Journal of
Applied & Computational Mathematics

Mishra et al., J Appl Computat Math 2016, 5:4
DOI: 10.4172/2168-9679.1000316

Research Article Open Access

The Generalized Semi Normed Difference of χ^3 Sequence Spaces Defined by Orlicz Function

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Abstract

In this paper we introduced generalized semi normed difference of triple gai sequence spaces defined by an Orlicz function. We study their different properties and obtain some inclusion relations involving these semi normed difference triple gai sequence spaces.

2010 Mathematics Subject Classification: 40A05; 40C05; 40D05.

Keywords: Gai sequence; Analytic sequence; Triple sequence; Difference sequence

Introduction

Throughout the paper w,χ and Λ denote the classes of all, gai and analytic scalar valued single sequences, respectively. We write w^3 for the set of all complex triple sequences (x_{mnk}), where $m,n,k\in\mathbb{N}$, the set of positive integers. Then, w^3 is a linear space under the coordinatewise addition and scalar multiplication.

Some initial work on double series is found in Apostol [1] and double sequence spaces is found in Hardy [2], Subramanian et al. [3-9], and many others. Later on, some work on triple sequence spaces can also be found in Sahiner et al. [10] , Esi et al. [11-15], Subramanian et al. [16-19], Prakash et al. [20-24] and many others.

Let (x_{mnk}) be a triple sequence of real or complex numbers. Then the series $\sum_{m,n,k=1}^{\infty} x_{mnk}$ is called a triple series. The triple series $\sum_{m,n,k=1}^{\infty} x_{mnk}$ is said to be convergent if and only if the triple sequence (S_{mnk}) is convergent, where

$$S_{mnk}=\sum_{i,j,q=1}^{m,n,k} x_{ijq} \quad (m,n,k=1,2,3,\ldots).$$

A sequence $x=(x_{mnk})$ is said to be triple analytic if

$$sup_{m,n,k}\left|x_{mnk}\right|^{\frac{1}{m+n+k}}<\infty.$$

The vector space of all triple analytic sequences are usually denoted by Λ^3. A sequence $x=(x_{mnk})$ is called triple entire sequence if

$$\left|x_{mnk}\right|^{\frac{1}{m+n+k}}\to 0 \text{ as } m,n,k\to\infty.$$

The vector space of all triple entire sequences are usually denoted by Γ^3. The spaces Λ^3 and Γ^3 are metric spaces with the metric

$$d(x,y)=sup_{m,n,k}\left\{\left|x_{mnk}-y_{mnk}\right|^{\frac{1}{m+n+k}}:m,n,k:1,2,3,\ldots\right\}, \quad (1)$$

for all $x=\{x_{mnk}\}$ and $y=\{y_{mnk}\}$ in Γ^3. Let φ be the set of finite sequences.

Consider a triple sequence $x=(x_{mnk})$. The $(m,n,k)^{th}$ section $x^{[m,n,k]}$ of the sequence is defined by $x^{[m,n,k]}=\sum_{i,j,q=0}^{m,n,k} x_{ijq}\Im_{ijq}$ for all $i,j,q\in\mathbb{N}$, where $\Im_{ijq}$ is a three dimensional matrix with 1 in the $(i,j,k)^{th}$ position and zero otherwise.

Let M and Φ be mutually complementary Orlicz functions. Then, we have:

(i) For all $u,y\geq 0$,

$$uy\leq M(u)+\Phi(y), \text{ (Young's inequality) [see [25]]} \quad (2)$$

(ii) For all $u\geq 0$,

$$u\eta(u)=M(u)+\Phi(\eta(u)). \quad (3)$$

(iii) For all $u\geq 0$, and $0<\lambda<1$,

$$M(\lambda u)\leq\lambda M(u) \quad (4)$$

Lindenstrauss and Tzafriri [26] used the idea of Orlicz function to construct Orlicz sequence space

$$\ell_M=\left\{x\in w:\sum_{k=1}^{\infty}M\left(\frac{|x_k|}{\rho}\right)<\infty, \quad for \quad some \quad \rho>0\right\}.$$

The space ℓ_M with the norm

$$\|x\|=inf\left\{\rho>0:\sum_{k=1}^{\infty}M\left(\frac{|x_k|}{\rho}\right)\leq 1\right\},$$

becomes a Banach space which is called an Orlicz sequence space. For $M(t)=t^p(1\leq p<\infty)$, the spaces ℓ_M coincide with the classical sequence space ℓ_p.

