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Symmetric 2-Step 4-Point Hybrid Method for the Solution of General Third Order Differential Equations

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Abstract

This research considers a symmetric hybrid continuous linear multistep method for the solution of general third order ordinary differential equations. The method is generated by interpolation and collocation approach using a combination of power series and exponential function as basis function. The approximate basis function is interpolated at both grid and off-grid points but the collocation of the differential function is only at the grid points. The derived method was found to be symmetric, consistent, zero stable and of order six with low error constant. Accuracy of the method was confirmed by implementing the method on linear and non-linear test problems. The results show better performance over known existing methods solved with the same third order problems. AMS 2010 Subject Classification: 65D05; 65L05; 65L06.

Keywords: Symmetric; Hybrid method; Power series and exponential function; Continuous predictor-corrector method

Introduction

In this paper, we considered the solution of initial value problems for general third order ordinary differential equations of the form

$$y'''=f(t, y, y', y''), y(t_0)=y_0, y'(t_0)=y_1, y''(t_0)=y_2. \qquad (1)$$

Where $t, y, f \in R$.

The numerical and theoretical studies of eqn. (1) have appeared in literature severally. The direct approach for solving this type of ordinary differential equations have been studied and appeared in different literatures [1-7]. This direct approach has demonstrated advantages over the popular approach (reduction to system of first order approach) in terms of speed and accuracy [8,9]. Many authors have focused on direct solution of general second order ivps of odes of the form

$$y''=f(t, y, y') \qquad (2)$$

Majid et al. [10] proposed two point four step direct implicit block method for the solution of second order system of ordinary differential equations (ODEs), using variable step size. The method estimated the solutions of initial value problems at two points simultaneously by using four backward steps but with lower order of accuracies. Akinfenwa [11] presented ninth order hybrid block integrator for solving second order ordinary differential equations. In the paper, the proposed block integrator discretizes the problem using the main and the additional methods to generate system of equations. The resulting system was solved simultaneously in a block-by-block fashion but the order of accuracies is low compare to the order of the method. The authors came up with direct implementation of predictor-corrector methods [3,4,7]. The authors emphasized the need to develop the same order of accuracy of the main predictors and that of the correctors to ensure good accuracy of the method. The order of accuracies in these works improved significantly compare to the existing methods with lower order of their main predictors.

Attempts have also been made by these scholars [12-14,6,7,15]. Olabode [13] proposed a 5-step block scheme for the solution of special type of eqn. (1). The order of accuracy in Olabode et al. [13] improves more than that of Olabode et al [13]. Awoyemi et al. [6], developed a four-point implicit method for the numerical integration of third order ODEs using power series polynomial function [16]. Kuboye and Omar [7] proposed numerical solution of third order ordinary differential equations using a seven-step block method to improve on Awoyemi et al. [6] and Olabode [13] which are of lower order of accuracy. Furthermore, a symmetric hybrid linear multistep method of order six having two off-step points for the solution of eqn. (1) directly was presented by Obarhua and Kayode [15].

To improve on the study of Obarhua and Kayode [15] a symmetric of two-step four-point hybrid method for the solution of third order initial value problems of ordinary differential equations directly is therefore proposed using the combination of power series and exponential function as the approximate basis function [17].

Derivation of the Method

This research work considers the derivation of 2-step 4-point hybrid method for the solution of general third order initial value problems of ordinary differential equations. The approach is to solve eqn. (1) directly without reducing it to a system of first order differential equation. A combination of power series and exponential function is used as the basis function for eqn. (1). The approximate solution eqn. (1) and the resulting differential systems are respectively given as

$$y(t)=\sum_{j=0}^{r+s-1}\lambda_j t^j+\lambda_{r+s}\sum_{j=0}^{r+s}\frac{\alpha^j t^j}{j!}. \qquad (3)$$

Where r and s are the number of interpolation and collocation points respectively.

The third derivative of eqn. (3) as compared with eqn. (1) gives

$$f(t,\ y,\ y'\ y'')=\sum_{j=3}^{r+s-1}\left(j(j-1)(j-2)\lambda_j t^{j-3}\right)+\lambda_{r+s}\sum_{j=3}^{r+s-1}\frac{\alpha^j t^{j-3}}{(j-3)!}. \qquad (4)$$

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Collocating eqn. (4) at only the grid points, t_{n+j} , j=0(1)2, and interpolating (3) at both grid and off-grid points, $t_{n+j}\ j=0\left(\frac{1}{3}\right)2$, leads to the following system of equations.

