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Multi-objective design optimisation of standalone hybrid wind-PV-diesel systems under uncertainties

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ABSTRACT

Optimal design of a standalone wind-PV-diesel hybrid system is a multi-objective optimisation problem with conflicting objectives of cost and reliability. Uncertainties in renewable resources, demand load and power modelling make deterministic methods of multi-objective optimisation fall short in optimal design of standalone hybrid renewable energy systems (HRES). Firstly, deterministic methods of analysis, even in the absence of uncertainties in cost modelling, do not predict the levelised cost of energy accurately. Secondly, since these methods ignore the random variations in parameters, they cannot be used to quantify the second objective, reliability of the system in supplying power. It is shown that for a given site and uncertainties profile, there exist an optimum margin of safety, applicable to the peak load, which can be used to size the diesel generator towards designing a cost-effective and reliable system. However, this optimum value is problem dependent and cannot be obtained deterministically. For two design scenarios, namely, finding the most reliable system subject to a constraint on the cost and finding the most cost-effective system subject to constraints on reliability measures, two algorithms are proposed to find the optimum margin of safety. The robustness of the proposed design methodology is shown through carrying out two design case studies.

1. Introduction

In optimal design of standalone hybrid renewable energy systems (HRES), reliability of the system in supplying power for a demand load is as important as the levelised cost of energy (LCE) produced by the system. the system. Reliability of a standalone HRES in supplying power depends on various parameters, including, system configuration (e.g. wind-PV-battery, wind-diesel, etc), size of its components, reliability of each component in terms of operation and the availability of renewable resources. The availability of resources has the major influence on the reliability of a standalone HRES as stochastic nature of renewable resources imposes a great deal of uncertainty to the system operation and the power produced. Stochastic nature of renewable resource makes the reliability analysis of a standalone HRES impossible without employing probabilistic methods of analysis. In other words, multi-objective optimisation of standalone HRES (with cost and reliability as two objectives) cannot be performed deterministically.

Results of probabilistic analyses have random errors that can be reduced by increasing the size of sampling space. In order to achieve a desired level of accuracy in the results of probabilistic methods of analysis high computational time is required. This becomes a major concern within a design process, as evaluation of design candidates with respect to their cost and reliability becomes highly time-consuming. In practice, to circumvent this problem, adopting a deterministic approach, design of standalone HRES is carried out for a worst-case-scenario, while applying a load factor on the demand load. All calculations are based on the averaged values and the stochastic nature of demand load and renewable resources as well as the possible errors in the results due to employing low fidelity models are ignored. No reliability measure is calculated as part of the design candidate assessment. It is assumed that a suitable selection of the worst-case-scenario and safety factors will lead to reliable solutions. In fact, the multi-objective optimisation problem with two objectives of reliability and cost is reduced to a single-objective optimisation problem with the objective of cost only. In practice, normally, the size of the storage or backup/auxiliary components are determined based on a suitable worst-case-scenario to achieve a level of confidence in the expected power supply, while the remaining components are optimised for minimising the cost. After sizing the storage or

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backup/auxiliary components a single-objective optimisation search can be carried out to find the optimum size of the renewable components. Most of the literature on design of standalone HRES adopt this approach; for instance see Refs. [1–10].

In deterministic optimal sizing of a standalone wind-PV-diesel hybrid system, the margin of safety applied on the demand load affects the nominal size of the diesel generator and consequently the reliability of the power supply and the levelised cost of produced energy. Adopting high-enough margins of safety leads to reliable systems. However, as mentioned above since in deterministic design methods no actual reliability measure is calculated as part of the design candidate assessment, these methods cannot be used for quantifying the optimum value for margin of safety. A procedure including both deterministic and probabilistic analyses is required to find the margin of safety which corresponds to a desired reliability with minimal cost.

More recently, recognising the shortfall of deterministic methods in design of reliable and cost-effective standalone HRES, development of robust nondeterministic design methods has received increasing attention from the research community [11,12]. The aim of the present study is to develop a robust method of design under uncertainties for wind-PV-diesel configuration with minimal number of probabilistic analysis. Section 2 begins with definition of reliability measures used in this study, and then elaborates on power and cost modelling. Section 3 explains the fundamentals of the proposed design methodology and its development steps. Section 4 details two algorithms proposed for performing two design scenarios and the results of case studies delivered using the proposed design methodology.

2. Reliability assessment and system modelling

2.1. Reliability assessment measures

Performance of a standalone HRES in supplying power can be evaluated against different assessment criteria, amongst them total unmet load, blackout duration distribution and the mean-time between failures. For a standalone HRES the total unmet load is defined as:

$$U_t = \int_0^T (L(t) - P_a(t))dt \tag{1}$$

where, P_a and L are, respectively, the usable available power and the demand load ($0 \le P_a \le L$). Usable available power is defined as:

$$P_a = \min\{P_{t,a}, L\} \tag{2}$$

in which, $P_{t,a}$ stands for the total renewable and non-renewable available power. Using hourly-averaged load ($\overline{L}_h$) and hourly-averaged useable available power ($\overline{P}_{h,a}$), and a period of analysis of $T = 1\ year = 8760\ h$, Equation (1) can be rewritten as

$$U = \sum_{i=1}^{8760} \left(\overline{L}_h - \overline{P}_{h,a}\right)_i \tag{3}$$

Total, maximum and average blackout durations are three parameters which indicate the system downtime periods due to power deficiency irrespective of the amount of power deficiency. In contrast to the unmet load, assessment of design candidates based on blackout duration allows performing customer-need driven designs. Using hourly-averaged data, total blackout duration is defined as:

$$BO_t = \sum_{i=1}^{8760} \left[\left(1 - \overline{P}_{h,a}/\overline{L}_h\right)_i\right] \tag{4}$$

where, pair of square brackets [ ] stands for the integer value function. The information that can be extracted from the blackout distribution, such as the maximum blackout duration (the longest continuous blackout) BO_{max} and the average blackout duration BO_{av} (the average duration of each blackout), also can play an important role in evaluation of the system performance.

Mean time between failures (MTBF) is defined as the duration of the successful system operation over a period of time divided by the number of failures during that period. If the successful system operation is defined as the case when available usable power is greater than or equal to the load ($P_a \ge L$), using hourly-averaged quantities, the MTBF can be defined as:

$$\text{MTBF} = \frac{8760 - \sum_{i=1}^{8760}\left[\left(1 - \overline{P}_{h,a}/\overline{L}_h\right)_i\right]}{n_{fail}} \tag{5}$$

where n_{fail} is the number of blackout occurrences during period $T = 8760\ h$.

2.2. Power modelling and dispatch strategies

The power produced by a wind turbine is given by:

$$P_{WT} = \frac{1}{2}\rho V_{hub}^3 A_{WT} C_P \eta_{EG} \tag{6}$$

in which ρ is the air density, V_{hub} is the wind speed at hub elevation, A_{WT} is the rotor area, η_{EG} is the overall efficiency of the electrical components and the gearbox, and C_P is the rotor power coefficient given by:

$$\begin{aligned} C_P = & -2.025 \times 10^{-7} V_{hub}^6 + 1.926 \times 10^{-5} V_{hub}^5 \\ & -7.421 \times 10^{-4} V_{hub}^4 + 1.483 \times 10^{-2} V_{hub}^3 - 0.162 V_{hub}^2 \\ & +0.887 V_{hub} - 1.508 \end{aligned} \tag{7}$$

This model is extracted via curve fitting and using the power coefficient data of about 60 wind turbines within the range of 10–500 kW. The wind turbines used for developing this model are of both types of constant and variable speeds and also both types of pitch controlled and stall regulated. This model has a maximum relative error of 7% for the range of $3 \le V_{hub} \le 25\ m/s$.