A sequence $f=(f_{mnk})$ of Orlicz functions is called a Musielak-Orlicz function. A sequence $g=(g_{mnk})$ defined by

$$g_{mnk}(v)=sup\{|v|u-(f_{mnk})(u):u\geq 0\}, m,n,k=1,2,\cdots$$

is called the complementary function of a Musielak-Orlicz function f. For a given Musielak-Orlicz function f, the Musielak-Orlicz sequence space t_f is defined as follows

$$t_f=\left\{x\in w^3:M_f\left(|x_{mnk}|\right)^{1/m+n+k}\to 0, \quad as \quad m,n,k\to\infty\right\},$$

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Received June 08, 2016; **Accepted** July 18, 2016; **Published** July 22, 2016

Citation: Mishra Vishnu Narayan, Deepmala, Subramanian N, Mishra Lakshmi Narayan (2016) The Generalized Semi Normed Difference of χ^3 Sequence Spaces Defined by Orlicz Function. J Appl Computat Math 5: 316. doi: 10.4172/2168-9679.1000316

where M_f is a convex modular defined by

$$M_f(x)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty} f_{mnk}\left(|x_{mnk}|\right)^{1/m+n+k}, x=(x_{mnk})\in t_f.$$

We consider t_f equipped with the Luxemburg metric

$$d(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty} f_{mnk}\left(\frac{|x_{mnk}-y_{mnk}|^{1/m+n+k}}{mnk}\right)$$

is an extended real number.

If X is a sequence space, we give the following definitions:

(i) X= the continuous *dual of* X;

(ii) $X^{\alpha}=\left\{a=(a_{mnk}):\sum_{m,n,k=1}^{\infty}|a_{mnk}x_{mnk}|<\infty, \quad for \quad each \quad x\in X\right\};$

(iii) $\left\{ (_{mnk}):\sum_{m,n,k=1}^{\infty} {}_{mnk}\;{}_{mnk} \quad , \quad \in \right\};$

(iv) $X^{\gamma}=\left\{a=(a_{mnk}):sup_{mnk\geq 1}\left|\sum_{m,n,k=1}^{M,N,K}a_{mnk}x_{mnk}\right|<\infty, for \quad each \quad x\in X\right\};$

(v) $let \quad X \quad be \quad an \quad FK-space \quad \supset\phi; \quad then \quad X^{f}=\left\{f(\Im_{mnk}):f\in X'\right\};$

(vi) $X^{\delta}=\left\{a=(a_{mnk}):sup_{mnk}|a_{mnk}x_{mnk}|^{1/m+n+k}<\infty, \quad for \quad each \quad x\in X\right\};$

X, X, X are called α-(or Kothe-Toeplitz) dual of X, β-(or generalized-Kothe-Toeplitz) dual of X, γ-dual of X, δ-dual of X respectively.

The notion of difference sequence spaces (for single sequences) was introduced by Kizmaz [27] as follows

$$Z(\Delta)=\left\{x=(x_k)\in w:(\Delta x_k)\in Z\right\}$$

for $Z=c,c_0$ and ℓ_{∞}, where $\Delta x_k=x_k-x_{k+1}$ for all $k\in\mathbb{N}$.

Later on the notion was further investigated by many others. We now introduce the following difference double sequence spaces defined by

$$Z(\Delta)=\left\{x=(x_{mn})\in w^2:(\Delta x_{mn})\in Z\right\}$$

where $Z=\Lambda^2,\chi^2$ and $\Delta x_{mn}=(x_{mn}-x_{mn+1})-(x_{m+1n}-x_{m+1n+1})=x_{mn}-x_{mn+1}-x_{m+1n}+x_{m+1n+1}$ for all $m,n\in\mathbb{N}$.

