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Error Mitigation of Grover's Quantum Search Algorithm

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Grover's quantum search algorithm delivers quadratic speedup over classical counterparts for finding an item in unstructured database. But the accuracy of the algorithm degrades as the number of qubits are increased. Noises of various types affect the accuracy of the Grover's algorithm. The imperfect measurement on the qubit results in measurement errors appears to be one of the factors which limit the scalability of near-term quantum devices/algorithms. This paper is focused on the implementation of measurement error mitigation technique on Grover's algorithm up to 4-qubit. The measurement error mitigation model for Grover's algorithm is developed and implemented on the real-time quantum computer. The accuracy of Grover's algorithm up to 4-qubits with and without measurement errors is evaluated and compared. The results indicate that measurement error mitigation technique mitigates the measurement errors of the Grover's algorithm and improves its accuracy.

Keywords: Errors; Grover; Oracle; Measurement; Mitigation; Qubit, Noise

1 Introduction

Quantum computers and algorithms offer substantial enhancement of speedup for the problems which are hard to compute on the classical computers[1]. In the recent times, every electronic device is connected to the internet which leads to the accumulation of enormous data online. Classical computers and algorithms are up against a lot of challenges when it comes to managing big data[2-3]. The classical search algorithms, when searches for any marked item in the unstructured big data become inefficient as the data is composed of matrices and vectors. Quantum search algorithms efficiently perform the same task as they are driven from linear algebra[4]. Quantum computers opt a different methodology in order to process the information than classical computers. Classical computers are constructed using transistors which represent information in binary '0' and '1' whereas quantum computers are constructed using qubits which represent the information in $|0\rangle$, $|1\rangle$ states and their superposition[5-6]. The superposition[7] and entanglement[8] are the principles of quantum mechanics which makes quantum computers capable of managing several states at once. The quantum gate[9-10] perform such operations on qubits. Hadamard gate set up qubits into superposition and CNOT gate is used to entangle qubits. These principles of quantum mechanics when used for information processing referred to *Quantum computing*[11]. Quantum computing offers algorithms such as Grover's search algorithm[12] for finding any marked item in unstructured databases, Shor's algorithm [13] for factorization problems etc.

The quantum algorithms are implemented by generating their quantum circuit. The total amount of gates as well as depth of circuit describe the size of a quantum circuit. *Circuit depth* of a quantum circuit is the path comprises of maximum gates from input to output, moving forward in time[14]. In case of Grover's algorithm, as the circuit depth increases, errors arise throughout the quantum circuit because of its complexity. This leads to the deterioration of accuracy of Grover's algorithm when search for any marked data as signal to noise ratio is small[15]. The accuracy of these algorithms when run on a real-time quantum computer is substantially less than the theoretical accuracy. Also, the addition of noise because of the real-time quantum computers degrades the physical accuracy of the quantum algorithms. These noisy intermediate scale quantum computers (NISQ) go through various types of noises such as depolarization, rotation, decoherence, decay, measurement errors *etc*[16]. To eliminate the effect of noise, various error correction codes[17-18] and error mitigation techniques[19-23] are developed. The error mitigation techniques do not introduce any overhead in context of number of gates and qubits.

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This paper reports the measurement error mitigation model for Grover's search algorithm to mitigate the measurement errors. The accuracy of Grover's search algorithm with measurement errors is compared with the accuracy of the Grover's algorithm with measurement error mitigation. The accuracy investigation of Grover's algorithm is done using real-time available quantum computer[25]. The 2, 3 and 4-qubit implementations of Grover's algorithm with $round\ \left(\frac{\pi}{4}\sqrt{N}\right)$ iterations are taken for comparison.

The paper is structured as follows: Section 2 introduces Grover's search algorithm. Section 3 discusses the measurement error mitigation technique. Section 4 presents the experimental evaluation and measurement error mitigation model for Grover's algorithm. The 2-qubit to 4-qubit implementations of Grover's search algorithm with measurement errors and with measurement error mitigation are compared. Section 5 concludes the paper.

