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Excitation of beta-induced Alfvén eigenmodes by the coupling between geodesic acoustic mode and magnetic island

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Abstract

The mechanism of excitation of beta-induced Alfvén eigenmodes (BAEs) with magnetic islands larger than a threshold without energetic ions is studied. It is found that the nonlinear coupling between geodesic acoustic mode and magnetic islands can drive the pair of BAEs. The phase of BAEs to island should be $\pi/2$ to excite the BAEs and the magnetic island is larger than a threshold. The results are consistent with the experimental results shown in EAST(#86309). It implies that similar experimental results in other tokamaks, that BAEs excitation by magnetic islands without energetic ions, may be from the nonlinear coupling between islands and waves. It also implies that the existence of magnetic islands can make the excitation of BAEs easier in plasma with energetic ions, since the magnetic island can also increase the pressure gradient of energetic ions near the island separatrix. This predicts that BAEs may appear more frequently in the presence of magnetic islands in ITER.

Keywords: beta induced Alfvén eigenmodes, magnetic island, geodesic acoustic mode, tearing modes

(Some figures may appear in colour only in the online journal)

1. Introduction

Beta-induced Alfvén eigenmode (BAE) [1–3], one kind of the Alfvén eigenmodes, is expected to play an important role in the magnetic confined fusion device, such as ITER, since it can be driven by energetic particles and in turn cause the redistribution and loss of energetic particles. Then, it would affect the confinement of energetic particles and the heating efficiency. Thus, BAE is important for burning plasmas. BAE is always excited at the rational surface, and the mode structure of BAE is localized around the rational surface. Its frequency is inside the shear Alfvén continuous spectrum gap caused by the finite thermal plasma compressibility, and is close to that of geodesic acoustic mode (GAM) [4], which is thought of as an electrostatic mode with toroidal mode number $n = 0$. Tearing mode (including neoclassical tearing mode) [5, 6] is one of the most dangerous magnetohydrodynamics instabilities in tokamak discharge, which can lead to the formation of magnetic islands, increase local radial transport, and degrade plasma confinement. If the island becomes large enough, it can

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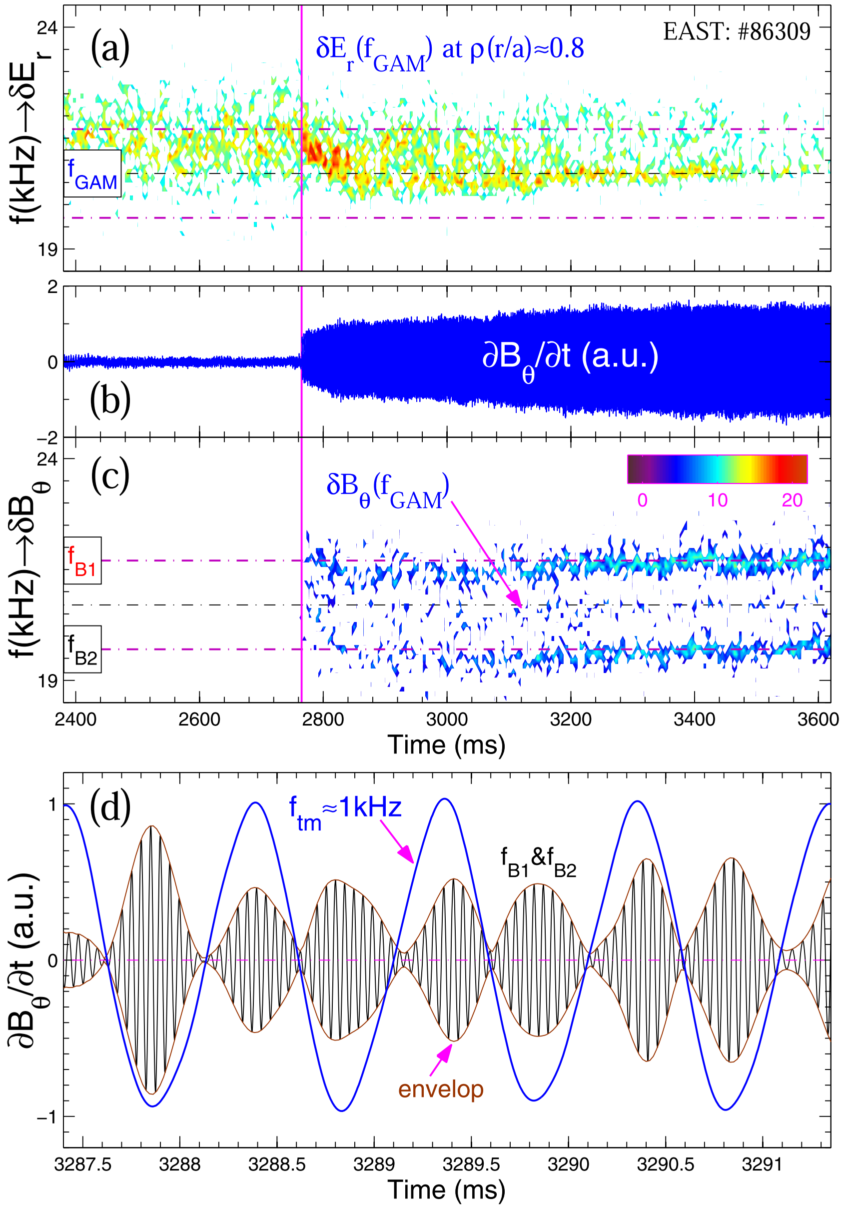