Let $w^3,\chi^3(\Delta_{mnk}),\Lambda^3(\Delta_{mnk})$ be denote the spaces of all, triple gai difference sequence space and triple analytic difference sequence space respectively and is defined as

$\Delta_{mnk}=x_{mnk}-x_{m,n+1,k}-x_{m,n,k+1}+x_{m,n+1,k+1}-x_{m+1,n,k}+x_{m+1,n+1,k}+x_{m+1,n,k+1}-x_{m+1,n+1,k+1}$ and $\Delta^0 x_{mnk}=\langle x_{mnk}\rangle$.

Definitions and Preliminaries

A sequence $x=(x_{mnk})$ is said to be triple analytic if $sup_{mnk}|x_{mnk}|^{\frac{1}{m+n+k}}<\infty$. The vector space of all triple analytic sequences is usually denoted by Λ^3. A sequence $x=(x_{mnk})$ is called triple entire sequence if $|x_{mnk}|^{\frac{1}{m+n+k}}\to 0$ as $m,n,k\to\infty$. The vector space of triple entire sequences is usually denoted by Γ^3. A sequence $x=(x_{mnk})$ is called triple gai sequence if $\left((m+n+k)!|x_{mnk}|\right)^{\frac{1}{m+n+k}}\to 0$ as $m,n,k\to\infty$. The vector space of triple gai sequences is usually denoted by χ^3. The space χ^3 is a metric space with the metric

$$d(x,y)=sup_{m,n,k}\left\{\left((m+n+k)!|x_{mnk}-y_{mnk}|\right)^{\frac{1}{m+n+k}}:m,n,k:1,2,3,...\right\} \tag{5}$$

for all $x=\{x_{mnk}\}$ and $y=\{y_{mnk}\}$ in χ^3.

Throughout the article $w^3,\chi^3(\Delta),\Lambda^3(\Delta)$ denote the spaces of all, triple gai difference sequence spaces and triple analytic difference sequence spaces respectively [28].

For a triple sequence $x\in w^3$, we define the sets

$$\chi^3(\Delta)=\left\{x\in w^3:\left((m+n+k)!|\Delta x_{mnk}|\right)^{1/m+n+k}\to 0, \quad as \quad m,n,k\to\infty\right\}$$

$$\Lambda^3(\Delta)=\left\{x\in w^3:sup_{m,n,k}|\Delta x_{mnk}|^{1/m+n+k}<\infty\right\}.$$

The space $\Lambda^3(\Delta)$ is a metric space with the metric

$$d(x,y)=sup_{m,n,k}\left\{|\Delta x_{mnk}-\Delta y_{mnk}|^{1/m+n}:m,n,k=1,2,\cdots\right\}$$

for all $x=(x_{mnk})$ and $y=(y_{mnk})$ in $\Lambda^3(\Delta)$.

The space $\chi^3(\Delta)$ is a metric space with the metric

$$d(x,y)=sup_{mnk}\left\{\left((m+n+k)!|\Delta x_{mnk}-\Delta y_{mnk}|\right)^{1/m+n+k}:m,n,k=1,2,\cdots\right\}$$

for all $x=(x_{mnk})$ and $y=(y_{mnk})$ in $\chi^3(\Delta)$.

Let $p=(p_{mnk})$ be a sequence of positive real numbers. We have the following well known inequality, which will be used throughout this paper:

$$|a_{mnk}+b_{mnk}|^{p_{mnk}}\leq D\left(|a_{mnk}|^{p_{mnk}}+|b_{mnk}|^{p_{mnk}}\right) \tag{6}$$

where a_{mnk} and b_{mnk} are complex numbers, $D=max\{1,2^{H-1}\}$ and $H=sup_{mnk}p_{mnk}<\infty$.

Spaces of strongly summable sequences were studied at the initial stage by Kuttner, Maddox and others. The class of sequences those are strongly Cesaro summable with respect to a modulus was introduced by Maddox as an extension of the definition of strongly Cesaro summable sequences. Jeff Connor further extended this definition to a definition of strongly A-summability with respect to a modulus when A is non-negative regular matrix.

