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Managerial Efficiency of Production System with Price-Sensitive Demand

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Abstract: The significance of forward and reverse supply chain management and remanufacturing operations have an important role in business and hence need additional attention from the researchers. In this paper, we have tackled the problem of how to induce pricing and collecting in a closed-loop supply chain, which consists of a manufacturer, a retailer, and a third party for a finite time horizon. We have developed a three-echelon supply chain policy consisting of a manufacturer, a retailer, and a third party to check the effects of product collection and remanufacturing mandates on the inducement and resulting profits in the closed-loop supply chains. We have shown analytically that the profit functions of the manufacturer, retailer, and third party are concave concerning their decision variables. Also, the profit function of the retailer and the third party linearly decreases due to the increasing price-sensitive parameter. Finally, we have presented a numerical example to illustrate the proposed model.

Keyword – Remanufacturing; inventory; pricing; closed-loop supply chain

AMS subject classification – 90B05, 90B30, 90B50

1. INTRODUCTION

The closed-loop supply chains (CLSC) admit traditional forward supply-chain operations and the additional operations (inspection, disassembly, component reprocessing or remanufacturing, reassembly, testing, and advertising) of the reverse supply chain. Generally, the forward chain concerns the flow of physical products from the manufacturer to the customers, while the reverse chain describes the flow of used physical products from the customers, then acting as supplier, to the remanufacturer. These flows are then "closed" by, for example, the remanufacturing operation. A closed-loop supply chain comprises the returns processes, and the manufacturer has the aim of capturing additional value and further combining all supply chain actions (Östlin *et al.*, 2008; Nuss *et al.*, 2014; Wu and Kao, 2018).

These additional operations include:

- product acquisition to obtain the products from the end users,
- reverse logistics to move the products from the points of use to a point(s) of disposition,
- testing, sorting, and disposition to determine the products condition and the most economically attractive reuse option,
- refurbishing to enable the most economically attractive of the options: direct reuse,repair, remanufacture, recycle, or disposal, and
- remarketing to create and exploit markets for refurbished goods and distribute them.

The situation is complicated in the reverse supply chain because users may return products during the product lifecycle (commercial returns: a result of liberal reseller policies that permit customers to return products for any reason during a 30-, 60-, or 90-day period after purchase, warranties, repairs), at the end of use, and end of life. Compared to manufacturing, remanufacturing has some general characteristics that complicate the supply chain.

One of the complicating issues is that the quality of the used products is usually not known (Guide, 2000, Guide and Jayaraman, 2000; Geyer and Jackson, 2004).

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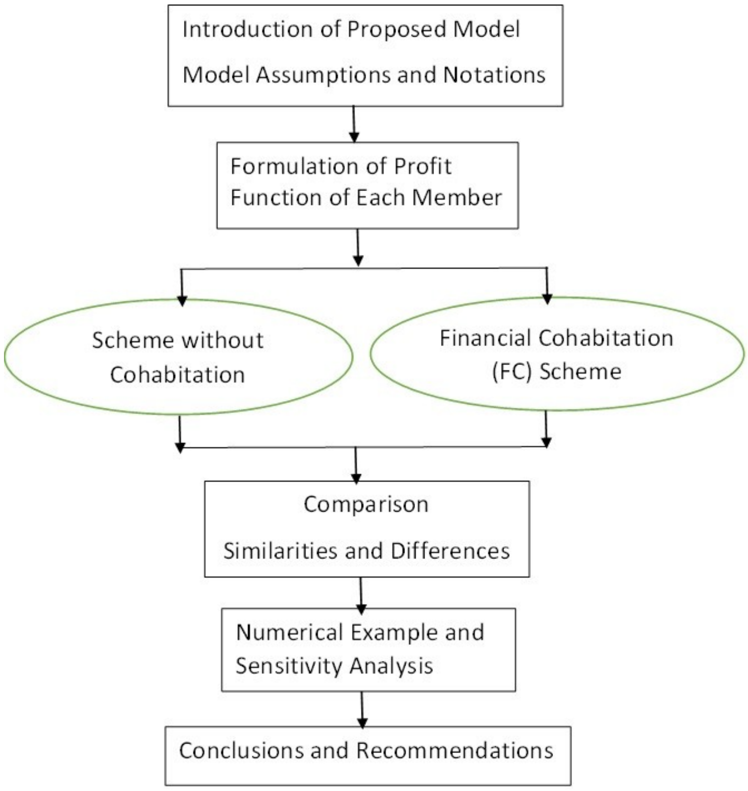

