

## Accepted Manuscript

Competition between injunctive social norms and conservation priorities gives rise to complex dynamics in a model of forest growth and opinion dynamics

Ram P. Sigdel, Madhur Anand, Chris T. Bauch

PII: S0022-5193(17)30360-0  
DOI: [10.1016/j.jtbi.2017.07.029](https://doi.org/10.1016/j.jtbi.2017.07.029)  
Reference: YJTBI 9164



To appear in: *Journal of Theoretical Biology*

Received date: 22 December 2016  
Revised date: 17 July 2017  
Accepted date: 31 July 2017

Please cite this article as: Ram P. Sigdel, Madhur Anand, Chris T. Bauch, Competition between injunctive social norms and conservation priorities gives rise to complex dynamics in a model of forest growth and opinion dynamics, *Journal of Theoretical Biology* (2017), doi: [10.1016/j.jtbi.2017.07.029](https://doi.org/10.1016/j.jtbi.2017.07.029)

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

---

**Highlights**

- A model with rarity-based conservation values and social norms is developed.
- Injunctive social norms drastically change the stability paradigm.
- Periodic orbits can be ruled out for certain parameter regimes.
- Global asymptotic stability can be established for some cases.

ACCEPTED MANUSCRIPT

# Competition between injunctive social norms and conservation priorities gives rise to complex dynamics in a model of forest growth and opinion dynamics

Ram P. Sigdel<sup>1</sup>, Madhur Anand<sup>2</sup>, Chris T. Bauch<sup>1,3</sup>

<sup>1</sup>Department of Mathematics and Statistics, University of Guelph

<sup>2</sup>School of Environmental Sciences, University of Guelph

<sup>3</sup>Department of Mathematics and Statistics, University of Waterloo

Correspondence: rsigdel@uoguelph.ca

---

## Abstract

*Human and environmental systems are often treated as existing in isolation from one another, whereas in fact they are often two parts of a single, coupled human-environment system. Developing theoretical models of coupled human-environment systems is a continuing area of research, although relatively few of these models are based on differential equations. Here we develop a simple differential equation coupled human-environment system model of forest growth dynamics and conservationist opinion dynamics in a human population. The model assumes logistic growth and harvesting in the forest. Opinion spread in the human population is based on the interplay between conservation values stimulated by forest rarity, and injunctive social norms that tend to support population conformity. We find that injunctive social norms drive the system to the boundaries of phase space, whereas rarity-based conservation priorities drive the system to the interior. The result is complex dynamics including limit cycles and alternative stable states that do not occur if injunctive social norms are absent. We found that the model with injunctive social norms had five possible observable outcomes, whereas the model without social norms had only two stable states. Thus social norms have dramatic influence in conservation dynamics. We also find that increasing*

*the conservation value of forests is the best way to boost and stabilize forest cover while also boosting conservationist opinion in the population, although for some parameter regimes it can also give rise to long-term oscillations in opinions and forest cover. We conclude that simple models can provide insights and reveal patterns that might be difficult to see with high-dimensional computational models, and therefore should be used more often in research on coupled human-environment systems.*

*Keywords:* Conservation value, Imitation dynamics, Socio-ecological system, Differential equation model, Alternative stable states, Forest transitions, Regime shift.

---

## **1. Introduction**

To meet the growing demand for food, fresh water, timber, fibre and fuel, human beings have been changing ecosystems for substantial gains to their well-being and economic development, but this has also degraded many ecosystem services (Marten (2001); Francis and Krishnamurthy (2014)). Many natural resources upon which human depends have been depleted (Assessment, 2005). According to some scholars, three of nine ‘planetary boundaries’ which should not be exceeded in order for humanity to operate safely have already been exceeded: biodiversity loss, interruption of nitrogen cycle and climate change (Rockström et al., 2009). Sustainable land use, including maintaining healthy forest ecosystems, is also one of the nine planetary boundaries, and deforestation is threatening to push us past this boundary as well (Rockström et al., 2009). Due to direct human activities, about 7.5 million hectares of North American forests (or 1% of total forest area) are disturbed each year (Masek et al., 2011). The Amazon forest has lost at least 17% of its forest cover, while the Sumatra island of Indonesia lost 85% of original forests in last five decades due

to human activities (WWF, 2013). Intensive ranching is an important culprit in deforestation: it is estimated that 70% of deforestation in Latin American forest has been caused by cattle ranching operations (Fearnside, 1993). At the global level, forest cover is predicted to continue to decline in the coming decades (Pagnutti et al., 2013).

Despite the declining trends at the global level, many countries and regions have seen a forest transition, an increase in forest cover after decades of decline (Lambin and Meyfroidt, 2011). In several cases, such as the case of Costa Rica, this has happened because people have changed their opinion of the value of forests, largely driven by the loss itself (the increasing rarity of the forests). In a more localized example, the city of Greater Sudbury, Canada began restoring forests previously destroyed by industry-generated acid rain in a massive effort starting in the 1970s (McCracken, 2013; Anand et al., 2005). There are several other examples of where humans can change the pathways of an ecosystem. Human activities lead to a decline in natural ecosystems, the decline in natural ecosystems can in turn stimulate human action to conserve endangered ecosystems or attempt to restore badly damaged ecosystems (Heinen and Yonzon, 1994). Recovery of ecosystems from changing human values and behaviour is still not widely seen, but must increase in order to lead to a sustainable future (Galvani et al., 2016).

One way to conceptualize these two-way influences is through a coupled human and environment system (HES or CHES), also known as coupled human and natural system. This is a system in which a human subsystem and an environment subsystem are connected through a coupled feedback loop of mutual influence. Such interactions can apply at local scales, global scales, or across multiple scales (Liu et al., 2007) and can also result in regime shifts between alternative stable states (Lade et al., 2013; Bauch et al., 2016; Henderson

et al., 2016).

In the past, various models have explored human-environment interactions in the context of forest dynamics, including models that account for socioeconomic and demographic factors (An et al., 2005); two-person games and their impact on deforestation (Rodrigues et al., 2009); the effect of landowner decision-making on landscape dynamics (Satake and Iwasa, 2006), Markov chain models to study the effect of social learning on landowner decisions (Satake et al., 2007); the role of governance in landowner decision-making and landscape dynamics (Henderson et al., 2013); tipping points (Bauch et al., 2016) and regime shifts (Henderson et al., 2016) in coupled forest HESs, among others. Previous research in modelling coupled human-environment systems has predominantly relied upon agent-based models (An et al., 2005). In the occasional case where differential equation models have been used, analysis has been primarily numerical, often due to the dimensionality of the system (Henderson et al., 2016), stochasticity (Barlow et al., 2014), nonlinear resource harvesting (Bauch et al., 2016) or use of nonlinear recruitment thresholds (Innes et al., 2013; Henderson et al., 2016).

In this paper we develop a simple, differential-equation coupled human-environment system model of a human population making conservation decisions about a forest ecosystem. We simplify this system to examine the relative importance of two aspects driving human behaviour: conservation priority and injunctive social norms. Social norms are the unwritten but socially acceptable rules and guidelines of a society. Such rules have been established based on their practice and may vary from society to society. The role and influence of injunctive social norms has been accounted for in models of coupled dynamics of vaccinating behaviour and disease dynamics (Oraby et al., 2014), as well as in coupled HES models

where it has been studied in the context of forest pest invasion due to human decisions to transport firewood (Barlow et al., 2014); harvesting of a renewable resources, in the form of social ostracism directed toward over-harvesters (Lade et al., 2013); and forest dynamics (Satake et al., 2007). However, to our knowledge, no study has explicitly examined the relative importance of injunctive social norms vs. conservation value. Given a simple and common ecological dynamic, such as logistic forest growth with harvesting, we ask: what is more important for forest conservation, the inherent value of those forests (e.g., rarity of forests increases conservation value (Capmourteres and Anand, 2016) or social norms to conform the opinions of the population? We evaluate the influence of injunctive social norms on coupled HES models by comparing models with and without social norms, and to gain general insights by applying the methods of qualitative analysis of ODEs to the system in order to generate more mathematical rigor than has been feasible before in models of coupled HESs.