Let $\eta=(\lambda_{abc})$ be a non-decreasing sequence of positive real numbers tending to infinity and $\lambda_{111}=1$ and $\lambda_{a+b+c+3}\leq\lambda_{a+b+c+3}+1$, for all $a,b,c\in\mathbb{N}$.

The generalized de la Vall ̀e e-Poussin means are defined by $t_{abc}(x)=\lambda_{abc}^{-1}\sum_{m,n,k\in I_{abc}}x_{mnk}$, where $I_{abc}=[abc-\lambda_{abc}+1,abc]$. A sequence $x=(x_{mnk})$ is said to (V,λ)-summable to a number L if $t_{abc}(x)\to L$, as $abc\to\infty$.

Throughout the article E will represent a semi normed space by a semi norm q. We define $w^3(E)$ to be the vector space of all E-valued sequences. Let f be an Orlicz function and $p=(p_{mnk})$ be any sequence of positive real numbers. Let $A=\left(a_{mn}^{jk}\right)$ be four dimensional infinite regular matrix of non-negative complex numbers such that $\sum_{m,n,k}a_{mn}^{jk}<\infty$.

We define the following sets of sequences in this article:

$$\left[V_{\lambda}^{E},A,\Delta_{\gamma}^{r},f,p\right]_{\chi^3}=\left\{x\in w^3(E):lim_{p,q,r\to\infty}\lambda_{pqr}^{-1}\sum_{mnk\in I_{pqr}}a_{mn}^{jk}\left[f\left(q\left((m+n+k)!\Delta_{\gamma}^{r}x_{mnk}\right)^{1/m+n+k}\right)\right]^{p_{mnk}}=0\right\}$$

uniformly in m,n,k.

$$\left[V_{\lambda}^{E},A,\Delta_{\gamma}^{r},f,p\right]_{\Gamma^3}=\left\{x\in w^3(E):lim_{p,q,r\to\infty}\lambda_{pqr}^{-1}\sum_{mnk\in I_{pqr}}a_{mn}^{jk}\left[f\left(q\left(\Delta_{\gamma}^{r}x_{mnk}\right)^{1/m+n+k}\right)\right]^{p_{mnk}}=0\right\}$$

uniformly in m,n,k

$$\left[V_{\lambda}^{E},A,\Delta_{\gamma}^{r},f,p\right]_{\Lambda^3}=\left\{x\in w^3(E):sup_{n,j,k}sup_{p,q,r}\lambda_{pqr}^{-1}\sum_{mnk\in I_{pqr}}a_{mn}^{jk}\left[f\left(q\left(\Delta_{\gamma}^{r}x_{mnk}\right)^{1/m+n+k}\right)\right]^{p_{mnk}}<\infty\right\}$$

For $\gamma=1$ these spaces are denoted by $\left[V_{\lambda}^{E},A,\Delta^{r},f,p\right]_{Z}$, for $Z=\chi^3,\Gamma^3$ and Λ^3 respectively. We define

$$\left[V_{\lambda}^{E},A,\Delta_{\gamma}^{r},f,p\right]_{\chi^3}=\left\{x\in w^3(E):lim_{p,q,r\to\infty}\lambda_{pqr}^{-1}\sum_{mnk\in I_{pqr}}\left[f\left(q\left((m+n+k)!\Delta_{\gamma}^{r}x_{mnk}\right)^{1/m+n+k}\right)\right]^{p_{mnk}}=0\right\}$$

Similarly $\left[V_{\lambda}^{E},\Delta_{\gamma}^{r},f,p\right]_{\Gamma^3}$ and $\left[V_{\lambda}^{E},\Delta_{\gamma}^{r},f,p\right]_{\Lambda^3}$ can be defined.