Figure 1: Research Flow Chart

To illustrate the importance of a close relationship, Seitz and Peattie (2004) give insight from a vehicle manufacturer: For vehicle manufacturers, a crucial issue is to maintain a relationship with customers so that when an engine fails, the customer returns to the retail network for a replacement. If the customer goes elsewhere, the loop will not be closed, and the manufacturer will not get access to the cores they need. Unfortunately, loyalty to OEM (Original Equipment Manufacturer) service schemes decreases noticeably over time. Zerang *et al.* (2018) observed that the manufacturer-Stackelberg case is the most effective in CLSC. He *et al.* (2018) analyzed the recovery efficiency, as well as the customer behavior, under channel inconvenience in a closed-loop supply chain. Zhau *et al.* (2020) developed a game-theoreticalmodel to understand the implications of consumer education upon a CLSC consisting of one manufacturer and one supplier. Shekarian (2020) investigated and reviewed the existing literature based on the closed-loop supply chain models that are structured based on the Game theory. Jian *et al.* (2021) established a green effect closed-loop supply chain with profit-sharing contracts and fairness concerns. There are many reasons for returning used products. In theory, there are four basic types of returns:

(1) End-of-Life Returns. These are returns that are taken back from the market to avoid environmental or commercial damage. These used products are often returned as a result of takeback laws.
(2) End-of-Use Returns. These are used products or components that have been returned after customer use. These used products are normally traded after the market or are remanufactured.
(3) Commercial Returns. These returns are linked to the sales process. Other reasons for the returns include problems with products under warranty, damage during transport, or product recalls.
(4) Re-Usable Components. These returns are related to the consumption, use, or distribution of the main product. The common characteristic is that they are not part of the product itself but contain the actual product; an example of this kind of return is remanufactured toner cartridges (Krikke *et al.*, 2004).

Table 1: Contribution of different authors

Authors	Demand pattern	Used product collection by the third party	Remanufacturing	CLSC	Time horizon
Wu and Kao 2017	price and level dependent	No	Yes	Yes	Finite
Zerang *et al.* 2018	selling price and marketing efforts dependent	Yes	Yes	Yes	Infinite
He *et al.* 2018	price dependent	Yes	Yes	Yes	Infinite
Feng *et al.* 2021	quality, quantity and willingness dependent	No	Yes	Yes	Infinite
Feng *et al.* 2022	retail price and willingness dependent	No	Yes	Yes	Infinite
This paper	price dependent	Yes	Yes	Yes	Finite

The issue of forecasting used product returns has proven to be a difficult challenge for the remanufacturing industry. The return of mainly mechanical products is dependent on factors such as age and use of the product, whereas electrical products tend to have a more random pattern of failure.

Remanufacturing is an industrial process where used products are restored (remanufactured) to useful life. Remanufacturing is a production strategy whose objective is to recuperate the residual value of used products. Used products can be remanufactured at a lower cost than the production cost. Product remanufacturing plays an important role in the environment and economic benefits. Currently, more and more countries are involved in product remanufacturing activity. The contribution of different authors is presented in Table 1. There is substantial research exploring the issues concerning pricing, remanufacturing, and management of supply chains (Sarkar *et al.,* 2015; Tiwari *et al.,* 2018). Remanufacturing is not new. Some industries have been remanufacturing since the 1920s (for example, automotive parts have long been remanufactured by third parties). Research on remanufacturing has increased since the early 1980s (Lund, 1983), with most published research appearing since 1990 and focusing on operational or engineering issues (Guide *et al.,* 2003*b*). There are various reasons for product remanufacturing; some are environmental measures to save the earth, legislation, increased profitability, ethical responsibility,secured spare part supply, business benefits, increased market share, and brand protection(Seitz and Peattie, 2004). The military has routinely remanufactured assets for decades. However, companies now remanufacture large volumes of low-value items, for example, mobile telephones and ink-jet printers. Companies in the US must also deal with the returns of commercial products during their life cycles. Many firms look at remanufacturing as a technical operational problem: how to turn an ill-functioning returned product into a functioning product that satisfies all the quality requirements of a new product. The profit of a supply chain is mainly affected by market demand. The market demand depends on certain crucial components; those are the retail price of the product, stock of the product, quality of the product and availability of the product, and many more. This paper is based on a price-sensitive demand rate.