Figure 1. (*a*) Electrostatic component of GAM at $\rho(r/a) \approx 0.8$ measured by DBS signal, (*b*) raw magnetic signal measured by edge magnetic pickup probe, (*c*) electromagnetic component of GAM and BAEs, (*d*) electromagnetic component of BAEs in the frequency range of 18 kHz $\leqslant f \leqslant$ 23 kHz are filtered, and the envelop is modulated by the oscillation of TMs. Note: in this experiment, there is no neutral beam injection. The magnetic island can be observed for $t \geqslant 2.76$ s, and the electromagnetic component of three branches are excited accordingly. The frequencies are $f_{\mathrm{GAM}} \approx 20.7$ kHz, $f_{\mathrm{B1}} \approx 21.7$ kHz, $f_{\mathrm{B2}} \approx 19.7$ kHz, and the relationship among GAM, BAEs and TMs are listed: $f_{\mathrm{GAM}} \equiv (f_{\mathrm{B1}} + f_{\mathrm{B2}})/2$, $f_{\mathrm{B1}} - f_{\mathrm{B2}} \equiv 2f_{\mathrm{tm}}$.

cause disruption. Tearing mode can also interact with energetic ions strongly. It would lead to the redistribution and loss of energetic ions, and energetic ions would affect tearing modes in turn [7]. Hence, the control of magnetic islands is one of the critical physics problems to achieve steady-state and high confinement plasmas [8].

The mode structures of BAE and tearing modes are both localized around the rational surface. Thus, they would interact with each other strongly. Recently, some experiments [9–16] have shown that BAEs can be excited in the presence of magnetic islands. BAEs were observed during tearing modes in FTU [9], TEXTOR [10], HL-2A [11, 12] and J-TEXT [13, 17] Ohmic plasmas, EAST with lower hybrid wave plasmas, HL-2A [18] and TJ-II stellarator [15] with energetic ions plasmas. In these experimental results, they have common characteristics: the excited BAEs during tearing modes are a pair waves, propagate in poloidally and toroidally with opposite direction. They only appear when the island width increases above a threshold. The absolute values of poloidal and toroidal mode numbers are the same with tearing modes, and the frequency difference between the pair of BAEs is twice the fundamental frequency of the tearing mode. Some theoretical works

[19, 20] tried to understand the physics. In reference [19], the BAE frequency with magnetic islands is solved and compared well with the experimental results. Actually, the Alfvén continuum spectrum in the island has been explored in reference [21, 22]. In reference [20], it was shown that the BAEs are excited by the plasma flow around the magnetic island. However, the excitation physics mechanism is still unclear. Recently, the experiment in EAST(#86309) showed that a GAM and magnetic island exist before the excitation of BAEs, and the pair of BAEs is excited when the magnetic island is larger than a threshold. It can be found in figure 1. It is clear that the excitation of the pair of BAEs are related to the interaction between GAM and the magnetic island. Based on these experiments, it is thought that the pair of BAEs may be excited by the nonlinear wave interaction without energetic ions. We try to explore the excitation mechanism of BAEs during tearing modes.

In section 2, a fluid model is presented, and the interaction between BAEs, GAM and magnetic island is analyzed and compared with the experimental results. Finally, the conclusion is given in section 3.