For $E=\mathbb{C}$, the set of complex numbers, $q(x)=|x|$; $f(x)=x^{1/m+n+k}$; $p_{mn}=1$, for all $m,n,k\in\mathbb{N}$, $r=0$, $\gamma=0$, the spaces $\left[V_{\lambda}^{E},\Delta_{\gamma}^{r},f,p\right]_{Z}$, for $Z=\chi^3,\Gamma^3$ and Λ^3

represent the spaces $[V,\lambda]_Z$, for $Z=\chi^3,\Gamma^3$ and Λ^3. These spaces are called as λ-strongly gai to zero, λ-strongly entire to zero and λ-strongly analytic by the de la Valle-Poussin method. In the special case, where $\lambda_{pqr}=pqr$, for all $p,q,r=1,2,3\ldots$ the sets $[V,\lambda]_{\chi^3}$, $[V,\lambda]_{\Gamma^3}$ and $[V,\lambda]_{\Lambda^3}$ reduce to the sets $w^3_{\chi^3}$, $w^3_{\Gamma^3}$ and $w^3_{\Lambda^3}$.

In this chapter we introduced generalized semi normed difference of triple gai sequence spaces defined by an Orlicz function. We study their different properties and obtain some inclusion relations involving these semi normed difference triple gai sequence spaces.

Main Results

Theorem 1

Let the sequence $p=(p_{mnk})$ be analytic. Then the sequence space $\left[V_\lambda^E, A, \Delta_\gamma^r, f, p\right]_Z$, are linear spaces over the complex field $\mathbb{C}$, for $Z=\chi^3$ and Λ^3.

Proof: It is easy. Therefore the proof is omitted.

Theorem 2

Let f be an Orlicz function, then $\left[V_\lambda^E, A, \Delta_\gamma^r, f, p\right]_{\chi^3} \subset \left[V_\lambda^E, A, \Delta_\gamma^r, f, p\right]_{\Lambda^3}$

Proof: Let $x=(x_{mnk}) \in \left[V_\lambda^E, A, \Delta_\gamma^r, f, p\right]_{\chi^3}$ will represent a semi normed space by a semi norm q. Here there exists a positive integer M_1 such that $q\leq M_1$. Then we have

$$\lambda_{pqr}^{-1}\sum_{mnk\in I_{pqr}} a_{mn}^{jk}\left[f\left(q\left(\Delta_\gamma^r x_{mnk}\right)^{1/m+n+k}\right)\right]^{p_{mnk}} \leq D \quad \lambda_{pqr}^{-1}\sum_{mn\in I_{pqr}} a_{mn}^{jk} \left[f\left(q\left((m+n+k)!\Delta_\gamma^r x_{mnk}\right)^{1/m+n+k}\right)\right]^{p_{mnk}} + D\left(M_1, f(1)\right)^H \lambda_{pqr}^{-1}\sum_{m,n,k\in I_{pqr}} a_{mn}^{jk}$$

Thus $x\in\left[V_\lambda^E, A, \Delta_\gamma^r, f, p\right]_{\Lambda^3}$. Since $x\in\left[V_\lambda^E, A, \Delta_\gamma^r, f, p\right]_{\chi^3}$. This completes the proof.

Theorem 3

Let $p=(p_{mnk})\in\chi^3$, then $\left[V_\lambda^E, A, \Delta_\gamma^r, f, p\right]_{\chi^3}$ is a paranormed space with

$$g(x)=sup_{pqr}\left(\lambda_{pqr}^{-1}\sum_{mnk\in I_{pqr}} a_{mn}^{jk}\left[f\left(q\left((m+n+k)!\Delta_\gamma^r x_{mnk}\right)^{1/m+n+k}\right)\right]^{p_{mnk}}\right)^{1/H}$$

where $H=max(1,sup_{mnk}p_{mnk})$

Proof: From Theorem 3.2, for each $x\in\left[V_\lambda^E, A, \Delta_\gamma^r, f, p\right]_{\chi^3}$, $g(x)$ exists. Clearly $g(-x)=g(x)$. It is trivial that $\left((m+n+k)!\Delta_\gamma^r x_{mnk}\right)^{1/m+n+k}=\theta$ for $x=\bar{\theta}$. Hence, we get $g(\bar{\theta})=0$. By Minkowski inequality, we have $g(x+y)\leq g(x)+g(y)$. Now we show that the scalar multiplication is continuous. Let α be any fixed complex number. By definition of f, we have $x\to\theta$ implies, $g(\alpha x)\to 0$. Similarly we have for fixed X and $\alpha\to 0$ implies $g(\alpha x)\to 0$. Finally $x\to\theta$ and $\alpha\to 0$ implies $g(\alpha x)\to 0$. This completes the proof.