## 2. Model

We let  $F$  denote the proportion of forest cover in a region, with the remaining land cover  $G = 1 - F$  being other land types including natural grassland, urban spaces, pasture, and other natural or anthropogenic types. Suppose individuals are divided only into two groups, where  $x$  is the proportion of forest conservationists and  $1 - x$  is the proportion of non-conservationists. Here an individual adopts one of the two opinions, either to favor forest conservation ( $\mathbb{F}$ ) or favor other land uses ( $\mathbb{G}$ ). Based on the two strategies, individuals get sources of information and sample one another at the rate  $k'$ . In this case, an individual adopting strategy  $\mathbb{F}$  samples other individuals adopting either of the strategies  $\mathbb{F}$  or  $\mathbb{G}$ . Let

$\pi(\mathbb{F})$  be the perceived value of forest, and  $\pi(\mathbb{G})$  be the perceived value of other land use.

Before switching their strategies, an individual compares utility gain and loss received by adopting the same strategy and may switch strategies if the utility for switching is attractive enough. If  $\pi(\mathbb{F}) > \pi(\mathbb{G})$  then the one adopting strategy  $\mathbb{G}$  switches into strategy  $\mathbb{F}$  with probability  $pU_F(F)$ , where  $U_F(F) = \pi(\mathbb{F}) - \pi(\mathbb{G}) > 0$  be the net gain in utility by adopting strategy  $\mathbb{F}$  than adopting strategy  $\mathbb{G}$  and  $p$  is the proportionality constant. Therefore,  $(1-x)$  non-conservationists at any given time become conservationists at the rate  $(1-x)k'xpU_F(F)$ . Similarly if  $U_G(F) = \pi(\mathbb{G}) - \pi(\mathbb{F}) > 0$  then  $x$  conservationists becomes non-conservationists at the rate  $xk'(1-x)\alpha pU_G(F)$  where,  $U_G(F)$  is the net gain in utility, by adopting strategy  $\mathbb{G}$  than adopting strategy  $\mathbb{F}$  and  $\alpha$  is the scaling constant. For convenience, we absorb  $\alpha$  into  $k'$ .

Combining the above two rates gives:

$$\begin{aligned} \frac{dx}{dt} &= (k'p)x(1-x)[U_F(F) - U_G(F)] \\ &= kx(1-x)[U_F(F) - U_G(F)], \end{aligned} \tag{1}$$

where  $k = pk'$ .

From Innes et al. (2013),  $U_F(F) = r_0G - q_0F$ , where  $r_0$  and  $q_0$  control the conservation value (or simply the value/importance) of forest and other land use respectively. In addition to the utility gain  $U_F(F)$  or  $U_G(F)$ , each member in each social group further feels a uniform pressure  $\delta_0$  to remain in the same group due to injunctive social norms. Hence an individual adopting a strategy  $\mathbb{F}$  experiences the uniform pressure at the rate  $\delta_0x$  and those adopting  $\mathbb{G}$  experience the uniform pressure at the rate  $\delta_0(1-x)$ . Therefore  $U_F(F)$  and  $U_G(F)$  in

equation (1), can be replaced by  $U_F(F) + \delta_0 x$  and  $-U_F(F) + \delta_0(1 - x)$  respectively to get,

$$\begin{aligned} \frac{dx}{dt} &= kx(1 - x)[U_F(F) + \delta_0 x + U_F(F) - \delta_0(1 - x)] \\ &= kx(1 - x)[r - mF + \delta_0(2x - 1)], \end{aligned} \quad (2)$$

where  $r = 2r_0$ ,  $q = 2q_0$  and  $m = r + q$ . This imitation dynamic has been used in previous models of coupled HESs (Innes et al., 2013; Barlow et al., 2014; Lade et al., 2013) and coupled behavior-disease systems (Oraby et al., 2014).

The corresponding equation for forest cover is a simple logistic population growth model with harvesting

$$\dot{F} = RFG - h(1 - x)F, \quad (3)$$

where  $R$  is the natural succession rate of forest from other land and  $h$  is the maximal harvesting rate. The realized harvesting rate is  $h(1 - x)$  and can be less than the maximal harvesting rate  $h$  due to population support for conservationism. Here, as the new forest depends upon the existing forest and the available land to grow new trees, we consider the new forest is created at the rate proportional to the product of existing forest and other land areas. Also, the harvesting rate is proportional to the product of existing forest cover and proportion of non-conservationists.

Therefore the complete system of equations can be written as

$$\begin{aligned} \dot{x} &= \kappa x(1 - x)[c - F + \delta(2x - 1)] \\ \dot{F} &= RF(1 - F) - h(1 - x)F, \end{aligned} \quad (4)$$

where  $\kappa \equiv km$ ,  $c \equiv \frac{r}{m}$  and  $\delta \equiv \frac{\delta_0}{m}$ .

The model variables, parameter definitions, and baseline parameter values appear in Table 1. Baseline parameter values were chosen to capture observed trends in old growth forest and public opinion regarding forest conservation in the Pacific Northwest United States in the twentieth century (Bolsinger and Waddell, 1993; Davis et al., 2001). In particular, when  $F(0) = 0.9$  and  $x(0) = 0.1$ , forest cover is initially reduced until it reaches a low point of 20 – 30%, at which time conservation opinion starts to grow more quickly, resulting in an inflection point and slow recovery in forest cover. A similar pattern is observed in data from the Pacific Northwest (see Bauch et al. (2016) for further details.)

Table 1: Variables/Parameters, their definition, ranges and baseline values.

Variables	Descriptions	Ranges
$x$	Proportion of conservationist	[0, 1]
$F$	Forest cover	[0, 1]
Parameters	Descriptions	Values
$\kappa$	Social learning rate	0.5/year
$c$	Conservation value of forest	0.6
$\delta$	Strength of injunctive social norms	0.01
$R$	Natural succession rate of forest	0.01/year
$h$	Maximal harvesting rate	0.02/year

### 3. Results

We first analyze the model with injunctive social norms included. In the following subsection, we will analyze the special case where injunctive social norms are not included, and then compare the two models.

## 3.1. Model with injunctive social norms

## 3.1.1. Model Steady States

From equation (4), there are two steady states  $(F^*, x^*)$  of full non-conservationism. One has no forest cover

$$A_1 = (0, 0)$$

and the second one has a mix of forest and other land cover

$$A_2 = \left(1 - \frac{h}{R}, 0\right).$$

$A_2$  is biologically meaningful when the natural succession rate of forest dominates the harvesting rate, that is  $h < R$ . In this case, lower the harvesting rate, higher the forest cover.

There are two steady states of full conservationism. One has no forest cover

$$A_3 = (0, 1)$$

and the second one has full forest cover

$$A_4 = (1, 1).$$

In the above steady states, the population has the homogeneous opinion of either conservation or non-conservation.

Finally, there are two steady states with mixtures of conservationists and non-conservationists.

One has no forest cover

$$A_5 = \left(0, \frac{\delta - c}{2\delta}\right),$$

whereas the second one has a mix of forest and other land cover

$$A_6 = (1 + hS, 1 + RS)$$

and this is the only interior steady state of the model. Here  $A_5$  is biologically meaningful when injunctive social norms dominate the conservation value of forest, that is  $c < \delta$ .  $A_6$  is biologically meaningful if  $0 > RS > -1$  and  $0 > hS > -1$ , where  $S = \frac{1-c-\delta}{2R\delta-h}$ . Here,  $A_6$  is biologically meaningful if the absolute value of  $S$  is larger than the reciprocal of both  $h$  and  $R$ .

### 3.1.2. Stability and dynamics

From local stability analysis,  $A_1 = (0, 0)$  is locally asymptotically stable (LAS) if the injunctive social norms dominate the conservation value of forest and harvesting dominates the natural succession rate, that is  $\frac{\delta}{c} > 1$  and  $\frac{h}{R} > 1$ . Here, in the population with a higher harvesting rate, stronger injunctive norms shift the population to the homogeneous opinion so that the state where the entire population are non-conservationists exists.

