

Computation by Intention and Electronic Image of the Brain

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Abstract

Neurons as active unities are connected one with the others by synapses in an electronic way. We argue that brain is not comparable with digital computer with algorithms because intention as software is introduced as transformation in the neural states without any digital reduction. Any electronic system has voltages and currents sources and complex interconnected impedances. By electronic system and neural network we have different possibilities to introduce Freeman intentional transformation in the brain. One is to use source voltages (sensor) to generate wanted behavior of currents (internal flows of the signals) with the same impedance network. We can also reverse the process: given the behavior of the currents we generate wanted voltages transformation (effectors as muscles) with the same impedance. Another possibility is to change the impedance network (memory) to generate wanted internal current. When intention is transformation of references, geometry changes and also the form of straight line (geodesic). Special reference and geometry can be modeled by the electrical power as metric. Different types of brain geometries as hyperbolic geometry of waves and elliptic geometry of stable states are discussed with examples. Because we have waves in brain, Karl Pribram created holographic model of brain that by scattering and transmitted matrix can be joined to electronic model. Mechanical system metrics are implemented in the neural network as electronic network.

Keywords: Intention; Neuron; Electrical circuit; Scattering matrix; Transmission matrix; Impedance matrix; Multi pendulum system

Introduction

This work gives a possible mathematical formulation of intentional brain dynamics following Freeman's half century-long dynamic systems approach [1-3] and the electrical behavior of the brain. In 1980 an artificial neural network was built that works but has high precision components, slow unstable learning, it is non-adaptive and needs an external control. Now we want low precision components, fast stable learning, adapt to environment and autonomous. How can we get this? We can make dynamical components, add feedback (positive and negative) and close the loop with the outside world. The ordinary differential equations or ODEs to control the neural dynamic are a stiff and nonlinear system. Why not just program this on a computer? We know that stiff and nonlinear dynamical systems are inefficient on a digital computer. An example is the IBM Blue Gene project with 4096 CPUs and 1000 Terabytes RAM, which, to simulate the Mouse cortex uses 8 106 neurons, 2 1010 synapses 109 Hz, 40 Kilowatts and digital. The brain uses 1010 neurons, 1014 synapses 10 Hz and 20 watts. Analogue system is more efficient than the digital system by many orders of magnitude. Snider [4] suggests to use analogue electrical circuit denoted CrossNet or neuromorphic computing with memristor [5-7] to solve the problem of the neural computation. Let's recall that for Turing the physical device is not computable by a Turing machine, which is the theoretical version of the digital computer. Carved [8] suggests that the physics or analogue computer is more efficient to solve the neural network problem. In fact, for analogue system we do not have algorithms to program the neurons. Rather, the digital program is substituted by the dynamics in the analogue computer. We can program the Cross Net Takashi Kohno, Rinzel electrical system as it was used by Snider to compute the parameters useful to generate the desired trajectories to solve problems. Physical description of the intentionality [3,4] is beyond any algorithmic or digital computation. To clarify better the new computation paradigm, we can refer the following principle: "Animals and humans use their finite brains to comprehend and adapt to infinitely complex environment."

To comprehend and adapt means to change the internal brain parameters (conductance of the synapse) to mimic the external transformation by suitable use of the sensors and effectors. The paper is divided in different parts. The first part of the paper shows the connection between neural network and electronic network. The second part explains the possibility by sensor voltages to produce wanted change in the brain currents. The third part explains how currents generate wanted effects on the muscle or other external element and the last part is the change of the internal impedance network or memory to control currents by fixed value of the sensor voltages. We take electrical power of the brain as metric geometry in the current or voltage space. Fixed the power, we can generate in the wanted transformed reference the straight line or geodesic that gives the best or minimum path in time of the point in the current or voltage space. Then we explain how intention (Freeman) can be introduced in the electronic model of the brain. After we take care of the Karl Pribram holographic model [9,10] of the brain according to the electronic model. So transmission, reflection waves can be modelled by the impedance matrix. The last part studies the connection between electronic brain model and mechanical system as pendulum and double pendulum.

Neuromorphic Computing by Neural Network

Biological information processing systems operate on completely different principles from those with which most engineers are familiar [11]. For many problems, particularly those in which the input data are ill-conditioned and the computation can be specified in a relative manner, biological solutions are many orders of magnitude more

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effective than those we have been able to implement using digital methods. This advantage can be attributed principally to the use of elementary physical phenomena as computational primitives, and to the representation of information by the relative values of analog signals, rather than by the absolute values of digital signals. This approach requires adaptive techniques to mitigate the effects of component differences. This kind of adaptation leads naturally to systems that learn about their environment. Large-scale adaptive analog systems are more robust to component degradation and failure than are more conventional systems, and they use far less power. For this reason, adaptive analog technology can be expected to utilize the full potential of wafer scale silicon fabrication. There is a myth that the nervous system is slow, is built out of slimy stuff, uses ions instead of electrons, and is therefore ineffective. When the Whirlwind computer was first built back at M.I.T., they made a movie about it, which was called "Faster than Thought." The Whirlwind did less computation than your wristwatch. We have evolved by a factor of about 10 million in the cost of computation since the Whirlwind. Yet we still cannot begin to do the simplest computations that can be done by the brains of insects, let alone handle the tasks routinely performed by the brains of humans. So we have finally come to the point where we can see what is difficult and what is easy. Multiplying numbers to balance a bank account is not that difficult. What is difficult is processing the poorly conditioned sensory information that comes in through the lens of an eye or through the eardrum. A typical microprocessor does about 10 million operations and uses about 1 W. In round numbers, it cost about 10^{-7} J to do one operation, the way we do it today, on a single chip. If we go off the chip to the box level, a whole computer uses about 10^{-5} J /operation. A whole computer is thus about two orders of magnitude less efficient than is a single chip. Back in the late 1960's we analyzed what would limit the electronic device technology as we know it; those calculations have held up quite well to the present. The standard integrated circuit fabrication processes available today allow us to build transistors that have minimum dimensions of about $1 (10^{-6} \text{ m})$. By ten years from now, we will have reduced these dimensions by another factor of 10, and we will be getting close to the fundamental physical limits: if we make the devices any smaller, they will stop working. It is conceivable that a whole new class of devices will be invented—devices that are not subject to the same limitations. But certainly the ones we have thought of up to now—including the superconducting ones—will not make our circuits more than about two orders of magnitude more dense than those we have today. The factor of 100 in density translates rather directly into a similar factor in computation efficiency. So the ultimate silicon technology that we can envision today will dissipate on the order of 10^{-9} J of energy for each operation at the single chip level, and will consume a factor of 100-1000 more energy at the box level. We can compare these numbers to the energy requirements of computing in the brain. There are about 1016 synapses in the brain. A nerve pulse arrives at each synapse about ten times/s, on average. So in rough numbers, the brain accomplishes 1016 complex operations/s. The power dissipation of the brain is a few watts, so each operation costs only 10^{-16} J. The brain is a factor of 1 billion more efficient than our present digital technology, and a factor of 10 million more efficient than the best digital technology that we can imagine.

From the first integrated circuit in 1959 until today, the cost of computation has improved by a factor about 1 million. We can count on an additional factor of 100 before fundamental limitations are encountered. At that point, a state-of-the-art digital system will still require 10 MW to process information at the rate that it is processed by a single human brain. The unavoidable conclusion, which I

reached about ten years ago, is that we have something fundamental to learn from the brain about a new and much more effective form of computation. Even the simplest brains of the simplest animals are awesome computational instruments. They do computations we do not know how to do, in ways we do not understand. We might think that this big disparity in the effectiveness of computation has to do with the fact that, down at the device level, the nerve membrane is actually working with single molecules. Perhaps manipulating single molecules is fundamentally more efficient than is using the continuum physics with which we build transistors. If that conjecture were true, we would have no hope that our silicon technology would ever compete with the nervous system. In fact, however, the conjecture is false. Nerve membranes use populations of channels, rather than individual channels, to change their conductance, in much the same way that transistors use populations of electrons rather than single electrons. It is certainly true that a single channel can exhibit much more complex behaviors than can a single electron in the active region of a transistor, but these channels are used in large populations, not in isolation.

We can compare the two technologies by asking how much energy is dissipated in charging up the gate of a transistor from a 0 to a 1. We might imagine that a transistor would compute a function that is loosely comparable to synaptic operation. In today's technology, it takes about 10^{-13} j to charge up the gate of a single minimum-size transistor. In ten years, the number will be about 10^{-15} j within shooting range of the kind of efficiency realized by nervous systems. So the disparity between the efficiency of computation in the nervous system and that in a computer is primarily attributable not to the individual device requirements, but rather to the way the devices are used in the system.