$$At=b \tag{5}$$

where $t=[\lambda_0 \ldots \lambda_8]^T$; $b=[y_n \ldots f_{n+2}]^T$

$$A=\begin{bmatrix} 1 & t_n & t_n^2 & \cdots & t_n^k & \delta_1 \\ 1 & t_{n+\frac{1}{3}} & t^2_{n+\frac{1}{3}} & \cdots & t^k_{n+\frac{1}{3}} & \delta_2 \\ 1 & t_{n+\frac{2}{3}} & t^2_{n+\frac{2}{3}} & \cdots & t^k_{n+\frac{2}{3}} & \delta_3 \\ 1 & t_{n+1} & t^2_{n+1} & \cdots & t^k_{n+1} & \delta_4 \\ 1 & t_{n+\frac{4}{3}} & t^2_{n+\frac{4}{3}} & \cdots & t^k_{n+\frac{4}{3}} & \delta_5 \\ 1 & t_{n+\frac{5}{3}} & t^2_{n+\frac{5}{3}} & \cdots & t^k_{n+\frac{5}{3}} & \delta_6 \\ 0 & 0\ 0\ 6 & 24t_n & \cdots & \psi t_n^4 & \lambda_1 \\ 0 & 0\ 0\ 6 & 24t_{n+1} & \cdots & \psi t^4_{n+1} & \lambda_2 \\ 0 & 0\ 0\ 6 & 24t_{n+2} & \cdots & \psi t^4_{n+2} & \lambda_3 \end{bmatrix} \quad \text{Where,}$$

$$\delta_1=[1+\alpha t_n+\frac{\alpha^2 t_n^2}{2!}+\ldots+\frac{\alpha^8 t_n^8}{8!}]$$

$$\delta_2=[1+\alpha t_{n+\frac{1}{3}}+\frac{\alpha^2 t^2_{n+\frac{1}{3}}}{2!}+\ldots+\frac{\alpha^8 t^8_{n+\frac{1}{3}}}{8!}]$$

$$\delta_3=[1+\alpha t_{n+\frac{2}{3}}+\frac{\alpha^2 t^2_{n+\frac{2}{3}}}{2!}+\ldots+\frac{\alpha^8 t^8_{n+\frac{2}{3}}}{8!}]$$

$$\delta_4=[1+\alpha t_{n+1}+\frac{\alpha^2 t^2_{n+1}}{2!}+\ldots+\frac{\alpha^8 t^8_{n+1}}{8!}]$$

$$\delta_5=[1+\alpha t_{n+\frac{4}{3}}+\frac{\alpha^2 t^2_{n+\frac{4}{3}}}{2!}+\ldots+\frac{\alpha^8 t^8_{n+\frac{4}{3}}}{8!}]$$

$$\delta_6=[1+\alpha t_{n+\frac{5}{3}}+\frac{\alpha^2 t^2_{n+\frac{5}{3}}}{2!}+\ldots+\frac{\alpha^8 t^8_{n+\frac{5}{3}}}{8!}]$$

$$\lambda_1=[\alpha^3+\alpha^4 t_n+\frac{\alpha^5 t_n^2}{2!}+\ldots+\frac{\alpha^8 t_n^5}{5!}]$$

$$\lambda_2=[\alpha^3+\alpha^4 t_{n+1}+\frac{\alpha^5 t^2_{n+1}}{2!}+\ldots+\frac{\alpha^8 t^5_{n+1}}{5!}]$$

$$\lambda_3=[\alpha^3+\alpha^4 t_{n+2}+\frac{\alpha^5 t^2_{n+2}}{2!}+\ldots+\frac{\alpha^8 t^5_{n+2}}{5!}]$$

$\psi=j\,(j-1)\,(j-2)$ as j=5,6.

Solving eqn. (5) for λ_j's and substituting back into eqn. (3), with some manipulation yields, a linear multistep method with continuous coefficients in the form:

$$y(t)=\sum_{j=0}^{k-1}\alpha_j(t)y_{n+j}+\{\tau_1(t)y_{n+r}+\tau_2(t)y_{n+s}+\tau_3(t)y_{n+u}+\tau_4(t)y_{n+v}\}+h^3\sum_{j=0}^{k}\beta_j(t)f_{n+j}\,. \tag{6}$$