2. Interaction between BAEs, GAM and magnetic island

In this work, a fluid model is used to describe the excitation of BAEs, for simplicity. The kinetic effects such as Landau damping, finite Larmor radius are not considered, since we focus on the mechanism of nonlinear wave coupling. To investigate the excitation of BAEs by the coupling between GAM and tearing mode, GAM is assumed to be a constant pump wave and tearing mode evolves. The fluctuations of fields are $\delta\phi = \delta\phi_{\rm B} + \delta\phi_{\rm G} + \delta\phi_{\rm t}$, $\delta A_\parallel = \delta A_{\parallel,\rm B} + \delta A_{\parallel,\rm t}$, and $\delta\phi_{\rm B} = \delta\phi_+ + \delta\phi_-$, $\delta A_{\parallel,\rm B} = \delta A_{\parallel,+} + \delta A_{\parallel,-}$, where the subscripts of B, G, t denote the BAE, GAM and tearing mode, respectively. The subscripts $\pm$ are the upper and lower frequencies of the pair of BAEs, respectively. Here, GAM is assumed to be predominantly electrostatic, as $\delta\phi_{\rm G} = \delta\hat{\phi}_{\rm G}$ $\exp(-{\rm i}\omega_{\rm G}t)$ ($\omega_{\rm G}$ is the frequency of GAM). For tearing mode, $\delta A_{\parallel,\rm t} = \delta\hat{A}_{\parallel,\rm t}(0,t)\cos\xi$, where the familiar constant-$\delta\psi_{\rm t}$ ($\delta A_{\parallel,\rm t} = -R\,\delta\psi_{\rm t}$) approximation is used, $\xi = m\,\theta - n\,\zeta - \omega_{\rm t}t$ is the helical angle. $\omega_{\rm t}$ is the frequency of tearing mode and is assumed to be constant with respect to t. m, n are the poloidal and toroidal mode numbers, respectively. For the pair of BAEs, the mode numbers are the same with those of tearing modes, and they form as a standing wave in the island rest frame. Then, the fluctuations of BAEs can be written as $(\delta\phi_\pm, \delta A_{\parallel,\pm}) = (\delta\hat{\phi}_\pm, \delta\hat{A}_{\parallel,\pm})\exp(\pm{\rm i}(m\theta - n\zeta + \varphi_0) - {\rm i}\omega_\pm t)$ ($\omega_\pm$ are the frequencies of the upper and lower frequencies of the pair of BAEs, respectively). Here, $\omega_\pm = \omega_{\rm G} \pm \omega_{\rm t}$, φ_0 is the phase shift to tearing mode. Then, the current conservation equation is

$$
\begin{aligned}
&\nabla\cdot\left(\frac{\rho}{B^2}\mathbf{B}\times\frac{\partial\delta\mathbf{u}_{\rm B}}{\partial t}\right) + \mathbf{B}\cdot\nabla\frac{\delta J_{\parallel,\rm B}}{B} \\
&+ \delta\mathbf{B}_{\rm B}\cdot\nabla\frac{J_\parallel}{\rm B} + \nabla\cdot\left(\frac{\mathbf{B}\times\nabla\delta p_{\rm B}}{B^2}\right) \\
&+ \sum_{\omega_{\rm G}\pm\omega_{\rm t}=\omega_\pm}\nabla\cdot\left(\frac{\delta\rho_{\rm t}}{B^2}\mathbf{B}\times\frac{\partial\delta\mathbf{u}_{\rm G}}{\partial t}\right) \\
&+ \sum_{\omega_{\rm G}\pm\omega_{\rm t}=\omega_\pm}\nabla\cdot\left(\frac{\delta\rho_{\rm G}}{B^2}\mathbf{B}\times\frac{\partial\delta\mathbf{u}_{\rm t}}{\partial t}\right) \\
&+ \sum_{\omega_{\rm G}\pm\omega_{\rm t}=\omega_\pm}\nabla\cdot\left(\frac{\rho}{B^2}\mathbf{B}\times\delta\mathbf{u}_{\rm t}\cdot\nabla\delta\mathbf{u}_{\rm G}\right) \\
&+ \sum_{\omega_{\rm G}\pm\omega_{\rm t}=\omega_\pm}\nabla\cdot\left(\frac{\rho}{B^2}\mathbf{B}\times\delta\mathbf{u}_{\rm G}\cdot\nabla\delta\mathbf{u}_t\right) = 0, \qquad (1)
\end{aligned}
$$

where the terms on the left-hand side represent plasma inertial, field line bending, kink, pressure-curvature coupling, and nonlinear wave coupling, respectively. Here, $\mathbf{B}\cdot\delta\mathbf{B} + \delta P \sim 0$ is made, namely the compressional Alfvén wave is not considered. The nonlinear wave coupling effect enters explicitly through the last two terms. Implicit nonlinear wave coupling contribution is from the field line bending term, kink term, and pressure-curvature coupling term via the nonlinear Ohm's law and pressure evolution equation, as