$A_2 = (1 - \frac{h}{R}, 0)$  is LAS if  $\frac{h}{R} < 1 + \delta - c < 1$  or  $\frac{h}{R} < 1 < 1 + \delta - c$ . In either case, harvesting is dominated by natural succession. Here, the population can consist entirely of non-conservationists and yet forest can persist at intermediate levels because harvesting is dominated by the rate of natural succession.

$A_3 = (0, 1)$  is always unstable and its instability is reasonable as total forest devastation

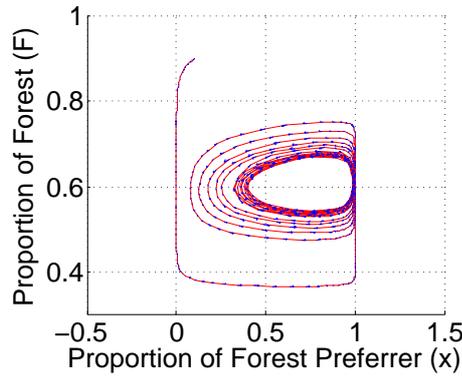


Figure 1: Appearance of weakly damped oscillations to an interior steady state when the social learning rate  $\kappa = 2/\text{year}$ . All other parameter values are as in Table 1.

is impossible in the presence of full forest conservationism, since harvesting always tends to zero as the proportion of conservationists approaches unity.

$A_4 = (1, 1)$  is LAS if injunctive social norms dominate the conservation value of other land use, that is  $c + \delta > 1$ . Here, stronger injunctive social norms force the population to a homogeneous opinion of full conservationism; it does not matter whether harvesting dominates natural succession rate or natural succession dominates harvesting rate.

$A_5 = (0, \frac{\delta-c}{2\delta})$  is unstable and the instability is reasonable except at  $\delta = c$  because total forest devastation is impossible in the presence of conservationism.

From the above results (and neglecting for the moment the interior steady state), we conclude that for sufficiently strong injunctive social norms, the population converges either to the state of full non-conservationism  $x = 0$  or the state of full conservationism  $x = 1$ . However, in the population where harvesting dominates natural succession with sufficiently strong injunctive social norms, we notice the co-existence of full non-conservationism  $A_1$  with full-conservationism  $A_4$ . Also, the state of full conservationism  $A_4$  can coexist with the state of full non-conservationism  $A_2$ .

The interior steady state  $A_6 = (1 + hS, 1 + RS)$  is LAS if  $1 + \delta - c < \frac{h}{R}$  or  $\frac{2\delta}{\delta+c} < \frac{h}{R}$ ,  $c + \delta < 1$ ,  $0 < 2\kappa\delta(1 - x^*)x^* < RF^*$ , where  $F^* = 1 + hS$  and  $x^* = 1 + RS$ . In this case, the injunctive social norms are dominated by the conservation value of other land use. Intuitively, the interior steady state is stable because injunctive social norms are not strong enough to force the population to either extreme of  $x = 0$  or  $x = 1$ , and the conservation value of forest is strong enough to support an intermediate level of both forest cover and conservationist opinion, without the support from injunctive social norms. We note that  $A_6$  cannot coexist with any other steady state.

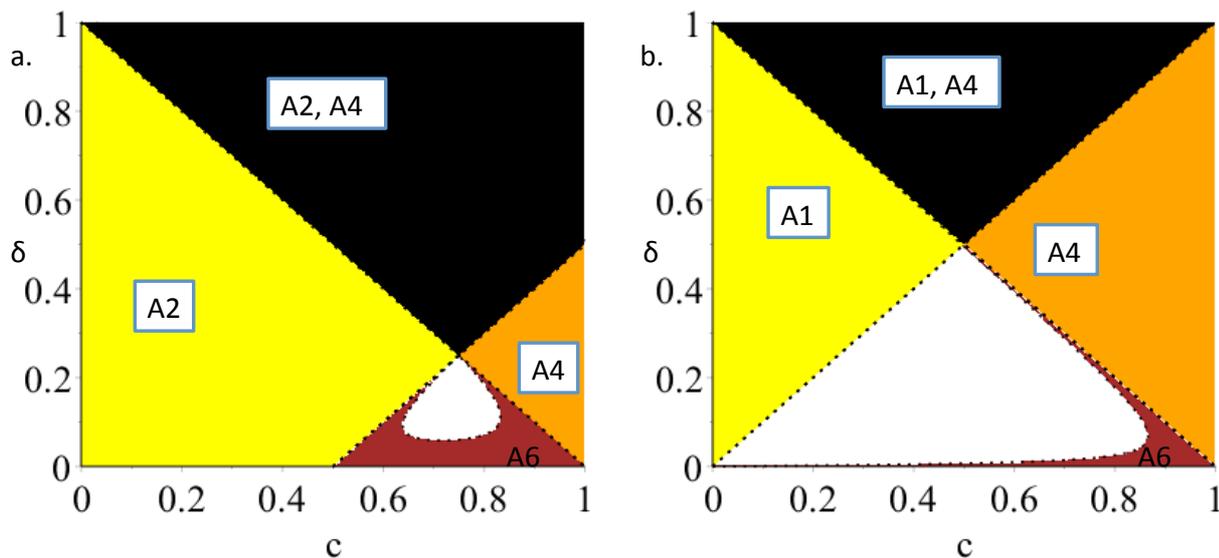


Figure 2: The relationship between harvesting rate and natural succession rate influences the region where all steady states are unstable. Subpanels show (a) succession dominating harvesting  $h = 0.01/\text{year}$ ,  $R = 0.02/\text{year}$  and (b) harvesting dominating succession  $h = 0.02/\text{year}$ ,  $R = 0.01/\text{year}$ . The steady states in the subpanels are stable in the corresponding region. White is the region of instability for all steady states and the presence of stable limit cycles. All other parameters except those being varied appear in Table 1.

Local stability analysis does not preclude the possibility of periodic solutions or other nontrivial dynamical behaviour away from steady states. At  $\kappa = 2/\text{year}$ , numerical simulations suggest the possibility of a stable limit cycle, as conservationism and forest cover appear to oscillate perpetually over very long timescales (Figure 1). The presence of oscil-

lations would be significant: oscillating forest cover is less desirable than stable forest cover since harvesting/restoration cycles could create perpetual disturbances in highly biodiverse climax forest communities. However, we were able to rule out the possibility of a stable limit cycles for certain parameter regimes: for  $R > h$  and sufficiently small value of  $\kappa\delta$ , periodicity in the interior region  $D = \{(F, x) : 0 < F < 1, 0 < x < 1\}$  can be ruled out by using Dulac's criterion (Hale and Koçak, 2012, p.373) (for detail see SI Appendix: Appendix A). Thus for  $R > h$  and sufficiently small value of  $\kappa\delta$ , using Poincaré–Bendixson theorem (Hale and Koçak, 2012, p.366), the interior equilibrium solution  $A_6$  is globally asymptotically stable in  $D$ . Nonetheless, the weakly damped nature of oscillations to the interior steady state are still undesirable due to their implication of harvesting/restoration cycles in the population and the ubiquitous presence of noise, which can sustained weakly damped oscillations indefinitely. We also note that stable limit cycles can occur at other parameter values (see below).