Where did all the energy go? There is a factor of 1 million unaccounted for between what it costs to make a transistor work and what is required to do an operation the way we do it in a digital computer. There are two primary causes of energy waste in the digital systems we build today.

1) We lose a factor of about 100 because, the way we build digital hardware, the capacitance of the gate is only a very small fraction of capacitance of the node. The node is mostly wire, so we spend most of our energy charging up the wires and not the gate.

2) We use far more than one transistor to do an operation; in a typical implementation, we switch about 10 000 transistors to do one operation. So altogether it costs 1 million times as much energy to make what we call an operation in a digital machine as it costs to operate a single transistor. I do not believe that there is any magic in the nervous system that there is a mysterious fluid in there that is not defined, some phenomenon that is orders of magnitude more effective than anything we can ever imagine.

There is nothing that is done in the nervous system that we cannot emulate with electronics if we understand the principles of neural information processing.

Now we are stuck with an artifact, so we must try to reverse engineer it. Let us consider the primitive operations and representations in the nervous system, and contrast them with their counterparts in a digital system. As we think back, many of us remember being confused when we were first learning about digital design. First, we decide on the information representation. There is only one kind of information, and that is the bit: It is either a 1 or a 0. We also decide the elementary operations we allow, usually AND, OR, and NOT or their equivalents. We start by confining ourselves to an incredibly impoverished world, and out of that, we try to build something that makes sense. The

miracle is that we can do it! But we pay the factor of 104 for taking all the beautiful physics that is built into those transistors, mashing it down into a 1 or a 0, and then painfully building it back up, with AND and OR gates to reinvent the multiply. We then string together those multiplications and additions to get more complex operations those that are useful in a system we wish to build. What kind of computation primitives are implemented by the device physics we have available in nervous tissue or in a silicon integrated circuit? In both cases, the state variables are represented by an electrical charge.

We do basic aggregation of information using the conservation of change. We can dump current onto an electrical node at any location, and it all ends up as charge on the node. Kirchhoff's law implements a distributed addition, and the capacitance of the node integrates the current into the node with respect to time.

In nervous tissue, ions are in thermal equilibrium with their surroundings, and hence their energies are Boltzmann distributed. This distribution, together with the presence of energy barriers, computes a current that is an exponential function of the barrier energy. If we modulate the barrier with an applied voltage, the current will be an exponential function of that voltage. The principle is used to create active devices (those that produce gain or amplification in signal level), both in the nervous system and in electronics. In addition to providing gain, an individual transistor computes a complex nonlinear function of its control and channel voltages. That function is not directly comparable to the functions that synapses evaluate using their presynaptic and postsynaptic potentials, but a few transistors can be connected strategically to compute remarkably competent synaptic functions. Most important, the nervous system contains mechanisms for long-term learning and memory. All higher animals undergo permanent changes in their brains as a result of life experiences. A silicon retina that does a rudimentary form of learning and long term memory. The ability to learn and retain analog information for long periods is thus a natural consequence of the structures created by modern silicon processing technology. The fact that we can build devices that implement the same basic operations as those the nervous system uses leads to the inevitable conclusion that we should be able to build entire systems based on the organizing principles used by the nervous system. We will refer to these systems generically as neuromorphic systems. We start by letting the device physics define our elementary operations. These functions provide a rich set of computational primitives, each a direct result of fundamental physical principles. They are not the operations out of which we are accustomed to building computers, but in many ways, they are much more interesting. They are more interesting than AND and OR. They are more interesting than multiplication and addition. But they are very different. If we try to fight them, to turn them into something with which we are familiar, we end up making a mess. So the real trick is to invent a representation that takes advantage of the inherent capabilities of the medium, such as the abilities to generate exponentials, to do integration with respect to time, and to implement a zero-cost addition using Kirchhoff's law. These are powerful primitives; using the nervous system as a guide, we will attempt to find a natural way to integrate them into an overall system-design strategy.

Neural System as a Complex Electrical Circuit

In opposition to actual digital sequential computers where computations are carried out by a single complex processor there are Cellular Neural/Non-linear Networks (CNN) [12] which are analog parallel machines with a high number of simple processors, which are disposed in a regular array, and each processor is connected to

the other processors in a reduced neighborhood. One of these analog processors is represented by the electrical activity of the synapse given by the electrical circuit (Figure 1).

The impedance matrix is

$$Z = \begin{bmatrix} R_{ins} + 3 & 1 & 0 \\ 1 & R_m + 3 & R_m \\ 0 & R_m & R_{syn} + R_m + 2 \end{bmatrix}$$

The geodesic trajectory [9] of the synapse activity is controlled by the relation $power = i^T Z i$ where Z is the impedance matrix in the currents space. In an extensive form we have

$$power = \left(\frac{ds}{dt} \right)^2 = (R_m + 3)i_5^2 + (R_m + R_{syn} + 2)i_6^2 + 2i_5 i_6 + 2R_m i_5 i_8 \\ = (R_m + 3) \left(\frac{dq_2}{dt} \right)^2 + (R_m + 3) \left(\frac{dq_3}{dt} \right)^2 + (R_m + R_{syn} + 2) \left(\frac{dq_8}{dt} \right)^2 + \left(\frac{dq_2}{dt} \right) \left(\frac{dq_3}{dt} \right) + 2R_m \left(\frac{dq_5}{dt} \right) \left(\frac{dq_8}{dt} \right)$$

Brain as an Electronic System

For the electrical circuit that simulates membrane electrical circuit (Figure 2).

For the Hindmarsh – Rose [13,14] model of neuron we have

$$\begin{cases} i_1 = v_2 + \phi(v_1) - 2v_3 + I \\ i_2 = \psi(v_1) - v_2 \\ i_3 = r(s(v_1 - v_R) - v_3) \end{cases}$$

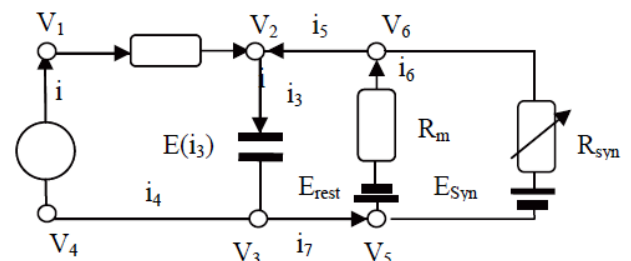


Figure 1: Electrical circuit of the synapse.

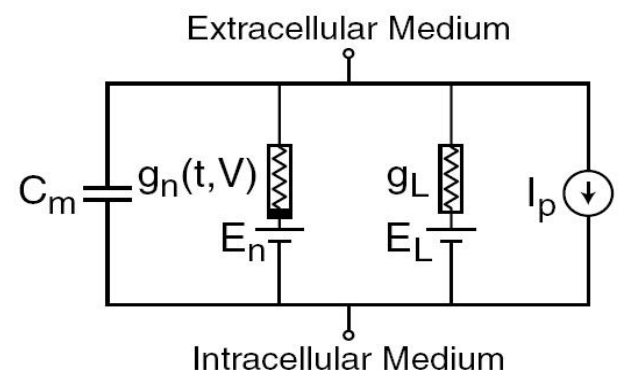


Figure 2: Membrane electrical circuit.

Where

$$i_1 = \frac{dv_1}{dt}, i_2 = \frac{dv_2}{dt}, i_3 = \frac{dv_3}{dt}$$

And

$$ds^2 = di_1^2 + di_2^2 + di_3^2$$

Now the metric tensor is

$$g_{\alpha,\beta} = \sum_j \frac{\partial i_j}{\partial v_\alpha} \frac{\partial i_j}{\partial v_\beta} = \begin{bmatrix} \frac{\partial i_1}{\partial v_1} & \frac{\partial i_1}{\partial v_2} & \frac{\partial i_1}{\partial v_3} \\ \frac{\partial i_2}{\partial v_1} & \frac{\partial i_2}{\partial v_2} & \frac{\partial i_2}{\partial v_3} \\ \frac{\partial i_3}{\partial v_1} & \frac{\partial i_3}{\partial v_2} & \frac{\partial i_3}{\partial v_3} \end{bmatrix}^T \begin{bmatrix} \frac{\partial i_1}{\partial v_1} & \frac{\partial i_1}{\partial v_2} & \frac{\partial i_1}{\partial v_3} \\ \frac{\partial i_2}{\partial v_1} & \frac{\partial i_2}{\partial v_2} & \frac{\partial i_2}{\partial v_3} \\ \frac{\partial i_3}{\partial v_1} & \frac{\partial i_3}{\partial v_2} & \frac{\partial i_3}{\partial v_3} \end{bmatrix}$$

$$= \begin{bmatrix} 2ax-3x^2 & 1 & -1 \\ -2bx & -1 & 0 \\ rs & 0 & -r \end{bmatrix}^T \begin{bmatrix} 2ax-3x^2 & 1 & -1 \\ -2bx & -1 & 0 \\ rs & 0 & -r \end{bmatrix}$$

$$= \begin{bmatrix} 4(a^2+b^2)v_1^2+r^2s^2 & 2(a+b)v_1 & -(2av_1+r^2s) \\ 2(a+b)v_1 & 2 & -1 \\ -(2av_1+r^2s) & -1 & 1+r^2 \end{bmatrix}$$

and

$$ds^2 = [4(a^2+b^2)v_1^2+r^2s^2]dv_1^2 + 2(a+b)dv_1^2 - (1+r^2)dv_3^2 + 4[(a+b)v_1]dv_1dv_2 - (2av_1+r^2s)dv_2dv_3$$

