Journal of Applied & Computational Mathematics

Elkholy et al., J Appl Computat Math 2015, 4:4
DOI: 10.4172/2168-9679.1000226

Research Article | Open Access

Folding of Digraphs

Elkholy EM[1]*, Sakr IA[2] and Ahmed H[2]

[1]*Department of Mathematics, Faculty of Science, Tanta University, Tanta, Egypt*
[2]*Department of Mathematics, Faculty of Shoubra Engineering, Banha University, Banha, Egypt*

Abstract

In this paper we introduced the definition of dibipartite graphs, complete dibipartite graphs and digraph folding, and then we proved that any dibipartite graph can be folded but the complete dibiparatite graph can be folded to an arc. By using adjacency matrices we described the digraph folding.

Keywords: Digraphs; Dibipartite graphs; Complete dibipartite graphs; Folding of dibipartite graphs; Adjacency matrices

Introduction

1) A digraph D consists of a set of elements, called vertices and a list of ordered pairs of these elements, called arcs. The set of vertices is called the vertex set of D, denoted by V(D), and the list of arcs is called the arc list of D, denoted by A(D). If v and ware vertices of D, then an arc of the form vw is said to be directed from v to w [1].

2) A dibipartite graph is a digraph whose vertex set can be split into sets A and B in such a way that each arc (directed edge) of the digraph runs from a vertex in A to a vertex in B (or a vertex of B to a vertex of A). we can distinguish the vertices in A from those in B by drawing the former in black and the latter in white, so that each arc is incident from a black (or white) vertex to a white (or black) vertex (Figure 1).

3) A complete dibipartite graph is a dibipartite graph in which each black (or white) vertex is joined to each white (or black) vertex by exactly one arc. The complete dibipartite graph with r black vertices and s white vertices is denoted by K_{rs}. We call a complete dibipartite graph of the form $K_{1,s}$ star sink or star source diagraphs (Figure 2).

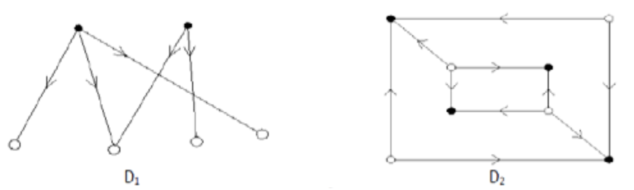

Figure 1: Arc incidence from a black (or white) vertex to a white.

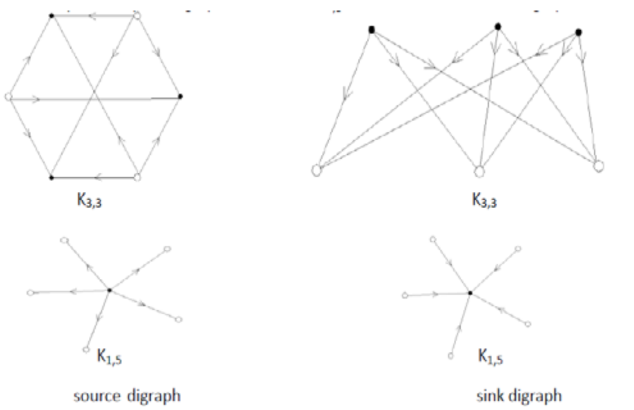

Figure 2: Star sink or star source diagraphs.

4) Let D_1 and D_2 be digraphs and $f: D \rightarrow D_2$ is a continous function. Then f is called a digraph map if,

a) For each vertex $v \in V(D_1)$, f (v) is a vertex in V (D_2).

b) For each arc $e \in A(D_1)$, dim (f (e)) ≤ dim (e).

Folding of Dibipartite Graphs

Definition

Let D_1 and D_2 be simple digraphs, we call a digraph map $f: D \rightarrow D_2$ a digraph folding if f maps vertices to vertices and arcs to arcs, i.e., for each $v \in V(D_1)$, $f(v) \in V(D_2)$ and for each $e \in A(D_1)$, $f(e) \in A(D_2)$.

If the digraph contains loops, then the digraph folding must send loops to loops but of the same direction .We denote the set of diagraph foldings between diagraphs D_1 and D_2 by D (D_1, D_2) and the set of diagraph foldings of D into itself by Ð (D). In the case of a diagraph folding f the set of singularities, $\sum f$, consists of vertices only [2]. The diagraph folding is non-trivial if $\sum f \neq \emptyset$. In this case the no.V (f (D_1)) ≤ no.V (D_1), also no.A (f (D_1)) ≤ no.A (D_1).

Example

Let D be the diagraph shown in Figure 3.Then the graph map $f: D \rightarrow D$ defined by $f(v_1,\ldots,v_4)=(v_3,v_2,v_3,v_4)$ and $f(e_1,e_2,e_3,e_4,e_5)=(e_4,e_3,e_4,e_5)$ is a diagraph folding. The image f (D) is shown in Figure 3. From now on the omitted vertices or arcs will be mapped into themselves.