The operations management literature comprises a growing and substantial number of papers pertaining to closed-loop supply chains with remanufacturing used products (Atasu *et al.* 2008*b*) and managing returned products (Guide and van Wassenhove, 2001; Ferrer and Swaminathan, 2006; Singh *et al.*, 2020). Majumder and Groenevelt (2001) outline the effect of competition between original equipment manufacturers and local remanufacturers for manufacturability. Atasu *et al.* (2008*a*) outline the guidelines for remanufacturing decisions. They have suggested that the market growth rate is the main source of profitable remanufacturing. Debo *et al.* (2005) describes the model in which the manufacturer determines the manufacturability level of the new product and the production technology selection problem in a market that comprises heterogeneous consumers. Savaskan and van Wassenhove (2006) highlighted an interesting trade-off between direct versus indirect collection systems. They investigated the interactions between the decisions of the manufacturer and retailer in the forward and reverse logistics channels. Khedlekar *et al.* (2018) determine the optimal selling price, replenishment cycle, and lot size of seasonal items. The focus is on determining the effect of preservation technology on the profit of manufacturers and retailers for deteriorating items. Nigwal and Khedlekar (2018) developed a reverse supply chain that consists of re-manufacturers, retailers, and collectors to optimize their advantages in incomplete information scenarios.

Duong *et al.* (2017) designed a model for seasonal demand, and they aimed to find the different factors influencing backlogs during busy periods. They have done a case study of a vehicle recovery company. Yamada (2020) proposed a supply-chain transport model which considered the uncertainty of product demands. Recently, Feng *et al.* (2021) discussed full-remanufacturing and partial-remanufacturing using two game models. It also investigated the effect of remanufacturing subsidies on these two models. Chandra (2021) has considered two warehouse inventory models (one rented warehouse RW and another owned warehouse OW) for deteriorating items with stock-dependent demand rates. This inventory model applies to seasonable fruits and vegetables, newly launched fashion items, etc. The inventory manager offers a price discount to customers who are willing to backorder their demand. Vithyadevi and Annadurai (2021) derived a single-vendor and a single-buyer integrated inventory model in a fuzzy environment. It helps to formulate the optimal order quantity and minimum integrated total cost near real value. Katariya and Shukla (2021) assumed the rate of demand to be a nonlinear function of price and time for new productsand a linear function of price and time for buying back used products. Using this model, they formulated the optimal price and quantity for new products and the optimal buyback quantity for used products. Feng *et al.* (2022) discussed a comparative study of remanufacturing in a closed-loop supply chain under different conditions, focused on the quality of recycled products, government participation policies, and revenue-sharing contract coordination.

Zheng *et al.* (2017) explored a reverse supply chain consisting of a remanufacturer and a collector with complete and incomplete information structures to optimize their profits. Guide *et al.* (2003*a*) analyze economically a market-driven recovery system for the calculation of the optimal selling price and the optimal acquisition prices for remanufactured products. Atasu *et al.* (2009, 2012), as well as Atasu and Subramanian (2012), document the challenges of take-back legislation under individual and collective systems. They investigate the environmental and economic impacts of such legislation. Roy *et al.* (2015) designed a two-echelon supply chain comprising one manufacturer and two competing retailers withrandom arrival of the customers. They considered sales price-dependent demand. They analyzed the profit functions of the manufacturer and two retailers and compared them with the following approaches- Stackelberg, Bertrand, Cournot-Bertrand, and integration. In the supply chain management literature, papers such as Atasu and van Wassenhove (2010), Govindan *et al.* (2013), Govindan and Popiuc (2014), Khedlekar and Singh (2019), and Yao *et al.* (2021) discuss an appropriate coordination contract among the supply chainmembers.

A revenue-sharing contract for a remanufacturing supply chain (RSC) with multi- uncertainties (stochastic manufacturability rate and random demand) can increase profit for the whole RSC as well as the remanufacturer and the retailer by eliminating double marginalization (Zhao and Zhu, 2015). In the remanufacturing literature, our work investigates coordination in members of the channel structure under a finite period. To address the above subjects, we develop and analyze a model for a three-echelon supply chain consisting of one manufacturer, one retailer, and one-third party. The pattern of the proposed model is shown in Fig. 2. In the proposed model third party collects the used product for remanufacturing under a finite time horizon, and the same manufacturer evolves in remanufacturing activity. The remanufacturer provides financial help to the third party. The paper aims to get financial profits from end-of-life products using remanufacturing and its effect on the economic and environmental performance of the supply chain.