$$
-\mathbf{b}\cdot\nabla\delta\phi_{\rm B} - \frac{1}{c}\frac{\partial\delta A_{\parallel,\rm B}}{\partial t} - \sum_{\omega_{\rm G}\pm\omega_{\rm t}=\omega_\pm}\delta\mathbf{B}_{\rm t}\cdot\nabla\delta\phi_{\rm G} = \eta J_{\parallel,\rm B}, \qquad (2)
$$

$$
\begin{aligned}
&\frac{\partial\delta p_{\rm B}}{\partial t} + \delta\mathbf{u}_{\rm B}\cdot\nabla p + \Gamma p\nabla\cdot\delta\mathbf{u}_{\rm B} + \sum_{\omega_{\rm G}\pm\omega_{\rm t}=\omega_\pm}\delta\mathbf{u}_{\rm G}\cdot\nabla\delta p_{\rm t} \\
&+ \sum_{\omega_{\rm G}\pm\omega_{\rm t}=\omega_\pm}\delta\mathbf{u}_{\rm t}\cdot\nabla\delta p_{\rm G} = 0. \qquad (3)
\end{aligned}
$$

Here, the modification of equilibrium current and pressure profiles due to the magnetic island is included. Given the equilibrium magnetic field $\mathbf{B} = I\nabla\zeta + \nabla\zeta\times\nabla\psi$, $\delta\mathbf{u} = \mathbf{B}\times\nabla\delta\phi/B^2$ is the electric drift, where $\delta\mathbf{A} = A_\parallel\mathbf{b}$ is taken for BAEs [3]. Then, considering one of BAEs, taking the partial derivative $\partial/\partial t$ on equation (1), and then operating $\langle\langle\ldots\rangle_\theta\rangle_\xi$, where $\langle\ldots\rangle_\theta = \oint{\rm d}\theta\,J(\ldots)\exp(-{\rm i}(m\,\theta - n\,\zeta + \varphi_0 - \omega_+t))/\oint J{\rm d}\theta$, $\langle\ldots\rangle_\xi = \oint{\rm d}\xi(\ldots)/\oint{\rm d}\xi$, one can obtain

$$
L_m(\delta\hat{\phi}_+) + L_N(\delta\hat{\phi}_{\rm G}, \delta\hat{A}_\parallel) = 0, \qquad (4)
$$

where the linear operator is

$$
\begin{aligned}
L_m(\delta\hat{\phi}_+) = &\frac{\partial}{\partial r}\left(r\left(\bar{\omega}_+^2 - 2\bar{\omega}_{\rm sou}^2 - k_\parallel^2\right)\frac{\partial\delta\hat{\phi}_+}{\partial r}\right) \\
&- \frac{m^2}{r}\left(\bar{\omega}_+^2 - 2\bar{\omega}_{\rm sou}^2 - k_\parallel^2 - \bar{\kappa}_r\alpha/q^2\right)\delta\hat{\phi}_+ \\
&+ \frac{{\rm d}k_\parallel^2}{{\rm d}r}\delta\hat{\phi}_+ + m\,k_\parallel R_0\langle\frac{\rm d}{{\rm d}r}\frac{J_\parallel - J_{\parallel,0}}{B_0}\rangle_\xi\delta\hat{\phi}_+ \\
&- \frac{m^2}{r}\bar{\kappa}_r R_0\langle\frac{\rm d}{{\rm d}r}(\beta - \beta_0)\rangle_\xi\delta\hat{\phi}_+, \qquad (5)
\end{aligned}
$$

where $\bar{\omega}_+ = \omega_+/\omega_A$, $\bar{\omega}_{\rm sou} = \omega_{\rm sou}/\omega_A$, $\omega_A = v_A/R_0, v_A = B_0/\sqrt{\mu_0\langle\rho\rangle_\xi}$, $\omega_{\rm sou}^2 = \Gamma\langle p\rangle_\xi/(\langle\rho\rangle_\xi R_0^2)$, $k_\parallel = m/q - n$, $\bar{\kappa} = \epsilon(1 - 1/q^2) + \alpha/2$, $\alpha = -R_0q^2{\rm d}\beta_0/{\rm d}r$, $\beta_0 = 2\mu_0 p_0/B_0^2$. The sideband coupling and resistivity are not considered. Here, the modified profiles of current density and pressure in the presence of islands are included, which are reflected at the total parallel current density $J_\parallel$ and pressure p. Without island modified effect, the operator (5) reduces to that in reference [23]. The nonlinear operator is