And

$$i_1 = \frac{dv_1}{dt}, i_2 = \frac{dv_2}{dt}, i_3 = \frac{dv_3}{dt}$$

Because the brain is a complex electrical circuit with capacity and nonlinear resistors, a network of neurons or an electronic network is a general transformation or MIMO (Figure 3).

We know that in any electronic system the voltages and currents are related in this way

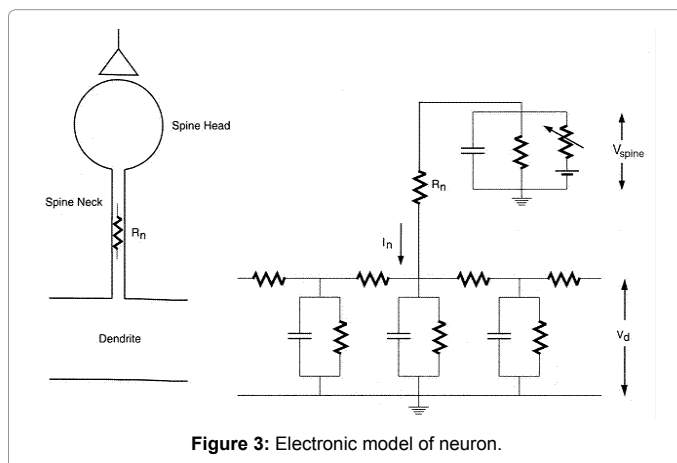


Figure 3: Electronic model of neuron.

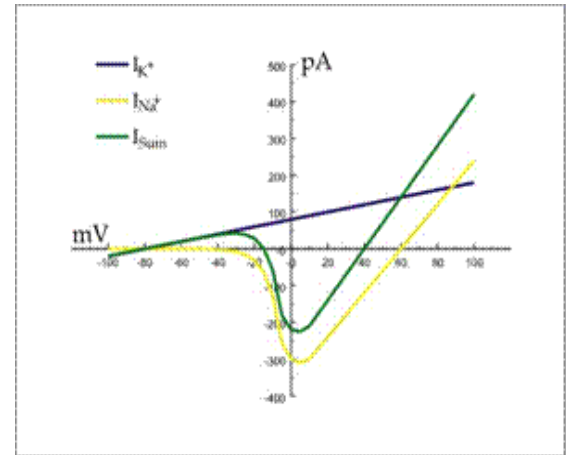


Figure 4: An approximation of the potassium and sodium ion components of a so-called "whole cell" I-V curve of a neuron.

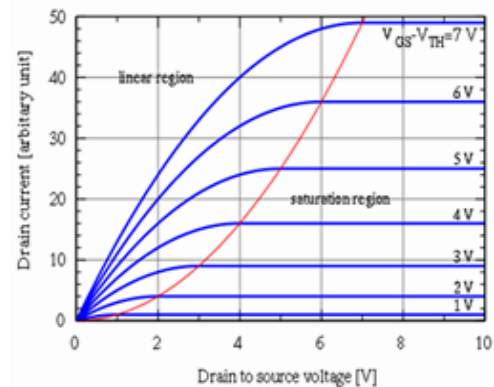


Figure 5: MOSFET drain current vs. drain-to-source voltage for several values of the overdrive voltage, $V_{GS} - V_{th}$; the boundary between linear (ohmic) and saturation (active) modes is indicated by the upward curving parabola.

$$\begin{cases} i_1 = f_1(v_1, v_2, \dots, v_p) \\ i_2 = f_2(v_1, v_2, \dots, v_p) \\ \dots \\ i_n = f_n(v_1, v_2, \dots, v_p) \end{cases}$$

where the voltages are the control variables and the currents are the controlled elements [5,6]. The Jacobian matrix is

$$J = \begin{bmatrix} \frac{\partial i_1}{\partial v_1} & \frac{\partial i_1}{\partial v_2} & \dots & \frac{\partial i_1}{\partial v_p} \\ \frac{\partial i_2}{\partial v_1} & \frac{\partial i_2}{\partial v_2} & \dots & \frac{\partial i_2}{\partial v_p} \\ \dots & \dots & \dots & \dots \\ \frac{\partial i_q}{\partial v_1} & \frac{\partial i_q}{\partial v_2} & \dots & \frac{\partial i_q}{\partial v_p} \end{bmatrix} = \begin{bmatrix} Y_{1,1} & Y_{1,2} & \dots & Y_{1,p} \\ Y_{2,1} & Y_{2,2} & \dots & Y_{2,p} \\ \dots & \dots & \dots & \dots \\ Y_{q,1} & Y_{q,2} & \dots & Y_{q,p} \end{bmatrix} = Y$$

where Y is the dynamic admittances in one point of the system voltage current function. For one dimension the current voltage relation is written in this form $i=f(v)$ that in electronics is denoted characteristic function. In Figures 4 and 5 we show two different cases of characteristic function in one dimension [15].

We remark that

$$I = YV \text{ So } V = Y^{-1} I = ZI$$

where Z is the multi - port impedance. Given the admittance matrix Y the diagonal elements are the self-admittance for any element of the N ports and the cross admittance in non-diagonal admittance is the mutual admittance for which we have a transfer of the power between ports. The transfer power is the bond between ports. For example given the electrical circuit in Figure 6 we have three currents and three voltages one for any resistance. The three currents are not independent, in fact we have

$$i_3 = i_1 + i_2$$

and the three voltages are not independent in fact we have

$$v_3 = R_3 \left(\frac{v_1}{R_1} + \frac{v_2}{R_2} \right)$$

Now the power of the electrical circuit is

$$\begin{aligned} P &= R_1(i_1)^2 + R_2(i_2)^2 + R_3(i_3)^2 \\ &= R_1(i_1)^2 + R_2(i_2)^2 + R_3(i_1 + i_2)^2 \\ &= (R_1 + R_3)(i_1)^2 + (R_2 + R_3)(i_2)^2 + 2R_3i_1i_2 \end{aligned}$$

The power can be written in a matrix way in this form

$$\begin{aligned} P &= \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}^T \begin{bmatrix} R_1 & 0 & 0 \\ 0 & R_2 & 0 \\ 0 & 0 & R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_1 + i_2 \end{bmatrix}^T \begin{bmatrix} R_1 & 0 & 0 \\ 0 & R_2 & 0 \\ 0 & 0 & R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_1 + i_2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}^T \begin{bmatrix} R_1 & 0 & 0 \\ 0 & R_2 & 0 \\ 0 & 0 & R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_1 + i_2 \end{bmatrix} \\ &= \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}^T \begin{bmatrix} R_1 & 0 & 0 \\ 0 & R_2 & 0 \\ 0 & 0 & R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_1 + i_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}^T \begin{bmatrix} R_1 + R_3 & R_3 \\ R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}^T \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \end{aligned}$$

So

$$\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} R_1 + R_3 & R_3 \\ R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

And

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} R_1 + R_3 & R_3 \\ R_3 & R_2 + R_3 \end{bmatrix}^{-1} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

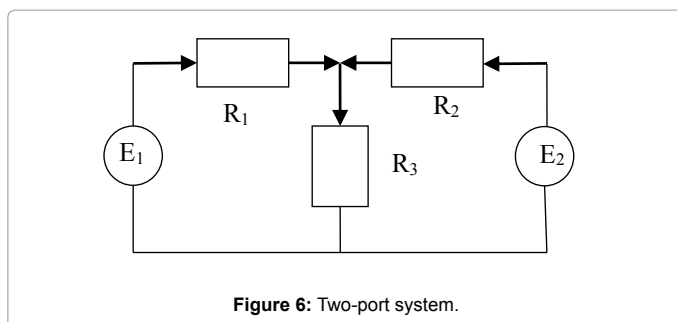


Figure 6: Two-port system.