Given wind speed V_{ref} at elevation h_{ref}, the wind speed at the hub elevation can be calculated by the logarithmic law:

$$V_{hub} = V_{ref} \ln\left(\frac{h_{hub}}{z_0}\right) \Big/ \ln\left(\frac{h_{ref}}{z_0}\right) \tag{8}$$

in which, z_0 stands for the site surface roughness length. The hub height h_{hub} depends on the size of the wind turbine, which is unknown prior to the design. For small to medium size wind turbines the hub height can be estimated via the rule of thumb:

$$h_{hub} = \max\{h_c + R,\ 2R\} \tag{9}$$

where h_c is the minimum blade tip-ground clearance and R is the rotor radius.

Power produced by PV panels is given by

$$P_{PV} = I A_{PV} \eta_{PV} \tag{10}$$

in which, I stands for the solar irradiance, A_{PV} is the PV panel area and η_{PV} is the overall PV unit efficiency.

Table 1
Cost modelling parameters.

	Wind turbine	PV panel	Diesel generator
S	Rotor area A_{WT} (m^2)	Panel area A_{PV} (m^2)	Nominal power $P_{D,nom}$ (W)
C_u	$480\$/m^2$	$830\$/m^2$	$0.4\$/W_{nom}$
α_{ins}	0.2	0.4	0
$\alpha_{O\&M}$	0.03	0.01	0.15
N_{nom}	20 years	20 years	15,000 h
$C_{O\&M,V}$	0	0	See Equation (19) $C_{fuel} = 1\$/l$

In this study, using hourly-averaged data, the following diesel dispatch strategy is used:

- Excess power $\overline{P}_{h,R} - \overline{L}_h \geq 0$: No need for diesel generator power $\overline{P}_{h,D} = 0$.
- Power deficit less than the nominal power of the diesel generator $0 \leq \overline{L}_h - \overline{P}_{h,R} \leq P_{D,nom}$: The power deficit is compensated by the diesel generator $\overline{P}_{h,D} = \overline{L}_h - \overline{P}_{h,R}$.
- Power deficit greater than the nominal power of the diesel generator $\overline{L}_h - \overline{P}_{h,R} > P_{D,nom}$: Blackout; The diesel generator works at its nominal power $\overline{P}_{h,D} = P_{D,nom}$.

Parameters $\overline{P}_{h,D}$ and $\overline{P}_{h,R}$, respectively, stand for the hourly-averaged diesel and renewable power and $P_{D,nom}$ stands for the diesel generator nominal power.

2.3. Cost modelling

Using levelised cost of energy allows design alternatives to be compared when different scales of operation and investment exist. For systems with constant annual output over the life-span of the system LCE, C_l, can be calculated as follows:

$$C_l = \frac{C_a}{P_t} \tag{11}$$

where P_t denotes the annual energy output and C_a stands for the annualised cost. Since the power produced by a standalone HRES excess to the demand load is dumped, in Equation (11), the usable amount of produced energy should be used instead of the system total energy output:

$$P_t = \sum_{j=1}^{8760} \min\{\overline{P}_h, \overline{L}_h\}_j \tag{12}$$

The annualised cost C_a is given by Ref. [13]:

$$C_a = C_t UCRF \tag{13}$$

parameters C_t and $UCRF$ in Equation (13) are, respectively, total life-span cost (TLSC) and uniform capital recovery factor, given by:

$$\mathrm{UCRF} = \frac{d(1+d)^{N_S}}{(1+d)^{N_S} - 1} \tag{14}$$

in which, d is the annual discount rate and N_S represents the life-span of the system in years. Assuming there is no escalation in the price of the components, the formula for calculating the present value of TLSC is as follows:

$$C_t = \sum_{j=0}^{N_S} \frac{C_j}{(1+d)^j} \tag{15}$$

where C_j is the cost in year j including capital cost C_c, fixed operation and maintenance (O&M) costs $C_{O\&M,F}$, variable O&M costs $C_{O\&M,V}$, and the replacement cost C_r. Case $j = 0$ represents the

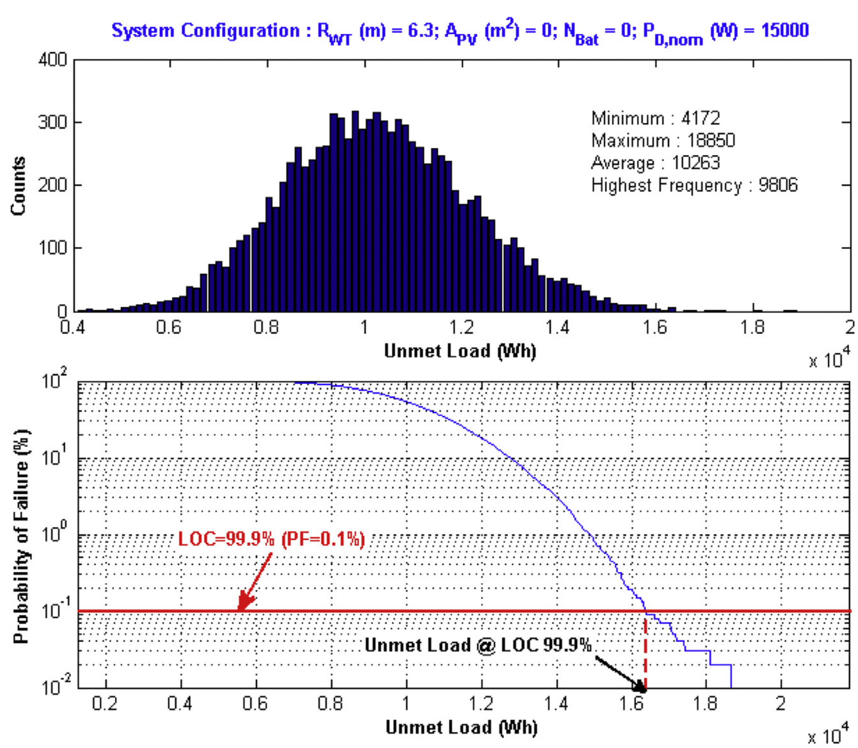

Fig. 1. Illustrative example of finding reliability measures at a given level of confidence.

Table 2
Uncertainties in resources, demand load and modelling.

Parameter/model	Distribution	
	Case U1	Case U2
Wind speed	Uniform (δ=±0.15)	Uniform (δ=±0.30)
Solar irradiance	Uniform (δ = ±0.05)	Uniform (δ = ±0.10)
Demand load	Uniform (δ = ±0.10)	Uniform (δ = ±0.20)
C_P model	Uniform (δ = ±0.07)	Uniform (δ = ±0.07)
PV array efficiency	Uniform (δ = ±0.05)	Uniform (δ = ±0.05)

beginning of the life span with its corresponding cost, C_0, standing for the capital cost only. The capital cost of the system (including installation cost) is given by:

$$C_c = \sum_{\text{comp}} C_{u,\text{comp}} S_{\text{comp}} \left(1 + \alpha_{\text{ins,comp}}\right) \tag{16}$$

in which S is the size of the component, C_u is the unit cost and α_{ins} is the installation cost as a fraction of the total cost of the component. Cost estimation at conceptual design phase of HRES can be based on either cost per unit of nominal power production or cost per unit of size. To be consistent with the power models, for wind turbine and PV array the cost per unit size is used whilst for the diesel generator the cost per nominal output power is used. The O&M cost includes fixed and variable parts:

$$C_{\text{O\&M}} = \sum_{\text{comp}} C_{\text{O\&M,F,comp}} + \sum_{\text{comp}} C_{\text{O\&M,V,comp}} \tag{17}$$

The fixed part can be represented by

$$C_{\text{O\&M,F,comp}} = \alpha_{\text{O\&M,comp}} C_{\text{c,comp}} \tag{18}$$

The variable part of the O&M cost for wind turbine and PV panel is zero. Using hourly-averaged data, the annual variable part of the O&M cost for diesel generator (the cost of consumed fuel) is given by Ref. [14]

$$C_{\text{O\&M,V,D}} = \frac{0.246 \sum_{i=1}^{8760} \overline{P}_{\text{h,D}_i} + 0.08145 P_{\text{D,nom}} T_{\text{D}}}{1000} C_{\text{fuel}} \tag{19}$$

in which T_D stands for the total number of hours that the diesel generator operates, $\overline{P}_{\text{h,D}}$ is the hourly-averaged diesel power and C_{fuel} is the fuel price.