2 Grover's Algorithm

Grover's algorithm, developed in 1996 by L.K. Grover is a quantum search algorithm which searches a particular item in unstructured database [12]. The Grover's algorithm searches a specific item in the unstructured database of size N in $O(\sqrt{N})$ steps whereas classical search algorithms do the same job in $O(N)$ steps [15]. It implies that Grover's algorithm offers a quadratic speedup when compared with its classical equivalents. The unstructured database comprises of states 2^n *i.e.*, $N = 2^n$ where N is database size and n signify number of qubits. The procedure to implement Grover's algorithm is given as

1. Start with $|0\rangle^{\otimes n}$ where $|0\rangle^{\otimes n}$ implies to $|0\rangle \otimes |0\rangle \otimes \dots\dots\dots \otimes |0\rangle$ for n times and $\otimes$ denotes tensor product. Afterwards algorithm is initialized with the superposition of 2^n states using Hadamard gate on $|0\rangle^{\otimes n}$ as $|\Psi\rangle = (H|0\rangle)^{\otimes n} = \frac{1}{\sqrt{N}}\sum_{x=0}^{N-1}|x\rangle$ where $|\Psi\rangle$ signifies superposition of all states and $\frac{1}{\sqrt{N}}$ is the amplitude [12].
2. The oracle of the form $U_w\,|x\rangle = (-1)^{f(x)}|x\rangle$ is applied [6] on $H^{\otimes n}|0\rangle^{\otimes n}$ to reflect the target where, x is target and

$$f(x) = \begin{cases} 1, if\ x\ is\ searched \\ 0, Otherwise \end{cases}$$

3. Next, Grover's diffusion operator *(D)* is applied after the oracle which amplifies the amplitude of target and represented as $2|\Psi\rangle\langle\Psi| - I$. Step -2, -3 is repeated for $round\ \left(\frac{\pi}{4}\sqrt{N}\right)$ times for better performance of the algorithm [28].
4. In the end, output is measured using measurement gate [6].

The circuit representation of the Grover's algorithm is revealed in Fig. 1.

3 Measurement Error Mitigation

The noise affects the output of a quantum system in terms of speed and accuracy. The noise can be mitigated by using various kind of error mitigation techniques[21-24]. The measurement error mitigation technique eliminates the errors which occurs at the time of measurement. The measurement when performed on a quantum circuit of n qubits, gives an output out of 2^n possibilities[22]. First, measurement chooses the output in noiseless manner and then noise perturbs the correct output. The measurement calibration plays an important role in the measurement error mitigation. The measurement calibration initializes the 2^n input basis states and calculate the likelihood of measuring rounds in other basis states. These calibrations help in correcting the average results of the experiment. In a single calibration round, n qubit register is initialized in $|x\rangle$ basis state followed by noisy measurement on every qubit and stores the measured outcome y. The number of rounds can be described as $m\ (y, x)$ where y and x are the measured output and input states respectively [29].

Firstly, a set of input states $C \subseteq \{0,1\}^n$ is taken and N_{cal} calibration rounds is performed on every input

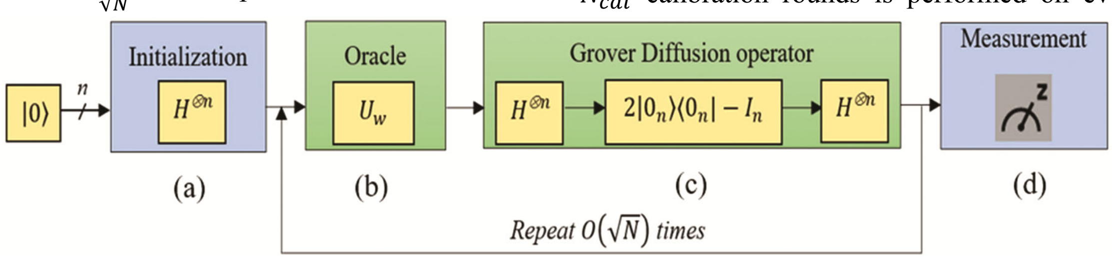

Fig. 1 — Circuit representation of Grover's algorithm for $N = 2^n$ states where (a) Initialization of the algorithm (b) Oracle construction for the marked state (c) Applying diffusion operator to get the correct output (d) Measurement of the output.

state $x \in C$. If $x \in C$ then $\sum_y m(y,x) = N_{cal}$ and $m(y,x) = 0$ if $x \notin C$. The measurement calibration entails a total of $N_{cal}.|C|$ experiments. In order to identify full noise matrix, calibration of all the possible input states is required. The empirical estimate of full noise matrix can be obtained by $\frac{m(y,x)}{N_{cal}}$ [29].

The measurement probability when the noise is present in the quantum system can be described in (1) as

$$P_{noisy} = MP_{ideal} \quad ...(1)$$

where M is the calibration matrix which is constructed by using the probabilities of noisy measurements of the input state and P_{ideal} is the perfect output when no noise is considered[30].