Parameter planes provide significant insight into the conditions of stability of the various steady states. From the  $c - \delta$  parameter plane, we observe major changes in the stability paradigm when we reverse the relative magnitude of the maximal harvesting rate  $h$  and the natural succession rate  $R$ . Recall that  $h$  is the maximal harvesting rate and therefore  $(1-x)h$  is the realized harvesting rate as limited by conservationism. When  $R > h$ , the forest can sustain itself even in the face of maximal harvesting. Therefore when  $R = 0.02$  and  $h = 0.01$ , we observe that the  $c - \delta$  parameter plane is dominated by full non-conservationism with intermediate forest cover ( $A_2$ ) and full conservationism and full forest cover ( $A_4$ ), often with both simultaneously stable (Figure 2a). A smaller part of the parameter plane allows for stability of the interior equilibrium ( $A_6$ ) and the presence of limit cycles. However, when

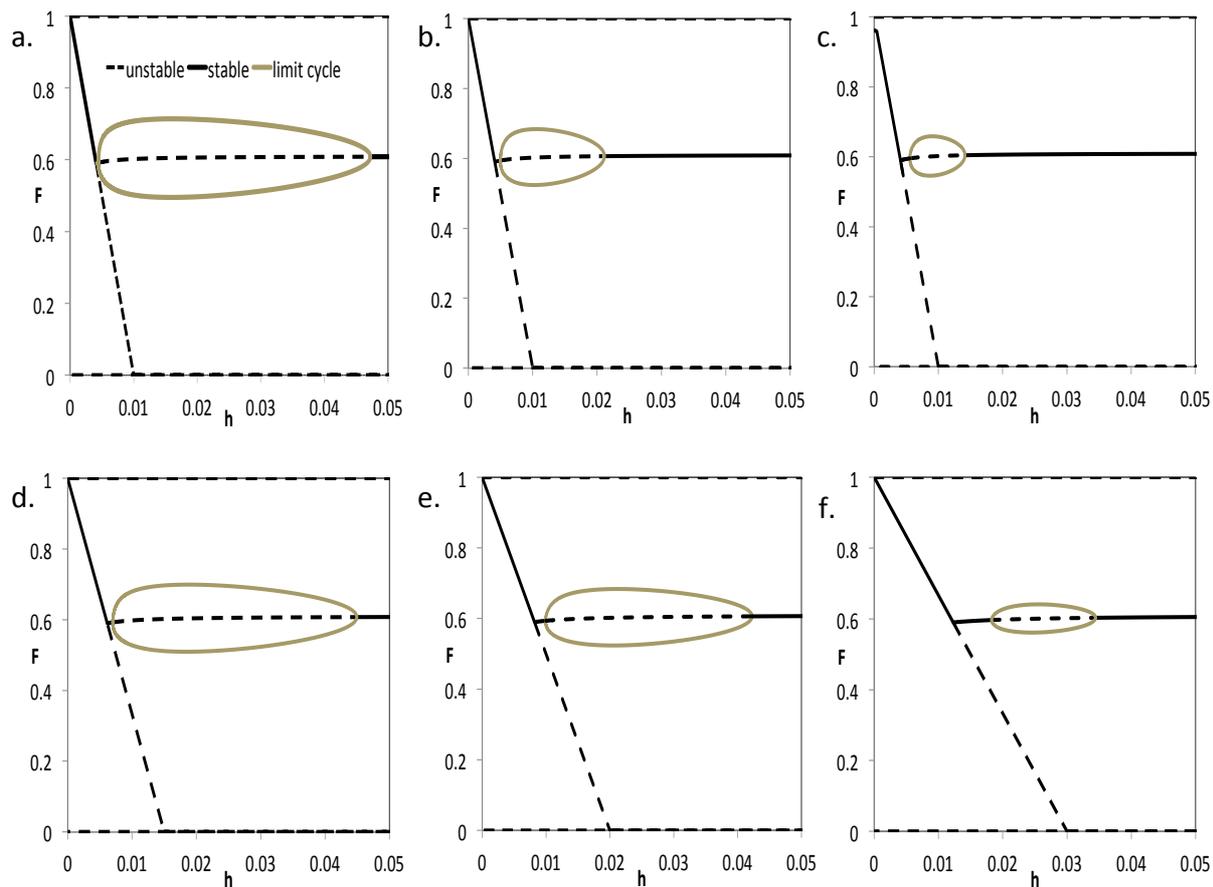


Figure 3: Decreasing the social learning rate  $\kappa$  or increasing the natural succession rate  $R$  helps reduce the tendency to oscillate. Subpanels show bifurcation diagrams of forest cover  $F$  versus maximal harvesting rate  $h$  for various values of  $\kappa$ , (a) 4/year (b) 2/year (c) 1.5/year, and various values of  $R$  (d) 0.015/year (e) 0.02/year (f) 0.03/year.  $\kappa = 4/\text{year}$  in subpanels (d-f); all other parameters except those being varied appear in Table 1.

$R < h$ , we obtain the more interesting scenario with respect to modelling social dynamics. In this parameter regime, conservationism is more important if the forest is to be conserved. When  $h = 0.02$  and  $R = 0.01$ ,  $A_2$  is destabilized and replaced by full non-conservationism with no forest cover ( $A_1$ ), while the region of stability for  $A_4$  expands (Figure 2b). Also the region of stability of the interior steady state  $A_6$  has been drastically decreased, resulting in a larger region of limit cycles.

We also note that limit cycles (oscillations) can appear either when  $R < h$  (Figure 3a-f) or when  $R > h$  (SI Appendix Figure S4). Limit cycles are more common when the maximal harvesting  $h$  and the natural succession rate  $R$  are relatively low, or when the social learning rate  $\kappa$  is high. Increasing the strength of injunctive social norms,  $\delta$ , can destabilize the interior steady state and generate limit cycles (SI Appendix Figure S1). As the value of  $\delta$  is further increased, the oscillations can become more extreme and the transition between high and low forest cover correspondingly becomes more sudden (SI Appendix Figure S1).

In the bifurcation diagrams we observe that even a small harvesting rate can cause oscillations in the system. Decreasing the social learning rate  $\kappa$  stabilizes the system by reducing the range of values of  $h$  for which oscillatory solutions are obtained (Figure 3a-c). Thus, surprisingly, higher social learning rates tend to destabilize the system. Similarly, increasing the natural succession rate  $R$  (Figure 3d-f), reduces the parameter range for which oscillations occur and eventually eliminates it (SI Appendix Figure S5,6). Values of  $R > 0.05$  are not relevant for many forest ecosystems. However, our model represents any population undergoing logistic growth and harvesting and therefore higher values of  $R$  represent natural populations with higher turnover than forests, such as some fisheries. The quenching of oscillations for higher values of  $R$  suggests that more rapid demographic turnover tends

to stabilize socio-ecological dynamics. Also the oscillations that appeared due to small conservation value can be reduced and removed with the increase in the conservation value (SI Appendix Figure S3 a-c). But an increase in injunctive social norms increases oscillations for a larger range of harvesting (SI Appendix Figure S3 d-f).

The time evolution of model dynamics helps illustrate the role of injunctive social norms in the model. For certain initial conditions, injunctive social norms do not favor conservationism. When  $x$  is initially small, human preference starts increasing during an early phase of forest destruction (Figure 4a). For higher values of injunctive social norms, it may take several years to centuries to manifest change in human preference and thus in forest cover (Figure 4b, c). Also based on our baseline parameter values, society is practicing a higher harvesting rate than the natural succession rate of forest. As a result, stronger social norms support harvesting and lead the forest to decline. Thus stronger social norms gradually increase the population opinion homogeneity, eventually dominating conservation priorities for the rare and endangered species. The situation is different over a larger time window, in which an increase in injunctive social norms compel the forest cover to accelerate in both direction as well as helps to reduce the oscillation (SI Appendix Figure S1). Note that the situation will be different when we change the initial conditions such that  $x$  is initially higher: in these cases, social norms will move population to greater conservationism (SI Appendix Figure S2 a-c).

