Neuropsychologia 153 (2021) 107771

Contents lists available at ScienceDirect

Neuropsychologia

journal homepage: http://www.elsevier.com/locate/neuropsychologia

Differences in event-related potential (ERP) responses to small tie, non-tie and 1-problems in addition and multiplication

Jeanne Bagnoud [*], Jasinta Dewi, Catherine Thevenot [*]

University of Lausanne, Institute of Psychology, Switzerland

ARTICLE INFO

Keywords:
Numerical cognition
Mental arithmetic
Cognitive strategies
Retrieval network
EEG

ABSTRACT

Using ERP, we investigated the cause of the tie advantage according to which problems with repeated operands are solved faster and more accurately than non-tie problems. We found no differences in early or N400 ERP components between problems, suggesting that tie problems are not encoded faster or suffer from less interference than non-tie problems. However, a lesser negative amplitude of the N2 component was found for tie than non-tie problems. This suggests more working-memory and attentional resource requirements for non-tie problems and therefore more frequent use of retrieval for tie than non-tie problems. The possible peculiarity of problems involving a 1 was also investigated. We showed less negative N2 amplitudes for these problems than for other non-tie problems, suggesting less working-memory resources for 1-problems than other non-tie problems. This could be explained either by higher reliance on memory retrieval for 1-problems than non-1 problems or by the application of non-arithmetical rules for 1-problems.

1. Introduction

In the domain of numerical cognition, and more specifically in the domain of mental arithmetic, it has been repeatedly shown that tie problems, which are constructed with repeated operands such as 6 + 6, are solved faster than non-tie problems, which are constructed with different operands such as 6 + 5. Moreover, whereas, even in adults, non-tie problems show substantial problem-size effect, or, in other words, longer solution times and more error rates for problems with larger than smaller operands, tie problems show little to no problem-size effect very early in development (e.g., Bagnoud et al., 2021; Campbell and Gunter, 2002; Groen and Parkman, 1972). Because they are not subjected to the same effects as non-tie problems, tie problems are often excluded from the material in behavioural (Cragg et al., 2017; De Smedt et al., 2007; Imbo et al., 2011; Newman, 2016; Rabinowitz and Wooley, 1995) and brain imaging studies about mental arithmetic (e.g., Núñez-Peña et al., 2011; Rütsche et al., 2015; Van Beek et al., 2014). However, contrasting the results obtained for tie and non-tie problems could shed light on the possible different cognitive mechanisms that underlie their solving process. This is precisely what is done in the present article. EEG was recorded in adults in a delayed verification task with the aim to investigate potential differences in brain activation between simple tie and non-tie addition and multiplication problems. On a more theoretical point of view, we aimed at examining the relevance of different hypotheses that have been formulated to explain the tie advantage (i.e., shorter solution times and smaller size effect for tie than non-tie problems) by recording electroencephalogram activities (EEG) and focusing our analyses on event related potential (ERP) differences between tie and non-tie problems.

Blankenberger (2001) explained the tie advantage by the fact that encoding two identical stimuli is quicker than encoding two different stimuli. Accordingly, the author showed that when problems are presented in mixed formats, two + 2 for example, the tie advantage vanishes. However, whereas this encoding hypothesis can explain that tie problems are solved faster than non-tie problems, it cannot explain why tie problems present a smaller problem-size effect than non-tie problems (LeFevre et al., 2004). If the difference between tie and non-tie problems is purely due to the encoding phase, a similar decrease in solution times for small and large tie problems should be observed and problem-size effects for tie and non-tie problems should therefore be the same. Additionally, replications of Blankenberger's study where small and large problems were analyzed separately showed that large tie problems presented in a mix format are still solved faster than large non-tie problems. This confirmed that the tie advantage cannot only be a consequence of encoding-based factors (Campbell and Gunter, 2002; LeFevre et al., 2004).

* Corresponding authors. University of Lausanne, SSP, Institute of Psychology, Géopolis Building, Room 4536, CH, 1015 Lausanne, Switzerland.
E-mail addresses: jeanne.bagnoud@unil.ch (J. Bagnoud), catherine.thevenot@unil.ch (C. Thevenot).

https://doi.org/10.1016/j.neuropsychologia.2021.107771
Received 20 August 2020; Received in revised form 25 January 2021; Accepted 30 January 2021
Available online 4 February 2021

A second hypothesis put forward to explain the tie advantage is that the two types of problems are solved by different strategies. More specifically, non-tie problems would be solved more often by procedural strategies than tie problems, which would more often be solved by retrieval of the answer from long-term memory (LeFevre et al., 1996a, 1996b). Large size effects in non-tie problems would be due to the fact that larger problems require more procedural steps than smaller ones (Groen and Parkman, 1972). Thus, tie problems would present smaller size effects because they are rarely solved by such procedural strategies. Accordingly, participants, who report using other strategies than retrieval, present larger problem size effects for non-tie than for tie problems, while participants who report using retrieval for all problems showed similar problem-size effects for the two types of problems (LeFevre et al., 2003). To sum up, the difference in problem-size effects and solution times for tie and non-tie problems could therefore be due to more memory retrieval of the answers for tie than non-tie problems. In the case of addition, this could be due to the salience of tie numerical patterns in children's environments, which could facilitate their memorization. For example, playing dominos or board games with dice could allow children to perceive two groups of four dots as "one eight" (Clements, 1999). On a more embodied level, when a child count five fingers on one of his hand and 5 on the other, the knowledge that he or she has 10 fingers easily allows the mental association of 5 and 5 with 10. Similarly, if a child knows that an egg box contains 12 eggs, counting the two rows of six eggs facilitate the association of 6 and 6 with 12 (Baroody and Purpura, 2017). Note that this explanation in terms of salience can account for the higher reliance on retrieval for tie than non-tie problems but also for the better quality of retrieval, which is the hypothesis presented in the next paragraph.

A third hypothesis accounting for the tie advantage is that retrieval is more efficient for tie than for non-tie problems. This was corroborated by Campbell and Gunter (2002) who found that even in trials where the reported strategy was retrieval, shorter solution times and smaller size effects were observed for tie than non-tie problems. They therefore concluded that this advantage was not only due to the use of different strategies but also to a difference in retrieval efficiency between tie and non-tie problems. In the literature, difference in retrieval efficiency can be explained in several ways. Retrieval efficiency can increase with the frequency of practice (Zbrodoff, 1995). However, Hamann and Ashcraft (1986) did not find evidence that tie problems are presented more frequently than non-tie problems in textbooks. Moreover, in an experimental task where children have to memorize arithmetic-like facts constructed with letters instead of numbers, the tie advantage can be observed even when the frequency of presentation of tie and non-tie problems is controlled (Graham and Campbell, 1992). Nevertheless, it could be that tie problems are indirectly solved more often than non-tie problems because they are learnt before non-tie problems and constitute the basis for more complex problems (Baroody et al., 2015). Another factor that could influence the efficiency of retrieval of a problem is how much it interferes with other problems. De Visscher and Noël (2014a) proposed a model in which an interference index was considered for each multiplication fact. This index was calculated using the number of already learnt problems sharing at least two digits with the given problem (e.g., 6 and 7 in $6 \times 7 = 42$ and $7 \times 8 = 56$). Because they are constructed with repeated operands, tie problems have fewer common digits with other problems than non-tie problems. Smaller interference index for tie than non-tie problems would therefore explain why they are solved more efficiently. Another explanation also based on interference was put forward by Campbell (Campbell, 1995; Campbell and Oliphant, 1992) in order to explain the tie advantage. The author noted that incorrect answers given by participants for tie problems often correspond to answers to other tie problems but only rarely to other non-tie problem answers. Therefore, it was suggested that tie and non-tie problems are represented in two different memory sub-networks. Because interference within a category would be higher than between categories and because tie problems are less numerous than non-tie problems, interference effects would be weaker in the tie than in the non-tie sub-network. As a consequence, tie problems would be retrieved more easily than non-tie problems. Finally, concerning interference effects, Graham and Campbell (1992) hypothesized that operands of a given problem activate table related neighbour problems, which would interfere with the given problem. Because tie problems are constructed with only one operand repeated twice, neighbour interferences are necessarily less important than for non-tie problems.

To summarize, three main explanations are put forward in the literature in order to account for the tie advantage. Compared to non-tie problems, tie problems could be more easily encoded, more often solved through retrieval or more easily retrieved from memory, mainly because they suffer from less interference. Those explanations are not necessarily mutually exclusive, and, as already stated above, our goal is to go further in their understanding and to test their plausibility by studying event-related potentials (ERP) in adults solving additions and multiplications.