Taking k=2, the coefficients $\alpha_j(t)$ and $\beta_j(t)$ are expressed as function of $v=\dfrac{t-t_{n+1}}{h}$ as follows:

$$\alpha_0(v)=\frac{913}{5530}v^2-\frac{1215}{632}v^4+\frac{12879}{3160}v^6-\frac{729}{553}v^8$$

$$\alpha_1(v)=1-\frac{28213}{2212}v^2+\frac{11907}{316}v^4-\frac{11421}{316}v^6+\frac{22577}{2212}v^8$$

$$\tau_1(v)=\frac{61}{1092}v-\frac{239437}{172536}v^2+\frac{124497}{8216}v^4-\frac{243}{52}v^5-\frac{204039}{8216}v^6+\frac{243}{182}v^7+\frac{447849}{57512}v^8$$

$$\tau_2(v)=-\frac{440}{273}v+\frac{172033}{21567}v^2-\frac{149769}{4108}v^4+\frac{243}{26}v^5+\frac{197721}{4108}v^6-\frac{243}{91}v^7-\frac{209709}{14378}v^8$$

$$\tau_3(v)=\frac{440}{273}v+\frac{271939}{43134}v^2-\frac{139563}{8216}v^4-\frac{243}{26}v^5+\frac{56295}{8216}v^6+\frac{243}{91}v^7-\frac{17739}{14378}v^8$$

$$\tau_4(v)=-\frac{61}{1092}v-\frac{262273}{862680}v^2+\frac{20817}{8216}v^4+\frac{243}{52}v^5+\frac{78813}{81080}v^6-\frac{243}{182}v^7-\frac{49815}{57512}v^8$$

$$\beta_0(v)=\frac{4}{331695}v+\frac{41359}{104815620}v^2-\frac{77}{16432}v^4-\frac{1}{780}v^5+\frac{1327}{123240}v^6+\frac{1}{364}v^7-\frac{575}{115024}v^8$$

$$\beta_1(v)=-\frac{4778}{331695}v-\frac{423632}{26203905}v^2+\frac{1}{6}v^3+\frac{580}{3081}v^4-\frac{67}{195}v^5-\frac{6148}{15405}v^6+\frac{17}{182}v^7+\frac{928}{7189}v^8 \tag{7}$$

$$\beta_2(v)=\frac{4}{331695}v+\frac{9769}{104815620}v^2-\frac{46}{49296}v^4-\frac{1}{780}v^5+\frac{157}{123240}v^6+\frac{1}{364}v^7+\frac{127}{115024}v^8$$

Evaluating eqn. (7) at v=1 gives the discrete method:

$$y_{n+2}=\frac{256}{39}y_{n+\frac{5}{3}}-\frac{395}{39}y_{n+\frac{4}{3}}+\frac{395}{39}y_{n+\frac{2}{3}}-\frac{256}{39}y_{n+\frac{1}{3}}+y_n+\frac{h^3}{9477}(28f_{n+2}-1856f_{n+1}+28f_n). \tag{8}$$

The first and second derivatives of eqn. (8) are:

$$hy'_{n+2}=\frac{100048}{35945}y_{n+\frac{5}{3}}-\frac{1452853}{28756}y_{n+\frac{4}{3}}-\frac{5480}{553}y_{n+1}+\frac{493522}{7189}y_{n+\frac{2}{3}}-\frac{307952}{7189}y_{n+\frac{1}{3}}+\frac{72421}{11060}y_n+\frac{h^3}{8734635}(222904f_{n+2}-10653608f_{n+1}+170254f_n). \tag{9}$$

$$h^2y''_{n+2}=\frac{8220428}{107835}y_{n+\frac{5}{3}}-\frac{11174753}{86268}y_{n+\frac{4}{3}}-\frac{47276}{553}y_{n+1}+\frac{12093337}{43134}y_{n+\frac{2}{3}}-\frac{3607720}{21567}y_{n+\frac{1}{3}}+\frac{284317}{11060}y_n+\frac{h^3}{52407810}(9331249f_{n+2}-234051128f_{n+1}+4055719f_n). \tag{10}$$

Applying the truncation error formula in Awoyemi et al. [6], associated with eqn. (6) by the difference operator eqn. (9) to determine the order and error constant of the methods:

$$L[y(t):h]=\sum_{j=0}^{k}\begin{bmatrix}\alpha_j y(t_n+jh)+\{\tau_1 y(t_n+jhr)+\tau_2 y(t_n+jhs)+\tau_3 y(t_n+jhu)+\tau_4 y(t_n+jhv)\}\\ -h^3\beta_j y'''(t_n+jh)\end{bmatrix} \tag{11}$$

Where $y(x)$ is assumed to be continuously differentiable of high order. Therefore, expanding eqn. (11) in Taylor's series and comparing the coefficient of h to give the expression

$$L\,[\,y(x):h]=c_0 y\,(x)+c_1 hy'(x)+c_2 h^2\,y'(x)+\ldots+c_p h^p\,y^{(p)}(x)+\ldots+c_{p+3}h^{p+3}\,y^{(p+3)}(x) \tag{12}$$