The main aim of this paper is to amend the paper of Atasu and Subramanian (2012) with an opinion on making the model more consistent and compatible in practice. We focused on the interaction among a manufacturer/remanufacturer, a retailer, and a third party within a finite time horizon to get the benefits of remanufacturing. We compared the model for two different cases. One is without providing any financial support to the collector by the manufacturer, and the second is by providing financial help to the collector. The scope of the paper is business economics of product reuse, improving financial performance, and return functions/ coordination mechanisms. In the proposed paper, we have formulated the profit functions of each member in both cases, and then we find the optimal solutions of decision variables with the optimal value of the profit function. The main contribution of this study to the extant research is an in-depth examination of the interactions among decisions of the manufacturer, retailer, and third party in the remanufacturing channel for a finite time horizon. The remainder of the paper is prepared as follows. Section 2 is devoted to the assumptions and notations used throughout the model. In section 3, starting with the closed-loop supply chain model, we present the general formulations and solutions to two models with remanufacturing. The profit functions of the individuals are demonstrated to be concave in the decision variables. Section 4 is devoted to a discussion of the results and numerical examples. In section 5, we conclude with possible directions for future research.

2. NOTATIONS AND ASSUMPTIONS

We designed the proposed model by using the following assumptions and notations:

2.1 Notations:

N	The finite time horizon,
δ	The unit cost saving from the remanufacturing $\delta = C_M - C_R$,
x	The cost cohabitation percentage in the FC scheme, which is determined by the manufacturer,
p	The retail price of the product, which is decided by the retailer $\$/unit$,
t	The transfer price paid to the collector for each unit of the used product returned $\$/cycle$,
w	The wholesale price of the product, decided by the manufacturer $\$/unit$,
r	The return rate of the used products from consumers,
C	The collecting cost coefficient of collector's firm,
C_R	The profit function of the retailer (\$),
P_{3P}	The profit function of the third party (\$),
P_M	The profit function of the manufacturer (\$),
O_R	The ordering cost of the retailer $\$/\,units\ period$,
h_R	The holding cost of the retailer $\$/\,unit\ per\ unit\ time$,
O_M	The ordering cost of the manufacturer $\$/\,unit\ per\ unit\ period$,
h_M	The holding cost of the manufacturer $\$/\,unit\ per\ unit\ time$,
C_M	The manufacturing cost of a unit new product $\$/unit$,
C_R	The remanufacturing cost of a unit used product $\$/unit$,
$I_R(n)$	The level of inventory of retailer, function of n, $0 \le n \le N$,
$D(p)$	The demand rate $D(p) = a - bp$, where $a, b > 0$ $(unit/period)$.

2.2 Assumptions:

For the development of a proposed model, we made the following assumptions:
Consistent with the existing literature (Ferrer and Swaminathan, 2006; Atasu *et al.*, 2013), we considered such types of products in which new and remanufactured products are perfectly substitutable. Assume that r is the fraction of product returned to the total product sold, $0 \le r \le 1$. We present the proposed model under the supposition that shortages are not allowed, the replenishment rate is infinite and lead time is zero. The transfer price should be less than the unit cost saving; that is, $t < \delta$. Moreover, the collection cost function of the third party is Cr^2. As a matter of fact, these assumptions are used in the literature of Atasu *et al.* (2013).

2. THE MATHEMATICAL MODEL

In this problem, we focused on the interaction among a manufacturer, a retailer and a third party, to get benefits of product remanufacturing within a finite time horizon. The manufacturer is manufacturing new products as well as remanufacturing of used products collected by the third party. The retailer is engaged in the selling of product and the third party is engaged in a collection of used products. Here, two cases are considered. First, each member of closed supply chain involved in their activities, and holding their revenue independently. Second, manufacturer provides some percentage of financial help to the third party to increase the engaged in collection activity.

2.1 Scheme without Cohabitation

In this scheme, our main analysis and discussion are for a three-echelon supply chain in which the manufacturer acts as the leader and the retailer and the third-party act as the followers. So, firstly manufacturer will decide the manufacturing cost w, then retailer and third party will decide the length of time horizon N, retail price p and return rate r of used products respectively.