$$L_N(\delta\hat{\phi}_{\rm G},\delta\hat{A}_\|)\sim\frac{\omega_+}{\omega_A^2}\frac{m}{B_0}\left\langle\frac{\partial}{\partial r}\left(\left(1+\frac{2\omega_{\rm sou}^2}{\omega_{\rm G}\omega_+}\right)\frac{\partial^2\delta\hat{\phi}_{\rm G}}{\partial r^2}\delta\phi_{\rm t}-\frac{\partial\delta\hat{\phi}_{\rm G}}{\partial r}\frac{\partial\delta\phi_{\rm t}}{\partial r}\right){\rm e}^{-{\rm i}(\xi+\varphi_0)}\right\rangle_\xi-\frac{1}{2}k_\|\frac{mR_0}{B_0}\delta\hat{A}_\|(0,t)\frac{\partial^3\delta\hat{\phi}_{\rm G}}{\partial r^3}{\rm e}^{-{\rm i}\varphi_0},\tag{6}$$

where the first term in the bracket on the right hand results from the nonlinear Reynolds stress and curvature-pressure coupling effects, the final term is from the nonlinear Ohm's law. Here, it is assumed that the mode structures are localized, satisfying $a\partial\ln\delta f/\partial r\gg1$ (a is the minor radius). Equation (4) can be solved perturbatively [20]. Then, the growth rate can be obtained as

$$\frac{\gamma}{\omega_A}={\rm e}^{-{\rm i}\pi/2}\frac{\omega_A}{2r_s\omega_{\rm G}}\int_{-\infty}^{\infty}{\rm d}x\,L_N\left(\delta\hat{\phi}_{\rm G},\delta\hat{A}_\|\right)\delta\hat{\phi}_+\left(\int_{-\infty}^{\infty}\left(\frac{\partial\delta\hat{\phi}_+}{\partial x}\right)^2\right)^{-1}.\tag{7}$$

where $x=r-r_s$, and the expression of $\delta\phi_{\rm t}$ is determined by tearing modes. The other branch of BAE (ω_-) can be obtained similarly.

For tearing modes, the Ohm's law can be written as

$$-\left(\mathbf{b}+\frac{\delta\mathbf{B}_{\rm t}}{B}\right)\cdot\nabla\delta\phi_{\rm t}-\frac{1}{c}\frac{\partial\delta A_{\|,{\rm t}}}{\partial t}-\frac{\delta\mathbf{B}_{\rm B}}{B}\cdot\nabla\delta\phi_{\rm G}=\eta J_{\|,{\rm t}},\tag{8}$$

where the last term on the left-hand side is the nonlinear coupling effect. It is convenient to transform the coordinates (r,θ,ξ) to the island coordinates (Ω,θ,ξ), where $\Omega=2(r-r_s)^2/w^2-\cos\xi$, $w=2\sqrt{r_s\delta\hat{\psi}_{\rm t}/(S\psi_s')}$ is island width (the prime denotes the derivative with respect to r, the subscript s represents the variables defined at rational surface), S is the magnetic shear, $\delta\psi_{\rm t}=-R\,\delta A_{\|,{\rm t}}$, then one can obtain

$$\delta\phi_{\rm t}=\frac{\omega_{\rm t}rB}{m}\left(r-r_s-h(\Omega)\right),\tag{9}$$

$$\frac{\partial\delta\hat{\psi}_{\rm t}}{\partial t}\langle\cos\xi\rangle_\Omega+\frac{2m}{rB_0}\frac{1}{\omega_{\rm G}}\langle k_\|\delta\hat{\phi}_{\rm B}\frac{\partial\delta\hat{\phi}_{\rm G}}{\partial r}\sin(\xi+\varphi_0)\rangle_\Omega=\eta R_0\langle J_{\|,{\rm t}}\rangle_\Omega,\tag{10}$$

where $\langle\ldots\rangle_\Omega=\oint{\rm d}\xi/(2\pi)(\ldots)/\sqrt{\Omega+\cos\xi}$. $h(\Omega)$ is determined by the effect of the island on the radial transport, as [7]

$$\frac{{\rm d}h}{{\rm d}\Omega}=\sigma_x\frac{\sqrt{2}w}{4}\frac{\pi}{2}\frac{1}{(\Omega+1)^{1/2}E(1/(\Omega+1))}H(\Omega-1)\tag{11}$$

where σ_x denotes the sign of x, $H(\Omega-1)$ is the Heaviside function and $E(1/(\Omega+1))$ is the elliptic function.

