R. Johansson, L.X. Cen, S. Ashhab, F. Nori, Phys. Rev. Lett. 100 (2008) 113601.
- [3] J.I. Cirac, P. Zoller, Phys. Rev. Lett. 74 (1995) 4091.
- [4] C. Monroe, D.M. Meekhof, B.E. King, W.M. Itano, D.J. Wineland, Phys. Rev. Lett. 75 (1995) 4714.
- [5] A. Steane, App. Phys. B 64 (1997); D.F.V. James, App. Phys. B 66 (1998); J. Benhelm, G. Kirchmair, C.F. Roos, R. Blatt, Phys. Rev. A 77 (2008) 062306; A. Sørensen, K. Mølmer, Phys. Rev. Lett. 82 (1999) 1971; D. Jonathan, M.B. Plenio, P.L. Knight, Phys. Rev. A 62 (2000) 042307.
- [6] F. Schmidt-Kaler, H. Häfner, M. Riebe, S. Gulde, G.P.T. Lancaster, T. Deuschle, C. Becher, C.F. Roos, J. Eschner, R. Blatt, Nature 422 (2003) 408.
- [7] D. Leibfried, E. Knill, S. Seidelin, J. Britton, R.B. Blakestad, J. Chiaverini, D.B. Hume, W.M. Itano, J.D. Jost, C. Langer, R. Ozeri, R. Reichle, D.J. Wineland, Nature 438 (2005) 639; H. Häfner, W. Hänsel, C.F. Roos, J. Benhelm, D. Chek-al-kar, M. Chwalla, T. Körber, U.D. Rapol, M. Riebe, P.O. Schmidt, C. Becher, O. Gühne, W. Dür, R. Blatt, Nature 438 (2005) 643.

- [8] L.F. Wei, Y.X. Liu, F. Nori, Phys. Rev. A 70 (2004) 063801;
M. Zhang, H.Y. Jia, Acta Phys. Sin. 57 (2008) 0880;
G. Morigi, J.I. Cirac, M. Lewenstein, P. Zoller, Europhys. Lett. 39 (1997) 13.
- [9] D. Stevens, J. Brochard, A.M. Steane, Phys. Rev. A 58 (1998) 2750;
A. Steane, Appl. Phys. B 64 (1997) 623.
- [10] L.F. Wei, S.Y. Liu, X.L. Lei, Phys. Rev. A 65 (2002) 062316;
L.F. Wei, S.Y. Liu, X.L. Lei, Chin. Phys. Lett. 18 (2001) 1533;
L.F. Wei, X.L. Lei, J. Opt. B 2 (2000) 581.
- [11] E. Solano, R.L. de Matos Filho, N. Zagury, Phys. Rev. A 59 (1999) R2539;
E. Solano, P. Milman, R.L. de Matos Filho, N. Zagury, Phys. Rev. A 62 (2000) 021401;
E. Solano, M. França Santos, P. Milman, Phys. Rev. A 64 (2001) 024304.
- [12] C. Monroe, D. Leibfried, B.E. King, D.M. Meekhof, W.M. Itano, D.J. Wineland, Phys. Rev. A 55 (1997) R2489.
- [13] B. Demarco, A. Ben-Kish, D. Leibfried, V. Meyer, M. Rowe, B.M. Jelenković, W.M. Itano, J. Britton, C. Langer, T. Rosenband, D.J. Wineland, Phys. Rev. Lett. 89 (2002) 267901.
- [14] L.F. Wei, M. Zhang, H.Y. Jia, Y. Zhao, Phys. Rev. A 78 (2008) 014306.
- [15] P.A. Barton, C.J.S. Donald, D.M. Lucas, D.A. Stevens, A.M. Steane, D.N. Stacey, Phys. Rev. A 62 (2000) 032503.
- [16] H.C. Nägerl, Ch. Roos, D. Leibfried, H. Rohde, G. Thalhammer, J. Eschner, F. Schmidt-Kaler, R. Blatt, Phys. Rev. A 61 (2000) 023405.
- [17] W. Vogel, R.L. de Matos Filho, Phys. Rev. A 52 (1995) 4214;
C.A. Blockley, D.F. Walls, H. Kisten, Europhys. Lett. 17 (1992) 509.
- [18] D. Leibfried, R. Blatt, C. Monroe, D. Wineland, Rev. Mod. Phys. 75 (2003) 281.
- [19] Ch. Roos, Th. Zeiger, H. Rohde, H.C. Nägerl, J. Eschner, D. Leibfried, F. Schmidt-Kaler, R. Blatt, Phys. Rev. Lett. 83 (1999) 4713.
- [20] D.M. Meekhof, C. Monroe, B.E. King, W.M. Itano, D.J. Wineland, Phys. Rev. Lett. 76 (1996) 1796.