Several components of ERPs have been used to study arithmetic in both adults and children (see Hinault and Lemaire, 2016 for a review). Very early components appearing in the first 250 ms (P1 and N1) are supposed to reflect physical identification of the stimuli (El Yagoubi et al., 2003; Iguchi and Hashimoto, 2000). From 250 ms after stimulus presentation, problem-solving processes beyond encoding can already be identified (El Yagoubi et al., 2003). Another component that is particularly investigated in mental arithmetic studies is the N2 component. It can be observed mainly in anterior and central regions around 300–500 ms after stimulus presentation (Luo et al., 2009; Van Beek et al., 2014). According to Luo et al. (2009), the N2 component is modulated by the strategy used to solve a problem. More precisely, they observed in a two-digits operand addition task that solving a problem using a rounding strategy (e.g., $19 + 63 = 20 + 63–1 = 82$) leads to a less negative component around 400 ms after stimulus presentation than performing the calculation without using this shortcut strategy. This difference could stem from lower working-memory resource mobilization for the rounding strategy. It has also been noted that N2 is influenced by the problem size in children, with higher amplitudes for larger problems compared to smaller ones (Van Beek et al., 2014). Moreover, the N2 component is modulated by expertise with greater amplitudes for children than adults (Prieto-Corona et al., 2010). These results lead to the interpretation that the N2 component reflects the attentional and working-memory resources necessary to solve a problem, with more negative amplitude reflecting more working-memory or attentional resources. However, N2 component has also been interpreted as being modulated by phonological processing. More precisely Zhou et al. (2006) found a more negative N2 component around 300 ms after stimulus presentation for multiplication than for addition and subtraction. Because the source localisation analysis indicated that the difference was probably around Broca's area, the authors suggested that solving multiplication, due to rote learning at school, involves more verbal memory resources than addition and subtraction. A last component has been observed in arithmetic studies for verification tasks. More precisely, the ERP amplitude in the centroparietal regions around 400 ms after the answer presentation is more negative for trials associated with a false answer than for trials associated with the correct answer (Jost et al., 2004; Niedeggen and Rösler, 1999; Szűcs and Csépe, 2005). Moreover, Niedeggen and Rösler (1999) noted that this difference in amplitude, which is usually defined as the N400 component, is modulated by the nature of the false answer associated with the problem. More precisely, N400 amplitude was smaller when the false answer was an answer of a neighbour problem, that is a close table-related problem (e.g., $3 \times 6 = 24$), than an answer of an unrelated or a far table-related problem (e.g., $3 \times 7 = 32$). This result was interpreted as an indication that the answers to neighbour problems were activated when the problem was presented.

In the present study, brain activity was recorded in adults during a verification task in which additions and multiplications were presented. In order to avoid plausibility checking, which is the evaluation of the plausibility of an answer without calculating it (Lemaire and Fayol, 1995), we introduced a 1500 ms delay between the presentation of the problem and the proposed answer. Early and N2 components were extracted before the proposed answer was presented, while N400 was extracted after answer presentation. Because ERP analyses require a large number of trials per category, we studied problems with operands ranging from 1 to 5. This restriction did not constitute a limitation for addressing our research question because differences in problem-size effects between tie and non-tie problems systematically appear when such problems are studied (e.g., Bagnoud et al., 2021; Groen and Parkman, 1972; Svenson, 1975; Uittenhove et al., 2016). Moreover, this restriction on small tie problems is probably necessary if we want to study a homogeneous category of problems in terms of strategy use. Outside this restricted range, non-negligible size effects for tie problems start to appear (see Uittenhove et al., 2016, Fig. 1 for example). In addition to ERP measures, verbal reports were collected in order to objectively identify the problems that participants solved using conscious procedures. Moreover, problem accuracy and solution times were collected. However, because of the 1500 ms delay between problem and answer presentation, solution times could not reflect problem-solving processes but only the time needed to compare the calculated answer with the proposed answer. The degree of interference between the calculated and the proposed answer can also modulate this variable. In addition to tie and non-tie problems, which were analyzed as two distinct categories of problems, we considered problems involving a 1 (i.e., 1-problems) as a third category. We already know that, in children and adults, 1-problems do not present the same solution time distributions as the other non-tie problems (Bagnoud et al., 2021). A possible explanation of this phenomenon could be that the way they are solved differs from the other problems (e.g., Campbell, 1995). More specifically, they could be solved by rule-based processes (e.g., Baroody, 1995). Instead of excluding 1-problems from our analyses, we thought that it would be interesting to study them as a special category, especially given the fact that, to our knowledge, 1-problem solving has never been investigated using EEG, which adds to the novelty of our study.

Precise predictions on ERP can be made according to the three hypotheses that accounts for the tie-problem advantage (Table 1). If tie problems benefit from an encoding advantage, early ERP components should be different for tie problems than for non-tie or 1-problems. If the hypothesis stating that answers to tie problems are more often retrieved from memory than the answers to non-tie problems is correct, tie problems should put less demand on working memory and should therefore present less negative amplitudes for the N2 component than non-tie problems. This difference should be more pronounced for individuals who sometimes report non-retrieval strategy than for individuals who report exclusive reliance on retrieval. Because multiplication is generally viewed as more often solved through retrieval than addition (e.g., Campbell and Xue, 2001), a smaller difference between tie and non-tie problems for multiplication than addition could be expected. Finally, the interference hypothesis would predict less negative N2 amplitudes for tie than non-tie problems. Indeed, if tie problems are subjected to less interference, they should require less attentional resources to be solved than non-tie problems. This difference should still be observable when only individuals who report an exclusive reliance on retrieval are selected. Lastly, interference differences between tie and non-tie problems caused by activation of neighbour problems should result in larger negative amplitudes of the N400 component for tie than non-tie problems. Independently on the theories explaining the tie effects, prediction can be made for 1-problems. First, their N2 amplitude should be shorter than for non-tie problems because these problems are very quickly and accurately solved by adults and should therefore require little attentional and working-memory resources to be solved compared to other non-tie problems (Bagnoud et al., 2021; Baroody, 1995; LeFevre et al., 1996a, 1996b; Uittenhove et al., 2016). Then, for the N400 component, because 1-problems are supposedly not part of the interferent network (e.g., Campbell, 1995), we can reasonably expect that 1-problems suffer from less interference and thus present more negative N400 amplitudes than other non-tie problems.

2. Method

2.1. Participants

Twenty-five students (18 females) took part in this study. Sixteen of them were undergraduates from the psychology department of the University of Lausanne and received credits for their participation. The 9 other participants were students from other faculties in Swiss universities or higher education institutions and volunteered to take part in the study. All participants were aged between 18 and 24 years (mean 21.24-years old). Three participants reported being left-handed and were thus not included in the study. Eight additional participants were not included because of recording failures. Approval from the ethic committee of the Canton de Vaud was obtained for this study and informed

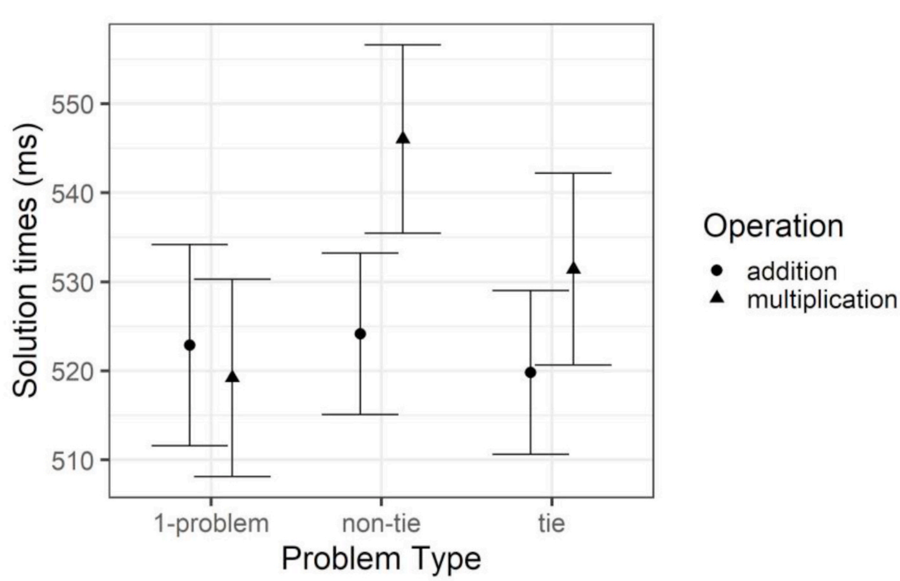

Fig. 1. Mean solution times (and standard errors) according to Problem Type and Operation.

Table 1
Summary of the predictions for the difference between tie and non-tie problems depending on the hypothesis for each component.

	Encoding hypothesis	Strategy hypothesis	Retrieval hypothesis
Early components	different	same	same
N2 component	same	different, larger effect for low than high retrievers and for multiplication than addition	different, equivalent effect for low and high retrievers
N400 component	same	same	different

consents were obtained from participants prior to the testing.

2.2. Material

In a delayed verification task, single-digit additions and multiplications involving operands from 1 to 5 were displayed on a computer screen and participants had to judge whether a given answer presented after the problem was correct or not. The 40 non-tie and 1-problems that can be constructed with operands from 1 to 5 (20 for addition, 20 for multiplication) were assigned to two sets. When a problem was included in Set 1 (e.g., $2 + 3$), the problem involving the same operands in the inverse order was included in Set 2 (e.g., $3 + 2$). Moreover, when an addition was included in a set (e.g., $2 + 3$), its multiplication counterpart was included in the same set (e.g., 2×3). Half of the problems in each set were constructed with the largest operand as the first operand. Each digit was presented 6 times as the first and 6 times as the second operand in each set. The list of the problems included in each set can be found in the supplementary material. Half of the participants were assigned to Set 1 (10 participants) and the other half to Set 2 (12 participants). All tie problems were presented in both sets. Therefore, each set contained 15 different problems (5 ties; 6 non-ties; and four 1-problems) for each operation. In order to record enough trials to extract ERP, each problem was presented 16 times in each set. Therefore, each participant solved a total of 480 problems.