Theorem

Any dibipartite graph D can be folded.

Proof:

Let D be a dibipartite graph, then the vertex set V (D) can be split into two sets A and B. Let $f: D \rightarrow D$ be a digraph map such that f maps vertices of A to vertex of A, say u, and vertices of B to a vertex of B, say v. Thus each arc e will be mapped to the arc f (e)=(u, v), where $u \in V(A)$ and $v \in V(B)$ and hence f is a digraph folding.

***Corresponding author:** Elkholy EM, Department of Mathematics, Faculty of Science, Tanta University, Tanta, Egypt, Tel: +20 40 3344352; E-mail: pro.entsarelkholy809@yahoo.com

Received April 11, 2015; **Accepted** June 13, 2015; **Published** June 18, 2015

Citation: Elkholy EM, Sakr IA, Ahmed H (2015) Folding of Digraphs. J Appl Computat Math 4: 226. doi:10.4172/2168-9679.1000226

Citation: Elkholy EM, Sakr IA, Ahmed H (2015) Folding of Digraphs. J Appl Computat Math 4: 226. doi:10.4172/2168-9679.1000226

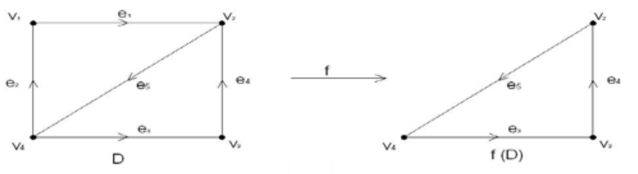

Figure 3: Diagraph.

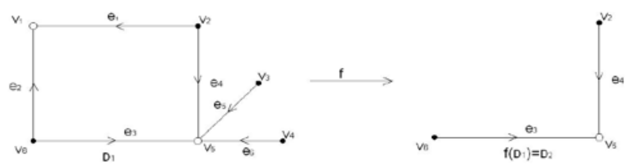

Figure 4: Dibipartite graph.

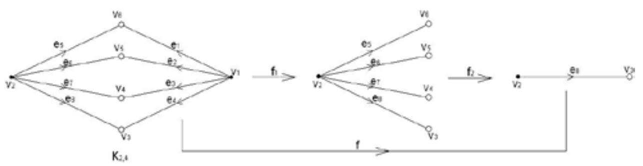

Figure 5: Digraph folding.

Example

Let D_1 be the dibipartite graph shown in Figure 4. A digraph folding $f \in Ð(D_1)$ can be defined as follows $f(v_1,v_3,v_4)=(v_5,v_2,v_2)$ and $f(e_1,e_2,e_5,e_6)=(e_4,e_3,e_4,e_4)$. The image $f(D_1)=D_2$ is shown in Figure 4.

Theorem

Any complete dibipartite graph D can be folded to an arc.

Proof:

Let D be a complete dibipartite graph with vertex set $V(D)=\{v_1,v_2,.....,v_{r_1},v_{r_1+1},....,v_r\}$ This set again can be split into two sets, $A=\{v_1,v_2,....,v_{r_1}\}$ and $B=\{v_{r_1+1},.....,v_r\}$, such that each vertex of A is joined to each vertex of B by exactly one arc.

Thus

$$A(D)=\{(v_1,v_{r_1+1}),(v_1,v_{r_1+2}),.....,(v_1,v_r),(v_2,v_{r_1+1}),(v_2,v_{r_2+1}),.....,(v_2,v_r),.....,(v_{r_1},v_{r_1+1}),(v_{r_1},v_{r_2+1}),.....,(v_{r_1},v_r)\}$$

Now let $f: D \to D$ be a diagraph map defined by

$$f(v_k)=\begin{cases} v_1, if\ k=1,.....,r \\ v_{r_1+1}, if\ k=r_1+1,......,r \end{cases}$$

Thus the image of any arc of A(D) will be the arc (v_1,v_{r_1+1}). Of course, this map is a digraph folding.

Example

Consider the complete dibipartite graph $K_{2,4}$ shown in Figure 5. A diagraph folding f of $K_{2,4}$ into itself may be defined as follows $f(v_1,v_4,v_5,v_6)=(v_2,v_3,v_3,v_3)$, $f(e_i)=e_8$, i=1,…,8. This may be done by the composition of the two digraph folding f_1 and f_2 shown in Figure 5.

The Diagraph Folding and Adjacency Matrix

Definition

Let D be a diagraph without loops, with n vertices labeled 1,2,3,…,n. The adjacency matrix M(D) is the nxn matrix in which the entry in row i and column j is the number of arcs from vertex i to vertex j [1]. For example if D is the diagraph shown in Figure 6, then the matrix M(D) will be given by

$$M(D)=\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array}\begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array}\!\!\!\!\begin{array}{c}\\ \end{array}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{pmatrix}$$

Note that every entry on the main diagonal (top- left to bottom-right) is **0**, since the digraph has no loops.