where

$$Z = \begin{bmatrix} Z_{1,1} & Z_{1,2} \\ Z_{2,1} & Z_{2,2} \end{bmatrix} = \begin{bmatrix} (R_1 + R_3) & R_3 \\ R_3 & (R_2 + R_3) \end{bmatrix}$$

$$Y = Z^{-1} = \begin{bmatrix} Y_{1,1} & Y_{1,2} \\ Y_{2,1} & Y_{2,2} \end{bmatrix} = \begin{bmatrix} (R_2 + R_3) & -R_3 \\ -R_3 & (R_1 + R_3) \end{bmatrix} \frac{1}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

And

$$P = i^T E = \begin{bmatrix} R_1 + R_3 & R_3 \\ R_3 & R_2 + R_3 \end{bmatrix}^{-1} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}^T \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}^T \begin{bmatrix} R_1 + R_3 & R_3 \\ R_3 & R_2 + R_3 \end{bmatrix}^{-1} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

Control of wanted transformation for currents and voltage

Given the relation between the current and voltages sources in the neural network as electrical circuit, we want to show how is possible to change the voltages sources to have wanted currents in the brain. For reverse how are the suitable currents to generate wanted sources. In the brain system the first problem is to adapt the sensor system to generate wanted behavior in the brain, the second is to adapt brain internal currents to fixed external sources of voltage.

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} R_1 + R_3 & R_3 \\ R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

or

$$e = Ri$$

And the reverse

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} R_1 + R_3 & R_3 \\ R_3 & R_2 + R_3 \end{bmatrix}^{-1} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

or

$$e = R^{-1}i$$

In the electrical circuit we have different types of control. The first type is to control current by voltages sources with the same impedance. If this is the wanted transformation of the current

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

or

$$I = Ti$$

the change of the voltages e to have wanted change of current is

$$E = R(Ti), i = R^{-1}e$$

so

$$E = RTR^{-1}e$$

In fact we have

$$I = R^{-1}E = R^{-1}(RTR^{-1}e) = TR^{-1}e = T i$$

The second type is to change the current to have wanted change of the voltages with the same impedance.

$$\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

or

$$E = qe$$

$$I = R^{-1}(Qe), e = Ri$$

So

$$I = R^{-1}QR i$$

And E is the set of voltages generators that change the current from

$$i \text{ to } T i = I$$

In the third type, we change the internal parameters of the brain or impedances in such a way that with the same voltages sources we have the wanted change of the current. In fact

$$e = Ri$$

$$e = R(T^{-1}T)i = RT^{-1}(Ti)$$

$$R' = RT^{-1}$$

and

$$I = (R')^{-1}e = (RT^{-1})^{-1}e = TR^{-1}e = Ti$$

In the fourth we change the internal parameters of the brain or impedance in such a way that with the same current we have the wanted change of the voltages.

$$R^{-1}e = i$$

$$R^{-1}(T^{-1}Te) = R^{-1}T^{-1}(Te) = i$$

$$(R')^{-1} = R^{-1}T^{-1}$$

In fact we have

$$E = R'i = TRi = Te$$

Change of electrical circuit variables with the same power

Because the power is the metric of the brain as electrical circuit, we are interested to change the brain variables as current and voltages with the same metric or power. So we have

$$p = i^T Ri, i = T^{-1}I = SI$$

$$p = I^T S^T RSI$$

$$R' = S^T RS$$

With the new impedance we have

$$E = R' I = S^T Ri = S^T e$$

$$i = SI, e = Ri$$

Reactive electrical elements in the electronic system

When the electrical circuit we adjoin capacitors and inductors, the relation between currents and voltages is given by impedances or admittances valuated by complex numbers. Now given the electrical circuit in Figure 7, we have the prototype relation between currents and voltages

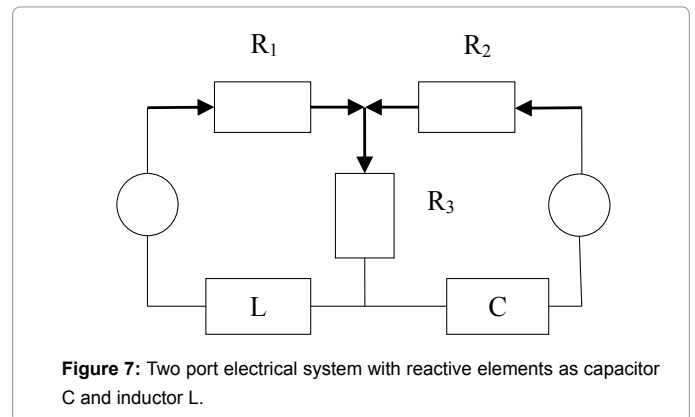


Figure 7: Two port electrical system with reactive elements as capacitor C and inductor L.

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}^T \begin{bmatrix} R_1 & 0 & 0 & 0 & 0 \\ 0 & Z_L & 0 & 0 & 0 \\ 0 & 0 & R_2 & 0 & 0 \\ 0 & 0 & 0 & Z_C & 0 \\ 0 & 0 & 0 & 0 & R_3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

That can be written in this way

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} R_1 + R_3 + Z_L & R_3 \\ R_3 & R_2 + R_3 + Z_C \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

or

$$e = Zi$$

And $E = ZTZ^{-1}i$. With the new potential sources we can change the current in a wanted way. For the power we have the expression

$$p = i^T Zi, i = T^{-1}I = SI, p = I^T S^T RZSI$$

$$R' = S^T ZS$$

With the new impedance we have

$$E = R' I = S^T Zi = S^T e$$

$$i = SI, e = Zi$$

Pribram Wave Holographic and Electronic Model of Brain

In the previous brain representation we consider the brain as an electronic circuit with currents, voltages, impedances and power as geometrical brain metric [9,16]. The aim of this chapter is to represent the electronic circuit as an optical mirror or an optical transmitter of energy. This image will be very useful to create a bridge from electronic parameters as impedances and optical property of the brain waves as scattering (reflection) and wave transmission from one point to another. In this way we can represent the experimental evidence of the signals movements into electronic internal properties. Now we begin to present this new type of brain image. Given two port system can be represented as in Figure 8.

In the electrical circuit the power is given by the generators and can be transmitted or reflected in accordance with the system impedance. For the electrical circuit we have the well-known impedance matrix for which

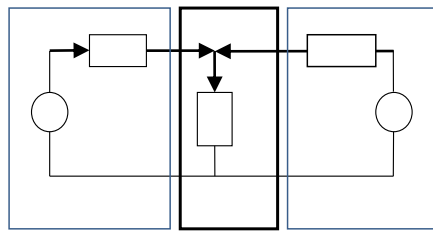


Figure 8: We divide the electrical circuit into two parts. One part is the external element and the other part is the internal element.

$$\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} Z_{1,1} & Z_{1,2} \\ Z_{2,1} & Z_{2,2} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Given the currents as sources or inputs in channels 1 and 2 we compute the voltages E or outputs in channels 1 and 2 by parameters Z. The transmission matrix is

$$\begin{bmatrix} E_2 \\ i_2 \end{bmatrix} = \begin{bmatrix} T_{1,1} & T_{1,2} \\ T_{2,1} & T_{2,2} \end{bmatrix} \begin{bmatrix} E_1 \\ i_1 \end{bmatrix}$$

Given channel 1 with its voltage E1 and current i1 we compute by T the voltage and current transmitted in channel 2. The scattering or reflected matrix is

$$\begin{bmatrix} i_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} S_{1,1} & S_{1,2} \\ S_{2,1} & S_{2,2} \end{bmatrix} \begin{bmatrix} E_1 \\ i_2 \end{bmatrix}$$