For each component the replacement cost is given by:

$$C_r = \sum_{\text{comp}} n_{\text{r,comp}} C_{\text{c,comp}} \tag{20}$$

where n_r is the number of replacements during the life-span of the system. Having the nominal life of system (N_S), wind turbine ($N_{\text{nom,WT}}$) and PV panel ($N_{\text{nom,PV}}$) in years and the nominal life of diesel generator $N_{\text{nom,D}}$ in hours of operation, the following equations can be used to find the number of replacements of these components.

Table 3
Resources and demand load.

	Site S1	Site S2
Wind speed, V_{ref}	Wind speed as in [15], ($h_{\text{ref}} = 3m$)	$^3/_4$ of the wind speed of [15], ($h_{\text{ref}} = 3$ m)
Solar irradiance, I	Solar irradiance as in [15]	Solar irradiance as in [15]
Demand load, L	Three times of the demand load of [15]	Three times of the demand load of [15]

Table 4
Results of deterministic designs for different MoS and reliability analysis for site S1.

Deterministic								Monte Carlo simulation @99.99% LOC											
								Uncertainties U1						Uncertainties U2					
Design case	MoS	WT rotor radius (m)	PV panel area (m^2)	Diesel nom. Power (kW)	Penetration (%)	TLSC ($)	LCE (cent/kWh)	Total BO (h)	Maximum BO (h)	Average BO (h)	U_t (kWh)	MTBF (h)	LCE (cent/kWh)	Total BO (h)	Maximum BO (h)	Average BO (h)	U_t (kWh)	MTBF (h)	LCE (cent/kWh)
D1	0.00	6.3	0	15	126	239,200	41.3	32	1	1	5.0	274	42.9	51	1	1	18.7	171	44.9
D2	0.05	7.0	0	15.75	160	243,850	42.1	8	1	1	0.5	1096	43.6	34	1	1	8.7	259	46.2
D3	0.10	7.0	0	16.5	160	247,800	42.8	0	0	0	0	8760	44.3	20	1	1	3.6	446	47.1
D4	0.20	7.0	0	18	160	255,720	44.2	0	0	0	0	8760	45.8	0	0	0	0	8760	48.7
D5	0.50	7.0	0	22.5	160	279,450	48.3	0	0	0	0	8760	50.2	0	0	0	0	8760	53.6
D6	1.00	7.9	0	30	209	313,020	54.1	0	0	0	0	8760	56.7	0	0	0	0	8760	61.7
D7	2.00	8.1	0	45	221	373,980	64.6	0	0	0	0	8760	68.8	0	0	0	0	8760	75.7
D8	N/A	6.3	0	14.2	126	234,150	40.4	62	1	1	16.1	140	42.0	71	1	1	30.1	140	43.9

Table 5
Results of deterministic designs for different MoS and reliability analysis for site S2.

Deterministic								Monte Carlo simulation @99.99% LOC											
								Uncertainties U1						Uncertainties U2					
Design case	MoS	WT rotor radius (m)	PV panel area (m^2)	Diesel nom. Power (kW)	Penetration (%)	TLSC ($)	LCE (cent/kWh)	Total BO (h)	Maximum BO (h)	Average BO (h)	U_t (kWh)	MTBF (h)	LCE (cent/kWh)	Total BO (h)	Maximum BO (h)	Average BO (h)	U_t (kWh)	MTBF (h)	LCE (cent/kWh)
D9	0.00	6.2	26	15	61	341,030	58.9	65	1	1	14.4	134	61.3	104	1	1	37.1	83	61.8
D10	0.05	6.2	26	15.75	61	347,720	60.1	36	1	1	4.1	243	62.6	73	1	1	20.0	119	63.1
D11	0.10	6.2	41	16.5	69	354,320	61.2	0	0	0	0	8760	63.7	46	1	1	8.7	190	64.4
D12	0.20	6.2	41	18	69	366,230	63.3	0	0	0	0	8760	66.0	0	0	0	0	8760	66.8
D13	0.50	6.2	41	22.5	69	401,960	69.4	0	0	0	0	8760	72.8	0	0	0	0	8760	73.7
D14	1.00	6.8	37	30	78	459,940	79.5	0	0	0	0	8760	84.3	0	0	0	0	8760	85.8
D15	2.00	10.2	12	45	162	568,300	98.2	0	0	0	0	8760	105.0	0	0	0	0	8760	108.2
D16	N/A	6.2	26	15	61	341,030	58.9	65	1	1	14.4	134	61.3	104	1	1	37.1	83	61.8

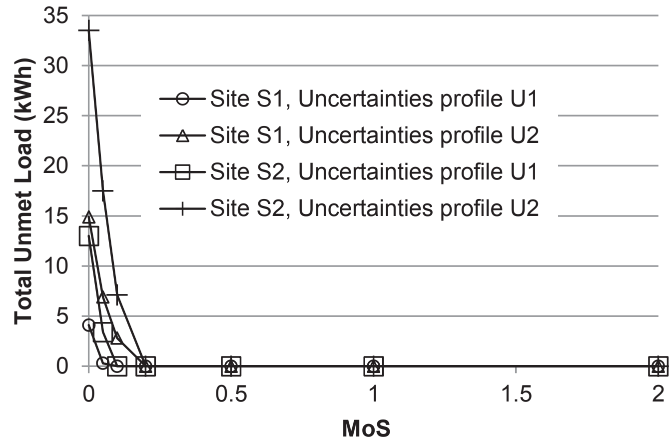

Fig. 2. Total unmet load versus MoS.

$$n_{r,comp} = \left[\frac{N_S}{N_{nom,comp}}\right] \quad \text{for wind turbine and PV panel} \tag{21}$$

$$n_{r,D} = \left[\frac{N_S T_D}{N_{nom,D}}\right] \quad \text{for diesel generator} \tag{22}$$

In this study the following parameters are used: air density $\rho = 1.225\ kg/m^3$; wind turbine electrical and gearbox efficiency $\eta_{EG} = 0.9$; surface roughness length $z_0{=}0.03$; minimum blade tip-ground clearance $h_c = 8\ m$; overall PV unit efficiency $\eta_{PV} = 12\%$; the life-span of the system $N = 20 years$ and the real discount rate $d = 4\%$. Table 1 summarises other parameters required for the cost analysis.

3. Design methodology development

Probabilistic analyses are highly time-consuming. A robust design method must include minimal number of probabilistic analyses. In order to develop such a method, the effect of margin of safety (MoS) used in the deterministic design method on the reliability measures is first investigated. The deterministic design method encompasses two steps. In the first step, size of diesel generator is found assuming that the diesel generator can cover the maximum peak load with a reasonable margin of safety *MoS* without any contribution from the renewable resources. Using

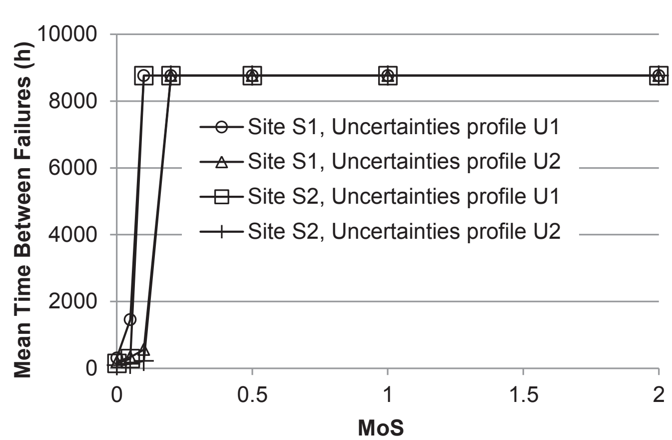

Fig. 3. Mean time between failures versus MoS.