For retailer: The inventory system goes like this: Q units of item are ordered by the retailer at the beginning of cycle. During the time-interval $[0, N]$, the inventory level is decreasing only owing to price-dependent demand rate of customers. The inventory level dropping to zero at $n = N$. We begin by characterizing the best-response function of the retailer. If I_R be the inventory level at time t. The differential equation governing for retailer is

$$\frac{dI_R(n)}{dn} = -(a-bp), \quad 0 \le n \le N$$

BC: $I_R(n) = Q$ at $n=0$ and: $I_R(n) = 0$ at $n=N$. On solving, we get

$$I_R(n) = (a-bp)(N-n)$$

For given w, the retailer's profit is consisting of sales revenue, purchasing cost, ordering cost and holding cost.

$$P_R(N,p) = (p-w)(a-bp)N - O_R - \frac{h_R(a-bp)N^2}{2} \tag{3.1}$$

Lemma 3.1 The retailer's profit is jointly concave in selling price p, and time horizon N, if, $2b(p-w)(a-bp) > (a-bp) > (a-bp)^2 + b^2(p-w)^2$.

Proof. The first partial derivative of retailer's profit P_R with respect to N and p respectively are

$$\frac{dP_R(N,p)}{dN} = (p-w)(a-bp) - h_R(a-bp)N$$

and

$$\frac{dP_R(N,p)}{dp} = (a+bw-2bp)N + \frac{h_R bN^2}{2}$$

By utilizing the first-order optimality condition

$$\frac{dP_R(N,p)}{dN} = 0, \frac{dP_R(N,p)}{dp} = 0$$

We obtain the retailer's optimal time horizon and retail pricing as

$$N^* = \frac{2(a-bw)}{3bh_k}, \text{ and } p^* = \frac{2a+bw}{3b} \tag{3.2}$$

The second-order condition for the retailer is

$$\frac{\partial^2 P_N(\mathrm{N},\mathrm{p})}{\partial N^2} = -h_R(\mathrm{a}-\mathrm{bp}) < 0,$$

$$\frac{\partial^2 P_N(\mathrm{N},\mathrm{p})}{\partial p^2} = -2nN < 0,$$

$$\frac{\partial^2 P_N(\mathrm{N},\mathrm{p})}{\partial p \partial N} = a + bw - 2bp + bh_R N,$$

The Hessian matrix of P_R concerning p and N is

$$H_R = \begin{pmatrix} \frac{\partial^2 P_R}{\partial p^2} & \frac{\partial^2 P_R}{\partial p \partial N} \\ \frac{\partial^2 P_R}{\partial N \partial p} & \frac{\partial^2 P_R}{\partial N^2} \end{pmatrix}$$

and the determinant of the Hessian $\det H_R = \frac{\partial^2 P_R(\mathrm{N},\mathrm{p})}{\partial N^2}\frac{\partial^2 P_R(\mathrm{N},\mathrm{p})}{\partial p^2} - \left(\frac{\partial^2 P_R(\mathrm{N},\mathrm{p})}{\partial p \partial N}\right)^2 > 0$, provided $2b(\mathrm{p}-\mathrm{w})(\mathrm{a}-\mathrm{bp}) > (\mathrm{a}-\mathrm{bp})^2 + b^2(\mathrm{p}-\mathrm{w})^2$ i.e., negative definite Hessian. Hence, the retailer's profit P_R is concave in retail price p and finite time horizon N.

For the third party: The third party is responsible to collect the used products and return to the manufacturer for remanufacturing. In this model, the third party undertakes the used product collection effort. Suppose $I_{3P}(n)$ is on-hand inventory at time n. The demand function satisfies the following differential equation in time interval $[0, N]$.