Half of the problems were followed by the correct answer, and the other half was followed by an incorrect answer. In order to study difference in the N400 component depending on the category of problems, proposed answers of a given problem were answers of its neighbour table-related problems (Niedeggen and Rösler, 1999). More precisely, incorrect answers were constructed by subtracting or adding 1 to one of the operands of the problem and calculating the answer (e.g., one of the incorrect answers to 3×2 was $3 \text{ x } (2 + 1) = 9$, while one of the answers to $3 + 2$ was $3 + (2 + 1) = 6$). All the possible incorrect answers were presented to participants an equal number of times. For multiplication, four different incorrect answers could be constructed for non-tie and 1-problems, while only two could be constructed for tie problems. Therefore, for tie multiplications, each incorrect answer appeared four times, while for other multiplications each incorrect answer appeared twice. For addition, however, only two different incorrect answers could be constructed for each problem and thus each incorrect answer was presented twice.

The delayed verification task construction was inspired from the task of Zhou et al. (2006). More precisely, after a preparation screen (•) of 500 ms, the problem was presented for 200 ms. A 1300 ms blank screen was then displayed. During this time, participants were asked to solve the problem. Then, the proposed answer was presented on the screen and participants had to press one of two keys of a QWERTZ keyboard to indicate whether this answer was correct or not. Half of the participants had to press P when the answer was correct and Q when it was incorrect (10 participants), and half of the participant had to do the opposite (12 participants). Finally, a jittered inter-trial blank screen was then displayed for 800 ms–1000 ms.

For each participant, the problems were randomized and presented in 8 blocks of 60 problems, which were separated by 1-min breaks. After 1 min, participants could decide to continue the task or to take more rest. In order to avoid the switch costs caused by mixed presentation, additions and multiplications were presented in separated blocks (Campbell and Arbuthnott, 2010). Half of the participants solved all the addition blocks first (12 participants) while the other solved the multiplication blocks first (10 participants). A processing-speed task was performed between the two sets of operation blocks as a filler task with the idea that its results could be exploited, but it was finally not used for the present paper. Before EEG recording and in order to familiarize the participant with the procedure, 8 warm-up problems that were not presented in the experimental phase were presented.

At the end of the task, a paper-and-pencil questionnaire was given to each participant. They had to indicate which strategy they used most of the time to solve each problem they were previously presented. They could indicate if they retrieved the answer from memory, counted, or used another strategy to solve the problem. When needed, an explanation of what we meant by retrieval or counting was given. Solving a problem using retrieval was described as knowing the answer of that problem by heart. An example of counting strategy was also given ($3 + 2 = 3, 4, 5$), when necessary. If they used other strategies to solve the problem, they were asked to describe the strategy. Participants also had to indicate if they thought they changed strategy over the task and to add precision if they did. For the analyses, only the percentage of retrieval reported was considered.

2.3. Procedure

The computerized task was constructed using Psychopy (Peirce et al., 2019). Stimuli were displayed in black on a white background in the center of the screen. EEG signal was recorded in a dimly lighted electromagnetically shielded room using ActiveTwo system from Biosemi with 64 electrodes (10–20 layout). Before the two computerized tasks, resting state was recorded for each participant for 5 min. During this time, participants were asked to close their eyes and relax.

Preprocessing of the EEG data was performed using the mne package on python (Gramfort et al., 2013, 2014). For each participant, malfunctioning electrodes were interpolated (less than 10% of the electrodes) and eye movements were removed using independent component analyses. Data were then filtered using a notch filter at 50 Hz, a high pass filter of 0.01 and a low pass filter of 60 Hz. Data were re-referenced using the average reference.

For early and N2 components, epochs of correctly solved problems for which the maximum peak to peak amplitude for each electrode was below 100 μV were extracted from 100 ms before problem apparition to 650 ms after. The 100 ms before stimulus apparition were used as baseline correction. On average participants had 418 valid epochs, with a minimum of 315 epochs. Epochs were then average for each participant and each condition (Problem Type: tie, non-tie and 1-problems, and Operation: multiplication and addition) and each electrode.

For the N400 component, epochs from 100 ms before answer presentation to 450 ms after were extracted. Trials constructed with the correct answer were considered separately than trials constructed with the incorrect answer. The 100 ms before stimulus apparition were used as baseline correction. On average participants had 441 valid epochs, with a minimum of 381 epochs. Then for each participant, each condition and each electrode, averaged epochs of the trials with correct answers were subtracted from averaged epochs of the trials with incorrect answers.

2.4. Statistical analyses

Data were then extracted, and analyses were performed in R (R Core Team, 2017). For the analyses, electrodes were grouped according to the area and the laterality they were on. Nine different areas were considered for the analyses: Occipital (O1, O2, Oz), ParietoOccipital (PO3, PO4, PO7, PO8, POz), Parietal (P1, P10, P2, P3, P4, P5, P6, P7, P8, P9, Pz), CentroParietal (CP1, CP2, CP3, CP4, CP5, CP6, CPz), Central (C1, C2, C3, C4, C5, C6, Cz), FrontoCentral (FC1, FC2, FC3, FC4, FC5, FC6, FCz), Frontal (Fz, F1, F2, F3, F4, F5, F6, F7, F8), AnteroFrontal (AF3, AF4, AF7, AF8, AFz) and Prefrontal (Fpz, Fp1, Fp2). For the laterality, three different zones were considered. Electrodes with an even number were situated on the right and therefore considered as being in the right hemisphere while electrodes with an odd number were on the left hemisphere. Electrodes ending with a "z" (e.g., Cz) were on the midline.

Following previous studies on ERP in arithmetic (Prieto-Corona et al., 2010; Van Beek et al., 2014) and visual inspection of our data, we considered three different time windows after the stimulus presentation: 100–150 ms (P1), 150–250 ms (N1) and 250–400 ms (N2) and one after the response presentation: 250–350 ms (N400). The selection of electrodes for the analyses of each component was also made following previous literature and visual inspection. Concerning the choice of the electrode, for the analyses of early components, Occipital, ParietoOccipital and Parietal areas were chosen. For the N2 component, pre-frontal to central electrodes were selected with the Central, FrontoCentral, Frontal, AnteroFrontal and PreFrontal areas. Finally, for the N400 component Frontal, FrontoCentral, Central, CentroParietal, and Parietal areas were selected. Analyses were performed on the mean amplitude, the peak amplitude and the peak latency. Mean amplitude, peak amplitude and peak latency were calculated in the time window of interest for each electrode separately. These values were then averaged for each area and each laterality (midline, right and left hemispheres).

Analyses were conducted in R (R Core Team, 2017). ANOVAs were conducted using the afex (Singmann et al., 2018) and the emmeans library (Lenth, 2018). For each component, ANOVAs were performed within the corresponding time window on the mean amplitude, the peak amplitude or on the peak latency depending on the analysis performed. ANOVAs were performed with Problem Type (tie, non-tie and 1-problems), Operation (addition and multiplication), Area (which vary according to the component as described in the previous paragraph) and Laterality (left, right, midline) as within factors. To simplify the result section, for ERP analyses, only main effects and significant interactions containing at least one of variable of interest were reported. Those variables of interest were Problem Type and Operation. Results were corrected using the Greenhouse-Geisser correction, when needed.

For the N2 component, additional exploratory Bayesian paired t-tests with the default prior (Cauchy with scale 0.707) were performed to compare tie and non-tie problems using the BayesFactor library (Morey and Rouder, 2018). For information, a BF_{10} above 1 implies that more evidence for the alternative than the null hypothesis. The evidence in favor of the alternative hypothesis is anecdotal when BF_{10} is between 1 and 3, moderate when BF_{01} is between 3 and 10 and strong when BF_{01} is more than 10. When BF_{10} is smaller than 1 there is more evidence in favor of the null than the alternative hypothesis. Evidence for the null hypothesis with a BF_{10} between 1/3 and 1 is anecdotal, between 1/10 and 1/3 is moderate and less than 1/10 is strong.

For solution times, only correct trials were considered. Because solution times usually presented skewed distribution, the median of this variable was calculated for each problem and each participant prior to applying any analyses. Behavioural analyses were performed with Problem Type (tie, non-tie and 1-problems) and Operation (addition and multiplication) as within variables.

3. Results

3.1. Behavioural data

3.1.1. Accuracy

An ANOVA on the percentages of correct answers was performed with Problem Type (tie, non-tie and 1-problems) and Operation (addition and multiplication) as within variables. Results indicated no main effect of Problem Type, $F(1.93, 40.52) = 1.84$, $p = .17$, or Operation, $F(1, 21) = 1.26$, $p = .27$. The interaction between the two variables was not significant, $F(1.64, 34.51) = 3.05$, $p = .07$. On average problems were correct in 97.47% of the trials.