The expected result i.e., P_{ideal} can be calculated by finding the inverse of matrix M.

$$P_{ideal} = M^{-1}P_{noisy} \quad ...(2)$$

The M^{-1} when applied to the P_{noisy} gives the mitigated results $P_{mitigated}$, which is nothing but P_{ideal}[21,30].

4 Experimental Evaluation

The experiment is performed using the measurement error mitigation on the Grover's algorithm. The superconducting qubit-based quantum computer is used for the computation. The Grover's algorithm up to 4 qubits is implemented with and without measurement error mitigation on the 5-qubit real-time quantum computer backend, *ibmq_quito*[25] using QISKIT[31]. The total of 8192 number of shots are used for the computation. In this paper, the Grover's algorithm is implemented with measurement error mitigation up to 4-qubits. The effect of measurement error mitigation on the accuracy of Grover's algorithm implementations is investigated and compared with the Grover's algorithm implementations which comprises of measurement errors.

The proposed measurement error mitigation model for Grover's search algorithm depicted in Fig. 2, is created to mitigate the measurement errors. Initially, in this model Grover's search algorithm is implemented in four steps *initialization, oracle, diffusion and measurement.* In the initialization step Hadamard gate is used, which puts qubits into superposition. Oracle inverts the marked state and keeps rest of the states as it is. Diffusion operator inverts the marked state again and amplify its amplitude. Measurement is taken in the last step of Grover's algorithm. The Grover's algorithm is then run on a real-time quantum computer. The real-time quantum computer evaluates the accuracy of the Grover's algorithm implementations up to 4-qubits.

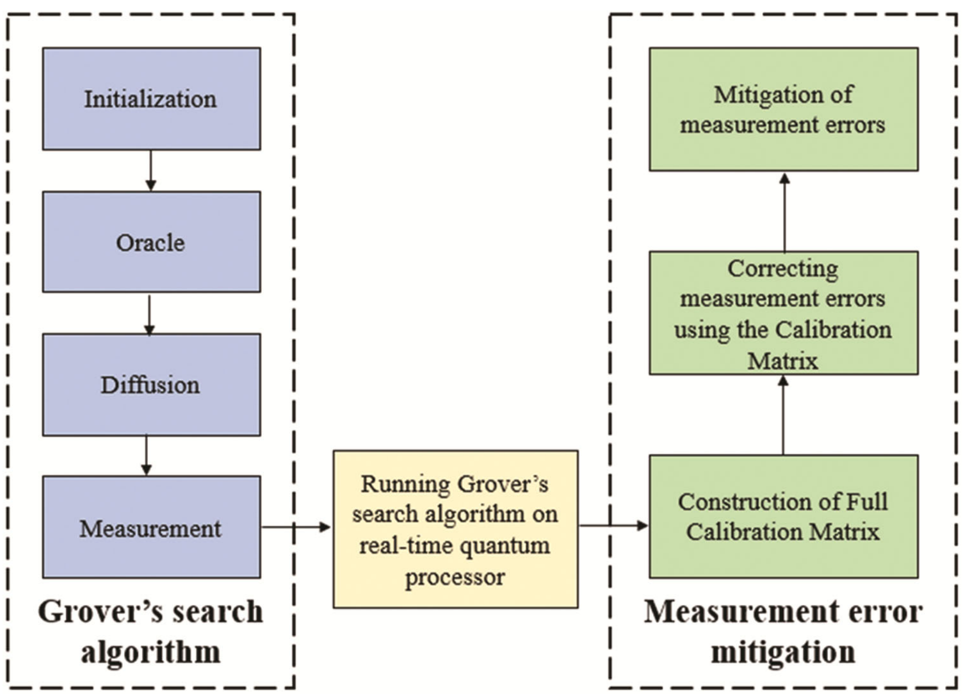

Fig. 2 — Measurement error mitigation model for Grover's search algorithm

The accuracy of Grover's algorithm degrades when run on real-time quantum computer. The measurement errors are one of reason which degrades the accuracy of Grover's algorithm.

