ORIGINAL RESEARCH

Submitted: 24.10.2022; Accepted: 18.11.2022; Published online: 20.11.2022

Analyzing the Financial Statements of Companies listed on the National Stock Exchange using the Benford's Law

[1]*Madhura Ranade, [2]Mahak Gandhi

[1,2]Symbiosis Institute of International Business, Symbiosis International (Deemed University), Pune, Maharashtra, India

*Corresponding author:
madhura.ranade@siib.ac.in

Abstract

Accounting numbers normally follow Benford's mathematical rule. These findings are so unpredictable that knowledge manipulators typically refuse to comply with the law. Based on that information, accounting data that are submitted fraudulently can be identified. The law provides, however, for instances where data containing errors are presented to be detected. This paper examines the Indian companies ' financial statements of the last 10 years listed on the "National stock exchange of India" and testing whether they adhere to a statistical theorem called The Benford Law. The rule of Benford notes that the numbers are not uniformly distributed in a sequence of numerical data but instead obey a certain logarithmic rule such that the numbers beginning with a smaller digit have a greater frequency to occur than those beginning with larger digits. They use the first and the second digit test of Benford law to check whether businesses are following the logarithmic distribution.

Keywords

Benford's Law; fraud; National Stock Exchange; Financial statements; First digit test; Second digit test; Chi-square Test

Imprint

Madhura Ranade, Mahak Gandhi. Analyzing the Financial Statements of Companies listed on the National Stock Exchange using the Benford's Law. Cardiometry; Issue 24; November 2022; p. 940-947; DOI: 10.18137/cardiometry.2022.24.940947; Available from: http://www.cardiometry.net/issues/no24-november-2022/analyzing-financial-statements

1. Introduction

We are all aware of the Normal Distribution and its omnipresent nature in all kinds of phenomena and observations. There are various other laws related to numbers. One law is not getting much attention but is constantly popping up everywhere- from the population of various nations to stock market volumes to companies' financial statements and the domain of physical constants. [1] Graphical representation of Benford's Law is presented in Figure 1.

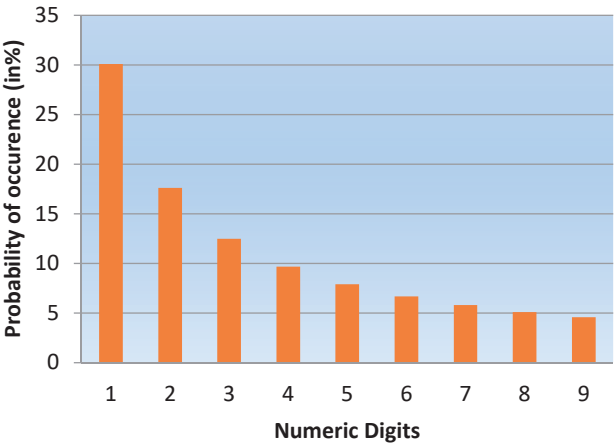

Figure 1: Graphical Representation of Benford's law

The rule of the Benford or the rule of the first digits indicates that the first digit of the numbers found in different sources does not follow a uniform distribution but is arranged in such a way that the Digit '1' occurs most frequently followed by '2', '3' and so on to '9.'

Several numerical documents have typically been found to obey Benford's Law:

a. Reflects the degree of events or phenomena, such as human populations, river water flow, or heavenly organisms;
b. have no minimum or maximum limits set beforehand;
c. may not contain the name or social security numbers, bank accounts, the telephone number used for identification purposes and
d. Have a mean less than the median, and the data are not clustered around the average.
e. We currently use the Benford Act to review Indian companies ' financial statements on the National Stock Exchange. The analysis of anomalies in the financial statements has often been used abroad. This research is original as none of the companies on the National Stock Exchange has ever checked their financial statements with this check. [2]