3.1.2. Solution times

An ANOVA on solution times associated with correctly solved problems was performed with Problem Type (tie, non-tie and 1-problems) and Operation (addition and multiplication) as within variables (Fig. 1). Results indicated a significant main effect of Problem Type, $F(1.55, 32.48) = 3.59$, $\eta^2_p = .15$, $p = .0497$. A series of Holm corrected contrasts indicated that solution times for tie problems (526 ms) were not significantly different than for non-tie (535 ms), $t(42) = -1.77$, $p = .16$, and 1-problems (521 ms), $t(42) = 0.85$, $p = .40$, but that solution times for non-tie problems were longer than for 1-problems, $t(42) = 2.63$, $p = .04$. There was no significant main effect of Operation, $F < 1$, and no interaction between Problem Type and Operation, $F(1.74, 36.60) = 2.82$, $p = .08$.

3.1.3. Percentages of reported retrieval

An ANOVA on the percentages of reported retrieval was performed with Problem Type (tie, non-tie and 1-problems) and Operation (addition and multiplication) as within variables (Fig. 2). The main effect of Operation was significant, $F(1, 21) = 4.60$, $\eta^2_p = .18$, $p = .04$, with a higher percentage of reported retrieval for multiplication (95%) than for addition (82%). There was also a significant main effect of Problem Type, $F(1.99, 41.69) = 4.82$, $\eta^2_p = .19$, $p = .01$. A series of Holm corrected contrasts indicated that the percentage of reported retrieval for tie problems (95%) was significantly higher than for non-tie problems (83%), $t(42) = 3.08$, $p = .01$. However, the percentage of reported retrieval was not significantly different between 1-problems (88%) and non-tie, $t(42) = -1.22$, $p = .23$, or tie problems, $t(42) = 1.86$, $p = .14$. Finally, the interaction between Problem Type and Operation was significant, $F(1.89, 39.60) = 4.43$, $\eta^2_p = .17$, $p = .02$. A series of Holm corrected contrasts indicated that for addition the difference in percentage of retrieval was significantly higher for tie than for non-tie problems, $t(84) = 3.41$, $p = .01$, and than for 1-problems, $t(84) = 3.41$, $p = .01$, (see Fig. 2 for the percentages of retrieval in each condition). The other contrasts were not significant ($p = 1$, except for multiplication, non-tie – 1-problems: $t(84) = -1.72$, $p = .36$).

In order to take the impact of reported strategies on the N2 component into account in exploratory analyses, participants were separated in 3 groups following LeFevre's distinction (2003). Eight participants who indicated that they systematically used retrieval were considered as high retrievers. Four participants indicated using alternative strategies for 1 or 2 problems over all problems (95% of retrieval on average). Finally, 10 participants who indicated that they did not use retrieval for at least 3 problems over all problems (69% of retrieval on average) were considered as low retrievers. Because the second group only contained 4 participants, only the high and low retriever groups were considered for the exploratory analyses that took the strategies into account. However, all participants, irrespective of the strategy they reported, were considered for all the other analyses.

3.2. ERP data

3.2.1. Early component analyses

Concerning P1 mean amplitude, the ANOVA revealed no main effect

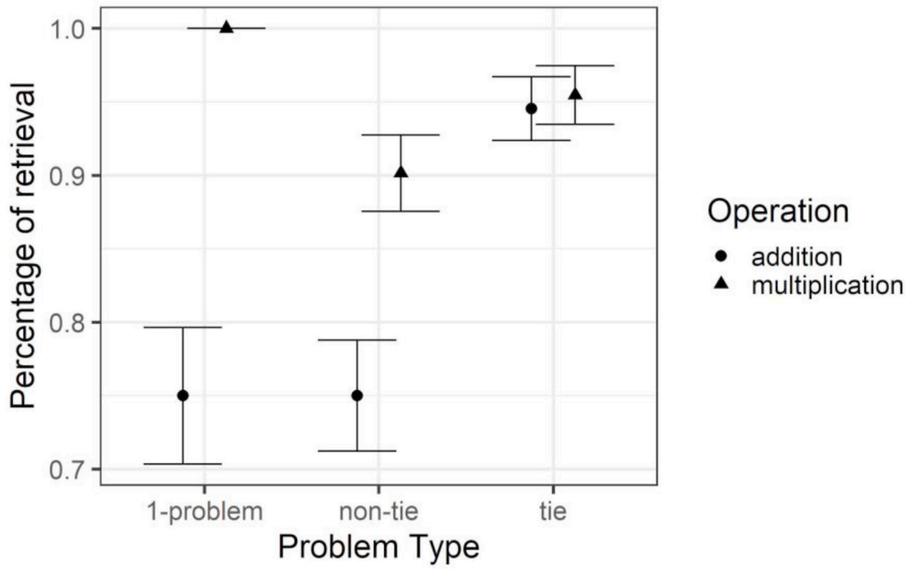

Fig. 2. Mean percentage of reported retrieval (and standard errors) according to Problem Type and Operation.

of Type, $F < 1$, or Operation, $F(1, 21) = 1.81$, $p = .19$. The interaction between operation and laterality was significant, $F(1.95, 40.86) = 4.08$, $\eta^2_p = .16$, $p = .03$ and indicated that the amplitude was higher for addition than multiplication in the midline and right hemisphere but that it was the inverse in the left hemisphere (Fig. 3). However, a series of Holm corrected contrasts indicated that the differences between addition and multiplication did not reach significance whatever the laterality (left: $t(38.8) = -0.40$, $p = .69$, midline: $t(38.8) = 2.12$, $p = .12$, right: $t(38.8) = 1.66$, $p = .21$).

Concerning P1 peak amplitude, the ANOVA indicated no significant main effect of Problem Type, $F < 1$. The main effect of Operation was significant, $F(1, 21) = 6.78$, $\eta^2_p = .24$, $p = .02$, with lower amplitude of the peak for multiplication (2.19 μV) than for addition (2.54 μV). The interaction between Operation and Area was significant, $F(1.41, 29.67) = 4.45$, $\eta^2_p = .17$, $p = .03$. A series of Holm corrected contrasts indicated that the difference between addition and multiplication was significant in the Occipital, $t(39.2) = 3.27$, $p = .01$, and ParietoOccipital, $t(39.2) = 2.69$, $p = .02$, but not in the Parietal area, $t(39.2) = 0.58$, $p = .57$.

Concerning P1 peak latency, the ANOVA revealed no main effect of Problem Type, $F(1.65, 34.71) = 1.29$, $p = .28$. The main effect of Operation was significant, $F(1, 21) = 38.65$, $\eta^2_p = .65$, $p < .001$, with earlier peak for multiplication (129 ms) than for addition (134 ms).

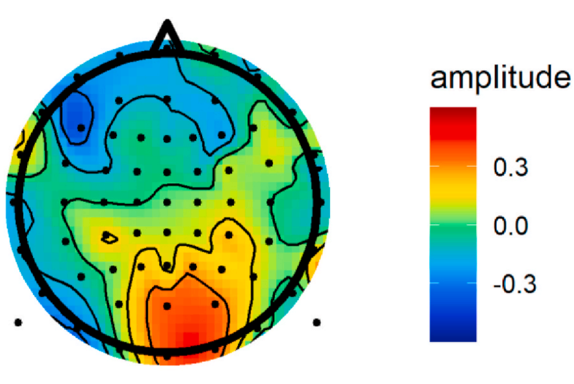

Fig. 3. (in color) Difference in mean amplitude (μV) between addition and multiplication on the whole head during the P1 time window (100–150 ms). (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of this article.)

Concerning N1 mean amplitude, the ANOVA indicated no significant main effect of either Problem Type, $F(1.90, 39.95) = 1.82$, $p = .18$, or Operation, $F < 1$. None of the interaction with Problem Type or Operation reach significance.

Concerning N1 peak amplitude, the ANOVA indicated no main effect of Problem Type, $F(1.69, 35.56) = 1.64$, $p = .21$, or of Operation, $F < 1$. The interaction between Operation, Problem Type and Area was significant, $F(2.60, 54.58) = 3.11$, $\eta^2_p = .13$, $p = .04$. A series of Holm corrected contrasts showed, in the occipital area for addition, more negative amplitude for 1-problems compared to non-tie problems, $t(116.3) = 3.73$, $p = .01$ (Fig. 4). All other contrasts were not significant ($p = 1$, except for additions in the Occipital area: 1-problems - tie: $t(116.3) = 2.63$, $p = .16$, and in the ParietoOccipital area: 1-problems – non-tie $t(116.3) = 2.35$, $p = .32$, 1-problems - tie: $t(116.3) = 2.02$, $p = .69$).