In order to mitigate these measurement errors, measurement error mitigation is applied on the Grover's algorithm. The measurement error mitigation is applied to the Grover's algorithm in two steps. The first step is the calibration of measurement errors using QISKIT Ignis. QISKIT[30-31] a framework is used to study and mitigate the noises from the noisy quantum devices and circuits. A full calibration matrix, M is constructed to mitigate the measurement errors. Fig. 3 reveals the calibration matrices for 2 and 3-qubit implementation of Grover's algorithm for states $|11\rangle$ and $|111\rangle$. In second step, after constructing the calibration matrix, measurement errors are mitigated by computing the inverse of the calibration matrix, M^{-1}. In the end, the model results in the mitigation of measurement errors from the Grover's algorithm.

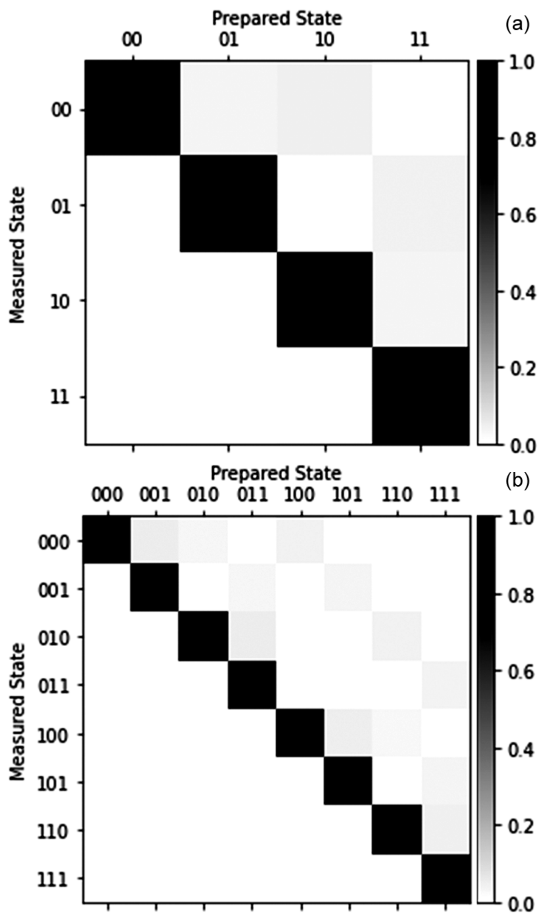

Fig. 3 — A full calibration matrix for (a) marked state $|11\rangle$ (b) marked state $|111\rangle$.

Table 1 depicts that there is a significant difference in the accuracies of Grover's algorithm when performed on the physical device with measurement errors and on the same device with measurement error mitigation. The accuracy of the algorithm degrades as the number of qubits are increased from 2-qubit to 4-qubit. The marked states taken for the experiment are $|11\rangle, |111\rangle$ and $|1111\rangle$. The number of iterations used for the better performance of Grover's algorithm are $round\left(\frac{\pi}{4}\sqrt{N}\right)$. Under the influence of measurement errors, the accuracy of the algorithm for 3 qubit implementation and 4-qubit implementation is degraded by 58.71% and 93.32% when compared with the 2-qubit implementation of the Grover's algorithm. When measurement error mitigation is applied to the Grover's search algorithm, the degradation of 57.46% and 92.99% in the accuracy is observed for 3-qubit and 4-qubit implementations upon comparing them with 2-qubit implementation. The measurement error mitigation when applied on the Grover's algorithm results in the improved accuracy of the algorithm.

It is also observed form the Table 1 that the accuracy of 2-qubit implementation of the Grover's algorithm when measurement error mitigation is applied on it, improves by 11.42% when compared with the 2-qubit implementation with measurement errors. The improvement in the accuracy of 2 -qubit implementation of Grover's algorithm for searching the marked state $|11\rangle$ can be seen in the Fig. 4.

In case of 3-qubit implementation of Grover's algorithm for searching the marked state $|111\rangle$, the measurement error mitigation improves the accuracy by 14.79% when compared with the same implementation but with measurement errors. Fig. 5 reveals the improvement in the accuracy of 3-qubit implementation of Grover's algorithm for searching the marked state $|111\rangle$.

Table 1 — Accuracy assessment of Grover's algorithm up to 4-qubits.