$$\frac{dI_{3P}(n)}{dn} = (a-bp), \qquad 0 \le n \le N$$

BC: $I_{3P}(\mathrm{n}) = 0$ at $n = 0$, and $I_{3P}(\mathrm{n}) = \mathrm{Q}$ at $n = N$. We can formulate the problem by using BC, we get

$$I_{3P}(n) = (a-bp)n$$

The profit function of the third party comprises sales revenue, collection cost, and holding cost

$$P_{3P}(r) = rt(a-bp)N - r^2C - \frac{h_C(a-bp)N^2}{2} \tag{3.3}$$

Our objective is to maximize $P_{3P}(\mathrm{r})$. By solving the first condition

$$\frac{dP_{3P}(r)}{dr} = t(a-bp)N - 2rC = 0,$$

we obtain $r = \frac{t(\mathrm{a-bw})^2}{9bCh_R}$, The second derivative of P_{3P} in (3.3) with respect to return rate r is $\frac{d^2P_{3P}(\mathrm{r})}{dr^2} = -2C < 0$, and thus P_{3P} is concave in r. So, the optimal return rate is

$$r^* = \frac{t(a-bw)^2}{9bCh_R} \tag{3.4}$$

Clearly, r^* is increasing in t, meaning that the higher the transfer price, the higher the return rate of used products provided by the third party.

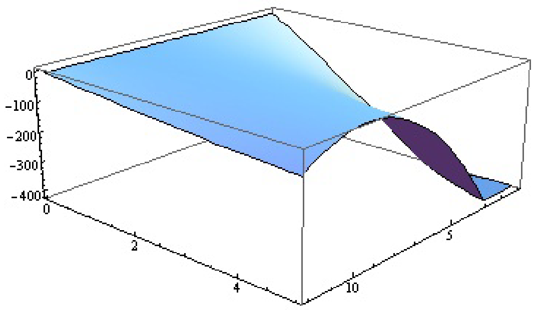

Figure 2: Concavity of retailer's profit $P_R(N, p)$ with respect to N and p

For Manufacturer: The third party collects the used products from consumers and then returns them to the manufacturer for remanufacturing. After that, the remanufactured products, along with the fresh products produced by raw materials, are delivered to the retailer for reselling. Let I_M be the inventory level of the manufacturer at a time t. Then, the described differential equation is

$$\frac{dI_M(n)}{dn} = (a-bp),\ 0 \le n \le N \tag{3.5}$$

BC: $I_M(\mathrm{n}) = 0$ at $n = 0$, and $I_M(\mathrm{n}) = \mathrm{Q}$ at $n = N$.
On solving Eq. (3.5) using BC, we get

$$I_M(n) = (a-bp)n$$

To take the product back, the manufacturer pays a transfer price t per product returned by the third party. Then, the manufacturer's profit function consists of sales revenue, manufacturing cost, ordering cost, and holding cost is

$$P_M(w) = (w + \delta r - C_M - tr)(a-bp)N - O_M - \frac{h_M(a-bp)N^2}{2}$$

Substituting the values of Eq. (3.2) and Eq. (3.4) into the manufacturer's objective function $P_M(\mathrm{w})$. We obtain

$$P_M(\mathrm{w}) = \frac{2(\mathrm{a-bw})}{9bh_R}\left[w - C_M + \frac{t(\delta-\mathrm{t})(\mathrm{a-bw})^2}{9bh_RC}\right] - O_M - \frac{2h_M(\mathrm{a-bw})^3}{27b^2h_R^2}, \tag{3.6}$$

and utilizing the first-order optimality condition, we obtain the manufacturer's optimal wholesale price as

$$w^* = \frac{8ab\delta t - 8abt^2 - 9bCh_M - 27bCh_R + 3(9b^2C^2h_M^2 - 32ab^2C\delta th_R + 32ab^2Ct^2h_R + 54b^2C^2h_Mh_R + 32b^3C\delta tC_Mh_R - 32b^3Ct^2C_Mh_R + 81b^2C^2h_R^2)^{1/2}}{8b^2\delta t - 8b^2t^2} \tag{3.7}$$

The second-order derivative of eq. (3.6) is

$$\frac{d^2P_M(w)}{dw^2} = -\frac{4h_M(a-bw)}{9h_R^2} + \frac{4t(\delta-t)(a-bw)^2}{81Ch_R^2} - \frac{8(a-bw)\left(1 - \frac{2t(\delta-t)(a-bw)}{9Ch_R}\right)}{9h_R} + \frac{4b\left(w - C_M + \frac{t(\delta-t)(a-bw)^2}{9h_R}\right)}{9h_R}$$

Therefore, the second-order condition for the manufacturer is

$$\frac{d^2P_M(w)}{dw^2} = -\frac{4h_M(a-bw)}{9h_R^2} - \frac{4(2a-b(3w-C_M))}{9h_R} + \frac{8t(\delta-t)\left(a-bw\right)^2}{27Ch_R^2} < 0$$