f. There are many hazards in the corporate world, including those where financial statements pose other forms of knowledge risks. Given income, liquidity, and capital expenditure, a financial statement presents the financial situation. From the perspective of stakeholders, these factors are very important. As a fundamental information source for executives, financial statements are the main sources of information for researchers throughout the world that research on the financial aspects of a field such as a bank financing, the payment of awards, taxation, auditors' opinions, etc., which shows how the risk is connected with any company's financial statements and how important it is, in particular for the risk of fraud in financial statements, to mitigate this risk as much as possible. The definition of financial fraud is not new. For other research, for example, for 2014, Mohamed and Handley Schachler, Kanapickienė and Grundienė, for 2015, this issue is discussed. Misreporting happens as a consequence of the misappropriation of properties. The probability that an auditor will not notice a major error is below the chance that a major fraud mistake is not found. Fraudulent financial reporting involuntarily misrepresents and/or refuses to deceive the consumers of financial statements. Financial fraud is not a widespread form of fraud, but the worst damage is done by far. Our reviews investigate how the Benford Law Review complies with the financial statements of the exchange-listed firms. The study reveals how beneficial the law is only by using this statistical knowledge of the financial statements and by taking a final review of the Benford Law Test. [3] In other countries like China, Brazil, India, the Russian Federation, Mexico, and Saudi Arabia, a similar study was conducted. Our analysis is unique, as nobody has ever tested the financial statements of a corporation listed on the National Stock Exchange through the Benford Law Review, and our research's second original contribution is results distinct from previous studies.

2. Literature Review

2.1. Benford's Law

Simon Newcomb (the American astronomer) found that "lower-digit pages were worn out more than last pages on first pages during a review of the library copies of several logarithmic tables." Based on these findings, he formulated a theory that the 10-digit incidence rate is not, in fact, identical. The numbers beginning with lower digits have a higher frequency than the numbers beginning with higher digits. Therefore, the natural figures starting from digit number 1 would occur at about 30 % of total observations. Those numbers are beginning in digit 2, at around 18% of overall observations, with the occurrence of leading numeric digit falling off to the digit 9, at around 4.6% of the overall observations, [4] which was later called "the Law of Benford." Until 1938, when Frank Benford found the trend, there was no theoretical explanation. He stated that there were twenty different types of natural numbers, describing 21,000 findings ranging from the atomic weight of compounds to American Scientific Men's street addresses.

The rule of Benford was considered very obscure for a very long time. While the statistical proof of Benford law is complex, the natural relation between growth rates and the original digit's logarithm gives us some insight into the fact that the growth factor is mathematically equal to the increase in variable logarithm. Suppose an organization's revenue is $100,000. It will rise to 100% to hit $200,000 (first digit 2) of revenue, 50% to the first digit of 3, and then 12.5%, to lift the top digit to 8. /9. It is said that "the leading figure 1 is higher than every other lead digit." [5] After many decades since its rediscovery by Frank Benford, Benford's law was usually referred to as the first digit law or the first digit phenomena (Macron and Moser, 1950; Ellentuck and Bumby 1969; Hill 1988). In some of the above-mentioned papers, the word "Benford" was not at all used. Instead, they kept on using it as the initial digit law. Hill in 1988 pointed out that "Benford's law included not only the initial digits but also the second digit (which can be 0), and many higher digits too. Also, it includes the joint distribution of digits." [6]

Various proofs show that many different factors lead to "Benford's law" and its distribution. There are also a significant number of data not consistent with this law. A binding minimum or maximum value in a dataset is one of the reasons causing non-conformances. Second, allocated numbers such as check numbers, zip codes are distributed in a single way rather than Benford. Thirdly, individual numbers such as prices or the quantity that must be taken out of the ATM do not adhere to the rule of Benford. Besides, the Benford Law review does not identify fraud involving the

omission of transactions. However, only a few studies in this field recommend that the numbers composed do not conform to the law. [7]

2.2. Detecting Fraud through Benford's law

Benford's law also assists many investigators and auditors in recognizing transactions with the possibility of fraud and error. Mark Nigrini also showed how digital analysis is an easy and cost-efficient method to detect fraud and errors. The digital research process has been combined with a software system to develop a new approach to fraud detection and prevention, in the name of Fletcher Lu, J. Boriz, D. Covvey. Using the automobile insurance data and real health statistics, they successfully detected actual fraudsters in their Adaptive fraud detection methods. [8]

Benford's law also established validity in international trade, where swindling involves huge sums of money and can also have a huge impact on the country's national budget. Identifying foreign trade fraud is an extremely important activity; statistical tools were developed for fraud detection in the customs declaration. With the help of an example, Tableau showed that a visual analytics tool could be used to do the first digit test of Benford's law to detect potential fraud in accounting data. The law's application for discovering the abnormalities in the number patterns because of the rising costs associated with business fraud was discussed. Benford's law can identify the irregularities in the number patterns that occur due to the fraudulence in the companies' financial reporting has been recognized and agreed upon by both the firms and the court. This paper also demonstrated a systematic method that can be used to apply the law in Microsoft Excel to a comparatively large dataset containing hundreds and thousands of transactions. The methodology explained can be used by many forensic accountants and auditors. [9]