Concerning N1 peak latency, the ANOVA revealed no significant main effect of Problem Type, $F < 1$. There was a main of Operation, $F(1, 21) = 7.00$, $\eta^2_p = .25$, $p = .02$, with earlier peak appearance for multiplication (193 ms) than for addition (197 ms). The interaction between Operation and Laterality was significant, $F(1.84, 38.55) = 5.19$, $\eta^2_p = .20$, $p = .01$. A series of Holm corrected contrasts indicated that the peak for multiplication appeared significantly earlier than the one for addition in the midline, $t(42.3) = 4.02$, $p < .001$, but not in the left, $t(42.3) = 1.38$, $p = .35$, or right hemisphere, $t(42.3) = 1.07$, $p = .35$. Finally, the interaction between Operation, Area and Laterality was significant, $F(1.92, 40.42) = 3.42$, $\eta^2_p = .14$, $p = .04$. A series of Holm corrected contrasts indicated a difference in peak latency between addition and multiplication in the midlines only in the Parietal area, $t(121.7) = 2.27$, $p < .001$ ($p = 1$ for all other contrasts, except in the midline in the Occipital area, $t(121.7) = 2.38$, $p = .15$).

To sum up, looking at the mean amplitude and the peak amplitude and latency of the P1 and N1 components, no difference between tie and non-tie problems was found. All of the significant results involving Problem Type were related to differences between 1-problems and non-tie problems. More precisely, 1-problems presented a more negative N1 peak amplitude than non-tie problems in occipital area for addition. Analyses on early components also revealed differences between addition and multiplication. More specifically, P1 and N1 peaks appeared earlier for multiplication than addition, especially in the midline of the Parietal Area for the N1 component. However, these differences were extremely small, with only 3 or 4 ms on average. Moreover, a difference in the P1 peak amplitude was observed with a higher peak for addition

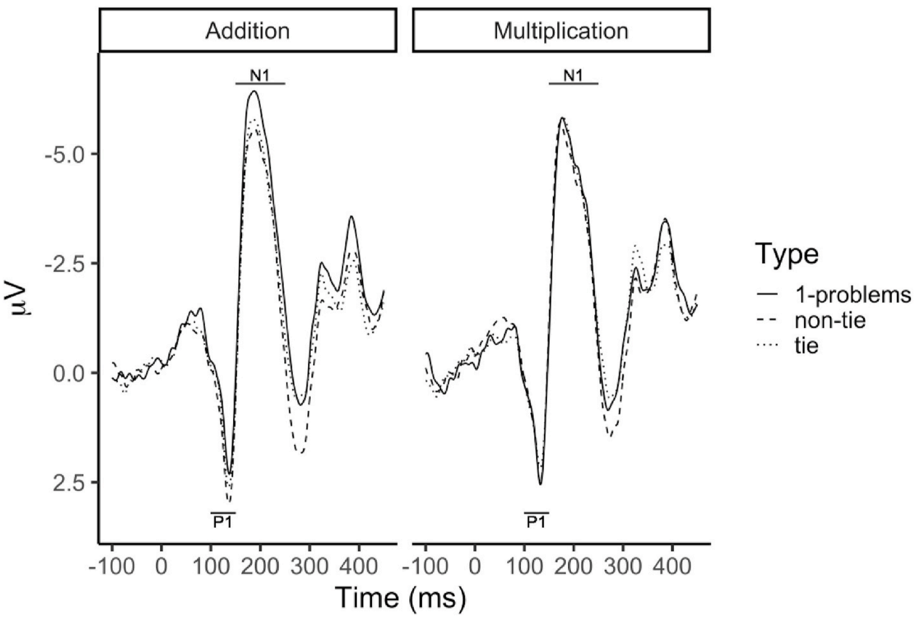

Fig. 4. Mean ERP for the electrode O1 (left occipital) for addition and multiplication for the three Problem Types.

than multiplication, particularly in the Occipital and ParietoOccipital area and we observed a difference in topography between addition and multiplication in P1 mean amplitude.

3.2.2. N2 component analyses

Concerning N2 mean amplitude, the ANOVA indicated no main effect of Operation, $F < 1$, but a significant main effect of Type, $F(1.88, 39.58) = 5.18$, $\eta^2_p = .20$, $p = .01$. A series of Holm corrected contrasts indicated that mean amplitude for non-tie problems (−0.42 µV) was significantly lower than for tie (−0.12 µV), $t(42) = 2.98$, $p = .01$, and 1-problems (−0.17 µV), $t(42) = 2.54$, $p = .03$, but that it did not significantly differ between tie and 1-problems, $t(42) = 0.44$, $p = .66$. The interaction between Problem Type and Laterality was significant, $F(2.57, 53.94) = 3.07$, $\eta^2_p = .13$, $p = .04$. A series of Holm corrected contrasts indicated that there was a significant difference in mean amplitude in the midline between tie and non-tie problems, $t(83.3) = 4.17$, $p < .001$, but that all of the other difference did not reach significance ($p = 1$, except in the midline: 1-problems – non-tie: $t(83.3) = 2.73$, $p = .06$ and right hemisphere: tie - non-tie: $t(83.3) = 2.07$, $p = .25$, and 1-problems – non-tie: $t(83.3) = 2.62$, $p = .07$).

Concerning N2 peak amplitude, the ANOVA indicated no main effect of Problem Type, $F(1.71, 35.93) = 2.44$, $p = .11$, or of Operation, $F < 1$. No interaction involving either Operation or Problem Type was significant.

Concerning N2 peak latency, the ANOVA revealed no main effect of Problem Type, $F(1.69, 35.54) = 2.27$, $p = .13$, or Operation, $F < 1$. The interaction between Problem Type and Area was significant, $F(4.09, 85.86) = 2.68$, $\eta^2_p = .11$, $p = .04$, and indicated that the peak for tie problems appeared later than the one of 1-problems and non-tie problems for FrontoCentral, Frontal and AnteroFrontal areas, but not for Central or Prefrontal areas. However, a series of Holm corrected contrasts did not indicate significant differences between tie, non-tie and 1-problems in any areas ($p = 1$, except for Frontocentral: tie – non-tie: $t(180.3) = 2.46$, $p = .21$, tie – 1-problems: $t(180.3) = 2.61$, $p = .15$; for Frontal: tie – non-tie: $t(180.3) = 2.38$, $p = .24$, tie – 1-problems: $t(180.3) = 2.00$, $p = .51$; for AnteroFrontal: tie – non-tie: $t(180.3) = 2.30$, $p = .27$; and for PreFrontal: tie – 1-problems: $t(180.3) = -1.93$, $p = .55$). Finally, the interaction between Operation, Area and Laterality was significant, $F(4.86, 101.96) = 2.36$, $\eta^2_p = .10$, $p = .047$. However, none of the Holm corrected contrasts were significant ($p = 1$ for all other contrasts, except in right hemisphere in the Central area, $t(146.6) = 1.97$, $p = .75$).

For this N2 component, differential results were expected for low and high retrievers, depending on theories that explained the difference between tie and non-tie problems. More specifically, if non-retrieval strategies are used more often for non-tie than tie problems, the difference between problem type for the N2 component should be larger for low than high retrievers. In contrast, if tie problems are easier to retrieve than non-tie problems, results on the N2 component should be similar for low and high retrievers. To further study the N2 component exploratory Bayesian paired t-tests were conducted to contrast tie and non-tie problems in high and low retrievers (Fig. 5). Note that those analyses were conducted on small sample sizes and should thus be considered with care.

Concerning the mean amplitude, Bayesian paired *t*-test results indicated that the evidence for a difference between tie and non-tie problems was anecdotal for high retrievers (−0.40 µV and −0.75 µV respectively), $BF_{10} = 2.89$, but strong for low retrievers (0.17 µV and −0.25 µV respectively), $BF_{10} = 10.95$. Concerning the peak amplitude, anecdotal evidence for the null hypothesis was found for high retrievers (−2.39 µV and −2.62 µV for tie and non-tie problems respectively), $BF_{10} = 0.99$, while strong evidence for a difference between tie (−1.62 µV) and non-tie problems (−2.04 µV) was observed for low retrievers, $BF_{10} = 24.06$. Finally, concerning the peak latency, results indicated anecdotal evidence in favor of the null hypothesis for high retrievers (316 ms for both tie and non-tie problems), $BF_{10} = 0.34$, but in favor of the alternative hypothesis for low retrievers (315 ms and 308 ms for tie and non-tie problems respectively), $BF_{10} = 1.10$.

To sum up, analyses of the N2 component indicated more negative mean amplitude for non-tie problems than for tie and 1-problems especially in the midline. Exploratory Bayesian t-tests indicated strong evidence of a difference in mean and peak amplitude between tie and non-tie problems for low retrievers. However, there was no conclusive evidence for such a difference for high retrievers. No differences were found between operations, and no interaction between Operation and Problem Type was found for the N2 component.

3.2.3. N400 component analyses

Concerning N400 mean amplitude, the ANOVA indicated no main effect of Problem Type or Operation, $Fs < 1$. No interaction involving either Operation or Problem Type was significant.