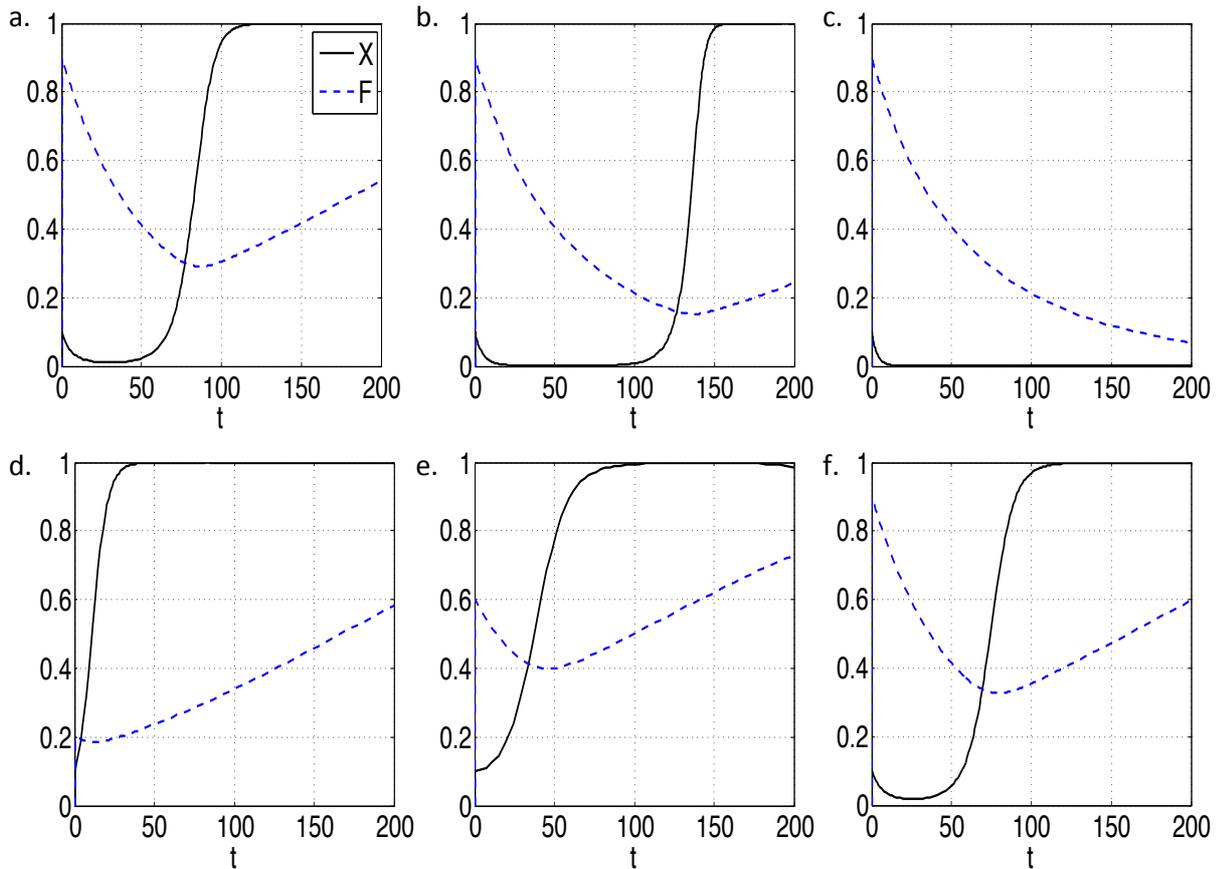


Figure 4: For the initial condition  $(F_0, x_0) = (0.9, 0.1)$ , increasing the strength of injunctive social norms  $\delta$ , (a) 0.05 (b) 0.2 (c) 0.4, decrease conservationism and thus forest cover. Varying the initial value of forest cover  $F_0$ , (d) 0.2 (e) 0.6 (f) 0.9 does not affect much the forest cover in longer time window. All other parameters except those being varied appear in Table 1.

Because of conservation priorities to protect forest ecosystems, forest cover can be maintained at an optimum level even when only 20% of forest cover remains initially (Figure 4d). Also if there is a larger difference between initial prevalence of conservationism and initial forest cover, forest cover will initially decline before it starts to recover (Figure 4e,f).

## 3.2. Model without injunctive social norms

## 3.2.1. Model Steady States

Substituting  $\delta = 0$  in equation (4), the coupled HES model reduces to a model with the following system of equations:

$$\begin{aligned}\dot{x} &= \kappa x(1-x)(c-F) \\ \dot{F} &= RF(1-F) - h(1-x)F.\end{aligned}\tag{5}$$

From equation (5), there are two steady states  $(F^*, x^*)$  of full non-conservationism of forest. One has no forest cover

$$B_1 = (0, 0)$$

and the second has a mix of forest and other land cover

$$B_2 = \left(1 - \frac{h}{R}, 0\right).$$

$B_2$  is biologically meaningful when natural succession rate dominates the harvesting rate, that is,  $\frac{h}{R} < 1$ . In this case, a lower harvesting rate results in higher forest cover.

There are two steady states of full conservationism. One has no forest cover

$$B_3 = (0, 1)$$

and the second one has full forest cover

$$B_4 = (1, 1).$$

In the above steady states, the population has the homogeneous opinion of either full conservationism or full non-conservationism.

Note that steady states  $B_1, B_2, B_3, B_4$  are the same as steady states  $A_1, A_2, A_3, A_4$  of the model with injunctive social norms, and  $A_5$  of the social norms model is no longer meaningful in this case.

The interior steady state  $A_6$  of the model with social norms reduces to:

$$B_5 = \left( c, 1 - \frac{R}{h}(1 - c) \right)$$

and it is biologically meaningful when the ratio of the harvesting rate to the succession rate dominates the relative conservation value of other land cover, that is  $\frac{h}{R} > 1 - c$ . Here, larger conservation values result in higher forest cover and prevalence of conservationism.

### 3.2.2. Stability and dynamics

The absence of injunctive social norms has a large impact on the stability of the coupled HES. The steady states of full non-conservationism and no forest cover, and full conservationism and full forest cover, can no longer be stable, making conservation priorities based on the rarity of forest more important for dynamics.

$B_1 = (0, 0)$  is unstable and its instability in the absence of injunctive social norms is reasonable as extreme rarity of forest should cause  $x$  to diverge away from zero, causing  $F$  to rise as well. This contrasts with the finding that  $A_1 = (0, 0)$  can be stable for certain parameter regimes in the model with injunctive norms.

$B_2 = (1 - \frac{h}{R}, 0)$  is LAS if the ratio of harvesting rate to natural succession rate is less than the relative conservation value of other land, that is  $\frac{h}{R} < 1 - c$ .  $x = 0$  implies  $\dot{x} = 0$

and thus the line  $x = 0$  is invariant. Thus, the only one locally asymptotically stable steady state  $B_2$  at the boundary is globally asymptotically stable (GAS) at  $x = 0$ .

$B_3 = (0, 1)$  is unstable and its instability is reasonable because forest cover cannot remain zero in the presence of full conservationism.  $B_4 = (1, 1)$  is also unstable because full conservationism cannot remain stable in the presence of full forest cover, due to competing needs for other uses of land. This contrasts with the finding that  $A_4$  can be stable for certain parameter regimes in the model with social norms.

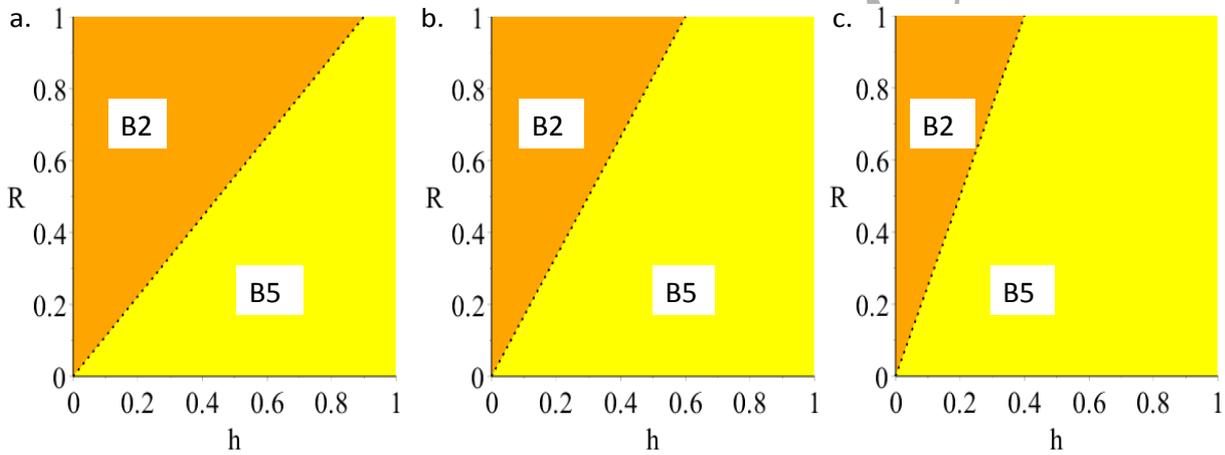


Figure 5: An increase in conservation value  $c$  increases the region of stability of the interior steady state  $B_5$  and thus decreases the stability region for the boundary steady state  $B_2$ . For (a)  $c = 0.1$  (b)  $c = 0.4$  (c)  $c = 0.6$ ,  $B_5$  and  $B_2$  are stable in the labeled regions. All other parameters except those being varied appear in Table 1.