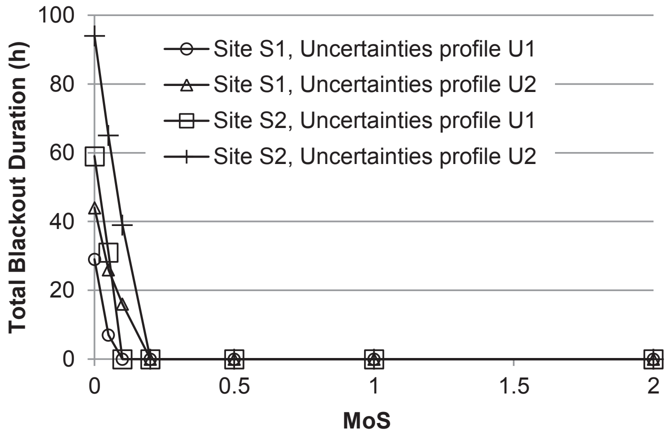

Fig. 4. Total blackout duration versus MoS.

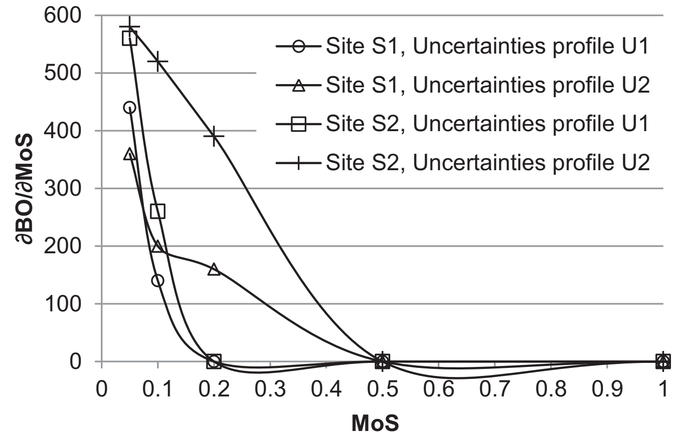

Fig. 6. Variation of total blackout duration with respect to MoS versus MoS.

hourly-averaged data the nominal size of the diesel generator $P_{\mathrm{D,nom}}$ is obtained by:

$$P_{\mathrm{D,nom}} = \overline{L}_{\mathrm{h,max}}(1 + MoS) \tag{23}$$

in which $\overline{L}_{\mathrm{h,max}}$ stands for the maximum hourly-averaged demand load. In the second step of the deterministic design method, using a single-objective optimisation, the size of wind turbine and PV panel which minimise LCE are determined. Using this method, for different margins of safety the optimal size of wind-PV-diesel components are obtained.

A genetic algorithm (GA) was developed to find the optimal size of components. The solution space for hybrid systems is clustered with multiple local optima. This can impact the search performance of an ordinary GA. Special care has been therefore made in design of reproduction operators for the developed GA. In order to increase the exploratory behaviour of the GA, avoiding stagnation in local optima, a dynamic mutation operator combined with a mixed parent selection strategy has been used. At earlier generations, identified by $fit_{\mathrm{av}}/fit_{\mathrm{max}} \le 0.9$, the GA explores the design space towards finding the cluster of the global optima by using a high mutation rate ($P_{\mathrm{m}} = 0.7$) and a random parent selection strategy (irrespective of the individual fitness). At latest generations ($fit_{\mathrm{av}}/fit_{\mathrm{max}} > 0.9$) when the GA has found the cluster of the global optima, the algorithm exploits the design space towards finding the global optima itself by adopting a parent selection based on the individual fitness. In this stage still a high mutation rate is used but the mutation effect is limited. The random perturbation of the i-th design variable x_i is selected from a shrinking interval $I_{i,\mathrm{m}} = (1 - (fit_{\mathrm{av}}/fit_{\mathrm{max}}))(x_{i,\mathrm{u}} - x_{i,\mathrm{l}})$, where $x_{i,\mathrm{l}}$ and $x_{i,\mathrm{u}}$ are, respectively, the lower and the upper limit of design variable x_i. This is aimed at a refine search in the vicinity of the global optima. Individual fitness in this algorithm is defined as the reciprocal of individual LCE. In the developed GA an arithmetic crossover operator is used. The infeasible solutions are defined as those with nonzero total blackout duration and are rejected on creation. The algorithm terminates when $fit_{\mathrm{max}} - fit_{\mathrm{av}} \le 1 \times 10^{-5}$.

For each deterministic design case, employing the Monte Carlo simulation method of Algorithm 1 below, the reliability of the system is evaluated.

Algorithm 1. Monte Carlo simulation for reliability and cost analysis

Given:

- $x_i = \tilde{x}_i + \hat{x}_i$; $i = 1,2,\ldots,n_{\mathrm{u}}$ the set of n_{u} uncertain parameters and their range and form of distributions ($\tilde{x}_i$ stands for the known mean value of parameter x_i and $\hat{x}_i$ is the random variation of x_i with known distribution).
- The desired level of confidence (LOC) corresponding to each one of the evaluated reliability assessment criteria $\{BO_{\mathrm{t}}, BO_{\mathrm{av}}, BO_{\mathrm{max}}, \mathrm{MTBF}, U_{\mathrm{t}}\}$ and LCE.
- The design candidate $\{A_{\mathrm{WT}}, A_{\mathrm{PV}}, P_{\mathrm{D,nom}}\}$ to be assessed

1. For $j = 1,2,\ldots,n_{\mathrm{sim}}$
   1.1. For each x_i; $i = 1,2,\ldots,n_{\mathrm{u}}$, select a random value $\hat{x}_i$ in the range consistent with its corresponding distribution.
   1.2. Find the value of the assessment measures $\{BO_{\mathrm{t}}, BO_{\mathrm{av}}, BO_{\mathrm{max}}, \mathrm{MTBF}, U_{\mathrm{t}}\}_j$ and LCE_j.
2. For each assessment criterion
   2.1. Using a histogram, find the probability of failure distribution.
   2.2. Find the value of assessment measure corresponding to the probability of failure of $\mathrm{PF} = 1 - \mathrm{LOC}$

Fig. 1 illustrates how Step 2 of Algorithm 1 is carried out to find the assessment measures at a given LOC (here total unmet load, U_{t}, at LOC 99.9%): First the range of the unmet load is divided into n_{seg} segments (here, $n_{\mathrm{seg}} = 1000$). Then, for each segment $k = 1,2,\ldots,n_{\mathrm{seg}}$ the probability of failure is found: PF_{k} = Probability of having a U_{t} greater than or equal to $U_{\mathrm{t,k}}$ = the total number of counts to the right of $U_{\mathrm{t,k}}$ divided by n_{sim} (for MTBF: PF_{k} = Probability of having a MTBF less than or equal to $\mathrm{MTBF}_{\mathrm{k}}$ = the total number of counts to the right of $\mathrm{MTBF}_{\mathrm{k}}$ divided by n_{sim}). In this study $n_{\mathrm{sim}}=10^4$ is used.

