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A limit theorem for the time of ruin in a Gaussian ruin problem

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Abstract

For certain Gaussian processes $X(t)$ with trend $-ct^{\beta}$ and variance $V^2(t)$, the ruin time is analyzed where the ruin time is defined as the first time point t such that $X(t) - ct^{\beta} \geq u$. The ruin time is of interest in finance and actuarial subjects. But the ruin time is also of interest in other applications, e.g. in telecommunications where it indicates the first time of an overflow. We derive the asymptotic distribution of the ruin time as $u \to \infty$ showing that the limiting distribution depends on the parameters β, $V(t)$ and the correlation function of $X(t)$.

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1. Introduction

Let $X(t), t \geq 0$, be a Gaussian process with mean 0 and variance $V^2(t)$, assuming that $V^2(t)$ is regularly varying at infinity with index $2H, 0 < H < 1$. Let the trajectories of X be a.s. continuous and $X(0) = 0$ a.s. Take $\beta > H$ and $c > 0$. In [9] we considered under additional

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restrictions the probability of ruin

$$P\{\sup_{t\geq 0}(X(t) - ct^{\beta}) > u\}. \tag{1}$$

For $u \to \infty$, the limiting probability of (1) is derived for a certain class of Gaussian processes which includes self-similar Gaussian processes and fractional Brownian motions. Other cases are discussed for instance in [6,11] (see also references in this paper).

In recent years, research in ruin theory has focused also on properties of distribution of the time to ruin. See for example [3,5,7,15] for classical risk models as well as risk models perturbed by diffusion. The aim of this note is to demonstrate that asymptotic methods in the theory of Gaussian processes allow us to get approximations not only for the ruin probability (1) but also for the time of ruin,

$$\tau_u = \inf\{t \geq 0 : u + ct^{\beta} - X(t) \leq 0\} \leq \infty.$$

If ruin happens, then one wants to know when it happens. Hence, continuing the considerations of Hüsler and Piterbarg [9], we prove a rather general conditional limit theorem for τ_u as $u \to \infty$, given that the ruin occurs, i.e. $\tau_u < \infty$.

In other contexts, e.g. in telecommunications, such models $X(t) - ct^{\beta}$ are considered for the storage at time t. Hence the ruin time indicates the first time of an overflow with storage space u.

2. Main result

As discussed in [9], it is more convenient to study the family of zero-mean Gaussian processes

$$X^{(u)}(s) = \frac{X(su^{1/\beta})}{V(u^{1/\beta})(1 + cs^{\beta})}, \quad s > 0.$$

By changing time $t = su^{1/\beta}$, we have

$$P\left\{\sup_{t\geq 0}(X(t) - ct^{\beta}) > u\right\} = P\left\{\sup_{s\geq 0} X^{(u)}(s) > \frac{u}{V(u^{1/\beta})}\right\},$$

and $\tau_u = u^{1/\beta}\tau$, where

$$\tau := \inf\left\{s \geq 0 : \frac{u}{V(u^{1/\beta})} - X^{(u)}(s) \leq 0\right\}, \tag{2}$$

i.e. τ denotes the ruin time in the transformed time. The process $X^{(u)}(s)$ with mean zero is not standardized; its variance equals $v_u^{-2}(s)$, where

$$v_u(s) = \frac{s^H V(u^{1/\beta})}{V(su^{1/\beta})} v(s) \quad \text{with } v(s) = s^{-H} + cs^{\beta - H},$$

and by assumption,

$$\frac{s^H V(u^{1/\beta})}{V(su^{1/\beta})} \to 1 \tag{3}$$

as $u \to \infty$, uniformly in s in any bounded interval not containing 0. We need a stronger assumption on V; see assumption A1 below. This condition is similar to other second-order conditions on regular varying functions. Define

$$A(u) = \min_s v_u(s) \quad \text{and} \quad s_0(u) = \text{argmin}_s v_u(s),$$

where the last definition means the first time point of hitting $A(u)$. The point

$$s_0 = \left(\frac{H}{c(\beta - H)} \right)^{1/\beta}$$

is the (unique) minimum point of the function $v(s)$. Notice that

$$v(s) = A + \frac{1}{2} B(s - s_0)^2 + o((s - s_0)^2), \quad \text{for } s \to s_0,$$

with

$$A := v(s_0) = \left(\frac{H}{c(\beta - H)} \right)^{-H/\beta} \frac{\beta}{\beta - H}$$

and

$$B := v''(s_0) = \left(\frac{H}{c(\beta - H)} \right)^{-(H+2)/\beta} H\beta.$$

Clearly, (3) implies $s_0(u) \to s_0$ as $u \to \infty$. For our result on the ruin time, we assume the following conditions on the Gaussian process $X(t)$.