2.3. Applying the Law of Benford on Companies ' Financial Statements

Numbers are used for anything and everything in the entire financial world. Most of these numbers represent some types of currency like the Indian Rupee or the US Dollar in the face of many financial reports. Numbers are used to determining the strength of companies' financial statements and assess the reliability and quality of financial reports. Amongst that one method is "Benford's law."

Carslaw (1988) studies the recurrence in the existence of second digits in the revenue numbers of around 220 listed firms in New Zealand and finds out that the digit 0 has a far greater frequency than the digit 9, which may explain the rounding up habit of the companies for earnings which lie beneath the reference points or below some cognitive threshold value. In an identical line, Thomas (1989) explained "similar differences from the planned ones in the quarterly earnings of firms operating in the US while a contrary behavior for losses (more nines and fewer zeros)." Took a much more comprehensive view and studied the "observations of income from about 22,000 firms operating in 18 countries". Compatible with the foregoing studies, they discovered an upward trend of rounding off the surpluses and also discovered a reversed pattern for the losses.

Tilden and Janes (2012) illustrated that "business accounting data comply with the law under normal financial and economic circumstances." However, in times of economic stagnation and recession, certain estimates such as net profit and the allocation of debt allowances do not correspond. Recently, frauds and scams have become a constant companion for all the business-related activity, which can happen on a scale ranging from small to large. Management has the prime responsibility of discovering fraud and preventing it. For this purpose, external and internal auditors are helpful. Also, the forensic accountant comes in handy and examined the reliability of companies' financial statements, which are listed on the "Ljubljana Stock Exchange." They conducted the first- and second-digit tests on the sample data. Also, they performed the Chi-square test using a hypothesis test with a 5% level of risk. The listed companies passed Benford's law test. [10]

3. Data Collection and Research Methodology

In the world of everyday business, we come across large amounts of facts and figures. With the emergence of new technologies, data handling and processing have become easier and less complicated. The test of Benford's law (a tool for digital analysis) has become progressively trendy. It is used in company finance as a tool to detect error and fraud and for scientific research.

Benford's law test is a single computerized process for review of data to detect irregularities, deviations, and frauds in large sets of data. Many tools can be used for executing Benford's law, like StatCrunch, Paradox,

dBase, or Excel. In this paper, we have made use of StatCrunch and Excel. Some auditors often implement Benford's law in their assignments and projects, while others are still unaware of its concept. The auditor's work is to determine the deviations in the data from the stated deviations in Benford's law test. [11]

The following formula determines the likelihood of the first digit:

$$P\{d\} = \frac{\ln\left(1+\frac{1}{d}\right)}{\ln(10)}. \qquad [1]$$

Source: (*Benford's Law: Applications for Forensic Accounting, Auditing, and Fraud Detection | Wiley*, n.d.)

d = {1, 2, 3, 4, 5, 6, 7, 8, 9}

Our numeric system has digits from 0-9. Suppose if the number is 413. The likelihood that in the first four positions, the number appears is shown below.

Table 1 above indicates that it is likely that number 1 is in the first place, at 30.103%, and the number is in the second position, at 11.389%. Furthermore, the numerical digit 1 is expected to take third place in several 10.139%. There exists a 17.609% probability that the numeric digit 2 is the first of a number and a 10.882% probability that a number is second. In comparison, the number 2 will be in the third position in 10.097% times. If the distribution of the data set is different from Benford's law, there may be any reason to assume. Most people claim that the difference can be triggered by human interference in the organization's financial statements, leading to further studies and inquiries. [12]

Table 1
Predicted Percentage Distribution for Each of the Figures 0-9 in Each of the Numbers in the First Four Positions

Count		in %		
	1st place	2nd place	3rd place	4th place
0	-	11.968	10.178	10.018
1	30.103	11.389	10.139	10.014
2	17.609	10.882	10.097	10.010
3	12.494	10.433	10.057	10.006
4	9.691	10.031	10.018	10.002
5	7.918	9.668	9.979	9.998
6	6.695	9.337	9.940	9.994
7	5.799	9.035	9.902	9.990
8	5.115	8.757	9.864	9.986
9	4.576	8.500	9.827	9.982
Together	100	100	100	100

Source: ((PDF) Benford's Law and Its Application in Auditing, n.d.)