This shows that the profit function of the manufacturer is concave in w. With the solutions given in (3.2), (3.4), and (3.7), it is easy to obtain the optimal profits of the retailer, third party, and manufacturer respectively.

$$\max P_R^* = \frac{2(a-bw^*)^3}{27b^2h_R} - O_R \tag{3.8}$$

$$\max P_C^* = \frac{t^2(a-bw^*)^4}{81b^2h_R^2C} - \frac{2(a-bw^*)^3h_C}{27b^2h_R^2} \tag{3.9}$$

$$\max P_M^* = \frac{2(a-bw^*)^2}{9bh_R}\left[w^* - C_M + \frac{t(\delta-t)(a-bw^*)^2}{9bh_RC}\right] - O_M - \frac{2h_M(a-bw^*)^3}{27b^2h_R^2} \tag{3.10}$$

where the optimal wholesale price is

$$w^* = \frac{8ab\delta t - 8abt^2 - 9bCh_M - 27bCh_R + 3(9b^2C^2h_M^2 - 32ab^2C\delta th_R + 32ab^2Ct^2h_R + 54b^2C^2h_Mh_R + 32b^3C\delta C_Mh_R - 32b^3Ct^2C_Mh_R + 81b^2C^2h_R^2)^{1/2}}{8b^2\delta t - 8b^2t^2} \tag{3.11}$$

2.2 Financial Cohabitation (FC) Scheme

With Scheme FC, the third party is liable for collecting the used products from customers. Here, the manufacturer will share some part of the collection cost by offering financial help. x is the FC percentage decided by the manufacturer. Therefore, firstly manufacturer will decide the manufacturing cost w and FC percentage x, then the retailer and third party will decide the length of time horizon N retail price p and return rate r of used products respectively.

For Retailer: The profit function of the retailer is given by

$$P_R(N,p) = (p-w)(a-bp)N - O_R - \frac{h_R(a-bp)N^2}{2} \tag{3.12}$$

Now, the first-order derivatives of Eq. (3.12) with respect to p and N are

$$\frac{\partial P_R(N,p)}{\partial p} = (a-bp)N + \frac{1}{2}bh_RN^2 - bN(p-w)$$

$$\frac{\partial P_R(N,p)}{\partial N} = -(a-bp)h_RN + (a-bp)(p-w)$$

By solving Eqs. $\frac{\partial P_R(N,p)}{\partial p} = 0, \frac{\partial P_R(N,p)}{\partial N} = 0$, we get

$$N^* = \frac{2(a-bw)}{3bh_R} \text{ and } p^* = \frac{2a+bw}{3b} \tag{3.13}$$

Taking the second-order derivatives of Eq. (3.12)

$$\frac{\partial^2 P_R(N,p)}{\partial p^2} = -2bN < 0,$$

$$\frac{\partial^2 P_R(N,p)}{\partial N^2} = -(a-bp)h_R < 0,$$

$$\frac{\partial^2 P_R(N,p)}{\partial p \partial N} = a - bp + bh_RN - b(\mathrm{p-w})$$

Hence,

$$\left(\frac{\partial^2 P_R(N,p)}{\partial p^2}\right)\left(\frac{\partial^2 P_R(N,p)}{\partial N^2}\right) - \left(\frac{\partial^2 P_R(N,p)}{\partial p \partial N}\right)^2 = 2b(a-bp)h_RN - (a-bp+bh_RN-b(p-w))^2 > 0,$$

provided $2b(a-bp)h_RN > (a-bp+bh_RN-b(p-w))^2$.

This shows that the retailer's profit $P_R(N,p)$ is concave with respect to N and p (see Fig. 3).

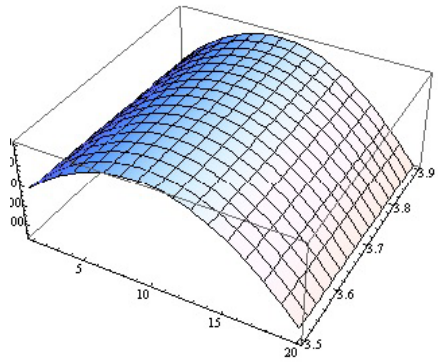

Figure 3: Concavity of retailer's profit $P_R(N,p)$ with respect to N and p

For the third party: The profit function of the third party is given by

$$P_{3P}^{FC}(r) = rt(a-bp)N - r^2C(1-x) - \frac{h_C(a-bp)N^2}{2} \tag{3.14}$$

Now, differentiating Eq. (3.14) with respect to (3.14) r, we get

$$\frac{dP_{3P}(r)}{dr} = \frac{1}{3bh_R}\left[2t(a-bw)(a+\frac{1}{3}(-2a-bw))\right] - 2C_r(1-x)$$