A1

$$\frac{v_u(s) - A(u)}{(s - s_0(u))^2} \to \frac{1}{2} B. \tag{4}$$

as $u \to \infty$ uniformly for s in a neighborhood of s_0.

Observe that the functions $V(t) = t^H$, which is the assumed variance function in [9], and $V(t) = t^H$, for all large t only, satisfy condition A1. If a second-order condition holds for $V(t)$, e.g. $V(t) = Ct^H + O(t^\gamma)$ with $\gamma < H$, then one can easily verify that condition A1 holds.

A2 (Local stationarity) One can find a function $K^2(h)$, regularly varying at 0 with positive index $\alpha \in (0, 2]$, such that

$$\lim_{u \to \infty} \frac{E\left[X^{(u)}(s)v_u(s) - X^{(u)}(s')v_u(s')\right]^2}{K^2(|s - s'|)} = D \tag{5}$$

uniformly for any $s, s' \in S_u := [s_0(u) - \delta(u), s_0(u) + \delta(u)]$ with $\delta(u) = u^{-1} V(u^{1/\beta}) \log u$, with $D > 0$.

Note that $X^{(u)}(s)v_u(s)$ is a standardized Gaussian process.

A3 For some $G, \gamma > 0$ and all $s, s' > 0$,

$$\limsup_u E(X^{(u)}(s) - X^{(u)}(s'))^2 \le G|s - s'|^\gamma. \tag{6}$$

Under these conditions we derive the conditional limiting distribution of the ruin time given $\tau_u < \infty$, which is our main result.

Theorem 1. *Let $X(t), t \geq 0$, be a Gaussian process with mean 0 and variance $V^2(t)$, being regularly varying at infinity with index $2H, 0 < H < 1$. Let $\beta > H$ and $c > 0$. Assume* A1–A3, *then*

$$P((\tau_u - s_0(u)u^{1/\beta})/\sigma(u) < x \mid \tau_u < \infty) \to \Phi(x)$$

as $u \to \infty$, for all x where $\sigma(u) := (AB)^{-1/2}u^{-1+1/\beta}V(u^{1/\beta})$ and Φ is the standard normal distribution function.

Remarks. (1) From the proof it follows that the same result holds also for the $\operatorname{argmax}_{t\geq 0}(X(t) - ct^\beta)$, taking into account Lemma 2.6 of Kim and Pollard [10], which states that the point of absolute maximum of a Gaussian process is a.s. unique.

(2) Note that $\sigma(u)$ may tend to infinity or zero depending on the sign of $-1 + 1/\beta + H/\beta$.

(3) In the case $H = \beta$ the behavior of $V(t)$ could be such that the probability of $\tau_u < \infty$ is still positive for any u. The analysis of this case is probably rather different and other tools are necessary.

Example (*Fractional Brownian Motion with Linear Trend*). Theorem 1 can be applied to the process $B_H(t) - ct$, where B_H is the fractional Brownian motion with Hurst parameter H, which is considered in finance and communication models. We have $\beta = 1$, $V(t) = t^H$, $s_0(u) = s_0$, $A(u) = A$, so A1–A3 hold, implying by Theorem 1 that

$$P(Cu^{-H}(\tau_u - s_0 u) < x \mid \tau_u < \infty) \to \Phi(x)$$

as $u \to \infty$, for any x, where

$$C = c^{H+1}\left(\frac{1-H}{H}\right)^{H+1/2}, \qquad s_0 = \frac{H}{c(1-H)}.$$

We believe that applying from general considerations of [6], e.g. transformations, one can get, in the same way, corresponding limit results for the time of ruin in more general Gaussian models. Here we briefly describe a possible result in the situation of Piterbarg and Prisyazhnuk [14] (see also [13]), assuming a different behavior of the variance in the neighborhood of the maximum variance point. They derived an asymptotic result for the exceedance of the level u for a particular Gaussian process without a trend. Let $X(t), t \in [0, T]$, be a zero-mean Gaussian process with continuous paths, variance $\sigma^2(t)$ and correlation function $r(t, s)$. Let $\sigma(t), t \in [0, T]$, be such that it has a unique point of maximum at $t = t_0$, $0 < t_0 < T$, such that for some positive a and $\zeta < 2$

$$\sigma(t) = 1 - a|t - t_0|^\zeta(1 + o(1)) \quad \text{as } t \to t_0.$$

Assuming the trend, the local stationarity and a condition analogous to A3, then the conditional distribution of the ruin time defined as in Theorem 1 can be derived. The details will analyzed, and given in a forthcoming paper.