Grover's Algorithm (Number of qubits)	Marked States	Accuracy		% Improvement in the accuracy (mitigated) compared with accuracy (noisy)
		With measurement errors	*With measurement error mittigattion*	
2-qubit	$|11\rangle$	88.40	98.50	11.42
3-qubit	$|111\rangle$	36.50	41.90	14.79
4-qubit	$|1111\rangle$	5.9	6.9	16.94

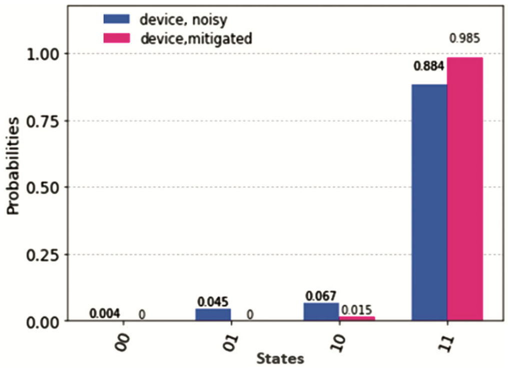

Fig. 4 — Accuracy of 2-qubit implementation of Grover's algorithm for searching state |11⟩ with measurement errors and mitigated measurement errors.

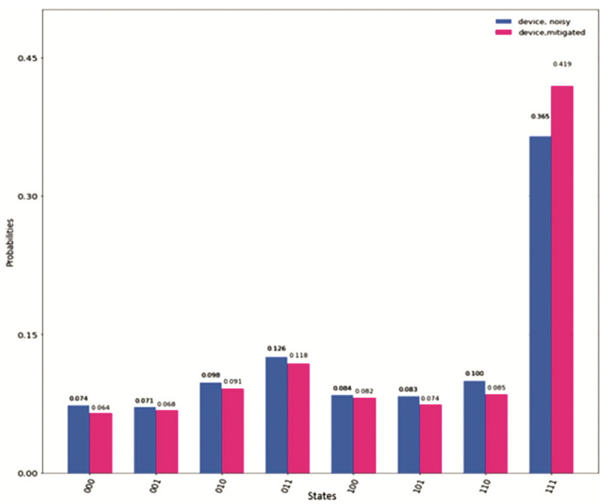

Fig. 5 — Accuracy of 3-qubit implementation of Grover's algorithm for searching state |111⟩ with measurement errors and mitigated measurement errors.

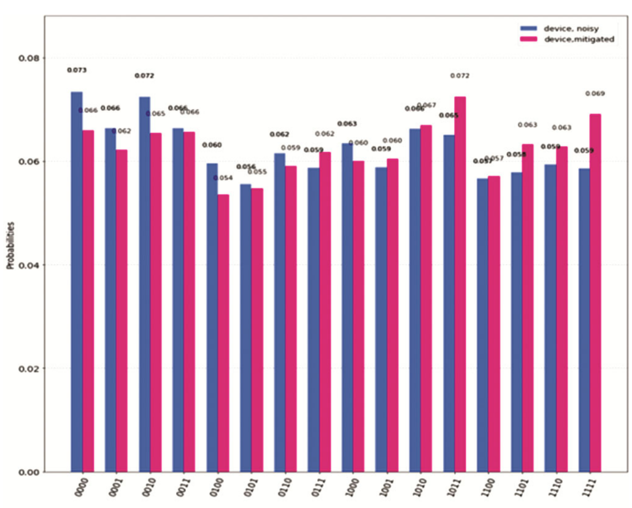

Fig. 6 — Accuracy of 3-qubit implementation of Grover's algorithm for searching |1111⟩ state with measurement errors and mitigated measurement errors.

Similarly, a significant improvement in the accuracy of Grover's algorithm is observed from Table 1 for 4-qubit implementation for searching marked state |1111⟩ when measurement error mitigation is applied on it. The accuracy of 4-qubit implementation of Grover's algorithm with measurement error mitigation is increased by 16.94% when compared with the same implementation with measurement errors. Fig. 6 shows the improvement in the accuracy of the Grover's algorithm for 4-qubit implementation.

5 Conclusion

In this paper, measurement error mitigation model is applied to Grover's quantum search algorithm. The algorithm is implemented up to 4-qubits with and without measurement error mitigation and compared. As the number of qubits increases from 2 to 4-qubit, the circuit complexity increases which in turn increase the measurement errors at the time of measurement. A significant improvement is observed in the accuracy of the Grover's algorithm when implemented with measurement error mitigation technique compared to Grover's algorithm implementations with measurement errors. The % improvement of 11.42 %, 14.79 % and 16.94 % is seen in the accuracy of 2-, 3- and 4-qubit implementations of Grover's algorithm; respectively when measurement errors are mitigated. It could be concluded that the measurement mitigation model for Grover's algorithm efficiently mitigates the measurement errors and improves the performance of the Grover's algorithm.

6 Declarations

Conflict of interest: Authors declare that there is no conflict of interests.

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