The linear operator L and the associated methods are said to be of order P if $c_0=c_1=c_2=\ldots=c_p=\ldots=c_{p+2}=0$, $c_{p+3}\neq 0$ c_{p+3}. is equal to the error constant. For the purpose of this work, expanding methods (8), (9) and (10) in Taylor's series and comparing the coefficient of h gives both methods of order p=6 and error constant, $c_{p+3}=3.633772\times10^{-6}$, $c_{p+3}=1.1804754\times10^{-3}$ and $c_{p+3}=1.029201\times10^{-4}$ respectively.

Equations (8-10) are of order six, symmetric, consistent, low error constants and capable of handling oscillatory problems.

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Implementation of the Method

To implement the implicit linear 2-step 4-point discrete scheme eqn. (8) and its first and second derivatives eqns.(9) and (10), respectively, the following symmetric explicit schemes and their derivatives are also developed by the same procedure for the evaluation of y_{n+2}, y'_{n+2} and y''_{n+2}.

$$y_{n+2} = \frac{10392}{2629} y_{n+\frac{5}{3}} - \frac{15753}{2629} y_{n+\frac{4}{3}} + \frac{11312}{2629} y_{n+1} - \frac{3903}{2629} y_{n+\frac{1}{3}} - \frac{115}{2629} y_n + \frac{h^3}{354915}\left(6636 f_{n+\frac{5}{3}} - 4780 f_{n+1} - 56 f_n\right). \quad (13)$$

$p=6$ $c_{p+3}=3.107634 \times 10^{-7}$.

$$hy'_{n+2} = \frac{46152}{8365} y_{n+\frac{5}{3}} - \frac{70731}{4780} y_{n+\frac{4}{3}} + \frac{227992}{8365} y_{n+1} - \frac{264798}{8365} y_{n+\frac{2}{3}} + \frac{19296}{1195} y_{n+\frac{1}{3}} - \frac{16511}{6692} y_n + \frac{h^3}{376425}\left(60792 f_{n+\frac{5}{3}} + 133840 f_{n+1} - 2782 f_n\right). \quad (14)$$

$p=6$, $c_{p+3}=9.0 \times 10^{-6}$.

$$h^2 y''_{n+2} = -\frac{29861988}{368060} y_{n+\frac{5}{3}} + \frac{44066883}{368060} y_{n+\frac{4}{3}} - \frac{63973128}{368060} y_{n+1} - \frac{154388262}{368060} y_{n+\frac{2}{3}} + \frac{89899548}{368060} y_{n+\frac{1}{3}} - \frac{13689309}{368060} y_n + h^3\left(\frac{9331249}{8281350} f_{n+\frac{5}{3}} + \frac{135647}{20790} f_{n+1} - \frac{1955008}{1774575} f_n\right). \quad (15)$$

$p=6$, $c_{p+3}=1.35 \times 10^{-4}$.

The methods eqns. (13), (14) and (15) are of order $p=6$ and error constant, $c_{p+3}=3.107634 \times 10^{-7}$, $c_{p+3}=9.0 \times 10^{-6}$ and $c_{p+3}=1.35 \times 10^{-4}$ respectively.

Other explicit schemes were also generated to evaluate other starting values and Taylor's series was used to evaluate the values for y_{n+i}, $i = \frac{1}{3}, \frac{2}{3}, 1$, as

$$y_{n+i} = y(x_n + ih) = y_n + ihy'_n + \frac{(ih)^2}{2!} y''_n + \frac{(ih)^3}{3!} f_n + \frac{(ih)^4}{4!} f'_n + \frac{(ih)^5}{5!} f''_n + \frac{(ih)^6}{6!} f'''_n.$$

$$y'_{n+i} = y'(x_n + ih) = y'_n + ihy''_n + \frac{(ih)^2}{2!} f_n + \frac{(ih)^3}{3!} f'_n + \frac{(ih)^4}{4!} f''_n + \frac{(ih)^5}{5!} f'''_n.$$

and

$$y''_{n+i} = y''(x_n + ih) = y''_n + ihf_n + \frac{(ih)^2}{2!} f'_n + \frac{(ih)^3}{3!} f''_n + \frac{(ih)^4}{4!} f'''_n. \quad (16)$$

Numerical experiments

Three third order problems out of which one is linear and two are non-linear with exact solutions are solved with our method to test the effectiveness and its accuracy.