However, Benford law is not a lie-detector application, so we cannot conclude that any time the knowledge that does not adhere to the law is false. The test is not a perfect instrument for detecting fraud. All that leads to further analysis is this. Nigrini classifies Benford's test into three groups; the basic test or the primary test, the advanced tests & other similar measures.

Primary tests, being of the utmost significance, are of three types. In our research, we have used the following two types of tests:

3.1. First Digit

The first digit check compares the real distribution of the initial digit frequency of a data set with Benford. It is an incredibly high-level test and only reveals apparent abnormalities, i.e., it just points you in the right direction. It is a very high-level test and is usually applied to huge sets of data. [13] The explanation of the first digit test can also be done graphically, as given in Table 2.

As we can see in the above Figure 2 graph, the data set A is not in line with Benford Law. However, we also

Table 2
Benford's Distribution And Some Other Distribution of Data Set A Consisting of Around 8000 Invoices of Different Vendors.

First Digit	Benford's distribution (In %)	Data Set A (In %)	Deviation from Benford's distribution (approx.)
1	30.103	23.00	0.07
2	17.609	19.00	0.01
3	12.494	25.00	-0.13
4	9.691	12.00	-0.2
5	7.918	4.00	0.03
6	6.695	7.00	0.00
7	5.799	6.00	-0.007
8	5.115	2.00	0.031
9	4.576	2.00	0.026

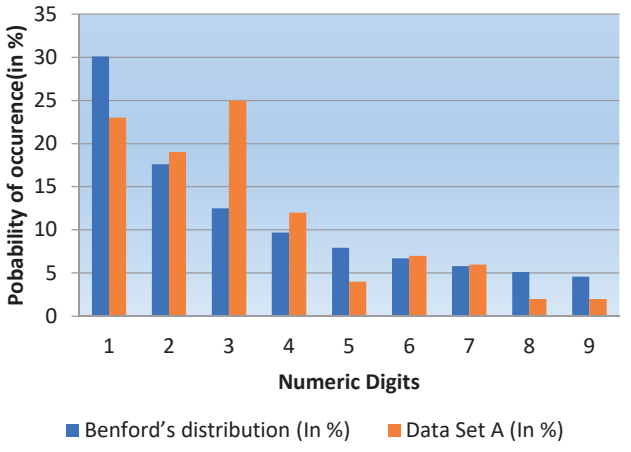

Figure 2: Graphical Representation of the Above Example

notice that the greatest difference is when the initial digit is 3 (25% in data instead of 12.4% in Benford's distribution).

3.2. Second Digit

The second digit check compares Benford's real second digit frequency distribution for a data set, which is also used to eliminate prejudices in collecting data. It is also a measure of reasonability at a high level. It shows whether or not the data is accurate. The second digit distribution of Benford's law is less skewed than the distribution of the first digits. This test cannot be used to pick out audit samples. Also, this test assists the professionals in forensic accounting to scrutinize the data.

The Advanced test can or cannot implement without the primary test. These are usually of two types; a test of second-order and a test of the total sum. Both of these tests are usually applied to discern irregularities. Lastly are the related tests, which are not the true benchmark for Benford's law test. They are mostly correlated to patterns of numbers.

The business accounts are openly accessible and are frequently subject to scrutiny to furnish an impartial and independent opinion in conformity with all the rules and regulations. From the National Stock exchange website and moneycontrol.com, we acquired a list of companies. We imported their financials, for the last ten years, to excel. These companies were selected because they depicted a medium-high-level performance in May and June (when the respective data was analyzed).[14] The imported data was separated into three heads- The Balance Sheet, The Profit & Loss Statement, and The Cash Flow Statement. The companies under scrutiny were:

1. Wipro
2. Vodafone Idea
3. Tata Motors
4. Eicher Motors
5. Alok Industries
6. Infosys
7. ITC
8. Reliance
9. Tata Steel
10. Nestle
11. Larsen and Toubro
12. Maruti Suzuki
13. Bharti Airtel
14. Britannia Industries