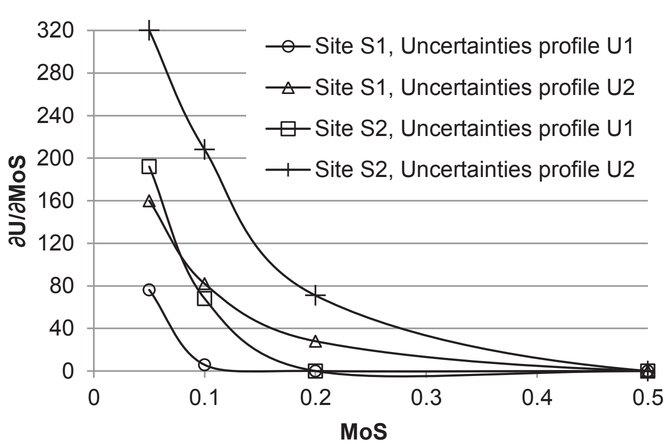

Fig. 5. Variation of total unmet load with respect to MoS versus MoS.

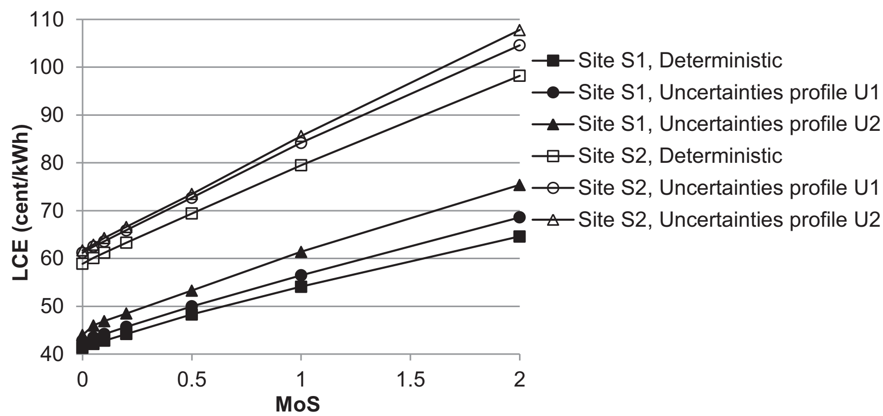

Fig. 7. LCE versus MoS.

In reliability analysis, uncertainties in resources (wind speed and solar irradiance), demand load and modelling (wind turbine power coefficient C_P and PV array efficiency) are considered. Table 2 shows two cases considered in this study. In this table δ represent the variation limit as a fraction of the mean value. In this study two sets of resource and demand load data are used. Table 3 compares the site data for these two sites.

Tables 4 and 5 show the results of deterministic designs for different margins of safety as well as the results of probabilistic reliability analysis. The last row of these tables includes the results of optimisation without considering a margin of safety, in which the size of the diesel generator is determined along with the other design variables.

Figs. 2–4 show three reliability measures: total unmet load, mean time between failures and total blackout duration against MoS. Figs 5 and 6 show trends of the variations of reliability measures with respect to MoS versus MoS. Fig. 7 shows LCE obtained deterministically and the LCE obtained using Monte Carlo simulation @ 99.99% LOC versus MoS. Solution spaces in two planes of LCE-total unmet load and LCE-total blackout duration are shown in Figs. 8 and 9.

These figures show

(i) Strong dependency of the reliability measures on the site data and their associated uncertainties (Figs. 1–6).
(ii) Regardless of the site data and their associated uncertainties, using a large-enough *MoS* leads to reliable designs (Figs. 1–4). That is, optimisation for reliability is equivalent to maximisation of MoS.
(iii) Probabilistic LCE deviates from deterministic LCE and this deviation increases with MoS (Fig. 7). In other words, the LCE calculated using deterministic methods is not accurate and should be found via probabilistic methods.
(iv) Parameter MoS used in deterministic design has significant effect on the LCE, and that both deterministic and probabilistic LCE vary linearly with MoS (Fig. 7). In other words, optimisation for cost is equivalent to minimisation of MoS.
(v) The LCE calculated using probabilistic methods depends on both site data and uncertainties profile (Fig. 7).
(vi) Predictable effect of increasing/decreasing MoS on the direction of forming Pareto Front in 2D solution space (Figs. 8 and 9).

Observations (ii), (iv) and (vi) lead us to the conclusion that MoS used in deterministic design is a key design parameter which can be used for directing the design towards solutions with desired reliability or cost. However, referring to observation (i), this key parameter is highly problem dependent and cannot be obtained deterministically. Moreover, according to observation (iii) and (v), even in the absent of uncertainties in cost modelling, design candidate assessment with respect to cost must be based on probabilistic cost analysis.

In summary, for each design problem, there exists an optimum MoS that can be used to produce a Pareto solution. Hence, the original multi-objective optimisation problem in which the

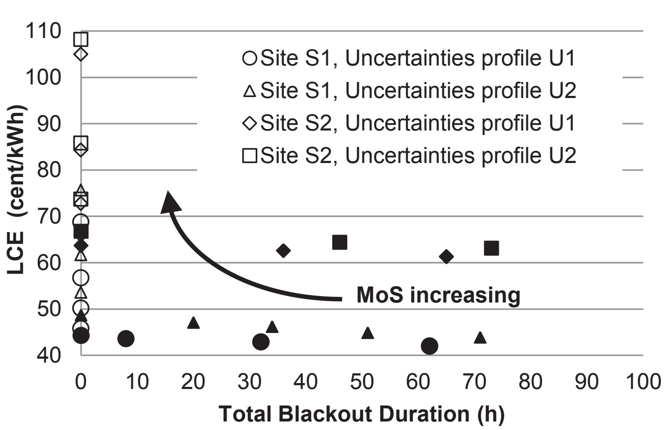

Fig. 8. Solution space in plane of LCE-Total blackout duration (solid markers represent Pareto solutions).

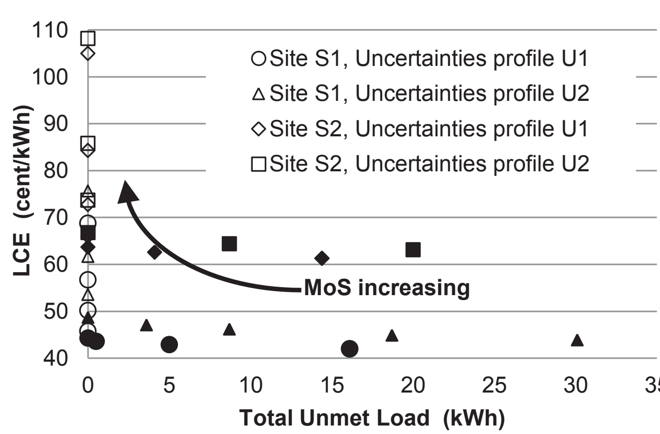

Fig. 9. Solution space in plane of LCE-Total unmet load (solid markers represent Pareto solutions).

Table 6
Results of case study 1.

>MoS	Diesel nom. Power (kW)	WT rotor radius (m)/deterministic optimisation for LCE	PV panel area (m^2)/deterministic optimisation for LCE	LCE @ 99.99% LOC (cent/kWh)
0 (1st initial point)	15.00	6.3	0	44.9
1 (2nd initial point)	30.00	7.9	0	61.7
0.036 (1st iteration)	15.54	6.3	0	45.5

optimum size of the system components are to be found through probabilistic analysis, can be reduced to a single-objective problem in which the optimum MoS is to be determined via probabilistic analysis and a single-objective optimisation in which the optimum size of system components are to be found deterministically.