The interior steady state  $B_5 = (c, 1 - \frac{R}{h}(1 - c))$  is LAS if the ratio of the harvesting rate to the natural succession rate is greater than the relative value of other land uses, that is  $\frac{h}{R} > 1 - c$ .  $B_5$  is LAS if it is biologically meaningful. Periodicity in the interior region  $D = \{(F, x) : 0 < F < 1, 0 < x < 1\}$  can be ruled out without any parameter restriction by using Dulac's criterion (Hale and Koçak, 2012, p.373) (for detail see SI Appendix: Appendix B). Thus using Poincaré–Bendixson theorem (Hale and Koçak, 2012, p.366), the interior steady state  $B_5$  is GAS in  $D$  when it exists. Note that the stability analysis of the above

five steady states is conducted in detail in SI Appendix: Appendix B.

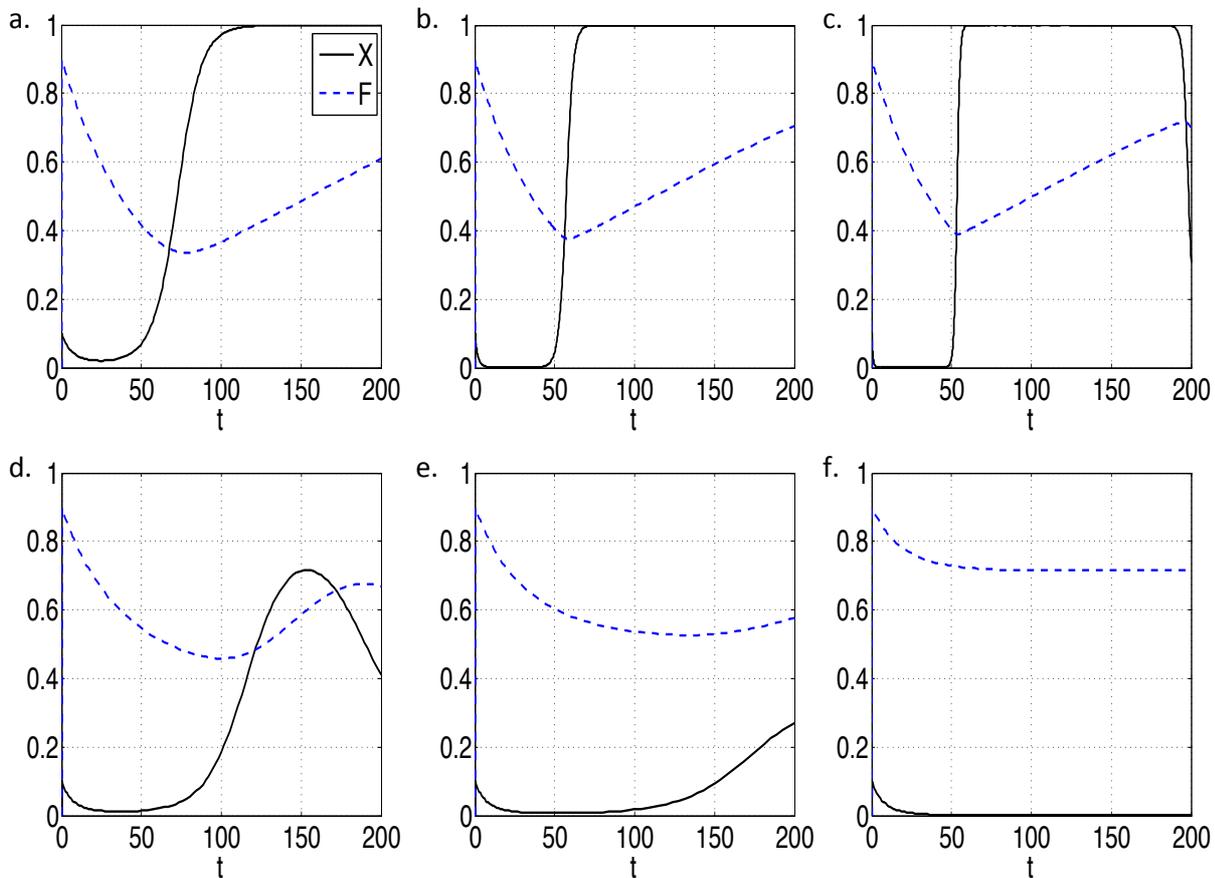


Figure 6: Increasing the social learning rate  $\kappa$  results in more rapid turnover, while increasing the natural succession rate  $R$  increases forest cover but not conservationism. Impact of variation in social learning rate  $\kappa/\text{year}$ , (a) 0.5 (b) 2 (c) 5 and natural succession rate  $R/\text{year}$ , (d) 0.03 (e) 0.04 (f) 0.07; all other parameters except those being varied appear in the table 1.

In the  $h - R$  parameter plane (Figure 5a-c), we observe significant differences from the model with social norms included. We only observe two stable steady states,  $B_2$  and  $B_6$ . Increasing the conservation value of forest shrinks the region of stability of pure non-conservationism  $B_2$  and thus increases the region of stability for the interior steady state  $B_5$ . Also, as the forest cover in the interior steady state directly depends upon the conservation value  $c$ , forest cover increases with the increase in  $c$ .