Theorem 4

If $r\geq 1$ then the inclusion $\left[V_\lambda^E, A, \Delta_\gamma^{r-1}, f, p\right]_{\chi^3} \subset \left[V_\lambda^E, A, \Delta_\gamma^r, f, p\right]_{\chi^3}$ is strict. In general $\left[V_\lambda^E, A, \Delta_\gamma^j, f, p\right]_{\chi^3} \subset \left[V_\lambda^E, A, \Delta_\gamma^r, f, p\right]_{\chi^3}$ for $j=0,1,2,\ldots r-1$ and the inclusions are strict.

Proof: The result follows from the following inequality

$$\lambda_{pqr}^{-1}\sum_{mnk\in I_{pqr}} a_{mn}^{jk}\left[f\left(q\left((m+n+k)!\Delta_\gamma^r x_{mnk}\right)^{1/m+n+k}\right)\right]^{p_{mnk}} \leq$$
$$D\quad \lambda_{pqr}^{-1}\sum_{mnk\in I_{pqr}} a_{mn}^{jk}\left[f\left(q\left((m+n+k)!x_{mnk}\right)^{1/m+n+k}\right)\right]^{p_{mnk}} +$$
$$D\quad \lambda_{pqr}^{-1}\sum_{mnk\in I_{pqr}} a_{mn}^{jk}\left[f\left(q\left((m+n+k+1)!x_{mnk+1}\right)^{1/m+n+k+1}\right)\right]^{p_{mnk}} +$$
$$D\quad \lambda_{pqr}^{-1}\sum_{mnk\in I_{pqr}} a_{mn}^{jk}\left[f\left(q\left((m+n+1+k)!x_{mn+1k}\right)^{1/m+n+1+k}\right)\right]^{p_{mnk}}$$
$$D\quad \lambda_{pqr}^{-1}\sum_{mnk\in I_{pqr}} a_{mn}^{jk}\left[f\left(q\left((m+1+n+k)!x_{m+1n}\right)^{1/m+1+n+k}\right)\right]^{p_{mnk}} +$$
$$D\quad \lambda_{pqr}^{-1}\sum_{mnk\in I_{pqr}} a_{mn}^{jk}\left[f\left(q\left((m+n+k+3)!x_{m+1n+1k+1}\right)^{1/m+n+k+3}\right)\right]^{p_{mnk}}$$

proceeding inductively, we have $\left[V_\lambda^E, A, \Delta_\gamma^j, f, p\right]_{\chi^3} \subset \left[V_\lambda^E, A, \Delta_\gamma^r, f, p\right]_{\chi^3}$ for $j=0,1,2,\ldots r-1$. The inclusion is strict and it follows from the following example.

Example 1: Let $E=C$, $q(x)=|x|$; $\lambda_{pqr}=1$ for all $p,q,r\in\mathbb{N}$, $p_{mnk}=3$ for all $m,n,k\in\mathbb{N}$. Let $f(x)=x$, for all $x\in[0,\infty)$; $a_{mn}^{jk}=m^{-3}n^{-3}k^{-3}$ for all $m,n,k,j\in\mathbb{N}$; $\gamma=1$, $r\geq 1$. Then consider the sequence $x=(x_{mnk})$ defined by $x_{mnk}=\frac{1}{(m+n+k)!}(mnk)^{r(m+n+k)}$ for all $m,n,k\in\mathbb{N}$. Hence $(x_{mnk})\in\left[V_\lambda^C, A, \Delta^r, f, p\right]_{\chi^3}$ but $(x_{mnk})\notin\left[V_\lambda^C, A, \Delta^{r-1}, f, p\right]_{\chi^3}$