Given E1 by the matrix S we compute the reflected value i1. For i2 we compute again with S the reflected value E2. Given the impedance matrix it is possible to compute the transmitted matrix as follows:

$$\begin{bmatrix} E_2 \\ i_2 \end{bmatrix} = \begin{bmatrix} T_{1,1}E_1 + T_{1,2}i_1 \\ T_{2,1}E_1 + T_{2,2}i_1 \end{bmatrix} = \begin{bmatrix} T_{1,1}(Z_{1,1}i_1 + Z_{1,2}E_2) + T_{1,2}i_1 \\ T_{2,1}(Z_{1,1}i_1 + Z_{1,2}E_2) + T_{2,2}i_1 \end{bmatrix}$$

$$T_{1,1}(Z_{1,1}i_1 + Z_{1,2}E_2) + T_{1,2}i_1 = Z_{2,1}i_1 + Z_{2,2}E_2 = E_2$$

$$\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} Z_{1,1}i_1 + Z_{1,2}E_2 \\ Z_{2,1}i_1 + Z_{2,2}E_2 \end{bmatrix} = \begin{bmatrix} Z_{1,1}i_1 + Z_{1,2}(T_{2,1}E_1 + T_{2,2}i_1) \\ Z_{2,1}i_1 + Z_{2,2}(T_{2,1}E_1 + T_{2,2}i_1) \end{bmatrix}$$

$$T_{2,1}(Z_{1,1}i_1 + Z_{1,2}E_2) + T_{2,2}i_1 = i_2$$

So

$$\begin{cases} T_{1,1}Z_{1,1} = Z_{2,1} \\ T_{1,1}Z_{1,2} + T_{1,2} = Z_{2,2} \\ Z_{1,2}T_{2,1} = I \\ Z_{1,1} + Z_{1,2}T_{2,2} = 0 \end{cases}$$

$$\begin{cases} T_{1,1}(R_1 + R_3) = R_3 \\ T_{1,1}R_3 + T_{1,2} = R_2 + R_3 \\ R_3T_{2,1} = I \\ (R_1 + R_3) + R_3T_{2,2} = 0 \end{cases}$$

For the circuit in Figure 5 we have

$$\text{and } \begin{bmatrix} E_2 \\ i_2 \end{bmatrix} = \begin{bmatrix} \frac{R_3}{R_1 + R_3} & \frac{R_1R_2 + R_1R_3 + R_2R_4}{R_1 + R_3} \\ \frac{1}{R_3} & -\frac{R_1 + R_3}{R_3} \end{bmatrix} \begin{bmatrix} E_1 \\ i_1 \end{bmatrix}$$

When we solve the system for the transfer matrix we have the relation between the transfer matrix and the impedance matrix. Now we can also find the relation between the scattering matrix and the impedance matrix in this way

$$\begin{bmatrix} i_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} S_{1,1}E_1 + S_{1,2}i_2 \\ S_{2,1}E_1 + S_{2,2}i_2 \end{bmatrix} = \begin{bmatrix} S_{1,1}(Z_{1,1}i_1 + Z_{1,2}E_2) + S_{1,2}i_2 \\ S_{2,1}(Z_{1,1}i_1 + Z_{1,2}E_2) + S_{2,2}i_2 \end{bmatrix}$$

$$S_{2,1}(Z_{1,1}i_1 + Z_{1,2}E_2) + S_{2,2}i_2 = Z_{2,1}i_1 + Z_{2,2}E_2 = E_2$$

$$\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} Z_{1,1}i_1 + Z_{1,2}E_2 \\ Z_{2,1}i_1 + Z_{2,2}E_2 \end{bmatrix} = \begin{bmatrix} Z_{1,1}(S_{1,1}E_1 + S_{1,2}i_2) + Z_{1,2}E_2 \\ Z_{2,1}(S_{1,1}E_1 + S_{1,2}i_2) + Z_{2,2}E_2 \end{bmatrix}$$

$$Z_{1,1}(S_{1,1}E_1 + S_{1,2}i_2) + Z_{1,2}E_2 = E_1$$

$$\text{So } \begin{cases} S_{2,1}Z_{1,1} = Z_{2,1} \\ S_{2,1}Z_{1,2} + S_{2,2} = Z_{2,2} \\ Z_{1,1}S_{1,1} = I \\ Z_{1,1}S_{1,2} + Z_{2,2} = 0 \end{cases}$$

$$\text{In our example we have } \begin{cases} S_{2,1}(R_1 + R_3) = R_3 \\ S_{2,1}R_3 + S_{2,2} = (R_2 + R_3) \\ (R_1 + R_3)S_{1,1} = I \\ (R_1 + R_3)S_{1,2} + R_2 + R_3 = 0 \end{cases}$$

$$\text{And } \begin{bmatrix} i_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1 + R_3} & -\frac{R_2 + R_3}{R_1 + R_3} \\ \frac{R_3}{R_1 + R_3} & \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1 + R_3} \end{bmatrix} \begin{bmatrix} E_1 \\ i_2 \end{bmatrix}$$

The relation between the transfer matrix and the scattering

$$\text{matrix is } \begin{cases} T_{1,2}S_{1,2} = S_{2,2} \\ T_{1,2}S_{1,1} + T_{1,1} = S_{2,1} \\ T_{2,2}S_{1,2} = I \\ T_{2,2}S_{1,1} + T_{2,1} = 0 \end{cases}$$

We can use the matrices also in the neuron shown in Figure 8. We connect impedance matrix Z with the transmission and scattering matrix by which we can study the propagation of the waves that we show in Figure 9. So in this way we establish a bridge between Pribram holographic model of the brain [9,10] and the electronic image of the brain that we study in this paper (Figure 9).

Scattering and transmitted signals in the brain. The input waves of the voltages and currents are similar to the light in the brain (Figure 10).

Each electronic system is a medium with N ports. Each port has one input and one reflected output. By the electronic circuit the N ports

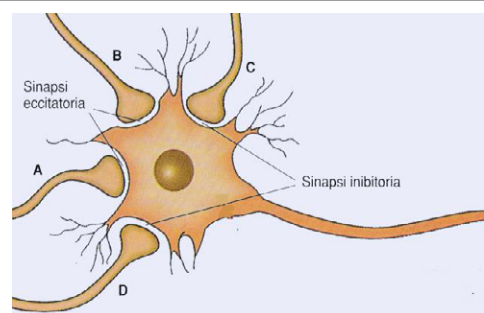


Figure 9: Neuron as an electronic system with four channels with their proper electrical parameters. For this neuron we can compute the impedance matrix, the transmission matrix and the scattering matrix.

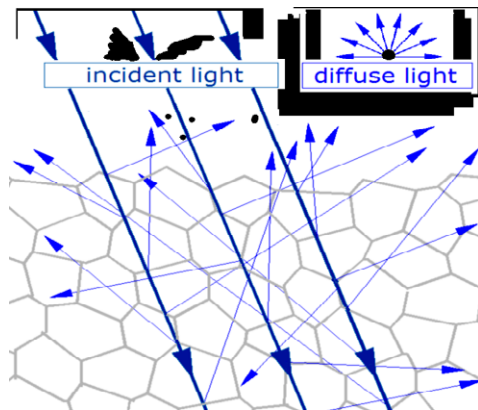


Figure 10: Scattering and scattered waves of voltages and currents as light in the brain as the pribram holographic model.

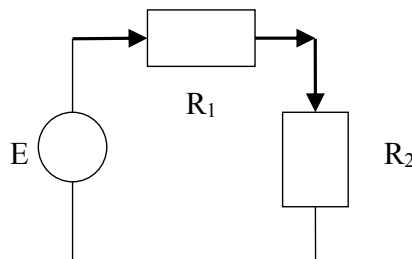


Figure 11: Simple electrical circuit.

are connected in a complex way. So when on one port are impressed voltages, these can be transformed in the current as in the case of the antenna or can be scattered in other ports by the transfer process or a reflection process. So power can be dissipated and absorbed as an antenna by a non-linear resistor network. Inside the electronic system the power can be transmitted to the other ports or reflected (scattering).

A Simple example of scattering of an elementary electrical circuit

Now given the electronic circuit (Figure 11).

The power in the reference resistor (reflection) R1 is given by the expression

$$P_{refl} = R_1 I^2 = R_1 \left(\frac{E}{R_1 + R_2} \right)^2,$$

$$P_{load} = R_2 I^2 = R_2 \left(\frac{E}{R_1 + R_2} \right)^2$$

and

$$\frac{dP_{refl}}{dR_1} = \left(\frac{E}{R_1 + R_2} \right)^2 \frac{R_2 - R_1}{R_1 + R_2} = \frac{R_2 - R_1}{R_1 + R_2} I^2 = S I^2$$

where S is the scattering coefficient. The behaviour of the power and S for $R_2=0.5$ is shown in Figure 12.

When $R_1 < R_2$ and R_1 increases, the power in R_1 (reflection) increases at the maximum value and $S > 0$. When $R_1 > R_2$ and R_1 increases, the power in the reference impedance decreases from the maximum value and $S < 0$. The scatter S measures the variation of the reflected power when we change the value of the reference impedance R_1 .