4. Design scenarios

There are three main approaches being adopted in performing a multi-objective optimisation. In the first approach, known as a priori method, a multi-objective optimisation problem is transformed to a single objective problem by combining all design objectives using a weighting system and forming a single aggregate or cost function. Weighting systems comprise of a set of weighting factors and/or tuning exponents representing the relative degree of importance of design objectives. At the end of a successful search process, the design alternative that minimises the cost function is entitled the optimum solution. This solution is a single point on the Pareto frontier of the corresponding original problem. In the second approach, known as a posteriori methods, no weighting system is used and the search process forms the Pareto frontier itself, or its best viable approximation. Here the first goal is to find Pareto front solutions. The designer evaluates the generated design alternatives against the assessment criteria and looks for trade-off solution. This is the chief advantage of this method compared to the first approach. However, the high computational time required to produce enough uniformly distributed Pareto solutions is the main drawback of this approach when adopted for design optimisation problems including probabilistic analyses. In the third approach of multi-objective optimisation, by treating all-but-one design objectives as constraints, the multi-objective optimisation problem is transformed to a single objective one. This method is most suited for cases in which one objective is dominant and other objectives either have known target values or have known upper and/or lower bounds. In case of conflicting objectives, solution obtained by this method is again a single point on the Pareto frontier of the original problem, while unlike the first approach the designer actually directly imposes constraints on the locus of the solution prior to commencing the optimisation. Adopting the third approach, the following two design scenarios are developed.

4.1. Design scenario 1

In this design scenario the most reliable hybrid wind-PV-diesel system subject to the constraint $\mathrm{LCE} \le \mathrm{LCE_g}$ is obtained. Here, LCE is calculated using the probabilistic analysis method of Algorithm 1 and therefore a LOC must be associated to $\mathrm{LCE_g}$. Algorithm 2 below details the design method for this design scenario. The optimum MoS which maximises the reliability subject to the constraint $\mathrm{LCE} \le \mathrm{LCE_g}$ is represented by $\mathrm{MoS_g}$ and is calculated through Steps 1 and 2 of this algorithm.

Algorithm 2. Most reliable system subject to a constraint on the cost

Given:

- Goal levelised cost of energy $\mathrm{LCE_g}$ and its corresponding LOC
- Tolerance ε: $\mathrm{LCE} \le \mathrm{LCE_g} + \varepsilon$; $\varepsilon \ge 0$
- Site data
- The set of uncertain parameters and their range and form of distributions ($x_i = \tilde{x}_i + \hat{x}_i$; $i = 1,2,\ldots,n_u$)

Step 1. For two arbitrary $\mathrm{MoS_1}$ and $\mathrm{MoS_2}$ do:

1.1. Using Equation (23), calculate the nominal size of diesel generator $P_{\mathrm{D,nom}}$.

1.2. Use a deterministic optimisation method to find the optimum size of other components.

1.3. For the obtained optimal solution run the Monte Carlo simulation of Algorithm 1 to find its corresponding *LCE*.

Step 2. Calculate the corrsponding *MoS* to the goal *LCE* using Equation (24)

$$\mathrm{MoS_g} = c_1 \mathrm{LCE_g} + c_2 \tag{24}$$

Step 3. For $MoS = MoS_g$ do:

3.1. Employ Equation (23) to calculate the nominal size of diesel generator $P_{\mathrm{D,nom}}$.

3.2. Use a deterministic optimisation method to find the optimum size of the other components.

3.3. For the obtained optimal solution run the Monte Carlo simulation of Algorithm 1 to find its corresponding LCE and reliability measures.

3.4. If $\mathrm{LCE} \le \mathrm{LCE_g} + \varepsilon$ stop the search; otherwise: update coefficients c_1 and c_2; go to Step 2.

For the first time in Step 2 parameters c_1 and c_2 are found using two points ($\mathrm{MoS_1}$, $\mathrm{LCE_1}$) and ($\mathrm{MoS_2}$, $\mathrm{LCE_2}$) in MoS − LCE plane:

Table 7
Results of Steps 1 and 4 of Algorithm 3 for case study 2.

MoS	Diesel nom. Power (kW)	WT rotor radius (m)/ Deterministic optimisation for LCE	PV panel area (m^2)/ Deterministic optimisation for LCE	LCE @ 99.99% LOC (cent/kWh)	$\mathrm{BO_t}$ @ 99.99% LOC (h)	U_t @ 99.99% LOC (kWh)	MTBF@ 99.99% LOC (h)
0 (1st initial point)	15.0	6.2	26	61.8	104	37.1	83
0.05 (2nd initial point)	15.8	6.2	26	63.1	73	20.0	119
0.1 (3rd initial point)	16.5	6.2	41	64.4	46	8.7	190
0.1215 (1st iteration)	16.8	6.2	41	64.9	37	5.45	243
0.1219 (2nd iteration)	16.82	6.2	41	64.9	36	5.03	243

Table 8
Results of Steps 2 and 3 of Algorithm 3 for case study 2.

Iteration	R_i	$[c_3,c_4,c_5]$ as in Eq. (4)	MoS_{g,R_i}	MoS_g
1	$BO_t(h)$	[+4E-6, −0.0023, +0.199]	0.1134	0.1215
	$U_t(kWh)$	[+5E-5, −0.0059,+0.1477]	0.11945	
	$MTBF(h)$	[−6E-6, +0.0027, −0.1785]	0.1215	
2	$BO_t(h)$	[+3E-6, −0.0021, +0.1914]	0.1122	0.1219
	$U_t(kWh)$	[+6E-5, −0.0064, +0.1524]	0.1219	
	$MTBF(h)$	[−4E-6, +0.002, −0.1372]	0.1028	

$$c_1 = \frac{MoS_2 - MoS_1}{LCE_2 - LCE_1} \quad (25.a)$$

$$c_2 = \frac{MoS_1 LCE_2 - MoS_2 LCE_1}{LCE_2 - LCE_1} \quad (25.b)$$

Updating coefficients c_1 and c_2 in Step 3.4 can be carried out either via Equation (25) by using the new point (MoS, LCE) from latest iteration and one of the previous points or via data regression (e.g. least square method) using all points. It should be noted that in case of a perfect linear correlation between probabilistic LCE and MoS, the first iteration should lead to the final solution.

4.1.1. Case study 1

It is desired to find the most reliable hybrid wind-PV-diesel system for site S1 with uncertainty profile U2 subject to LCE $\leq$ 45.5 cent/kWh @ LOC 99.99% ($LCE_g = 45.5$ *cent/kWh*).

A tolerance of $\varepsilon = 0.01$ *cent/kWh* is used. By selecting $MoS_1 = 0$ and $MoS_2 = 1$, Step 1 of Algorithm 2 leads to the results shown in the first two rows of Table 6. The genetic algorithm optimisation explained in Section 3 is used for performing the deterministic optimisation of Steps 1.2 and 3.2. Using Equations (24) and (25) the goal MoS is calculated as: $MoS_g = 0.036$. Using this value Step 3 of Algorithm 2 leads to the results shown in the third row of Table 6.

As it can be observed the first iteration leads to the final solution. The reliability measures for this solution are: $BO_t = 38\ h$, $BO_{av} = 1\ h$, $BO_{max} = 1h$, MTBF = 237 *h* and $U_t = 1.2\ kWh$ (all at a LOC of 99.99%).

For this case by performing only three Monte Carlo simulations a multi-objective optimal design under uncertainty is carried out. This highlights the robustness of this design method.

4.2. Design scenario 2

In this design scenario the most cost-effective hybrid wind-PV-diesel system subject to satisfying some goal reliability measures $R = \{R_i\} \subset \{BO_{t,g}, BO_{av,g}, BO_{max,g}, MTBF_g, U_{t,g}\}$ is obtained. Each goal reliability measure considered for the assessment is associated with a LOC. Algorithm 3 details the design method for this design scenario. The optimum MoS which minimises the LCE subject to the constraints $R_i \leq R_{i,g}$ is represented by MoS_g and is calculated through Steps 1 to 3 of this algorithm.