Problem 1: $y'''=-e^x$, $y(0)=1$, $y'(0)=-1$, $y''(0)=3$, $h=0.1$

Theoretical solution: $y(x)=2+2x^2-e^x$.

Table 1 shows the maximum absolute error of our predictor-corrector method and that of Olabode [13] block method for Problem 1. It reveals that the new method performed creditably well than that of Olabode [13] of higher order.

Problem 2:

$$y''' = y'(2xy'' + y'),\ y(0)=1,\ y'(0)=\frac{1}{2},\ y''(0)=0,\ h=0.1.$$

Theoretical solution: $y(x) = 1 + \frac{1}{2}\ln\left[\frac{2+x}{2-x}\right]$. In Table 2, y-exact, the y-computed, the errors of the new method and the time(s) of iteration for Problem 2 are shown.

Problem 3: $y'''=-y)^4$, $y(1)=-1$, $y'(1)=-1$, $y''(1)=-2$. $h=0.05$.

Theoretical solution: $y(x) = \frac{1}{x-2}$

Table 3 shows the y-exact, the y-computed, and the errors of the new method and the time of iteration for Problem 3.

Conclusion

This paper has produced 2-step 4-point hybrid method for direct solution of higher order ordinary differential equations. The method developed is symmetric, consistent and convergent which can handle

x	y_{exact}	$y_{computed}$	Error in Olabode, (2009), p=7, k=5	Error in new scheme, p=6, k=2
0.1	0.9148290819243523	0.9148290819245347	7.56477e-11	1.82410e-13
0.2	0.8585972418398302	0.8585972418415010	2.60170e-10	1.67078e-12
0.3	0.8301411924239970	0.8301411924299984	5.76003e-10	6.00142e-12
0.4	0.8281753023587299	0.8281753023735897	8.41270e-10	1.48598e-11
0.5	0.8512787292998718	0.8512787293299923	1.00013e-09	3.01205e-11
0.6	0.8978811996094913	0.8978811996633331	1.09051e-09	5.38418e-11
0.7	0.9662472925295238	0.9662472926178395	1.07048e-09	8.83157e-11
0.8	1.0544590715075328	1.0544590716435931	1.49247e-09	1.36060e-10
0.9	1.1603968888430511	1.1603968890429206	3.15695e-09	1.99870e-10
1.0	1.2817181715409554	1.2817181718237693	4.45905e-09	2.82814e-10

Table 1: The numerical solution of our methods of order 6 compared with the method of Olabode, (2009), of order 7.

x	y_{exact}	$y_{computed}$	Error in new scheme, p=6, k=2	Time(s)
0.1	1.0500417292784914	1.0500418242095606	9.49E-08	0.0027
0.2	1.1003353477310756	1.1003366644736043	1.32E-06	0.0248
0.3	1.1511404359364668	1.1511460842057299	5.65E-06	0.0256
0.4	1.2027325540540821	1.2027483597246535	1.58E-05	0.0261
0.5	1.2554128118829952	1.2554482979429176	3.55E-05	0.0266
0.6	1.3095196042031119	1.3095893044089619	6.97E-05	0.0272
0.7	1.3654437542713962	1.3655690060595540	1.25E-04	0.0278
0.8	1.4236489301936017	1.4238603658481614	2.11E-04	0.0283
0.9	1.4847002785940517	1.4850413496636110	3.41E-04	0.0288
10	1.5493061443340548	1.5498382025385242	5.32E-04	0.0293

Table 2: Numerical solution and errors for problem 2.

x	y_{exact}	$y_{computed}$	Error in new scheme, p=6, k=2	Time(s)
1.05	1.0526315789473684	1.0526315779293796	1.017989e-09	0.0291
1.10	1.1111111111111112	1.1111110853730750	2.573804e-08	0.0313
1.15	1.1764705882352944	1.1764704664689964	1.217663e-07	0.0316
1.20	1.2500000000000002	1.2499996375184190	3.624816e-07	0.0320
1.25	1.3333333333333337	1.3333324687923525	8.645410e-07	0.0323
1.30	1.4285714285714290	1.4285696093065769	1.819265e-06	0.0325
1.35	1.5384615384615392	1.5384579871365800	3.551325e-06	0.0328
1.40	1.6666666666666676	1.6666600337776776	6.632889e-06	0.0332
1.45	1.8181818181818195	1.8181697022691894	1.211591e-05	0.0335
1.50	2.0000000000000018	1.9999779695058428	2.20E-05	0.0338

Table 3: Numerical solution and errors for problem 3.

oscillatory type of problems. The numerical tests results obtained were compared with block method of Olabode [13] which was found to perform favorably than the existing method.

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