Theorem 5

Let f be an Orlicz function, then

(a) Let $0\leq p_{mnk}\leq q_{mnk}$, for all $m,n,k\in\mathbb{N}$ and $\left(\frac{q_{mnk}}{p_{mnk}}\right)$ be analytic, then $\left[V_\lambda^E, A, \Delta_\gamma^r, f, p\right]_{\chi^3} \subset \left[V_\lambda^E, A, \Delta_\gamma^r, f, p\right]_{\chi^3}$

(b) If $0<inf_{mnk}p_{mnk}<p_{mnk}\leq 1$ for all $m,n,k\in\mathbb{N}$ then $\left[V_\lambda^E, A, \Delta_\gamma^r, f, p\right]_{\chi^3} \subset \left[V_\lambda^E, A, \Delta_\gamma^r, f\right]_{\chi^3}$

(c) If $1\leq p_{mnk}\leq sup_{mnk}p_{mnk}<\infty$, then $\left[V_\lambda^E, A, \Delta_\gamma^r, f\right]_{\chi^3} \subset \left[V_\lambda^E, A, \Delta_\gamma^r, f, p\right]_{\chi^3}$

Theorem 6

Let f be an Orlicz function and s be a positive integer. Then, $\left[V_\lambda^E, A, \Delta_\gamma^r, f, q\right]_{\Lambda^3} \subset \left[V_\lambda^E, A, \Delta_\gamma^r, f, p\right]_{\Lambda^3}$

Proof: Let $\varepsilon>0$ be given and choose δ with $0<\delta<1$ such that $f(t)<\varepsilon$ for $0\leq t\leq\delta$. Write $y_{mnk}=f^{s^{-1}}\left(q\left(\Delta_\gamma^r x_{mnk}\right)^{1/m+n+k}-M\right)$ and consider $\sum_{mnk\in I_r} a_{mn}^{jk}\left[f(y_{mnk})\right]^{p_{mnk}} = \sum_{mnk\in I_r,\ y_{mnk}\leq\delta} a_{mn}^{jk}\left[f(y_{mnk})\right]^{p_{mnk}} + \sum_{mnk\in I_r} a_{mn}^{jk}\left[f(y_{mnk})\right]^{p_{mnk}}$.

Since f is continuous, we have

$$\sum_{mnk\in I_r,\ y_{mnk}\leq\delta} a_{mn}^{jk}\left[f(y_{mnk})\right]^{p_{mnk}} \leq \varepsilon^H \sum_{mnk\in I_r,\ y_{mnk}\leq\delta} a_{mn}^{jk} \quad (7)$$

and for $y_{mnk}>\delta$, we use the fact that, $y_{mnk}<\frac{y_{mnk}}{\delta}\leq 1+\frac{y_{mnk}}{\delta}$ and so, by the definition of f, we have for $y_{mnk}>\delta$,

$$f(y_{mnk})<2f(1)\frac{y_{mnk}}{\delta}$$

Hence

$$\frac{1}{\lambda_{pqr}}\sum_{mnk\in I_r,\ y_{mnk}\leq\delta} a_{mn}^{jk}\left[f(y_{mnk})\right]^{p_{mnk}} \leq max\left(1,\left(2f(1)\delta^{-1}\right)^H\right)\frac{1}{\lambda_{pqr}}\sum_{mnk\in I_r,\ y_{mnk}\leq\delta} a_{mn}^{jk} y_{mnk}^{p_{mnk}} \quad (8)$$

From (7) and (8) we obtain $\left[V_\lambda^E, A, \Delta_\gamma^r, f, q\right]_{\Lambda^3} \subset \left[V_\lambda^E, A, \Delta_\gamma^r, f, p\right]_{\Lambda^3}$. This completes the proof.

Acknowledgement

The authors are extremely grateful to the anonymous learned referee(s) for their keen reading, valuable suggestion and constructive comments for the improvement of the manuscript. The authors are thankful to the editor(s) and reviewers of J Appl Comp Math.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this research paper.

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