As reference [7], $J_{\|,{\rm t}}=\langle J_{\|,{\rm t}}\rangle_\Omega/\langle1\rangle_\Omega$ without including the contribution of neoclassical polarization current, bootstrap current and so on. Then, substituting the expression of $J_{\|,{\rm t}}$ into the island evolution equation [7]

$$\Delta'=\frac{4\sqrt{2}q_sR_0}{q_s'\psi'w}\int_{-1}^{\infty}{\rm d}\Omega\langle J_{\|,t}\cos\xi\rangle_\Omega,\tag{12}$$

one can obtain

$$\frac{2I_1}{\eta}\frac{{\rm d}w}{{\rm d}t}=\Delta'+\Delta_c,\tag{13}$$

$$\Delta_{\rm c}=8\frac{m^2\rho_i^2}{r_s^2}\frac{R_0^2}{a^2}\frac{\tau_{\rm R}}{\tau_{\rm A}}\frac{v_{\rm thi}^2}{v_A^2}\frac{1}{(\Gamma\beta)^{1/2}}\frac{1}{\Delta_{\rm G}}\frac{{\rm e}^2A_{\rm B}A_{\rm G}}{T_i^2}I_{T0},\tag{14}$$

where $\Delta'=(\partial\ln\delta A_\|/\partial r)|_{r_s^-}^{r_s^+}$ is the stability criterion of tearing modes [5, 6]. $\Delta_{\rm c}$ results from the coupling from GAM and BAE. The numerical value $I_1=1.66$,

$$I_{T0}=\int_{-1}^{\infty}{\rm d}\Omega\left\langle(\Omega+\cos\xi)^{1/2}\sin(\xi+\varphi_0)\delta\bar{\phi}_{\rm B}\times\frac{\partial\delta\bar{\phi}_{\rm G}(x/\Delta_{\rm G})}{\partial(x/\Delta_{\rm G})}\right\rangle_\Omega\frac{\langle\cos\xi\rangle_\Omega}{\langle1\rangle_\Omega},$$

$\delta\hat{\phi}_{\rm B,G}=A_{\rm B,G}\delta\bar{\phi}_{\rm B,G}$ is given, $\Delta_{\rm G}$ is the characteristic scale length of $\delta\bar{\phi}_{\rm G}$, $v_{\rm thi}$ is the ion thermal velocity, $\tau_{\rm R},\tau_{\rm A}$ are the resistivity diffusion time and Alfvén time, respectively. The stable or unstable effect of coupling between BAEs and GAM on tearing mode depends on the sign of I_{T0}. It can be found that $I_{T0}\neq0$ if the phase shift of BAE to tearing mode $\varphi_0=\pi/2+l\pi$ (l is an integer).

Now, for the growth rate of BAE, substituting equation (6) and the expression (9) into equation (7), one can obtain

$$\frac{\gamma}{\omega_+}A_{\rm B}=-{\rm e}^{-{\rm i}(\pi/2+\varphi_0)}\left(\frac{\omega_A^2}{\omega_+^2}\frac{\Delta_{\rm B}^2n^2S^2}{16r_s^2}\frac{w^2}{\Delta_{\rm B}^2}I_{\rm B3}-\frac{\omega_t}{2\omega_+}(I_{\rm B1}+I_{\rm B2})\right)\times\frac{1}{I_{\rm B0}}A_{\rm G},\tag{15}$$

$$I_{\rm B0}=\int_{-\infty}^{\infty}{\rm d}y\left(\frac{\partial\delta\bar{\phi}_{\rm B}}{\partial y}\right)^2,\tag{16}$$

$$I_{\rm B1}=\frac{w}{\sqrt{2}\Delta_{\rm G}}\int_1^{\infty}{\rm d}\Omega\left\langle3\frac{\partial\delta\bar{\phi}_{\rm G}(x/\Delta_{\rm G})}{\partial(x/\Delta_{\rm G})}\frac{\partial\delta\bar{\phi}_{\rm B}}{\partial y}\left(1-(\Omega+\cos\xi)^{1/2}\frac{2\sqrt{2}}{w}\frac{{\rm d}h}{{\rm d}\Omega}\right){\rm e}^{-{\rm i}\xi}\right\rangle_\xi,\tag{17}$$