Geometry and Conceptual Part in Neural Network

The electrical power gives us the material aspect of intentionality. The other part of intentionality is the conceptual one which is given by the wanted transformation

$$\begin{cases} x_1 = x_1(y_1, y_2, \dots, y_p) \\ x_2 = x_2(y_1, y_2, \dots, y_p) \\ \dots \\ x_q = x_q(y_1, y_2, \dots, y_p) \end{cases}$$

where (x_1, x_2, \dots, x_p) are the wanted variables and (y_1, y_2, \dots, y_n) are the initial variables. We implement the wanted transformation into the current space in this way

$$\begin{cases} i_1 = i_1(I_1, I_2, \dots, I_p) \\ i_2 = i_2(I_1, I_2, \dots, I_p) \\ \dots \\ i_q = i_q(I_1, I_2, \dots, I_p) \end{cases}$$

where the current substitutes the variables x and y without changing the relation between x and y. With the Jacobian of the previous transformation we give a linear local form for the transformation

$$S = \begin{bmatrix} \frac{\partial i_1}{\partial I_1} & \frac{\partial i_1}{\partial I_2} & \dots & \frac{\partial i_1}{\partial I_p} \\ \frac{\partial i_2}{\partial I_1} & \frac{\partial i_2}{\partial I_2} & \dots & \frac{\partial i_2}{\partial I_p} \\ \dots & \dots & \dots & \dots \\ \frac{\partial i_q}{\partial I_1} & \frac{\partial i_q}{\partial I_2} & \dots & \frac{\partial i_q}{\partial I_p} \end{bmatrix} = T^{-1}$$

Given the reference electronic system where the power is

$$p = i^T Z i,$$

$$i = T^{-1} I = S I$$

With substitution of i we have

$$p = I^T S^T Z S I$$

$$Z' = S^T Z S$$

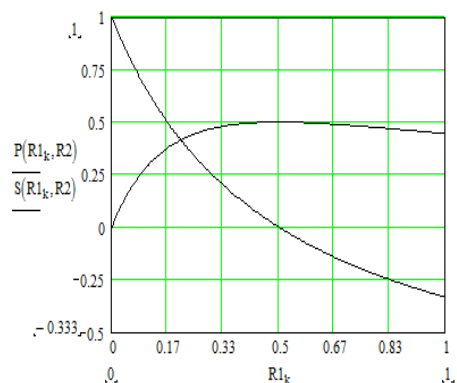


Figure 12: Power goes to the maximum value for $R_1=R_2$ and decreases. The scattering value when we change the resistor R_1 always decreases.

The power p is invariant for the transformation S and can be compared with the metrics and the metric tensor g .

$$s^2 = p = i^T g i = \sum_{r,s} g^{r,s} i_r i_s$$

$$g = Z$$

$$s^2 = p = I^T S^T Z S I = I^T G I = \sum_{r,s} G^{r,s} I_r I_s$$

$$R' = S^T Z S$$

And

$$G = S^T g S = S^T Z S$$

The identity between the geometric metric tensor G and the electrical circuit metric Z is the fundamental equation that connects conceptual metric tensor G with the impedance metric of the electronic image of the brain. The metric tensor is the conceptual part of the brain and the impedance the physical part of the brain. In conclusion we have the fundamental coherence principle between conceptual and material parts given by the following expression

$$G_{i,j} = Z_{i,j}$$

With the fundamental equation we can compute the power quadratic form in the electronic image of the brain.

Example:

Let's begin with an example. When the conceptual intention moves on a sphere given by the simple equation

$$x_1^2 + x_2^2 + x_3^2 = r^2$$

in elliptic geometry for stable situation this geometry can be obtained by the quadratic form of the power given by resistors without capacitor and inductors (Figure 13).

And we have the transformations (conceptual intention)

$$\begin{cases} x_1 = r \sin(\alpha) \cos(\beta) \\ x_2 = r \sin(\alpha) \sin(\beta) \\ x_3 = r \cos(\alpha) \end{cases}$$

Let's compute the geodesic in the space (x_1, x_2, x_3) . So we have

$$ds^2(\alpha, \beta) = \left(\frac{dx_1}{dt} \right)^2 + \left(\frac{dx_2}{dt} \right)^2 + \left(\frac{dx_3}{dt} \right)^2 = \left(\frac{dx_1}{d\alpha} \frac{d\alpha}{dt} + \frac{dx_1}{d\beta} \frac{d\beta}{dt} \right)^2 + \left(\frac{dx_2}{d\alpha} \frac{d\alpha}{dt} + \frac{dx_2}{d\beta} \frac{d\beta}{dt} \right)^2 + \left(\frac{dx_3}{d\alpha} \frac{d\alpha}{dt} + \frac{dx_3}{d\beta} \frac{d\beta}{dt} \right)^2 = r^2 \left(\frac{d\alpha}{dt} \right)^2 + r^2 \sin^2 \alpha \left(\frac{d\beta}{dt} \right)^2$$

for the fundamental equation $G_{ij} = Z_{ij}$ we have

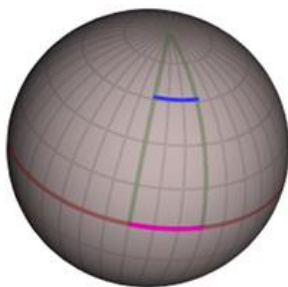


Figure 13: Sphere where the green, red and blue lines are geodesic.

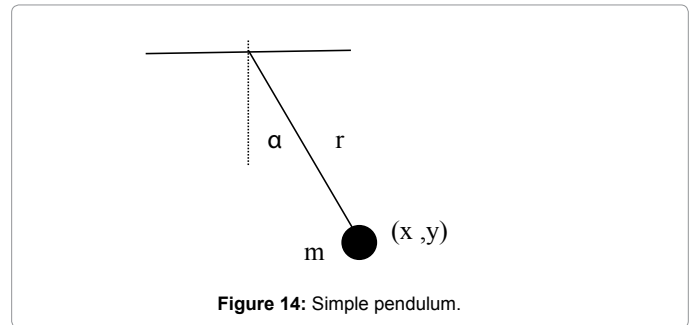


Figure 14: Simple pendulum.

$$z = \begin{bmatrix} r^2 & 0 \\ 0 & r^2 \sin^2(\alpha) \end{bmatrix}$$

When the new variables $\alpha = q_1, \beta = q_2$ the currents are

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \frac{d\alpha}{dt} \\ \frac{d\beta}{dt} \end{bmatrix}, \text{ power } r^2 \dot{i}_1^2 + r^2 \sin^2(q_1) \dot{i}_2^2$$

We remark that the resistors are

$$R_1 = r^2, R_2 = r^2 \sin^2(\alpha) = r^2 \sin^2(q_1)$$

And R_2 is a function that we can realize by a memristor. For capacitor and inductor the currents can have imaginary values. In this case it is possible to represent hyperbolic geometry of waves as in the space time geometry

$$s^2 = x^2 + y^2 + z^2 - c^2 t^2 = \begin{bmatrix} x \\ y \\ z \\ ict \end{bmatrix}^T \begin{bmatrix} x \\ y \\ z \\ ict \end{bmatrix} = s^2 = x^2 + y^2 + z^2 + (ict)^2,$$

Where

$$i^2 = -1$$

with complex value for the currents, we can have the virtual power p

$$p = I_1^2 + I_2^2 + I_3^2 + (iI_4)^2 = I_1^2 + I_2^2 + I_3^2 - I_4^2$$

as metric that represent the hyperbolic geometry of the waves.

Mechanical Geometry as Conceptual Intention in Neural Network

To understand the meaning of geometry in the neural dynamical process, we study the mechanical dynamic transformation of the two dimensional space reference from ordinary Cartesian coordinates (x, y) into polar coordinates (r, α) by pendulum system [9,15].