In 2013, Goulding said that "three statistical approaches can be used to check that a collection of data complies with Benford legislation. The methods are 1) Chi-square test 2) statistics and 3) absolute variance". A chi-square (χ^2) statistic is a method used to calculate how data expectancy correlates with the real data observed. The data used to compute a statistical chi-square must be randomly exclusive, derive from independent variables, and taken from an exceptionally large sample. In Benford's Law, chi-square statistic measures how well the empirical results match expectations. We assume two hypotheses, in which the observed data distribution is compared and analyzed with Benford's theoretical distribution. We also find out the risk and characteristic level. Critical values are listed from the Chi-squared table of distribution. All of this is carried out in Microsoft Excel using functions. The chi-square statistic uses the following formula for the chi-square test:

$$\chi_o^2 = \sum \frac{(O_i - E_i)^2}{E_i}. \qquad [2]$$

To check that the actual data was in line with the Benford rule, we conducted the Benford rule test, the first- and second-digit tests, and measured the Chi-square.

4. Data Analysis

The financials of the different companies which are selected in the sample hold 9634 data. The data set is huge, which sanctions the greater credibility of the test of "Benford's law.» [15] Table 3 and Table 4 shows the following is the first digit and the second digit distribution for the companies selected:

The difference betwixt the actual and expected occurrence of the single digits is performed later. Below are the results for the test of the first digit and the test of the second digit.

Figure 3 and Table 5 shows that while performing this test for the data set under study, the most substantial deflection of the dataset from Benford was digit 1, which was 2.015% less than the expected frequency. After that, digit 4 had the most deflection. It initiates at 0.897% more numbers. The minimum deflection occurs when the digit 3 appears in the first place. There were no numbers in the first place, which deflected from the theoretical distribution by more than 3%, which shows a positive result. [16]

Table 3
First Digit Distribution of Companies Selected

Digit/ Company	1	2	3	4	5	6	7	8	9
Alok industries	206	109	76	49	45	41	33	39	45
Eicher Motors	145	111	60	72	50	59	46	33	32
Infosys	182	110	90	64	56	58	34	20	27
ITC	187	80	92	124	94	48	51	40	33
Nestle	213	111	93	64	50	37	57	46	62
L & T	181	112	87	86	69	61	60	36	27
Maruti Suzuki	191	116	85	60	39	34	47	41	22
Britannia	211	124	84	59	46	44	53	43	25
Reliance	176	172	105	59	61	40	31	27	24
Tata Motors	231	138	80	70	46	55	35	34	45
Tata Steel	234	108	79	85	56	50	33	41	41
Vodafone Idea	189	112	109	49	33	31	26	45	41
Wipro	153	124	91	102	82	40	41	45	30
Bharti Airtel	207	105	76	77	56	70	33	58	36
Total	2706	1632	1207	1020	783	668	580	548	490

Table 4
Second Digit Distribution of Companies Selected

Digit/ Company	0	1	2	3	4	5	6	7	8	9
Alok industries	67	69	75	91	56	64	50	71	61	43
Eicher Motors	50	63	68	57	37	49	42	44	41	52
Infosys	70	87	77	65	64	48	80	41	50	55
ITC	92	98	93	88	87	74	79	74	67	72
Nestle	111	90	83	80	74	64	89	55	59	47
L & T	83	53	77	81	97	75	56	61	71	61
Maruti Suzuki	73	84	76	70	83	63	68	48	45	57
Britannia	78	87	86	69	72	55	65	59	70	60
Reliance	91	78	62	59	58	57	62	76	55	90
Tata Motors	77	88	80	93	69	79	59	68	75	46
Tata Steel	96	90	62	82	67	89	60	86	44	47
Vodafone Idea	82	68	77	60	58	62	52	57	51	54
Wipro	72	86	83	54	63	81	61	74	58	78
Bharti Airtel	70	76	73	63	59	77	69	68	80	76
Total	1112	1117	1072	1012	944	937	892	882	827	838

Table 5
Difference between the Actual and Expected Occurrence of the Initial Digit

Count	Sample Frequency	Theoretical/Expected Frequency- (in %) Benford's law	Obtained/Actual Frequency (in %)	Deviation of Obtained Frequency from Theoretical Frequency
1	2706	30.103	28.088	-2.015
2	1632	17.609	16.940	-0.669
3	1207	12.494	12.529	0.035
4	1020	9.691	10.588	0.897
5	783	7.918	8.127	0.209
6	668	6.695	6.934	0.239
7	580	5.799	6.020	0.221
8	548	5.115	5.688	0.573
9	490	4.576	5.086	0.510
Together	9634	100	100	