Further results on such a ruin time are given in [8] dealing with fractional Brownian motions, weighted sums of fractional Brownian motions, physical fractional Brownian motions and locally stationary Gaussian processes. A trend is always added to these processes, as in the above case of Theorem 1.

3. Further results and proofs

Observe that since we assume A1, Lemma 3 of [9] is still valid under the present conditions. Indeed, in the case $|s - s_0(u)| \le \delta$ (see Assumption A1) one can directly use Lemma 3 of [9]. The corresponding bounds for $|s - s_0(u)| > \delta$ follow from Potter's bounds on $V(su^{1/\beta})/V(u^{1/\beta})$, with the following application of Fernique's inequality as appeared in [9]. Thus it is sufficient to consider the maxima of the process $X^{(u)}(s)$ in the interval $S_u := [s_0(u) - \delta(u), s_0(u) + \delta(u)]$, where $\delta(u) = u^{-1}V(u^{1/\beta})\log u$.

The distribution of this maximum can be approximated accurately, by restricting the local behavior of the covariance function of the standardized Gaussian process $Y^{(u)}(s) := X^{(u)}(s)v_u(s)$, with corresponding changes of the barrier $uv_u(s)/V(u^{1/\beta})$, by simpler functions using (4). The assumption (5) in A2 means that we assume $Y^{(u)}$ to be locally stationary at the point $s_0(u)$ (that is, $Y^{(u)}(s - s_0(u) + s_0)$ is locally stationary at s_0).

For simplicity define $\tilde{u} = u/V(u^{1/\beta})$. Theorem 1 is a consequence of the following theorem and its corollary. Because of the transformation $t = su^{1/\beta}$, we have to investigate only the left hand side of the following equality:

$$\{\sup_{s\in S_u(x)} X^{(u)}(s) > \tilde{u}\} = \{\sup_{t\in T_u(x)} (X(t) - ct^{\beta}) > u\},$$

where

$$S_u(x) = [s_0(u) - \delta(u), s_0(u) + x\tilde{u}^{-1}/\sqrt{A(u)B}]$$

and

$$T_u(x) = [s_0(u)u^{1/\beta} - u^{1/\beta}\delta(u), s_0(u)u^{1/\beta} + x\tilde{u}^{-1}u^{1/\beta}/\sqrt{A(u)B}].$$

Under the same assumptions as Theorem 1 we derive two additional results.

Theorem 2. *Under the assumptions of Theorem 1 we have for all x*

$$P\{\sup_{s\in S_u(x)} X^{(u)}(s) > \tilde{u}\} \sim \frac{D^{1/\alpha}A^{2/\alpha-3/2}H_\alpha 2^{-1/\alpha}e^{-\frac{1}{2}A^2(u)\tilde{u}^2}\,\Phi(x)}{\sqrt{B}K^{-1}(\tilde{u}^{-1})\tilde{u}^2}$$

as $u \to \infty$, with $\tilde{u} = u/V(u^{1/\beta})$.

Corollary 3. *The assumptions of Theorem 1 imply*

$$P\{\sup_{t\ge 0}(X(t) - ct^{\beta}) > u\} \sim \frac{D^{1/\alpha}A^{2/\alpha-3/2}H_\alpha 2^{-1/\alpha}e^{-\frac{1}{2}A^2(u)\tilde{u}^2}}{\sqrt{B}K^{-1}(\tilde{u}^{-1})\tilde{u}^2}$$

as $u \to \infty$, with $\tilde{u} = u/V(u^{1/\beta})$.

Proof of Theorem 2. From [9] we know that for the regularly varying function $K^2(t)$ and any positive $\tilde{D}$ there exists a standardized stationary (and therefore locally stationary) Gaussian process $\tilde{U}(s)$ such that

$$\lim_{h\downarrow 0}\frac{E\left[\tilde{U}(s+h) - \tilde{U}(s)\right]^2}{K^2(h)} = \tilde{D}$$

where $\tilde{D} = D + \tilde{\varepsilon}$ for any small positive $\tilde{\varepsilon}$, and that we have for any $s, s' \in S_u$ and u sufficiently large

$$\mathrm{corr}(X^{(u)}(s)v_u(s), X^{(u)}(s')v_u(s')) \geq \mathrm{corr}(\tilde{U}(s), \tilde{U}(s')). \tag{7}$$

By analogy there also exists a standardized stationary Gaussian process $\tilde{U}$ corresponding to $\tilde{D} = D - \tilde{\varepsilon}$ such that the reverse of the inequality (7) holds.

