

# Perfect Folding of Graphs

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## Abstract

In this paper we introduced the definition of perfect folding of graphs and we proved that cycle graphs of even number of edges can be perfectly folded while that of odd number of edges can be perfectly folded to  $C_3$ . Also we proved that wheel graphs of odd number of vertices can be perfectly folded to  $C$ . Finally we proved that if  $G$  is a graph of  $n$  vertices such that  $2 < \text{clique number} = \text{chromatic number} = k < n$ , then the graph can be perfectly folded to a clique of order  $k$ .

**Keywords:** Clique number • Chromatic number • Perfect graphs • Graph folding • Capillaries

## Introduction

Let  $G=(V, E)$  be a graph, where  $V$  is the set of its vertices and  $E$  is the set of its edges. Two distinct vertices  $u, v \in V$  are called independent if  $\{u, v\}$  is not an edge in  $G$ . Two vertices  $u, v$  are called neighbors (adjacent) if  $\{u, v\}$  is an edge in  $G$ . The degree (valency) of a vertex is the number of edges with the vertex as an end point. A graph with no loops or multiple edges is called a simple graph. A graph is said to be connected if every pair of vertices has a path connecting them otherwise the graph is disconnected. A graph  $H=(V', E')$  is called induced subgraph of  $G=(V, E)$  if  $V' \subseteq V$  and  $\{u, v\}$  is an edge in  $H$  whenever  $u$  and  $v$  are distinct vertices in  $V'$  and  $\{u, v\}$  is an edge in  $G$ ,  $H$  is called proper if  $H \neq G$ . A cycle graph is a graph that consists of a single cycle, or in other words, some number of distinct vertices connected in a closed chain [1-3].

The cycle graph with  $n$  vertices is denoted by  $C_n$ . The number of vertices in  $C_n$  equals the number of edges, and every vertex has degree 2. The wheel graph  $W_n$  or  $n$ -wheel is a graph that contains a cycle of order  $n-1$ , and for which every graph vertex in the cycle is connected to one other graph vertex which is called the hub. A bipartite graph is a graph whose vertex set can be split into two sets  $A$  and  $B$  in such a way that each edge of the graph joins a vertex in  $A$  to a vertex in  $B$ . A vertex coloring of a graph  $G=(V, E)$  is a way of coloring the vertices of the graph such that no two adjacent vertices share the same color. A clique of a graph  $G$  is a maximal complete subgraph. In this case each pair of vertices of the clique are adjacent. The clique number  $W(G)$  of a graph is the number of graph vertices in the largest clique of  $G$ . The clique number of a cycle graph  $C_n$ ,  $n$  odd is 3 and 2 otherwise. For a wheel graph  $W_n$ ,  $n$  is even the clique number is 4 and is 3 otherwise. The chromatic number of a graph  $G$  is the smallest number of colors needed to color the vertices of a graph  $G$  so that no two adjacent vertices share the same color, and is

often denoted by  $\chi(G)$ . A graph  $G$  is called perfect if for every induced subgraph  $H$  of  $G$ ,  $\chi(H) = W(H)$ . Note that if  $G$  is a perfect graph, then every induced subgraph of  $G$  is also perfect [4-6].

## Literature Review

SEM of VCCs is the method of choice in the study of venous portal circulations, where postcapillary venules form portal veins, course over shorter or longer distances, and then capillarize again. Such portal circulations studied in vertebrates are the hypothalamo-hypophysial portal system in the brain, the hepatic portal vein system, the renal portal vein system and the pancreas insulo-acinar portal system [7].

The embryonal and early larval development of the cardiovascular system is excellently visualized by confocal micro-angiography. This technique is well suited for optically clear (transparent) thin animals but fail in opaque and thick objects. Here SEM of VCCs can be applied. Our group focusses upon spatio-temporal aspects of growth and regression of blood vessels in the *Xenopus laevis* model organism. *Xenopus* is an anuran amphibian and undergoes drastic changes in basically every organ/tissue during metamorphosis [60]. Most obvious is the loss of larval-specific organs, like the gills and tail. These organs are highly vascularized in early stages of metamorphosis where the growth of blood vessels dominates. At the height of metamorphosis (climax), gills and tail are resorbed and regression of a highly differentiated, complex vascular system can be studied. But also the microvascular anatomies of chick embryos, mouse embryos, rat embryos and of isolated human fetal organs have been successfully studied by this technique. Angiogenesis research is another field of application of SEM of VCCs. Here blood vessels that undergo sprouting and/or non-sprouting angiogenesis can be identified and localized [8].

In corrosion casts, vascular sprouts impose as blind ending tapering vessels preferentially occurring at capillary and postcapillary

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venular sites. Their identification in vascular casts should always be related to the state of the tissue under observation (healthy vs. diseased; growing vs. fully differentiated vs. involuting). Non-sprouting angiogenesis (Intussusceptive Microvascular Growth, IMG) and its facets Intussusceptive Arborization (IA), Intussusceptive Branch Remodeling (IBR), and Intussusceptive Pruning (IP) can be identified. Signs of non-sprouting angiogenesis impose in vascular casts as shallow to deep, round, oval or longish impressions or as holes or slits of different sizes and shapes.

In biomedical research SEM of VCCs is applied in atherosclerosis research, diabetes research, nephrology research, ophthalmologic research, tissue engineering research and tumor research. Studies on tumor vascular casts show that the normal hierarchy of the blood vascular system can be highly disturbed and vascular patterns can extremely differ. In tumors, the positive identification of casted structures such as blood vessels is sometimes difficult since casts of tumor vascular beds differ greatly in their appearance from casted normal blood vessels. Within short distances they change their diameters, kink, out pouch, constrict, or end abruptly. In areas of vascular mimicry, imprints of tumor cells can be found on their surfaces and in necrotic areas casted structures are found that resemble extravasations. A clear differentiation of casted vascular structures from artifacts is difficult and needs supplemental techniques [9].

## Discussion

Like other techniques, vascular corrosion casting is also prone to artifacts. Incompletely casted blood vessels impose as blindly ending vessels with rounded tips. They can be positively differentiated from broken vessels, which show straight, sharp endings, and also from sprouting vessels, which impose with gradually tapering endings. In some cases, "plastic strips" are found around vascular casts. According to their shape and rather annular structure, they are considered to represent plastified vascular smooth muscle cells or pericytes [10].

## Conclusion

Vascular corrosion casts are increasingly investigated by micro-computer tomography ( $\mu$ -CT). To gain spatial resolution comparable to that of the conventional SEM, only very small specimens can be

studied, a disadvantage if vascular routes connecting areas far apart each other are in the focus of interest.

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