Algorithm 3. Most cost-effective system subject to constraints on reliability measures

Given:

- Goal values for a selected subset of the reliability measures $R = \{R_i\} \subset \{BO_{t,g}, BO_{av,g}, BO_{max,g}, MTBF_g, U_{t,g}\}$ and their corresponding LOC
- Set of tolerance $\varepsilon = \{\varepsilon_i\}$: $R_i \leq R_{i,g} + \varepsilon_i$ for each $R_i \in R$ to be minimised ($BO_t, BO_{av}, BO_{max}, U_t$) and $R_i \geq R_{i,g} - \varepsilon_i$ for each $R_i \in R$ to be maximised (MTBF) ($\varepsilon_i \geq 0$)
- Site data
- The set of uncertain parameters and their range and form of distributions ($x_i = \tilde{x}_i + \hat{x}_i$; $i = 1,2,\ldots,n_u$)

Step 1. For three arbitrary MoS do:

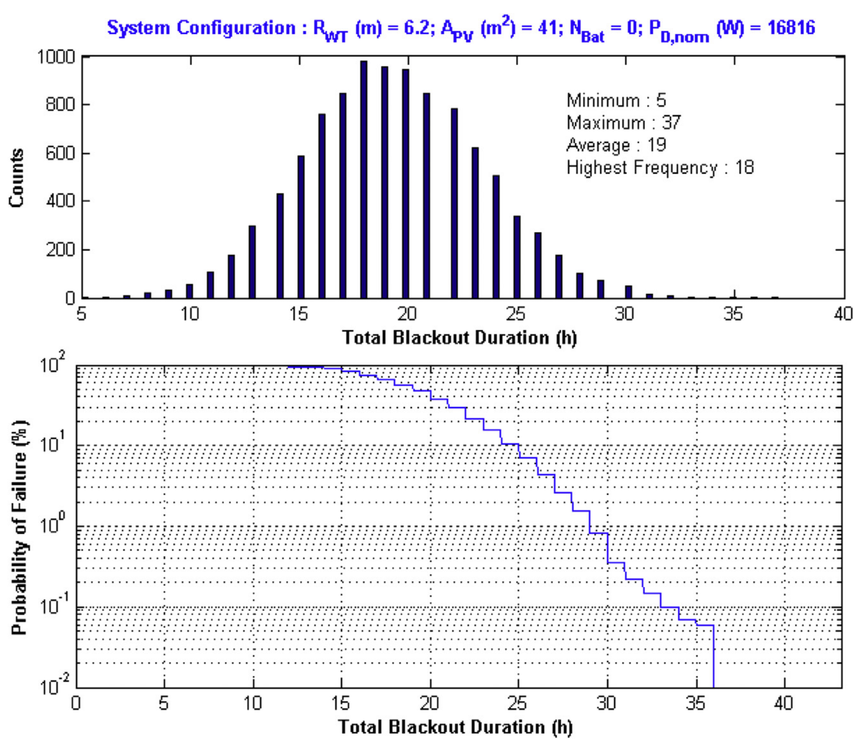

Fig. 10. Probability of failure distribution of the final solution-design quality: total blackout duration.

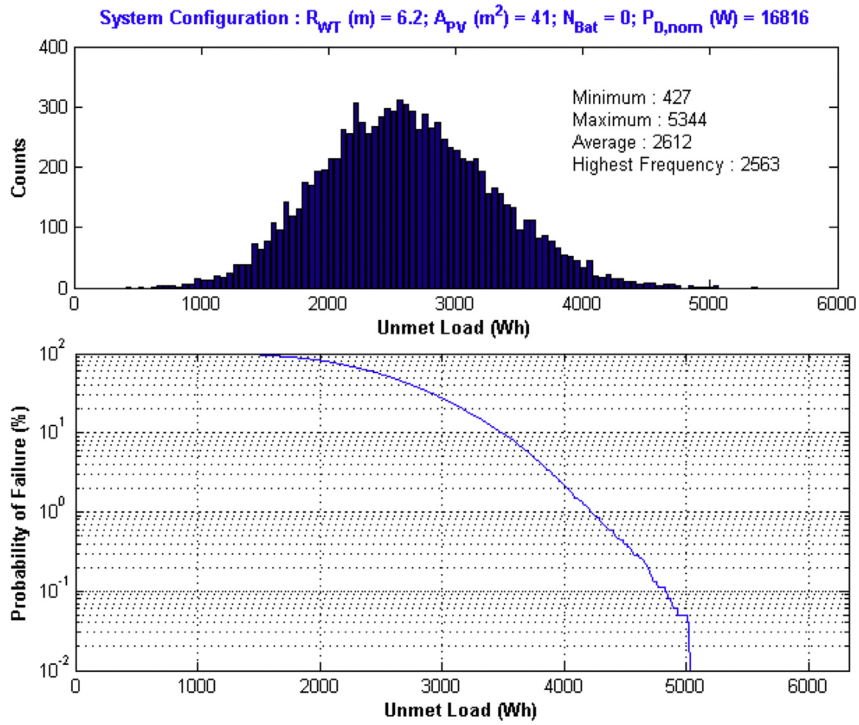

Fig. 11. Probability of failure distribution of the final solution-design quality: total unmet load.

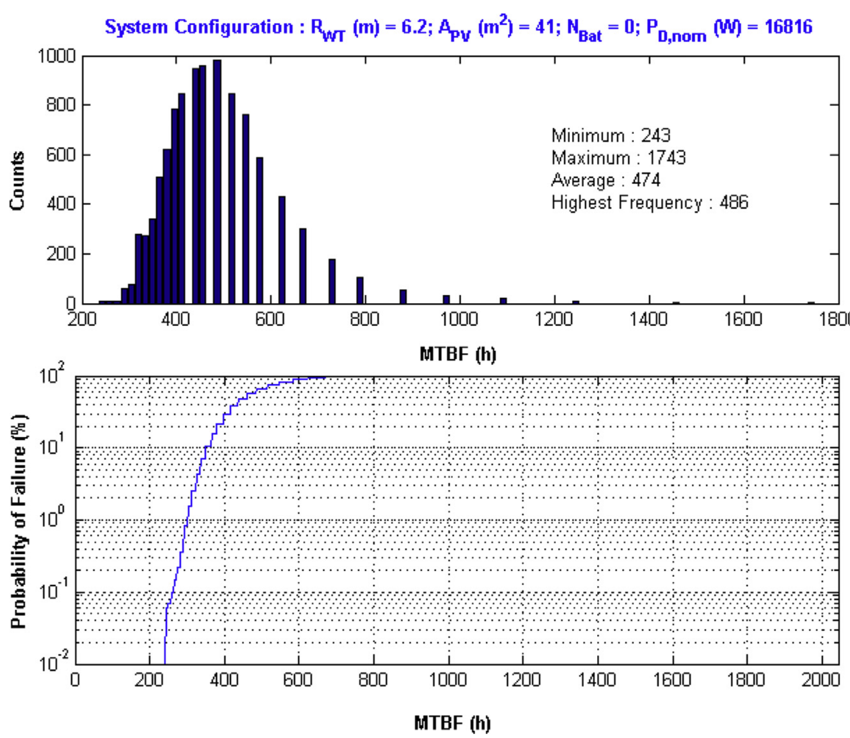

Fig. 12. Probability of failure distribution of the final solution-design quality: mean time between failures.