Hence this means that mainly the probability

$$P\left\{\exists s \in S_u(x) : \tilde{U}(s) > \tilde{u}v_u(s)\right\}$$

has to be investigated since by use of Slepian's inequality (see e.g. Leadbetter et al. [12]), we have for u sufficiently large and $\tilde{D} = D + \tilde{\varepsilon}$,

$$\begin{aligned} P\left\{\max_{s \in S_u(x)} X^{(u)}(s) > \tilde{u}\right\} &= P\left\{\exists s \in S_u(x) : X^{(u)}(s)v(s) > \tilde{u}v_u(s)\right\} \\ &\leq P\left\{\exists s \in S_u(x) : \tilde{U}(s) > \tilde{u}v_u(s)\right\}. \end{aligned} \tag{8}$$

Further, using (4), we get for arbitrarily small positive ε and all sufficiently large u,

$$\begin{aligned} &P\left\{\exists s \in S_u(x) : \tilde{U}(s) > \tilde{u}v_u(s)\right\} \\ &\quad \leq P\left\{\exists s \in S_u(x) : \tilde{U}(s) > \tilde{u}\left(A(u) + \frac{1}{2}(B - \varepsilon)(s - s_0(u))^2\right)\right\}. \end{aligned} \tag{9}$$

The lower bound for the left hand term in (8) is found by selecting $\tilde{D} = D - \tilde{\varepsilon}$ in the definition of $\tilde{U}(s)$, by applying the same arguments as above with Slepian's inequality and using (4), since

$$\begin{aligned} &P\left\{\max_{s \in S_u(x)} X^{(u)}(s) > \tilde{u}\right\} \\ &\quad \geq P\left\{\exists s \in S_u(x) : \tilde{U}(s) > \tilde{u}v_u(s)\right\} \\ &\quad \geq P\left\{\exists s \in S_u(x) : \tilde{U}(s) > \tilde{u}\left(A(u) + \frac{1}{2}(B + \varepsilon)(s - s_0(u))^2\right)\right\}. \end{aligned}$$

Define

$$w(s) = w_u(s) = A(u) + \frac{1}{2}\tilde{B}(s - s_0(u))^2, \quad \text{where } \tilde{B} = B - \varepsilon \text{ or } \tilde{B} = B + \varepsilon.$$

It remains to analyze

$$P\left\{\exists s \in S_u(x) : \tilde{U}(s) > \tilde{u}w(s)\right\}.$$

We are going to apply Cuzick's [4] result for stationary Gaussian processes. In [9] Bräker's result (see [1,2]) was applied by verifying his assumptions (f1) to (f5) (for nonstationary, but locally stationary Gaussian processes) for S_u in a slightly simpler situation ($A(u) \equiv A$). Bräker's conditions (f1) to (f5) in the case of a stationary process coincide with Cuzick's ones. However, we note that we apply Cuzick's theorem to a sequence of intervals $S_u(x)$ and S_u which are not increasing as assumed in the results of Cuzick or Bräker, but which are decreasing (to s_0). A careful investigation of Cuzick's and Bräker's proofs shows that only intervals S_u and $S_u(x)$ are necessary, which are infinitely longer than $\Delta = K^{-1}(1/\tilde{u})$ as $u \to \infty$. This holds obviously in our case. By a simple modification of the arguments in [9] we can show that the conditions f1–f5 of [4] or [2] hold also by assumptions A1, A2 and A3. Thus by *Cuzick's* or *Bräker's Theorem* we get

$$P\{\tilde{U}(s) > \tilde{u}w(s) \text{ for some } s \in S_u(x)\} \sim \Lambda_u(x) = \int_{S_u(x)} \lambda_u(s)\mathrm{d}s \tag{10}$$

where

$$\lambda_u(s) = H_\alpha 2^{-1/\alpha}\psi(\tilde{u}w(s))/K^{-1}\left(1/(\sqrt{\tilde{D}}w(s)\tilde{u})\right)$$

with $\psi(x) = \varphi(x)/x$ and φ the standard normal density. Note that in the cited papers of Cuzick and Bräker the constant H_α is defined with respect to a fractional Brownian motion with variance s^α whereas here we use the common definition of H_α, with respect to the fractional Brownian motion $\chi(s)$ with variance $2s^\alpha$. Therefore H_α (Cuzick–Bräker) $= H_\alpha 2^{-1/\alpha}$.