Figure 4 and Table 6 shows that while performing this test for the data set under study, the most substantial deflection of the data from Benford was the digit 0, which was 0.424% less than expected. After that, digit 2 had the most deflection. It initiates at 0.246% more numbers. The minimum deflection occurs when the digit 3 appears in the second place. There were numbers in the first place, which deflected from the theoretical distribution by more than 1%, showing a positive result. [17]

We put to the test the following hypotheses:

H1: "The financials of the companies listed on the *National Stock Exchange of India* pass the test of first digits of the Benford's law test.»

H2: "The financials of the companies listed on the *National Stock Exchange of India* pass the test of second digits of the Benford's law test.»

The null presumption (H0) has been set, which claimed that the whole test data is true of the Benford law in the first and second digits (there is no statistical difference between the theoretical and empirical results).

Table 7 shows p (risk level calculated) > α, which means we will embrace a null hypothesis that states the entire test data are valid in the Benford Law's first and second-digit studies. It means that Benford's first- and second-digit tests have been passed by the data

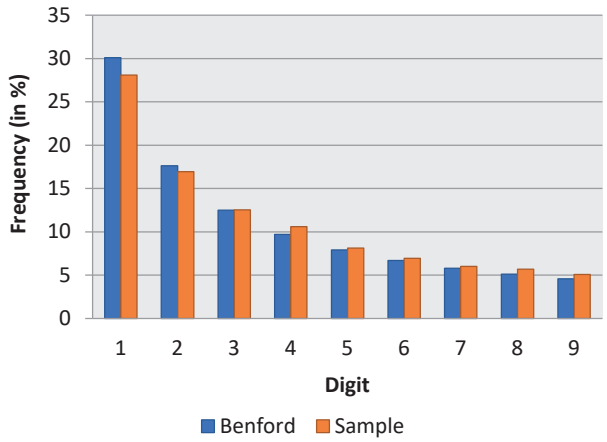

Figure 3: Diagrammatic Exhibit for the First Digit Test

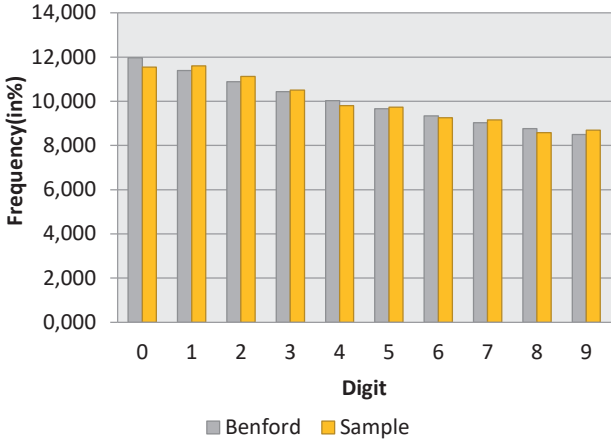

Figure 4: Diagrammatic Exhibit for the Second Digit Test

Table 6

Difference between the Actual and Expected Occurrence of the Initial Digit

Count	Sample Frequency	Theoretical/Expected Frequency- (in %) Benford's law	Obtained /Actual Frequency (in %)	Deviation of Obtained Frequency from Theoretical Frequency
0	1112	11.968	11.544	-0.424
1	1117	11.389	11.596	0.207
2	1072	10.882	11.128	0.246
3	1012	10.433	10.506	0.073
4	944	10.031	9.800	-0.231
5	937	9.668	9.727	0.059
6	892	9.337	9.260	-0.077
7	882	9.035	9.156	0.121
8	827	8.757	8.585	-0.172
9	838	8.500	8.699	0.199
Together	9633	100	100	

Table 7

Results of the Chi-Square Test

	First digit test(H1)	Second digit test(H2)
Rate of characteristic (α)	0.05	0.05
Theoretical χ^2	24.996	27.587
degree of freedom	16	18
Calculated χ^2	0.0000121157	0.915460133
Calculated risk level	1	1
Result	Calculated < Theoretical and p > 0.05 - > we accept &approve H0	

under review (previous 10 years of data for the companies listed on the National Stock Exchange).