Concerning N400 peak amplitude, the ANOVA indicated no main effect of Operation, $F < 1$, or Problem Type, $F(1.80, 37.80) = 1.88$, $p =$

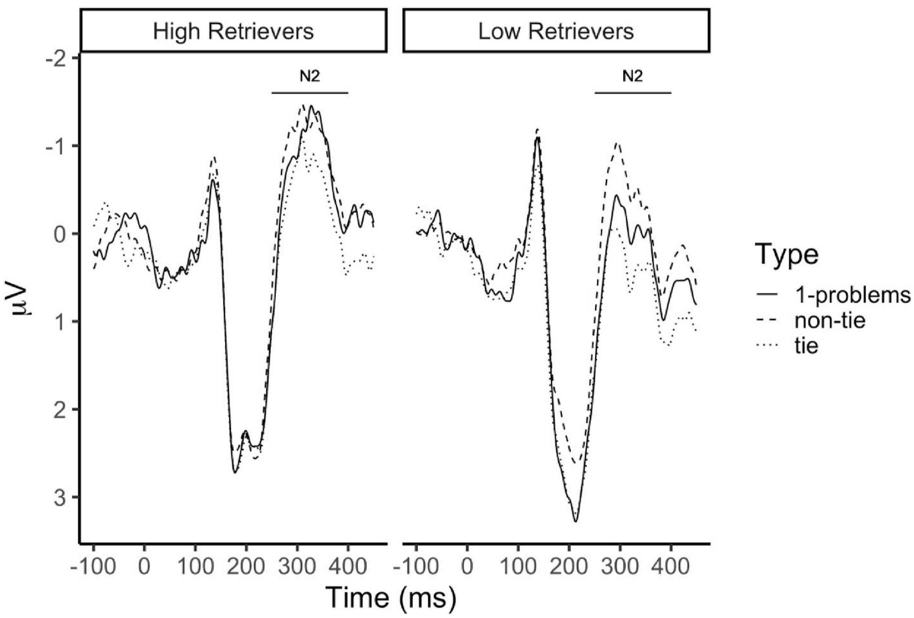

Fig. 5. Mean ERP over midline electrodes for all areas of interest for the N2 component, for the two Retriever groups and the three Problem Type separately.

.17. No interaction involving either Operation or Problem Type was significant.

Concerning N400 peak latency, the ANOVA indicated no main effect of Operation, $F < 1$, or Problem Type, $F(1.77, 37.22) = 1.24$, $p = .30$. The interaction between Problem Type and Area was significant, $F(4.81, 100.95) = 2.67$, $\eta^2_p = .11$, $p = .03$, and indicated that the N400 peak appeared faster for 1-problems than other problems in Parietal and CentroParietal areas but not in the Frontal, FrontoCentral and Central areas. However, none of the Holm corrected contrasts reached significance ($p = 1$, except in the Frontal area: tie – 1-problems: $t(199.6) = -2.00$, $p = .57$, FrontoCentral area: tie – non-tie: $t(199.6) = 2.86$, $p = .06$, and 1-problems – non-tie: $t(199.6) = 2.25$, $p = .36$, and Parietal area tie – 1-problems: $t(199.6) = 2.06$, $p = .53$, and 1-problems – non-tie: $t(199.6) = -1.93$, $p = .61$). Finally, the Problem Type × Operation × Area × Laterality interaction was significative, $F(8.55, 179.58) = 2.25$, $\eta^2_p = .10$, $p = .02$. However, none of the Holm corrected contrasts reached significance ($p = 1$, except in the midline in the FrontoCentral area for addition: tie – 1-problems: $t(1024.6) = 2.61$, $p = .80$, tie – non-tie: $t(1024.6) = 2.73$, $p = .57$, for multiplication: 1-problems – non-tie: $t(1024.6) = 3.22$, $p = .12$, tie – non-tie: $t(1024.6) = 2.90$, $p = .34$ and in the right hemisphere in the Parietal area for addition: tie – 1-problems: $t(1024.6) = 2.67$, $p = .66$).

To sum up, results indicated no significant differences of mean and peak amplitude of the N400 component. The peak latency for each problem type seemed to vary depending on the Area, the operation and the laterality, but none of the contrasts reached significance.

4. Discussion

In this ERP study, our first goal was to better understand the cognitive mechanisms underlying the fact that tie problems are solved quicker and more accurately than non-tie problems. We examined three hypotheses that could explain this tie advantage. First, tie problems could be encoded more easily than non-tie problems. However, we did not find difference between tie and non-tie problems in early ERP components, which does not confirm this hypothesis. Second, tie problems could be solved more often through memory retrieval than non-tie problems. In accordance with this hypothesis, we found a less negative amplitude for the N2 component for tie than non-tie problems, suggesting that tie problems, at least with operands up to 5, are more often solved through less working-memory demanding strategies than non-tie problems, such as retrieval. As in previous studies (e.g., Campbell and Gunter, 2002; LeFevre et al., 1996a, 1996b; Uittenhove et al., 2016), this interpretation was corroborated by verbal reports of our participants concerning their strategies. Nevertheless, this line of interpretation is questioned by the fact that multiplication problems do not present less negative amplitude of the N2 component than addition, whereas they were also expectedly reported as being solved more often by retrieval than addition (see also Campbell and Gunter, 2002). Thus, a change in the frequency of reported retrieval is not systematically associated with a change in amplitude of the N2 component. This observation could cast doubt on the fact that the difference in the N2 component between tie and non-tie problems stems from the use of different strategies. Alternatively, it could be that participants failed to accurately report the strategies they use to solve the problems (Kirk and Ashcraft, 2001; Lucidi and Thevenot, 2014; Thevenot et al., 2010) or that the way we collected verbal reports was not optimum. Indeed, instead of using a concurrent verbal report methodology where the strategy used is asked after each problem solving, we used a retrospective verbal report protocol where strategies are reported after the experiment. This choice prevented us from conducting analyses establishing a strict correspondence between strategy and N2 component modulation. Instead, we categorized participants on whether they only reported using retrieval or often report non-retrieval strategies. At least, this methodology ensured that the strategies used by participants during the experiment were not modified by verbal reports because of working memory additional demand or because of increased self-consciousness on the strategies used (Nisbett and Wilson, 1977). Our categorization allowed us to conduct analyses to explore whether the difference between tie and non-tie problems for the N2 component was similar for high and low retrievers. Results indicated strong evidence in favor of a difference between tie and non-tie problems in both peak and mean amplitude for low retrievers. In contrast, only anecdotal and contradictory evidence was found for high retrievers. Despite the fact that these results are highly coherent for low retrievers, they should be considered with care because they have been performed on small individual samples. Still, they suggest that verbal reports capture objective discrepancies in strategy use, which are observable in N2 component modulations. Finally, concerning the third hypothesis, tie problems could be subjected to less interference than non-tie problems. In this case, they should require less attentional resources to be solved and thus present less negative N2 amplitude compared to non-tie problems. We have already described that this result was observed in our exploratory analyses only in low retrievers, which could have corroborated De Visscher and Noël's conclusion (2014a, 2014b) that frequent retrievers show little sensitivity to interference compared with low retrievers. Nevertheless, this conclusion is contradicted by the fact that high retrievers did not seem to present a less negative N2 component than low retrievers. Moreover, within the interference hypothesis, larger negative amplitude of the N400 component for tie than for non-tie problems would also have been expected and it was not the case. All in all, our results do not provide strong evidence for the interference hypothesis, at least concerning the small range of problems that we studied.

The second goal of our study was to better understand the process and mechanisms underlying the resolution of problems involving a 1 (i.e., 1-problems). Those problems are sometimes solved using a rule, which consists for multiplication in giving the non-1 operand, namely the algebraic multiplicative-identity rule, and for addition in retrieving the number in the counting sequence just after the non-1 operand, namely the number-after rule (e.g., Baroody, 1995; 2018; Baroody et al., 2012). In adequation with previous conclusion of the literature showing that 1-problems are solved more efficiently than other non-tie problems (Bagnoud et al., 2021; Baroody, 1995; LeFevre et al., 1996a, 1996b; Uittenhove et al., 2016), we found a less negative N2 amplitude for 1-problems than other non-tie problems, which suggests that they require less working-memory and attentional resources to be solved than other problems. Following Grabner et al's. (2021).'s conclusions, this can lead to two different interpretations. Problems involving a 1 could be solved more often through retrieval strategies than other non-tie problems or, alternatively, they could be solved by the non-arithmetic rules described just above. In the case of addition, our results on the percentages of reported retrieval give credit to the second interpretation. Indeed, whereas 94% of tie problems were reported as having been solved by a retrieval strategy, this percentage fell to 75% for 1-problems. This difference was significant and suggests that tie and non-tie problems were not solved using the same strategies. Given that our behavioural and neuro-electrophysiological results and previous results of the literature strongly support the idea that retrieval is the dominant strategy for addition tie problems, it can be concluded that a different strategy was used for 1-problem. Because this strategy is less demanding than for non-tie problems, the hypothesis of the use of the number after rule is the more plausible one. Interestingly, for the 25% of problems of 1-problems that were not reported as retrieved, they were all reported as having been solved via counting whereas the category "other" was never used. This means that individuals do not have conscious access to their use of the number after rule. Finally and concerning the N400 component, a more negative peak and mean amplitude was not found for 1-problems than non-tie problems, suggesting that they are not less prone to interference than other problems. This challenges our interpretation that 1-problems are solved by a rule because in such case, they should not be part of the arithmetical network. As in Grabner's et al. Grabner et al., (2021), we are therefore left in a situation where the 1-problem advantage can be either explained by more reliance on retrieval or the use of a rule.