It remains now to evaluate the integral (10). We define

$$k(s) = k(s,u) = 1/K^{-1}\left(1/(\sqrt{\tilde{D}}w(s)\tilde{u})\right).$$

Notice that from A1 it follows that $s_0(u) \to s_0$. Therefore, we have by the regularity of K for $s \in S_u$, uniformly,

$$k(s) \sim k(s_0(u)) \sim \left(\sqrt{\tilde{D}}w(s_0)\right)^{2/\alpha}\left[K^{-1}(1/\tilde{u})\right]^{-1}.$$

Using $y = (A(u)B)^{-1/2}x$, the integral (10) can be written as

$$\begin{aligned}
\int_{S_u(x)} \lambda_u(s)\mathrm{d}s &= H_\alpha 2^{-1/\alpha}\int_{s_0(u)-\delta(u)}^{s_0(u)+y\tilde{u}^{-1}} k(s)\psi(\tilde{u}w(s))\,\mathrm{d}s \\
&\sim k(s_0)H_\alpha 2^{-1/\alpha}\int_{s_0(u)-\delta(u)}^{s_0(u)+y\tilde{u}^{-1}} \psi(\tilde{u}w_u(s))\,\mathrm{d}s \\
&\sim \frac{k(s_0)H_\alpha 2^{-1/\alpha}}{\sqrt{2\pi}\tilde{u}w_u(s_0)}\int_{s_0(u)-\delta(u)}^{s_0(u)+y\tilde{u}^{-1}} \mathrm{e}^{-\frac{1}{2}\tilde{u}^2 w_u^2(s)}\,\mathrm{d}s \\
&\sim \frac{k(s_0)H_\alpha 2^{-1/\alpha}}{\sqrt{2\pi}\tilde{u}w_u(s_0)}\int_{s_0(u)-\delta(u)}^{s_0(u)+y\tilde{u}^{-1}} \mathrm{e}^{-\frac{1}{2}\tilde{u}^2\left(A^2(u)+(A(u)\tilde{B}+o(1))(s-s_0(u))^2\right)}\,\mathrm{d}s
\end{aligned} \tag{11}$$

as $u \to \infty$. Now change variables

$$v = \sqrt{A(u)\tilde{B}}\tilde{u}(s - s_0(u))$$

and recall the definition of y. Since $\delta(u)\tilde{u} = \log u \to \infty$ as $u \to \infty$, we obtain that

$$\begin{aligned}
\int_{S_u(x)} \lambda_u(s)\mathrm{d}s &\sim \frac{k(s_0)H_\alpha 2^{-1/\alpha}}{\sqrt{A(u)\tilde{B}\tilde{u}^2}w_u(s_0)}\mathrm{e}^{-\frac{1}{2}A^2(u)\tilde{u}^2}\,\Phi(x) \\
&\sim \frac{(\sqrt{\tilde{D}}w_u(s_0))^{2/\alpha}H_\alpha 2^{-1/\alpha}}{\sqrt{A(u)\tilde{B}}K^{-1}(1/\tilde{u})\tilde{u}^2 w_u(s_0)}\mathrm{e}^{-\frac{1}{2}A^2(u)\tilde{u}^2}\,\Phi(x) \\
&\sim \frac{\tilde{D}^{1/\alpha}A^{2/\alpha-3/2}H_\alpha 2^{-1/\alpha}}{\sqrt{\tilde{B}}K^{-1}(1/\tilde{u})\tilde{u}^2}\mathrm{e}^{-\frac{1}{2}A^2(u)\tilde{u}^2}\,\Phi(x)
\end{aligned}$$

as $u \to \infty$. Since $\tilde{D} = D \pm \tilde{\varepsilon}$ and $\tilde{B} = B \pm \varepsilon$ for any $\tilde{\varepsilon}, \varepsilon > 0$, the proof of Theorem 2 is complete. □

Proof of Corollary 3. This result follows from the last relation by letting $x = \infty$. □

Proof of Theorem 1. As mentioned in the introduction of this section, it remains to analyze the crossings of the transformed process $X^{(u)}(s)$ in the interval $S_u(x)$ for $u \to \infty$. Noting that

$$P((\tau_u - s_0(u)u^{1/\beta})/\sigma(u) < x \mid \tau_u < \infty) \sim \frac{P(\sup_{s \in S_u(x)} X^{(u)}(s) > \tilde{u})}{P(\tau_u < \infty)}$$

the statement follows by applying Theorem 2 to the numerator and Corollary 3 to the denominator. □

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