5. Conclusion

Through Benford's law, we examined the reliability of the financials of 14 companies for the last 10 years. The sample in our analysis contained more than 1000 entries, which supports the credibility of our findings. We found out that the financials of the companies listed on NSE conform to the first digit test and the second digit test of Benford's law. Also, through the Chi-square test, this was proven that that p (the calculated risk level) > α, which means that the entire data of the test is true to the first- and second-digit tests of Benford's Law. A few deviations in the data have also been revealed, requiring research and analysis in detail.

I recommend introducing additional testing of the law by Benford in Excel and all testing in other functional programs to study this subject further. We also propose choosing a larger sample and also take up the study for a longer period. Every fraud or scam traced through analysis of the financials of a company contributes to genuine accounting reporting. So, I suggest anyone interested in managing the perils associated with honesty in financials use this law.

References

7. Burns, B. D. (2009). Sensitivity to statistical regularities: People (largely) follow Benford's law. Undefined.

8. Cerioli, A., Barabesi, L., Cerasa, A., Menegatti, M., & Perrotta, D. (2019). Newcomb–Benford law and the detection of frauds in international trade. Proceedings of the National Academy of Sciences of the United States of America. https://doi.org/10.1073/pnas.1806617115

9. Gauvrit, N., Houillon, J. C., & Delahaye, J. P. (2017). Generalized Benford's law as a lie detector. Advances in Cognitive Psychology, 13(2), 121–127. https://doi.org/10.5709/acp-0212-x

10. Goh, C. (2020). Applying visual analytics to fraud detection using Benford's law. Journal of Corporate Accounting & Finance. https://doi.org/10.1002/jcaf.22440

11. I have Got Your Number. (n.d.). Retrieved August 15, 2020, from https://www.journalofaccountancy.com/issues/1999/may/nigrini.html

12. Kinnunen, J., & Koskela, M. (2003). Who Is Miss World in Cosmetic Earnings Management? A Cross-National Comparison of Small Upward Rounding of Net Income Numbers among Eighteen Countries. Journal of International Accounting Research, 2(1), 39–68. https://doi.org/10.2308/jiar.2003.2.1.39

13. Kuruppu, N. (2019). The Application of Benford's Law in Fraud Detection: A Systematic Methodology. International Business Research, 12(10), 1. https://doi.org/10.5539/ibr.v12n10p1

14. Lu, F., Boritz, J. E., & Covvey, D. (2006). Adaptive fraud detection using Benford's law. Lecture Notes in Computer Science (Including Subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics), 4013 LNAI, 347–358. https://doi.org/10.1007/11766247_30

15. Omerzu, N., & Kolar, I. (2018). Do the Financial Statements of Listed Companies on the Ljubljana Stock Exchange Pass the Benford's Law Test? International Business Research, 12(1), 54. https://doi.org/10.5539/ibr.v12n1p54

16. Raimi, R. A. (1976). The First Digit Problem. The American Mathematical Monthly, 83(7), 521. https://doi.org/10.2307/2319349

17. Benford, F. (1938) The Law of Anomalous Numbers. Proceedings of the American Philosophical Society, 78, 551-572.

18. Macon, N. and L. Moser (1950). On the distribution of first digits of powers. Scripta

19. Mathematica 16, 290-291

20. Bumby, R. and E. Ellentuck (1969). Finitely additive measures and the first digit problem. Fundamenta Mathematicae 65, 33-42

21. Yang, Y. H., Chiang, Y. M., Liu, H. M., & Huang, L. (2021). Negotiating or regulating executive compensation: application of Benford's law. Applied Economics Letters, 1-5.

22. Wennberg, J., & Rosencrantz Ollén, E. (2021). Assessing practicalities of Benford's Law-A study of the law's potential to detect fraud in transactional data.

23. Barabesi, L., Cerasa, A., Cerioli, A., & Perrotta, D. (2021). On characterizations and tests of Benford's law. Journal of the American Statistical Association, 1-44.

24. da Silva Azevedo, C., Gonçalves, R. F., Gava, V. L., & de Mesquita Spinola, M. (2021). A Benford's Law based methodology for fraud detection in social welfare programs: Bolsa Familia analysis. Physica A: Statistical Mechanics and its Applications, 567, 125626.