In addition to the differences observed in ERPs depending on the type of problems, our analyses revealed some differences between operations. Concerning early components, the P1 and N1 peak appeared earlier for multiplication than for addition. However, the difference in peak appearance is very small, with only 3–5 ms, and it is therefore difficult to definitely interpret them as reflecting strategy difference. Still, in addition to timing differences, the topography of P1 was also different between the two operations with more positive mean amplitude in the midline and right hemisphere and less positive amplitude in the left hemisphere for addition than for multiplication. Moreover, the peak amplitude of P1 was higher for addition than multiplication, particularly in the Occipital and ParietoOccipital area. These differences in P1 amplitude could reflect differences in the complexity of the encoding of the operands depending on the operation (Thevenot et al., 2011; Thevenot & Barrouillet, 2006, 2010), which would indicate that multiplication and addition problem solving are not subjected to the same cognitive mechanisms (Díaz-Barriga Yáñez et al., 2020; Fayol and Thevenot, 2012; Mathieu et al., 2018; Zhou et al., 2006).

To sum up, results of the ERP analyses did not provide evidence for the fact that the tie advantage is due to easier encoding or less interference effects for tie than non-tie problems. Rather, ERP analyses reveal that less working-memory and attentional resources are needed to solve tie than non-tie problems. On a more theoretical level, our results suggest that contrary to Campbell and Gunter's assumption (Campbell and Gunter, 2002), the tie advantage is not due to more efficient retrieval of the associated answers for tie than non-tie problems but rather, as suggested by LeFevre (LeFevre et al., 2003; LeFevre et al., 1996a, 1996b), to the use of different solving strategies for these two categories of problems. Retrieval could be used more often for tie problems than non-tie problems, and costlier non-retrieval procedures could be used more often for non-tie problems. Concerning 1-problems, our ERP analyses also suggest less attentional and working-memory demands than for other non-tie problems. However, no evidence was found in favor of a difference in interference with non-tie problems. Further studies are then necessary to know whether 1-problems are represented in the same arithmetic fact network as other problems, whether they are solved using arithmetical procedures or whether they are solved through the use of a rule such as for addition finding the number just after the non-1 operand in an ordinal numerical sequence (e.g., Baroody, 1983; 1995; Baroody et al., 2012).

Declaration of competing interest

We declare no conflict of interest.

Acknowledgments

We would like to thank Amélie Wassenberg for her help in data collection.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.neuropsychologia.2021.107771.

Credit author statements

Jeanne Bagnoud, Conceptualization, Methodology, Software, Formal analysis, Investigation, Writing, Visualization. Jasinta Dewi, Conceptualization, Formal analysis. Catherine Thevenot, Conceptualization, Writing, Supervision, Funding acquisition.

Funding

This work was supported by the Swiss National Science Foundation [grant number 100019_169208].

References

Bagnoud, J., Dewi, J., Castel, C., Mathieu, R., Thevenot, C., 2021. Developmental changes in size effects for simple tie and non-tie addition problems in 6- to 12-year-old children and adults. J. Exp. Child Psychol. 201, 104987. https://doi.org/10.1016/j.jecp.2020.104987.

Baroody, A.J., 1983. The development of procedural knowledge: an alternative explanation for chronometric trends of mental arithmetic. Dev. Rev. 3 (2), 225–230. https://doi.org/10.1016/0273-2297(83)90031-X.

Baroody, A.J., 1995. The role of the number-after rule in the invention of computational shortcuts. Cognit. InStruct. 13 (2), 189–219. https://doi.org/10.1207/s1532690xci1302_2.

Baroody, A.J., 2018. A commentary on Chen and Campbell (2017) : is there a clear case for addition fact recall? Psychon. Bull. Rev. 25 (6), 2398–2405. https://doi.org/10.3758/s13423-018-1440-y.