Proposition

Let D be a connected digraph without loops with n vertices. Then a digraph folding of D into itself may be defined, if there is any, as a digraph map f of D to an image f (D) by mapping:

1) The multiple arcs into one of its arc.

2) a. The vertex v_i to the vertex v_j if the numbers appearing in the adjacency matrix in the i^{th} and j^{th} rows (or columns) are the same.

b. The vertex v_i to the vertex v_j if the entries of the i^{th} and j^{th} rows are zeros and if the i^{th} and j^{th} columns are the same, or there exists a row k which has numbers 1 in the i^{th} and j^{th} columns.

3) a. The arc (v_i,v_k) to the arc (v_j,v_k) if the $i^{th}$and j^{th} rows (or columns) are the same .

b. The arc (v_i,v_j) to the arc (v_i,v_k) if the j^{th} and k^{th} columns (or rows) are the same .

In general the arc (v_i,v_j) will be mapped to the arc (v_k,v_l) if v_i maped to v_k and v_j maped to v_l .

Examples

(a) Let D_1 be the digraph shown in Figure 7.

The adjacency matrix M (D_1) is given by Figure 7.

The adjacency matrix M(D1) is given by

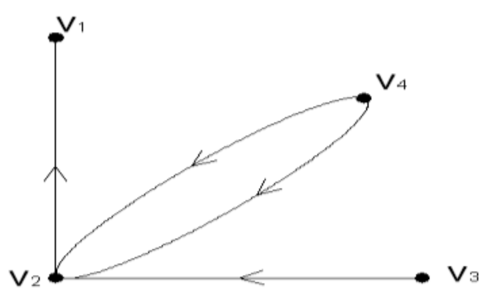

Figure 6: Diagraph without loops.

Citation: Elkholy EM, Sakr IA, Ahmed H (2015) Folding of Digraphs. J Appl Computat Math 4: 226. doi:10.4172/2168-9679.1000226

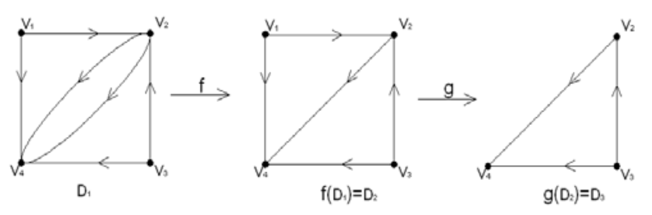

Figure 7: Digraph 7.

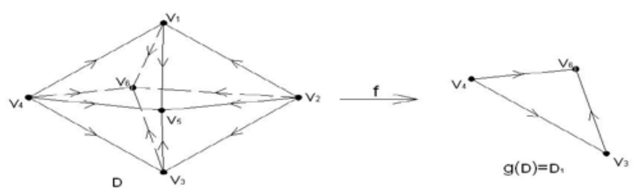

Figure 8: Folding the multiple arcs.

$$M(D_1)=\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array}\begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \left(\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right) \end{array}$$

Then we can fold first D_1 by folding the multiple arc into itself to get the digraph D_2.In this case $M(D_2)$ is nothing but $M(D_1)$ after replacing the number 2 by the number 1. Then a digraph folding $g \in Đ(D_2)$ can be defined by using $M(D_2)$ by mapping the vertex v_1 to the vertex v_3 since the first and the third row of $M(D_2)$ have the same entries. Thus the arcs (v_1,v_2) and (v_1,v_4) will be mapped to the arcs (v_3,v_2) and (v_3,v_4) respectively, since the first and the third row are the same.

(b) Let D be the digraph shown in Figure 8.

The adjacency matrix M(D) is given by

$$M(D_1)=\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array}\begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \left(\begin{array}{cccccc} 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{array}\right) \end{array}$$

Then a digraph folding $g: D \rightarrow D$ can be defined by using M (D) by mapping the vertices v_1, v_2 and v_5 to v_3, v_4 and v_6 respectively. Also the arcs (v_2,v_1) and (v_1,v_5) will be mapped to the arcs (v_4,v_3) and (v_3,v_6) respectively since $g(v_1)=v_3$, $g(v_2)= v_4$, $g(v_5)=v_6$. Also the image of the arc (v_4,v_1) is (v_4,v_3) since the first and third columns are the same. Finally the image of the arc (v_2, v_6) is (v_4, v_6) since the second and fourth rows are the same, and so on. See the adjacency matrix M (D).

References

1. Wilson RJ, Watking JJ (1990) Graphs: An introductory approach, John Wiley and Sons, Inc.
2. El-kholy E, Al-Esawy A (2005) Graph folding of some special graphs. J math and statistics 1: 66-70.