Simple pendulum and change of variables

Given the simple pendulum in (Figure 14) the pendulum coordinates are

$$x = r \cos(\alpha)$$

$$y = r \sin(\alpha)$$

With the derivative properties we have

$$\frac{dx}{dt} = \frac{dx}{dr} \frac{dr}{dt} + \frac{dx}{d\alpha} \frac{d\alpha}{dt} = \begin{bmatrix} \frac{dx}{dr} & \frac{dx}{d\alpha} \end{bmatrix} \begin{bmatrix} \frac{dr}{dt} \\ \frac{d\alpha}{dt} \end{bmatrix}$$

$$\frac{dy}{dt} = \frac{dy}{dr} \frac{dr}{dt} + \frac{dy}{d\alpha} \frac{d\alpha}{dt} = \begin{bmatrix} \frac{dy}{dr} & \frac{dy}{d\alpha} \end{bmatrix} \begin{bmatrix} \frac{dr}{dt} \\ \frac{d\alpha}{dt} \end{bmatrix}$$

and

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} = \begin{bmatrix} \frac{dx}{dr} & \frac{dx}{d\alpha} \\ \frac{dy}{dr} & \frac{dy}{d\alpha} \end{bmatrix} \begin{bmatrix} \frac{dr}{dt} \\ \frac{d\alpha}{dt} \end{bmatrix} \text{ and } V = JW,$$

In a graphic way we have (Figure 15)

$$\text{Where the Jacobian } J = \begin{bmatrix} \cos(\alpha) & -r \sin(\alpha) \\ \sin(\alpha) & r \cos(\alpha) \end{bmatrix}$$

is the connection matrix

$$\text{and } W = \begin{bmatrix} \frac{dr}{dt} \\ \frac{d\alpha}{dt} \end{bmatrix} = \begin{bmatrix} v_r \\ v_\alpha \end{bmatrix}$$

also the reverse connection matrix

$$\begin{bmatrix} \frac{dr}{dt} \\ \frac{d\alpha}{dt} \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & -r \sin(\alpha) \\ \sin(\alpha) & r \cos(\alpha) \end{bmatrix}^{-1} \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) \\ -\frac{1}{r} \sin(\alpha) & \frac{1}{r} \cos(\alpha) \end{bmatrix} \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix}$$

Now we compute the expression of the same intensity of velocity in the polar reference and in the Cartesian reference (Figure 16).

$$\left(\frac{ds}{dt}\right)^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix}^T \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} = V^T V = \left(\begin{bmatrix} \frac{dx}{dr} & \frac{dx}{d\alpha} \\ \frac{dy}{dr} & \frac{dy}{d\alpha} \end{bmatrix} \right)^T \left(\begin{bmatrix} \frac{dx}{dr} & \frac{dx}{d\alpha} \\ \frac{dy}{dr} & \frac{dy}{d\alpha} \end{bmatrix} \begin{bmatrix} \frac{dr}{dt} \\ \frac{d\alpha}{dt} \end{bmatrix} \right)$$

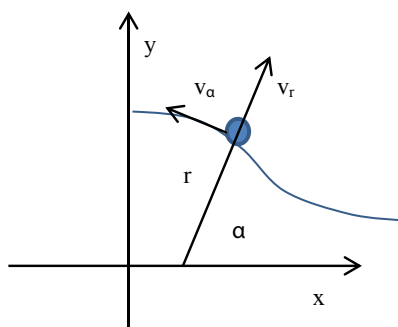


Figure 15: Velocity in polar coordinates.

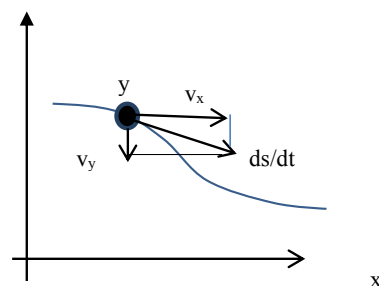


Figure 16: Velocity ds/dt in the Cartesian coordinates.

$$= \begin{bmatrix} \frac{dr}{dt} \\ \frac{d\alpha}{dt} \end{bmatrix}^T \begin{bmatrix} \frac{dx}{dr} & \frac{dx}{d\alpha} \\ \frac{dy}{dr} & \frac{dy}{d\alpha} \end{bmatrix}^T \begin{bmatrix} \frac{dx}{dr} & \frac{dx}{d\alpha} \\ \frac{dy}{dr} & \frac{dy}{d\alpha} \end{bmatrix} \begin{bmatrix} \frac{dr}{dt} \\ \frac{d\alpha}{dt} \end{bmatrix} = W^T J^T J W$$

$$= \left(\frac{dr}{dt}\right)^2 + r^2 \left(\frac{d\alpha}{dt}\right)^2 = V^T V = \alpha^2 + \beta^2$$

Kinetic energy T of the pendulum with r = constant

$$T = \frac{1}{2} m \left(r \frac{d\alpha}{dt} \right)^2 = \frac{1}{2} m r^2 \left(\frac{d\alpha}{dt} \right)^2$$

$$= \frac{1}{2} M(r) \left(\frac{d\alpha}{dt} \right)^2, \text{ where, } M = m r^2$$

is the inertial moment for a point with mass m. We remember that the inertial momentum M in the polar coordinates substitutes the mass m in the Cartesian reference. The velocity in polar coordinates is

$$v = r \frac{d\alpha}{dt}$$

In this example we show that the kinetic energy expression, as we know, changes with the change of the reference. Because the Lagrangiana is

$$L = T = \frac{1}{2} m \left(r \frac{d\alpha}{dt} \right)^2$$

and the momentum is

$$p_\alpha = \frac{\partial L}{\partial \left(\frac{d\alpha}{dt} \right)} = m r \frac{d\alpha}{dt}$$

we have

$$2T = v^\alpha p_\alpha = \left(r \frac{d\alpha}{dt} \right) m r \frac{d\alpha}{dt} = m \left(r \frac{d\alpha}{dt} \right)^2$$

Now given the general expression of the kinetic energy

$$2T = v^\alpha p_\alpha = \left(r \frac{d\alpha}{dt} \right) m r \frac{d\alpha}{dt} = m \left(r \frac{d\alpha}{dt} \right)^2$$

we show that the kinetic energy and the metric for the change of reference is the scalar product of two variables : one is the velocity and the other is the kinetic momentum. The velocity can be considered as the flux and the momentum as the force. In the pendulum natural dynamics the energy is invariant, so the pendulum movement has a geodetic as trajectory. Now for the general form of kinetic energy

$$T = \frac{1}{2} g_{i,j} v^i v^j, \text{ the momentum is,}$$

$$\frac{\partial T}{\partial v^j} = \frac{1}{2} (g_{i,j} + g_{j,i}) v^j = p_i$$

and

$$2T = v^\alpha p_\alpha = v^\alpha \frac{1}{2} (g_{\alpha,\beta} + g_{\beta,\alpha}) v^\beta$$

because

$$g_{\alpha,\beta} = g_{\beta,\alpha}, 2T = v^\alpha g_{\alpha,\beta} v^\beta = g_{\alpha,\beta} v^\alpha v^\beta$$

In this way we can give a more general interpretation of the kinetic momentum as dual variable of the velocity in classical mechanics.

Double pendulum

The double pendulum can be the model of two interaction

oscillating neurons as we show in Figures 17 and 18. To model a chain or path of pendulums (neurons) we use two main transformations: one is the set of multidimensional rotations and the other is the set of geometric translations. For simplicity we use the two dimension image. Rotation

$$\begin{bmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x\cos(\alpha) + y\sin(\alpha) \\ -x\sin(\alpha) + y\cos(\alpha) \\ 1 \end{bmatrix}$$

The simple pendulum is given by a simple rotation

$$\begin{bmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ L_1 \\ 1 \end{bmatrix} = \begin{bmatrix} L_1 \sin(\alpha) \\ L_1 \cos(\alpha) \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Behavior of the simple pendulum (Figure 19)

For the translation we have

$$\begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x+a \\ y+b \\ 1 \end{bmatrix}$$

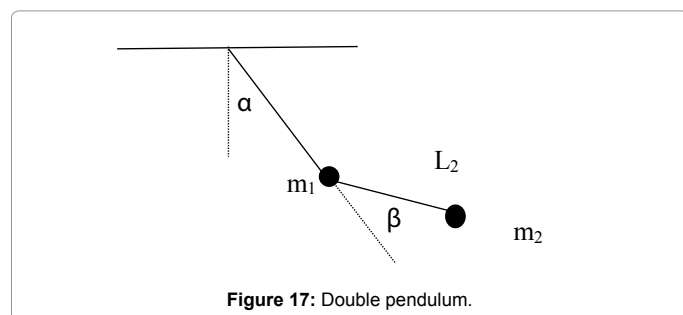


Figure 17: Double pendulum.

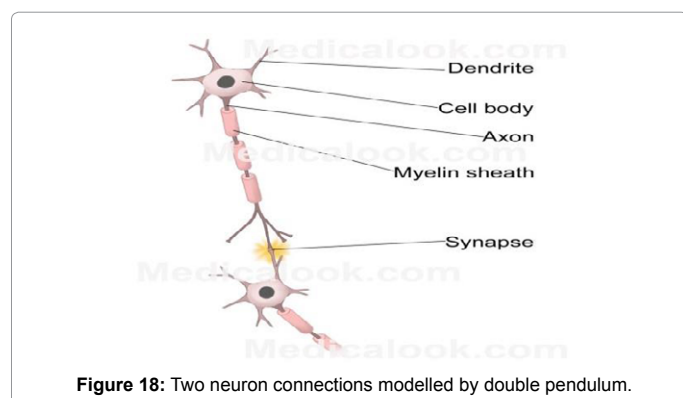


Figure 18: Two neuron connections modelled by double pendulum.