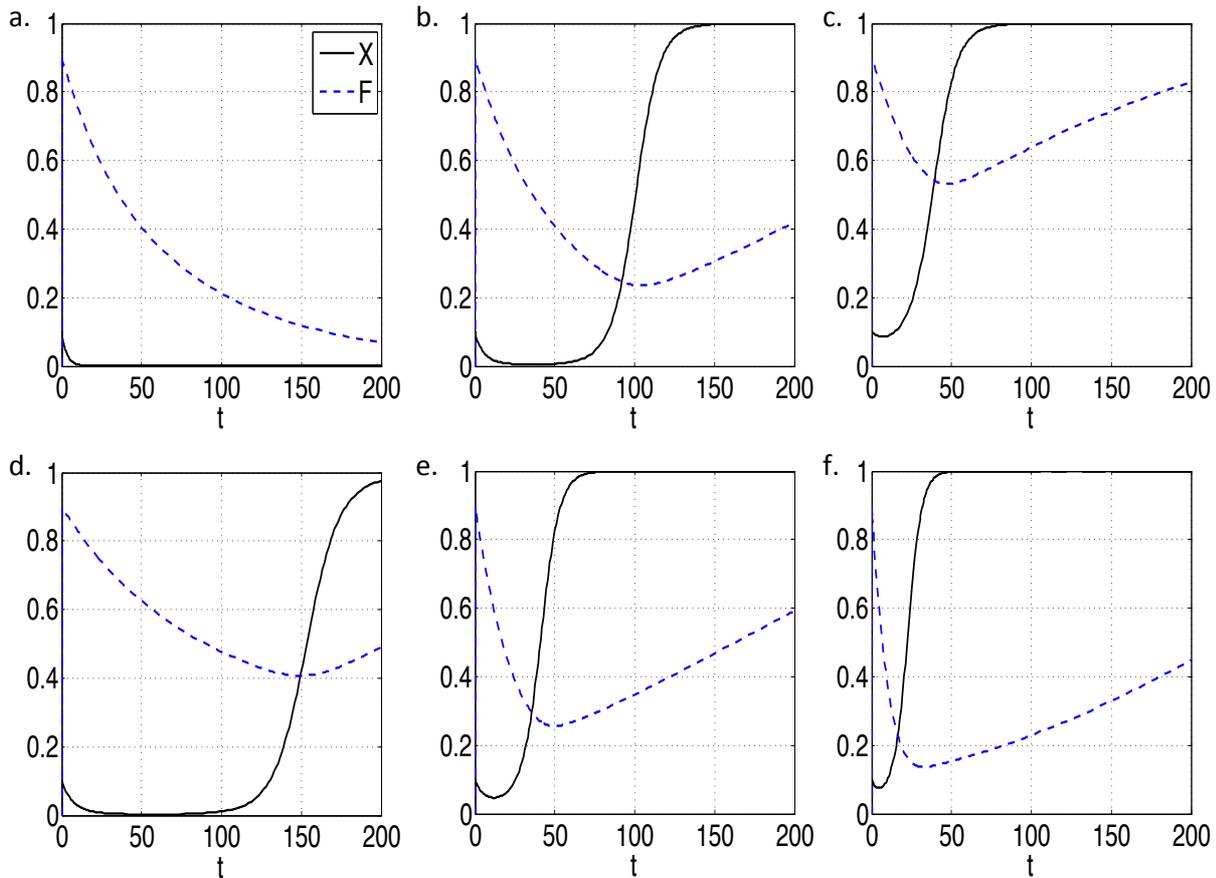


Figure 7: Increasing the conservation value of forest  $c$  supports forest cover and conservationism, while increasing the maximal harvesting rate  $h$  reduces forest cover but also stimulates conservationism and thus forest recovery. Impact of variation in relative conservation value of forest  $c$ , (a) 0.2 (b) 0.5 (c) 0.8 and harvesting rate  $h/year$ , (d) 0.01 (e) 0.04 (f) 0.1; all other parameters except those being varied appear in the table 1.

An increase in the social learning rate has both positive as well as negative impacts on the system. A low social learning rate results in slow decline of forest but also a slow recovery rate (Figure 6a), whereas a higher social learning rate increases both the rate of forest destruction as well as the rate of forest recovery, at least up unto its natural maximum  $R$  (Figure 6b, c). A higher social learning rate also results in rapid transitions between population states and decreases the period of damped oscillations. If the natural succession rate of forest is increased, forest cover dominates other land cover most of the time (Figure

6d-f). Also, a higher succession rate guarantees sustainable forest cover for a longer time window. Conservationism declines on account of the abundance of forest, but because  $R$  is high, forest cover can be maintained even in the absence of public support.

A higher conservation value of forest  $c$  helps to increase conservationism and thus forest cover (Figure 7a-c). It also stops the sharp increase in forest destruction by changing the human preference rapidly. A higher harvesting rate increases the rate of forest decline, but it also stimulates a more rapid response from conservationism, causing a decline in the effective harvesting rate  $h(1 - x)$  (Figure 7d-f). As a result, when  $h$  is large enough, full conservationism is stable and the forest can recover at its maximal rate  $R$ .

#### 4. Discussion

Here we compared dynamics of a coupled HES model for forest growth and conservationist opinion in a human population in the presence and absence of injunctive social norms. We found that the model with injunctive social norms had five possible observable outcomes (four stable equilibrium, plus stable limit cycles) whereas the model without social norms only had two stable steady states. In the case of the model with injunctive social norms, stable steady states could also co-exist, while in the case of the model without social norms, the two stable steady states are simply globally asymptotically stable whenever they exist in their respective parts of parameter space. The complexity in the case of the model with social norms arises because injunctive social norms drive dynamics to the boundaries of the phase space, while rarity-based conservation drives it back to the interior. These results were established with a mixture of numerical and qualitative analysis, including establishment of global asymptotic stability using the Dulac criterion and other methods.

The model yielded other interesting predictions. In general, if human opinion is based only the costs of conservationism weighed against the rarity of the ecosystem, the two extremes of complete forest destruction and full forest cover are unusual outcomes. However, injunctive social norms make these outcomes more probable because when a norm of conservationism or non-conservationism is universally accepted, it tends to persist for a long time (Helbing et al., 2010). In our model, this causes long-term implications for the ecological system. At this point the conservation value of rare ecosystems is overshadowed by social psychology. A similar fixation on boundary states due to injunctive social norms has been observed in coupled behaviour-disease models of vaccinating behaviour (Oraby et al., 2014) and in coupled forest pest outbreak-firewood transport models (Barlow et al., 2014).

For the model with injunctive social norms, the local and global stability of the interior steady state are tied to complex parameter restrictions, whereas the situation is simpler in the absence of social norms. As an example, limit cycles in the model with social norms can be ruled out only under specific parameter restrictions but they can be easily ruled out without parameter conditions when social norms are absent.

We opted to use a simple, two-dimensional differential equation model instead of a complicated agent-based model with high dimensionality. We believe the two approaches are complementary in the study of coupled human-environment systems, but that simpler models are relatively under-utilized at present. On one hand, simpler models run the risk of excluding important variables that higher-dimensional computational models can incorporate in principle (even when it is not possible to fully understand their impact in practice). On the other hand, simpler models enable a clearer and more comprehensive understanding of model dynamics and how they respond to changes in parameter values. This makes it

easier to see complex trade-offs between different parameter regimes, for instance. It also makes it possible to derive global stability results rigorously, as we did in this paper. The complementarity of simple and complicated models and the need for simple models is especially true in the study of coupled HESs since, by definition, they have higher dimension than their uncoupled counterparts. Thus they are more subject to the ‘curse of dimensionality’ of large parameter spaces and large phase spaces, and important dynamics can be missed if only complicated models are used.

As in previous models of coupled HESs (Lade et al., 2013; Bauch et al., 2016), we observed that a regime shift in an ecological system can be caused by social processes, in situations where a social and an ecological system are coupled. In our model, such a regime shift occurs because of the influence of injunctive social norms, which allow for the existence of alternative stable states where one or both states are boundary equilibria. The regime shift is associated with a transcritical bifurcation, instead of the fold bifurcation often observed in strictly ecological regime shifts Scheffer et al. (2009). Hence, important ecological outcomes can be missed if ecological models are studied in the absence of (our increasingly influential) human behaviour. Similar contrasts between predictions of coupled and uncoupled human-and-natural systems have been observed in models of dueling epidemiological and social contagions (Fu et al., 2017; Bauch and Galvani, 2013).

Our analysis also shows that without strong conservationist opinion, even the nominal rate of harvesting (2% as our baseline value, which is small compared to the rate under current practice in many low-income countries) can lead to removal of almost all forest cover. In a coupled HES setting, a population may eventually respond by demanding conservationism when forest cover becomes too low, leading to a partial counteracting of higher maximal

harvesting rates. However, this mechanism does not work perfectly and only conserves forest at the cusp of extinction.

We found that increasing the conservation value of forest ( $c$ ) increases and stabilizes forest cover and supports conservationist opinions in the population for most parameter regimes. However, from certain starting points (such as a steady state of pure non-conservationism,  $A_1$  and  $A_2$  in Figure 2), increasing  $c$  will cause the population to first move through a region of oscillations in conservationist opinion and forest cover before the population reaches a stable state of high forest cover and conservationism. This occurs because the population responds to changes in  $F$  under our assumption of rarity-based conservation priorities. The emergence of oscillations due to an increase in the conservation value of rare ecosystem states has also been observed in similar coupled HES models, such as in forest-grassland-opinion dynamics in the absence of social norms (Henderson et al., 2016). Hence, boosting the conservation value of forests through education, for instance, can be useful but may also have unintended consequences such as long-term oscillations in opinion and forest cover.

Coupled human-environment dynamics are often subject to separation of timescales in the environment versus human subsystem, and a better understanding of timescale differences has been identified as a priority area for future research in coupled HESs Rehmeyer (2010). In future work, singular perturbation theory could be used to conduct a timescale analysis of transient dynamics in our model and similar models, since the dynamics of  $x$  are faster than the dynamics of  $F$ . This could make it possible to better understand how the transition between forest decline and recovery temporally depends on model parameters.