$$I_{\rm B2}=\frac{w^2}{\Delta_{\rm B}\Delta_{\rm G}}\int_1^{\infty}{\rm d}\Omega\left\langle\frac{\partial\delta\bar{\phi}_{\rm G}(x/\Delta_{\rm G})}{\partial(x/\Delta_{\rm G})}\frac{\partial^2\delta\bar{\phi}_{\rm B}}{\partial y^2}\left((\Omega+\cos\xi)^{1/2}-\frac{\sqrt{2}}{w}h(\Omega)\right){\rm e}^{-{\rm i}\xi}\right\rangle_\xi,\tag{18}$$

$$I_{\rm B3}=\int_{-\infty}^{\infty}{\rm d}y\frac{\partial\delta\bar{\phi}_{\rm G}(y\Delta_{\rm B}/\Delta_{\rm G})}{\partial y}\left(2\frac{\partial\delta\bar{\phi}_{\rm B}}{\partial y}+y\frac{\partial^2\delta\bar{\phi}_{\rm B}}{\partial y^2}\right),\tag{19}$$

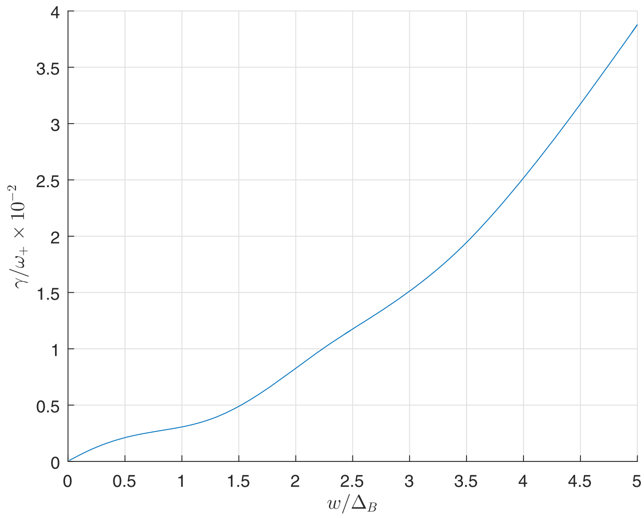

Figure 2. The growth rate γ of BAE against w/Δ_{B}.

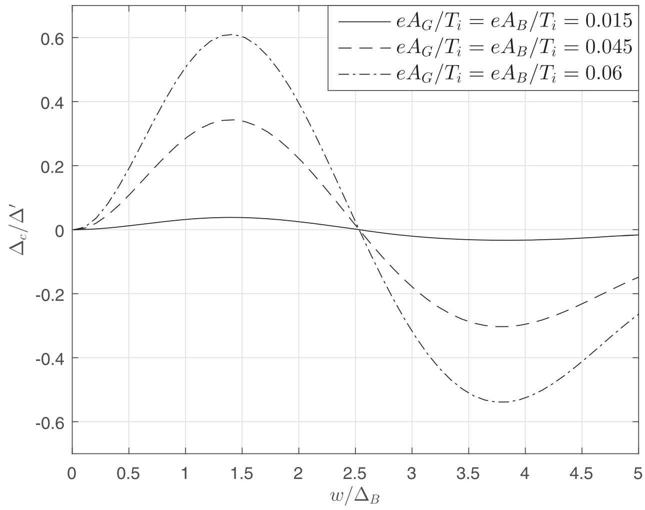

Figure 3. The ratio $\Delta_{\mathrm{c}}/\Delta'$ against w/Δ_{B}.

where $y = x/\Delta_{\mathrm{B}}$, Δ_{B} is the characteristic scale length of $\delta\bar{\phi}_{\mathrm{B}}$. As nonlinear coupling effect on tearing mode, γ is real when the phase shift $\varphi_0 = \pi/2 + l\pi$. Here, the damping rate of BAE is not included. When the growth rate in (15) is larger than the damping rate, the BAE is excited. It can be seen that γ depends on the island width. When the island width increases to a threshold, BAE would be excited.

To proceed, the profiles of $\delta\hat{\phi}_{\mathrm{G,B}}$ are given, as [20, 24]

$$\frac{\mathrm{d}\delta\hat{\phi}_{\mathrm{B}}}{\mathrm{d}y} = A_{\mathrm{B}}\left(1 - y\,\mathrm{e}^{-y^2/2}\int_0^y \exp(t^2/2)\mathrm{d}t\right), \quad (20)$$

$$\delta\hat{\phi}_{\mathrm{G}} = A_{\mathrm{G}}Airy(-x/\Delta_{\mathrm{G}}), \quad (21)$$

where

$$\Delta_{\mathrm{B}} = \left[\frac{3}{4}\left(\frac{\omega_{\mathrm{B}}\rho_i q R_0}{k_\theta\, v_A\, S}\right)^2\right]^{1/4},$$
$$\Delta_{\mathrm{G}} = \rho_i^{2/3}\left(\mathrm{d}\ \ln\ T_i/\mathrm{d}r\right)^{-1/3}.$$