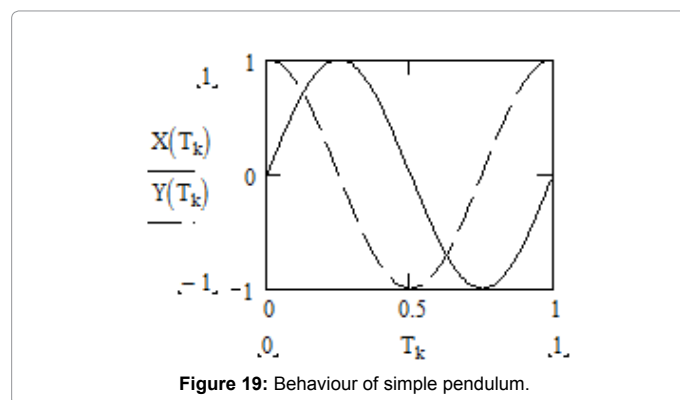


Figure 19: Behaviour of simple pendulum.

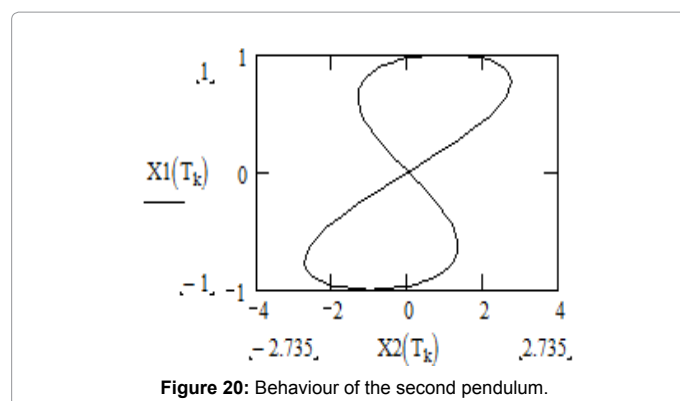


Figure 20: Behaviour of the second pendulum.

The translation is the transfer operator that moves from one pendulum to another (translation in the neurons is the connection from one neuron to another). With the two transformations we obtain the double pendulum coordinates of the masses m_1 and m_2 .

$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ L_1 \\ 1 \end{bmatrix} = \begin{bmatrix} L_1 \sin(\alpha) \\ L_1 \cos(\alpha) \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\beta) & \sin(\beta) & 0 \\ -\sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ L_1 \\ 1 \end{bmatrix} = \begin{bmatrix} L_1 \sin(\alpha) + L_2 \sin(\alpha + \beta) \\ L_1 \cos(\alpha) + L_2 \cos(\alpha + \beta) \\ 1 \end{bmatrix}$$

The behavior of the second pendulum (neuron) is the interference of the first pendulum (neuron) plus the conditioned value of the second pendulum that is equal to the first pendulum plus a new phase value β and with intensity L_2 . The second oscillator (neuron) behavior is shown in Figure 20.

The correlation of the first pendulum with the second or entanglement is shown in Figures 21 and 22.

For y we have

The kinetic energy (metric) of the double pendulum is

$$T = \frac{1}{2} \left(\frac{dA}{dt} \right)^T M \left(\frac{dA}{dt} \right) = \frac{1}{2} \left(J \frac{dB}{dt} \right)^T M \left(J \frac{dB}{dt} \right) - \left(\frac{dB}{dt} \right)^T (J^T M J) \left(\frac{dB}{dt} \right)$$

Where J is the Jacobian of the relation between A and B

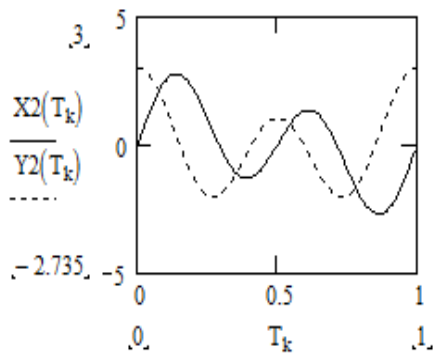


Figure 21: Correlation between the variables x1 and x2.

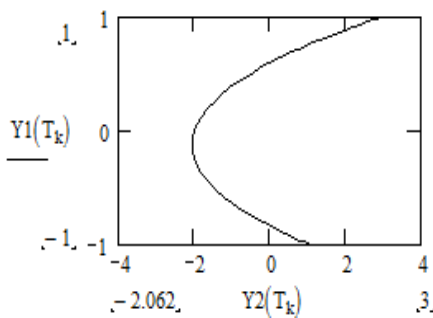


Figure 22: Correlation between the variables y1 and y2.

The kinetic energy as metric can be compared with electronic system in this way

$$T = power = \frac{1}{2} i^T (S^T Z S) i$$

$$i = \frac{dB}{dt}, S = J, Z = M$$

and

$$\frac{dA_i}{dt} = J_{i,j} \frac{dB_j}{dt}, J_{i,j} = \begin{bmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{bmatrix}, A = \begin{bmatrix} L_1 \\ L_2 \\ \alpha \\ \beta \end{bmatrix}$$

and

$$M = \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & m_2 \end{bmatrix}$$

We remark that the new mass matrix $J^T M J$ for the double pendulum is not diagonal but has cross elements that correlate variables. So the form of the kinetic energy is

$$T = \frac{1}{2} m_1 \left(\frac{dx_1}{dt} \right)^2 + m_1 \left(\frac{dy_1}{dt} \right)^2 + m_2 \left(\frac{dx_2}{dt} \right)^2 + m_2 \left(\frac{dy_2}{dt} \right)^2$$

and

$$T = \frac{1}{2} g_{1,1} \left(\frac{d\alpha}{dt} \right)^2 + g_{2,2} \left(\frac{d\beta}{dt} \right)^2 + 2g_{1,2} \frac{d\alpha}{dt} \frac{d\beta}{dt}$$

where

$$g_{1,1} = m_1 L_1^2$$

$$g_{2,2} = 2m_2 L_1 \cos(\beta) + (m_1 + m_2) L_1^2 + L_2^2 m_2$$

$$g_{1,2} = m_2 L_1 L_2 \cos(\beta) + 2L_2^2 m_2$$

where $g_{1,2}$ is the entangled bond between the two connected pendulum in the double pendulum. When we join two oscillators in one double oscillator the metric in the geometric space of the velocity moves from flat geometry where the cross term in the metric tensor is equal to zero to space with curvature where the cross term

$$m_2 L_1 L_2 \cos(\beta) + 2L_2^2 m_2$$

is different from zero. This means that there is dependence between the two oscillators (synchronization or entanglement between two neurons). The correlate double pendulum and two neurons system dynamics can generate this chaotic situation shown in Figure 23.

Now for the previous chapter we can compute the mechanical momentum

$$p_\alpha = \frac{1}{2} (g_{\alpha,\beta} + g_{\beta,\alpha}) v^\beta$$

$$v^\alpha = J^\alpha_\gamma v^\gamma$$

where J is the Jacobian and the v^i the velocity in the Cartesian reference.

Conclusion

In this paper we present brain as an electronic system with voltages sources as sensors, currents as internal variables in the brain and electrical power as metric in non- Euclidean space of the currents. The tensor metric in the brain is given by the impedances of the neurons. With this model we give suitable control for currents, voltages. Given a wanted sensor or voltages transformation we search with the same impedances the internal possible values of the currents, at the reverse we control the sensor or voltages sources to have wanted internal currents



Figure 23: Double pendulum and chaotic behavior.

transformations. We also can change the impedances or internal brain parameters to have with the same sensor voltages the wanted internal currents or reverse with the same internal currents wanted sources or voltages transformation. At the last we fixed the power or metric value in the current space and we change the internal parameters of the brain as impedances to have wanted internal currents. We can make the same for the voltages sources or sensors. Because is very difficult to measure the impedance in the brain, we study the brain as an optical system with scattering matrix (reflection) and transfer creator matrix for electronic system as the brain. When we measure the scattering matrix or the transmit with simple calculus is possible to compute the impedances matrix. At the end we connect geometrical transformation and mathematical metric with brain structure that in this way can implement any type of wanted transformation in non- Euclidean space with curvature. Because brain control mechanical part of the body we show that with the geometric representation of the brain system we can implement in the brain any type of the complex mechanical movements. We create also examples. The aim of this paper is not to solve particular problems by the electrical image of the brain but only to give the conceptual and mathematical instruments for future applications in medicine, in robotics, in computation theory and other possible applications. Intention in this model is relate to the abstract mathematical theory of wanted transformations and mathematical geometric image as curvature that we want to implement in the brain physical system.

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