Baroody, A.J., Eiland, M.D., Purpura, D.J., Reid, E.E., 2012. Fostering at-risk kindergarten children's number sense. Cognit. InStruct. 30 (4), 435–470. https://doi.org/10.1080/07370008.2012.720152.
Baroody, A.J., Purpura, D.J., 2017. Early number and operations: whole numbers. In: Cai, J. (Ed.), Compendium for Research in Mathematics Education. National Council of Teachers of Mathematics, pp. 308–354.
Baroody, A.J., Purpura, D.J., Eiland, M.D., Reid, E.E., 2015. The impact of highly and minimally guided discovery instruction on promoting the learning of reasoning strategies for basic add-1 and doubles combinations. Early Child. Res. Q. 30, 93–105. https://doi.org/10.1016/j.ecresq.2014.09.003.
Blankenberger, S., 2001. The arithmetic tie effect is mainly encoding-based. Cognition 82 (1), B15–B24. https://doi.org/10.1016/S0010-0277(01)00140-8.
Campbell, J.I.D., 1995. Mechanisms of simple addition and multiplication: a modified network-interference theory and simulation. Math. Cognit. 1 (2), 121–164.
Campbell, J.I.D., Arbuthnott, K.D., 2010. Effects of mixing and cueing simple addition and multiplication. Eur. J. Cognit. Psychol. 22 (3), 422–442. https://doi.org/10.1080/09541440902903629.
Campbell, J.I.D., Gunter, R., 2002. Calculation, culture, and the repeated operand effect. Cognition 86 (1), 71–96. https://doi.org/10.1016/S0010-0277(02)00138-5.
Campbell, J.I.D., Oliphant, M., 1992. Representation and retrieval of arithmetic facts: a network-interference model and simulation. In: Campbell, J.I.D. (Ed.), Advances in Psychology, 91. The Nature and Origins of Mathematical Skills, vol. 91, pp. 331–364.
Campbell, J.I.D., Xue, Q., 2001. Cognitive arithmetic across cultures. J. Exp. Psychol. Gen. 130 (2), 299–315. https://doi.org/10.1037/0096-3445.130.2.299.
Clements, D., 1999. Subitizing : what is it? Why teach it? Teach. Child. Math. 5, 400–405.
Cragg, L., Richardson, S., Hubber, P.J., Keeble, S., Gilmore, C., 2017. When is working memory important for arithmetic? The impact of strategy and age. PloS One 12 (12), e0188693. https://doi.org/10.1371/journal.pone.0188693.
De Smedt, B., Swillen, A., Devriendt, K., Fryns, J.P., Verschaffel, L., Ghesquière, P., 2007. Mathematical disabilities in children with velo-cardio-facial syndrome. Neuropsychologia 45 (5), 885–895. https://doi.org/10.1016/j.neuropsychologia.2006.08.024.
De Visscher, A., Noël, M.-P., 2014a. The detrimental effect of interference in multiplication facts storing: typical development and individual differences. J. Exp. Psychol. Gen. 143 (6), 2380–2400. https://doi.org/10.1037/xge0000029.
De Visscher, A., Noël, M.-P., 2014b. Arithmetic facts storage deficit: the hypersensitivity-to-interference in memory hypothesis. Dev. Sci. 17 (3), 434–442. https://doi.org/10.1111/desc.12135.
Díaz-Barriga Yáñez, A., Couderc, A., Longo, L., Merchie, A., Chesnokova, H., Langlois, E., Thevenot, C., Prado, J., 2020. Learning to run the number line: the development of attentional shifts during single-digit arithmetic. Ann. N. Y. Acad. Sci.
El Yagoubi, R., Lemaire, P., Besson, M., 2003. Different brain mechanisms mediate two strategies in arithmetic: evidence from Event-Related brain Potentials. Neuropsychologia 41 (7), 855–862. https://doi.org/10.1016/S0028-3932(02)00180-X.
Fayol, M., Thevenot, C., 2012. The use of procedural knowledge in simple addition and subtraction problems. Cognition 123 (3), 392–403. https://doi.org/10.1016/j.cognition.2012.02.008.
Grabner, R.H., Brunner, C., Lorenz, V., Vogel, S.E., Smedt, B.D., 2021. Fact retrieval or compacted procedures in arithmetic – a neurophysiological investigation of two hypotheses. J. Exp. Psychol. Learn. Mem. Cognit.
Graham, D.J., Campbell, J.I.D., 1992. Network interference and number-fact retrieval: evidence from children's alphaplication. Canadian Journal of Psychology/Revue Canadienne de Psychologie 46 (1), 65–91.
Gramfort, A., Luessi, M., Larson, E., Engemann, D.A., Strohmeier, D., Brodbeck, C., Goj, R., Jas, M., Brooks, T., Parkkonen, L., Hämäläinen, M., 2013. MEG and EEG data analysis with MNE-Python. Front. Neurosci. 7 https://doi.org/10.3389/fnins.2013.00267.
Gramfort, A., Luessi, M., Larson, E., Engemann, D.A., Strohmeier, D., Brodbeck, C., Parkkonen, L., Hämäläinen, M.S., 2014. MNE software for processing MEG and EEG data. Neuroimage 86, 446–460. https://doi.org/10.1016/j.neuroimage.2013.10.027.
Groen, G.J., Parkman, J.M., 1972. A chronometric analysis of simple addition. Psychol. Rev. 79 (4), 329–343. https://doi.org/10.1037/h0032950.
Hamann, M.S., Ashcraft, M.H., 1986. Textbook presentations of the basic addition facts. Cognit. InStruct. 3 (3), 173–192. https://doi.org/10.1207/s1532690xci0303_2.
Hinault, T., Lemaire, P., 2016. What does EEG tell us about arithmetic strategies? A review. Int. J. Psychophysiol. 106, 115–126. https://doi.org/10.1016/j.ijpsycho.2016.05.006.
Iguchi, Y., Hashimoto, I., 2000. Sequential information processing during a mental arithmetic is reflected in the time course of event-related brain potentials. Clin. Neurophysiol. 111 (2), 204–213. https://doi.org/10.1016/S1388-2457(99)00244-8.
Imbo, I., Vandierendonck, A., Fias, W., 2011. Passive hand movements disrupt adults' counting strategies. Front. Psychol. 2 https://doi.org/10.3389/fpsyg.2011.00201.
Jost, K., Hennighausen, E., Rösler, F., 2004. Comparing arithmetic and semantic fact retrieval: effects of problem size and sentence constraint on event-related brain potentials. Psychophysiology 41 (1), 46–59. https://doi.org/10.1111/1469-8986.00119_41_1.
Kirk, E.P., Ashcraft, M.H., 2001. Telling stories: the perils and promise of using verbal reports to study math strategies. J. Exp. Psychol. Learn. Mem. Cognit. 27 (1), 157–175. https://doi.org/10.1037/0278-7393.27.1.157.
LeFevre, J.-A., Bisanz, J., Daley, K.E., Buffone, L., Greenham, S.L., Sadesky, G.S., 1996a. Multiple routes to solution of single-digit multiplication problems. J. Exp. Psychol. Gen. 125 (3), 284–306. https://doi.org/10.1037/0096-3445.125.3.284.
LeFevre, J.-A., Sadesky, G.S., Bisanz, J., 1996b. Selection of procedures in mental addition: reassessing the problem size effect in adults. J. Exp. Psychol. Learn. Mem. Cognit. 22 (1), 216–230. https://doi.org/10.1037/0278-7393.22.1.216.
LeFevre, J.-A., Shanahan, T., DeStefano, D., 2004. The tie effect in simple arithmetic: an access-based account. Mem. Cognit. 32 (6), 1019–1031. https://doi.org/10.3758/BF03196878.
LeFevre, J.-A., Smith-Chant, B.L., Hiscock, K., Daley, K.E., Morris, J., 2003. Young adults' strategic choices in simple arithmetic: implications for the development of mathematical representations. In: Baroody, A.J., Dowker, A. (Eds.), The Development of Arithmetic Concepts and Skills: Constructing Adaptive Expertise. Lawrence Erlbaum Associates Publishers, pp. 203–228.
Lemaire, P., Fayol, M., 1995. When plausibility judgments supersede fact retrieval: the example of the odd-even effect on product verification. Mem. Cognit. 23 (1), 34–48. https://doi.org/10.3758/BF03210555.
Lenth, R., 2018. Emmeans: Estimated Marginal Means, Aka Least-Squares Means. https://CRAN.R-project.org/package=emmeans.
Lucidi, A., Thevenot, C., 2014. Do not count on me to imagine how I act: behavior contradicts questionnaire responses in the assessment of finger counting habits. Behav. Res. Methods 46 (4), 1079–1087. https://doi.org/10.3758/s13428-014-0447-1.
Luo, W., Liu, D., He, W., Tao, W., Luo, Y., 2009. Dissociated brain potentials for two calculation strategies. Neuroreport 20 (4), 360–364. https://doi.org/10.1097/WNR.0b013e328323d737.
Mathieu, R., Epinat-Duclos, J., Sigovan, M., Breton, A., Cheylus, A., Fayol, M., Thevenot, C., Prado, J., 2018. What's behind a "+" sign? Perceiving an arithmetic operator recruits brain circuits for spatial orienting. Cerebr. Cortex 28 (5), 1673–1684. https://doi.org/10.1093/cercor/bhx064.
Morey, R.D., Rouder, J.N., 2018. BayesFactor: Computation of Bayes Factors for Common Designs. https://CRAN.R-project.org/package=BayesFactor.
Newman, S.D., 2016. Does finger sense predict addition performance? Cognit. Process. 17 (2), 139–146. https://doi.org/10.1007/s10339-016-0756-7.
Niedeggen, M., Rösler, F., 1999. N400 effects reflect activation spread during retrieval of arithmetic facts. Psychol. Sci. 10 (3), 271–276. https://doi.org/10.1111/1467-9280.00149.
Nisbett, R.E., Wilson, T.D., 1977. Telling more than we can Know : verbal reports on mental processes. Psychol. Rev. 84, 231–259. https://doi.org/10.1037/0033-295X.84.3.231.
Núñez-Peña, M.I., Gracia-Bafalluy, M., Tubau, E., 2011. Individual differences in arithmetic skill reflected in event-related brain potentials. Int. J. Psychophysiol. 80 (2), 143–149. https://doi.org/10.1016/j.ijpsycho.2011.02.017.
Peirce, J., Gray, J.R., Simpson, S., MacAskill, M., Höchenberger, R., Sogo, H., Kastman, E., Lindeløv, J.K., 2019. PsychoPy2: experiments in behavior made easy. Behav. Res. Methods 51 (1), 195–203. https://doi.org/10.3758/s13428-018-01193-y.
Prieto-Corona, B., Rodríguez-Camacho, M., Silva-Pereyra, J., Marosi, E., Fernández, T., Guerrero, V., 2010. Event-related potentials findings differ between children and adults during arithmetic-fact retrieval. Neurosci. Lett. 468 (3), 220–224. https://doi.org/10.1016/j.neulet.2009.10.094.
R Core Team, 2017. R: A Language and Environment for Statistical Computing. R Foundation for Statistical Computing, Vienna, Austria. URL. https://www.R-project.org/.
Rabinowitz, M., Wooley, K.E., 1995. Much ado about nothing: the relation among computational skill, arithmetic word problem comprehension, and limited attentional resources. Cognit. InStruct. 13 (1), 51–71. https://doi.org/10.1207/s1532690xci1301_2.
Rütsche, B., Hauser, T.U., Jäncke, L., Grabner, R.H., 2015. When problem size matters: differential effects of brain stimulation on arithmetic problem solving and neural oscillations. PloS One 10 (3), e0120665. https://doi.org/10.1371/journal.pone.0120665.
Singmann, H., Bolker, B., Westfall, J., Aust, F., 2018. Afex: Analysis of Factorial Experiments. R Package Version 0.20-2. https://CRAN.R-project.org/package=afex.
Svenson, O., 1975. Analysis of time required by children for simple additions. Acta Psychol. 39 (4), 289–301. https://doi.org/10.1016/0001-6918(75)90013-X.
Szűcs, D., Csépe, V., 2005. The effect of numerical distance and stimulus probability on ERP components elicited by numerical incongruencies in mental addition. Cognit. Brain Res. 22 (2), 289–300. https://doi.org/10.1016/j.cogbrainres.2004.04.010.
Thevenot, C., Castel, C., Fanget, M., Fayol, M., 2010. Mental subtraction in high- and lower skilled arithmetic problem solvers: verbal report versus operand-recognition paradigms. J. Exp. Psychol. Learn. Mem. Cognit. 36 (5), 1242–1255. https://doi.org/10.1037/a0020447.
Thevenot, C., Barrouillet, P., 2006. Encoding numbers : behavioral evidence for processing-specific representations. Mem. Cognit. 34 (4), 938–948. https://doi.org/10.3758/BF03193439.
Thevenot, C., Barrouillet, P., 2010. Children's number processing is context dependent. Eur. J. Cognit. Psychol. 22 (3), 348–359. https://doi.org/10.1080/09541440903130743.
Thevenot, C., Barrouillet, P., Castel, C., Jimenez, S., 2011. Better elementary number processing in higher skill arithmetic problem Solvers : evidence from the encoding step. Q. J. Exp. Psychol. 64 (11), 2110–2124. https://doi.org/10.1080/17470218.2011.582129.
Uittenhove, K., Thevenot, C., Barrouillet, P., 2016. Fast automated counting procedures in addition problem solving: when are they used and why are they mistaken for retrieval ? Cognition 146, 289–303. https://doi.org/10.1016/j.cognition.2015.10.008.

Van Beek, L., Ghesquièr, P., De Smedt, B., Lagae, L., 2014. The arithmetic problem size effect in children: an event-related potential study. Front. Hum. Neurosci. 8 https://doi.org/10.3389/fnhum.2014.00756.

Zbrodoff, N.J., 1995. Why is 9+7 harder than 2+3? Strength and interference as explanations of the problem-size effect. Mem. Cognit. 23 (6), 689–700. https://doi.org/10.3758/BF03200922.

Zhou, X., Chen, C., Dong, Q., Zhang, H., Zhou, R., Zhao, H., Chen, C., Qiao, S., Jiang, T., Guo, Y., 2006. Event-related potentials of single-digit addition, subtraction, and multiplication. Neuropsychologia 44 (12), 2500–2507. https://doi.org/10.1016/j.neuropsychologia.2006.04.003.