Considered the main parameters of the experiment result in EAST, $R_0 = 1.89$ m, $a = 0.45$ m, $B = 2.05$ T, $n_0 = 1.0 \times 10^{19}$ m^{-3}, $T_{\mathrm{i}} = T_{\mathrm{e}} = 200$ eV, $f_{\mathrm{G}} \sim 20.7$ kHz, $f_{\mathrm{t}} \sim 1$ kHz, $q_s = 4$, $r_s = 0.8a$, and the deuterium plasma, $S = 0.1$ is taken, and $A_{\mathrm{B}} \sim A_{\mathrm{G}}$ is assumed, then the growth rate can be numerically calculated, as shown in figure 2, where $\varphi_0 = \pi/2$ is chosen. It can be seen that the growth rate without damping rate increases with island width. If the growth rate is greater than the damping rate, BAEs are excited. Here, it is needed to point out that the phase shift $\varphi_0 = \pi/2$ is taken, so that the growth rate is positive. It means that BAEs may be excited only when the phase shift is $\pi/2$. This is well consistent with the experimental result in EAST(#86309), where it was shown that the phase shift of BAE to tearing mode is about $\pi/2$, which can be found in figure 1. Thus, the nonlinear coupling between GAM and island seems to be responsible for the excitation of BAEs in EAST.

For tearing mode, the ratio $\Delta_{\mathrm{c}}/\Delta'$ can be seen in figure 3 where $r_s\Delta' = 1$ and $\varphi_0 = \pi/2$ are taken. It can be seen that $\Delta_{\mathrm{c}}/\Delta' < 1$, and it would become negative when w/Δ_{B} is larger enough, which is caused by the profiles of $\delta\bar{\phi}_{\mathrm{B}}$ and $\delta\bar{\phi}_{\mathrm{G}}$ chosen. It means that the nonlinear coupling effect from BAEs and GAM on island enhances the island evolution when the island width is small, and weakens the island evolution when the island is large. For EAST experiment, $\mathrm{e}A_{\mathrm{B}}/T_i \sim \mathrm{e}A_{\mathrm{G}}/T_i \sim 0.045$, so the nonlinear coupling effect from BAEs and GAM on the island is small, which is also consistent with the experimental result in EAST, where it was shown that the island was not changed much.

3. Conclusion and discussion

In conclusion, the excitation mechanism of BAEs during the island phase is explored. It is found that the nonlinear coupling between GAM and island could drive a pair of BAEs when the island is larger than a threshold. The phase of BAEs to island should be $\pi/2$. The result is consistent with the experimental results in EAST. In turn, the nonlinear coupling between BAEs and GAM would affect island evolution, although this effect is small. The excitation mechanism of BAEs by the island results from the nonlinear wave coupling. This excitation mechanism can be applied at other similar experiments [9–16]. It is thought that the excitation of BAEs by the island in these experiments may be from the nonlinear wave coupling based on the above results. In HL-2A experiment [12], it was shown that energetic-ions-induced GAM (EGAM), island and BAEs couples strongly, where the frequency of EGAM is close to BAE, so that mode coupling between EGAM, tearing modes and BAEs becomes easy. In general, the frequency of EGAM [25, 26] can be far from BAEs. In that case, the mode coupling between EGAM, tearing modes and BAEs may be difficult. In the presence of energetic ions, the physics would become complex due to the interaction between energetic ions and waves. In principle, if other mode (not just GAM), BAEs and (neoclassical) tearing mode satisfy three wave coupling condition, the physics described in this manuscript may be happen, although the model is different. It is known that energetic ions can drive BAE when the pressure of energetic ions is large enough. In the presence of energetic ions, when the island is large enough, the pressure of energetic ions near the island separatrix becomes steep, and can excite BAEs if the initial pressure of energetic ions is marginal. Thus, the effects of island on BAEs excitation are reflected in two sides if GAM and tearing mode exist. Tearing modes can couple with other mode to drive BAEs. On the other hand, it can increase the pressure gradient of energetic ions near the island separatrix to drive BAEs. Hence, it can be predicted that plasma with neutral beam injection or burning plasma in large tokamak like ITER, BAEs can be more easier to be excited in the presence of islands, and BAEs would affect the transport of energetic ions in turn.

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