Our model made simplifying assumptions in the name of analytical tractability, some of which might influence our predictions. For instance, we ignored stochastic effects, which

could be important in modelling the continued persistence or extinction of ecosystems at risk. We also assumed a single, closed population, whereas in fact both human populations and forest systems are open and connected to other populations from which species (or opinions) can be rescued when endangered in any one patch. Similarly, we did not include other aspects of real systems such as population or spatial heterogeneities. Our analysis focussed primarily on asymptotic behaviour such as limit cycles and equilibria, but transient dynamics can be more relevant to real-world coupled human-environment systems. For instance, whether a forest ecosystem collapses in 5 years or in 100 years matters since over the longer timescale, the assumption of constant model parameters is harder to justify. These simplifying assumptions can and should be relaxed in future research so that mathematical models of coupled human-environment system dynamics can provide more insights and better inform environmental policy.

**Acknowledgements:** The authors are grateful to two anonymous referees for comments on the manuscript. This research was supported by an NSERC Discovery Grant (CTB) and a McDonnell Foundation Complex Systems Scholar Award (MA).

## References

An, L., Linderman, M., Qi, J., Shortridge, A., Liu, J., 2005. Exploring complexity in a human-environment system: an agent-based spatial model for multidisciplinary and multiscale integration. *Annals of the association of American geographers* 95 (1), 54–79.

Anand, M., Laurence, S., Rayfield, B., 2005. Diversity relationships among taxonomic groups in recovering and restored forests. *Conservation Biology* 19 (3), 955–962.

## REFERENCES

## REFERENCES

- Assessment, M. E., 2005. Ecosystems and human well-being. Washington, DC.
- Barlow, L.-A., Cecile, J., Bauch, C. T., Anand, M., 2014. Modelling interactions between forest pest invasions and human decisions regarding firewood transport restrictions. *PloS one* 9 (4), e90511.
- Bauch, C. T., Galvani, A. P., 2013. Social factors in epidemiology. *Science* 342 (6154), 47–49.
- Bauch, C. T., Sigdel, R., Pharaon, J., Anand, M., 2016. Early warning signals of regime shifts in coupled human–environment systems. *Proceedings of the National Academy of Sciences*, 201604978.
- Bolsinger, C. L., Waddell, K. L., 1993. Area of old-growth forests in California, Oregon, and Washington. US Department of Agriculture, Forest Service, Pacific Northwest Research Station Portland, OR.
- Capmourteres, V., Anand, M., 2016. “conservation value”: a review of the concept and its quantification. *Ecosphere* 7 (10).
- Davis, A., et al., 2001. A forestry program for oregon: Oregonians discuss their opinions on forest management and sustainability. A Quantitative Research Project. Davis, Hibbits and McCaig Inc. Portland, Oregon.
- Fearnside, P. M., 1993. Deforestation in brazilian amazonia: the effect of population and land tenure. *Ambio-Journal of Human Environment Research and Management* 22 (8), 537–545.

## REFERENCES

## REFERENCES

- Francis, R. A., Krishnamurthy, K., 2014. Human conflict and ecosystem services: finding the environmental price of warfare. *International Affairs* 90 (4), 853–869.
- Fu, F., Christakis, N. A., Fowler, J. H., 2017. Dueling biological and social contagions. *Scientific Reports* 7.
- Galvani, A. P., Bauch, C. T., Anand, M., Singer, B. H., Levin, S. A., 2016. Human–environment interactions in population and ecosystem health.
- Hale, J. K., Koçak, H., 2012. Dynamics and bifurcations. Vol. 3. Springer Science & Business Media.
- Heinen, J. T., Yonzon, P. B., 1994. A review of conservation issues and programs in nepal: from a single species focus toward biodiversity protection. *Mountain Research and Development*, 61–76.
- Helbing, D., Yu, W., Opp, K.-D., Rauhut, H., 2010. The emergence of homogeneous norms in heterogeneous populations. *American Journal of Sociology*, submitted.
- Henderson, K. A., Anand, M., Bauch, C. T., 2013. Carrot or stick? modelling how landowner behavioural responses can cause incentive-based forest governance to backfire. *PloS one* 8 (10), e77735.
- Henderson, K. A., Bauch, C. T., Anand, M., 2016. Alternative stable states and the sustainability of forests, grasslands, and agriculture. *Proceedings of the National Academy of Sciences*, 201604987.

## REFERENCES

## REFERENCES

- Innes, C., Anand, M., Bauch, C. T., 2013. The impact of human-environment interactions on the stability of forest-grassland mosaic ecosystems. *Scientific reports* 3.
- Lade, S. J., Tavoni, A., Levin, S. A., Schlüter, M., 2013. Regime shifts in a social-ecological system. *Theoretical ecology* 6 (3), 359–372.
- Lambin, E. F., Meyfroidt, P., 2011. Global land use change, economic globalization, and the looming land scarcity. *Proceedings of the National Academy of Sciences* 108 (9), 3465–3472.
- Liu, J., Dietz, T., Carpenter, S. R., Folke, C., Alberti, M., Redman, C. L., Schneider, S. H., Ostrom, E., Pell, A. N., Lubchenco, J., et al., 2007. Coupled human and natural systems. *AMBIO: a journal of the human environment* 36 (8), 639–649.
- Marten, G. G., 2001. Human ecology: Basic concepts for sustainable development. *Earthscan*.
- Masek, J. G., Cohen, W. B., Leckie, D., Wulder, M. A., Vargas, R., de Jong, B., Healey, S., Law, B., Birdsey, R., Houghton, R., et al., 2011. Recent rates of forest harvest and conversion in north america. *Journal of Geophysical Research: Biogeosciences* 116 (G4).
- McCracken, K., 2013. Sudbury: The journey from moonscape to sustainably green. *Active-History.ca, History Matters*. <http://activehistory.ca>.
- Oraby, T., Thampi, V., Bauch, C. T., 2014. The influence of social norms on the dynamics of vaccinating behaviour for paediatric infectious diseases. In: *Proc. R. Soc. B. Vol. 281*. The Royal Society, p. 20133172.

## REFERENCES

## REFERENCES

- Pagnutti, C., Bauch, C. T., Anand, M., 2013. Outlook on a worldwide forest transition. *PLoS One* 8 (10), e75890.
- Rehmeier, J., 2010. Mathematical and statistical challenges for sustainability. In: Report of a DIMACS Workshop held November 2010.
- Rockström, J., Steffen, W., Noone, K., Persson, Å., Chapin, F. S., Lambin, E. F., Lenton, T. M., Scheffer, M., Folke, C., Schellnhuber, H. J., et al., 2009. A safe operating space for humanity. *Nature* 461 (7263), 472–475.
- Rodrigues, A., Koepl, H., Ohtsuki, H., Satake, A., 2009. A game theoretical model of deforestation in human–environment relationships. *Journal of theoretical biology* 258 (1), 127–134.
- Satake, A., Iwasa, Y., 2006. Coupled ecological and social dynamics in a forested landscape: the deviation of individual decisions from the social optimum. *Ecological Research* 21 (3), 370–379.
- Satake, A., Janssen, M. A., Levin, S. A., Iwasa, Y., 2007. Synchronized deforestation induced by social learning under uncertainty of forest-use value. *Ecological Economics* 63 (2), 452–462.
- Scheffer, M., Bascompte, J., Brock, W. A., Brovkin, V., Carpenter, S. R., Dakos, V., Held, H., Van Nes, E. H., Rietkerk, M., Sugihara, G., 2009. Early-warning signals for critical transitions. *Nature* 461 (7260), 53–59.
- WWF, 2013. World wildlife fund. [www.worldwildlife.org/threats/deforestation](http://www.worldwildlife.org/threats/deforestation), [www.worldwildlife.org/habitats/forest-habitat](http://www.worldwildlife.org/habitats/forest-habitat).