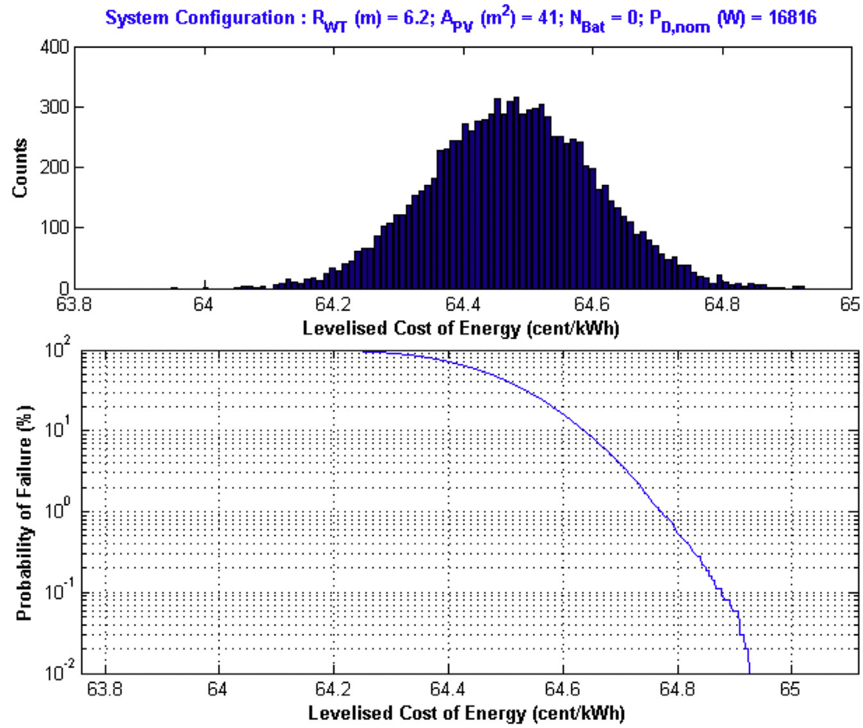

Fig. 13. Probability of failure distribution of the final solution-design quality: levelised cost of energy.

1.1. Using Equation (23), calculate the nominal diesel size $P_{D,nom}$.
1.2. Use a deterministic optimisation method to find the optimum size of other components.
1.3. For the obtained optimal solution run the Monte Carlo simulation of Algorithm 1 to find its corresponding LCE and reliability measures.

Step 2. For each $R_i \in R$, using Equation (26) find its corresponding MoS_{g,R_i}.

$$\mathrm{MoS}_{g,R_i} = c_3 R_i^2 + c_4 R_i + c_5 \quad (26)$$

Step 3. Assign $\mathrm{MoS}_g = \max\{\mathrm{MoS}_{g,R_i}\}$.
Step 4. For $\mathrm{MoS} = \mathrm{MoS}_g$ do:

4.1. Employ Equation (23) to calculate the nominal diesel size $P_{D,nom}$.
4.2. Use a deterministic optimisation method to find the optimum size of the other components.
4.3. For the obtained optimal solution run the Monte Carlo simulation of Algorithm 1 to find its corresponding LCE and the set of reliability measures R.
4.4. If desired reliability achieved end the search; otherwise updates parameters c_3 through c_5 and go to Step 2.

Calculating/updating coefficients c_3 through c_5 is carried out via data regression (e.g. least square method) using all available points $(\mathrm{MoS}_{g,R_i}, R_i)$. It should be noted that three arbitrary MoS of Step 1 should produce at least two distinct points in each R_i − MoS plane to be able to correlate R_i to MoS through Equation (26). Lower MoS are more likely to produce distinct points.

4.2.1. Case study 2

In this design case study it is desired to design a wind-PV-diesel system for site S2 under uncertainties U2. The reliability measures $BO_t \le 40\ h$, $\mathrm{MTBF} \ge 200\ h$ and $U_t \le 5\ kWh$ at a $\mathrm{LOC} = 99.99\%$ are desired.

Tolerances $\varepsilon = \{1\mathrm{h}\ 50\mathrm{Wh}\ 1\mathrm{h}\}$ for the reliability measures $R = \{BO_t\ U_t\ \mathrm{MTBF}\}$ are used. Results are shown in Table 7. The designed system in the second iteration satisfies all constraints within the tolerated margins. Table 8 summarises the results of Steps 2 and 3 leading to MoS_g for the first and second iterations.

Figs. 10–13 show the histograms and probability of failure distributions obtained via Monte Carlo simulation of Algorithm 1 for four design qualities (three reliability measures and LCE) of the final design.

5. Summary and concluding remark

Optimal design of a standalone wind-PV-diesel HRES is a multi-objective optimisation problem with conflicting objectives of cost and reliability. Due to uncertainties in renewable resources and demand load, probabilistic analysis methods such as Monte Carlo simulation are required to quantify the system reliability. Performing probabilistic analysis within a search process, in which tens of thousands of design candidates are produced and evaluated towards finding the global optima, is highly time-consuming and inefficient.

Uncertainties in renewable resources, demand load and power modelling make deterministic methods of multi-objective optimisation fall short in optimal design of standalone HRES. Firstly, deterministic methods of analysis, even in the absence of uncertainties in cost modelling, do not predict the LCE accurately. Secondly, since these methods ignore the random variations in parameters, they cannot be used to quantify the second objective, reliability of the system in supplying power. While it is well established that using safety factors and design for worst-case-scenarios leads to reliable solutions, it is also well known that deterministic designs can lead to non-optimal over-designed/under-designed systems as a result of employing improper safety factors.

Parameter MoS used in deterministic sizing of the diesel generator plays the key role in the development of the new design methodology. First it is shown that MoS has a major and predictable influence on both LCE and reliability-related design qualities. It is also shown that in the context of multi-objective optimisation with conflicting objectives of cost and reliability, for each design problem, there exists an optimum MoS that can be used to produce a Pareto solution. Hence, the original multi-objective optimisation problem in which the optimum size of the system components are to be found through tens of thousands of probabilistic analysis, can be reduced to a single-objective problem in which the optimum MoS is to be determined via few probabilistic analysis and a single-objective optimisation in which the optimum size of system components are to be found deterministically. As a result of this the number of probabilistic analysis reduces dramatically.

Optimum MoS depends on: (i) site data, (ii) uncertainties and (iii) desired (goal) design qualities in terms of the system cost and reliability of power supply (e.g. LCE $\leq$ 45.5 cent/kWh, $BO_t \leq 40\ h$, etc). For a given site and set of uncertainty profiles, different goal design qualities correspond to different optimum MoS, and consequently different Pareto solutions.

For two design scenarios, namely, most reliable system subject to a constraint on the cost and most cost-effective system subject to constraints on reliability measures, two algorithms are proposed to find the optimum MoS. The robustness of the proposed design methodology is shown through carrying out two design case studies. Design case study 2 also shows that how the proposed design methodology can be employed to design systems compatible with the end-user requirements.

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Glossary

A: area (m^2)
BO: blackout duration
C: cost ($)
C_u: unit cost ($/unit)
d: discount rate
h_c: ground-blade tip clearance (m)
I: solar irradiance (W/m^2)
L: demand load (W)
MoS: margin of safety
MTBF: mean time to failure
N: nominal life-span (years; hours of operation)
n: number
n_u: number of uncertain parameter
P: power (W)
PF: probability of failure (−)
S: size (various units)
U_t: total unmet load (Wh)
UCRF: uniform capital recovery factor
z_0: site surface roughness (m)
α: cost as a fraction of initial cost
η_{PV}: overall PV unit efficiency
ρ: air density (kg/m^3)
η_{EG}: wind turbine electrical and gearbox efficiency

Subscripts

a: available; usable available; annualised
av: average
c: capital
comp: HRES component (WT, PV, D)
D: diesel
d: daily
F: fixed
fail: failure
h: hourly
hub: hub elevation
ins: installation
max: maximum
min: minimum
nom: nominal
O&M: operation and maintenance
PV: photovoltaic
p: performance measures
R: renewable
r: replacement
S: system
sim: simulation
t: total
u: unit, uncertain parameter
V: variable
WT: wind turbine

Symbols

$\overline{\varphi}_T$: averaged value of quantity φ over time period T
$\tilde{\varphi}$: mean value of uncertain parameter φ
$\hat{\varphi}$: random part of uncertain parameter φ
$[\varphi]$: integer value of parameter φ

Abbreviations

HRES: hybrid renewable energy system
LCE: levelised cost of energy
LOC: level of confidence
O&M: operating and maintenance
TLSC: total life-span cost