By substituting the retailer's optimal decisions in the above equation of the third party and equating to zero, we can obtain the optimal return rate

$$r^* = \frac{t(a-bw)^2}{9bCh_R(1-x)} \tag{3.15}$$

The second-order derivative of Eq. (3.14) is

$$\frac{dP_{3P}^2(r)}{dr^2} = -2C(1-x) < 0.$$

This shows that the profit function of the third party P_{3P} is concave with respect to the return rate r (see Fig. 4).

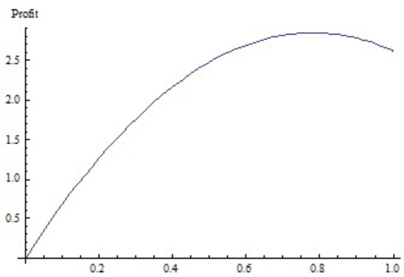

Figure 4: Graphical representation of P_{3P} with respect to r

For Manufacturer: The profit function consists of sales revenue, manufacturing cost, ordering cost, holding cost, and payment given to the third party is given by

$$P_M^{FC}(w,x) = (w+\delta r - C_M - tr)(a-bp)N - O_M - \frac{h_M(a-bp)N^2}{2} - xCr^2 \tag{3.16}$$

Now the first-order condition of optimality gives

$$x^* = \frac{3t-2\delta}{t-2\delta} \tag{3.17}$$

$$w^* = \frac{\begin{bmatrix} -9bCh_M - 27bCh_R + abt^2 - 4abt\delta + 4abt\delta^2 + 3(9b^2C^2h_M^2 + 54b^2C^2h_M h_R + 81b^2C^2h_R^2 - 4ab^2Ch_R t^2 + 4b^3CC_m h_R t^2 + 16ab^2Ch_R t\delta \\ -16b^3CC_m h_R t\delta - 16ab^2Ch_R\delta^2 + 16b^3CC_m h_R\delta^2)^{1/2} \end{bmatrix}}{b^2t^2 - 4b^2t\delta + 4b^2\delta^2} \quad (3.18)$$

The second-order derivatives of Eq. (3.16) are

$$\frac{\partial^2 P_M(w,x)}{\partial w^2} = \frac{4[a^2t\{t(x-2)-2(x-1)\delta\} - a\{3C(h_M+2h_R)(x-1)^2 + 2btw(t(x-2)-2(x-1)\delta)\} + b\{-3C(C_M h_R - (h_M+3h_R)w)(x-1)^2 + btw^2(t(x-2)-2(x-1)\delta)\}]}{27Ch_R^2(x-1)^2}$$

$$\frac{\partial^2 P_M(w,x)}{\partial x^2} = \frac{2t(a-bw)^4[t(x-4)-2(x-1)\delta]}{81b^2Ch_R^2(x-1)^4}$$

and,

$$\frac{\partial^2 P_M(w,x)}{\partial w\partial x} = \frac{4t(a-bw)^3\{t(x-3)-2(x-1)\delta\}}{81bCh_R^2(x-1)^3}$$

Therefore,

$$\left(\frac{\partial^2 P_M(w,x)}{\partial w^2}\right)\left(\frac{\partial^2 P_M(w,x)}{\partial x^2}\right) - \left(\frac{\partial^2 P_M(w,x)}{\partial w\partial x}\right)^2 = \frac{8t(a-bw)^4}{6561b^2C^2h_R^4(x-1)^6}\begin{bmatrix} -2t(a-bw)^2(t(x-3)-2(x-1)\delta)^2 + 3(t(x-4)-2(x-1)\delta) \\ \{a^2t(t(x-2)-2(x-1)\delta) - a\{2C(h_M+2h_R)(x-1)^2 + 2btw(t(x-2)-2(x-1)\delta)\} \\ +b\{-3C(C_m h_R - (h_M+3h_R)w)(x-1)^2 + btw^2(t(x-2)-2(x-1)\delta)\}\} \end{bmatrix}$$

Due to the complexity of the above equations, we obtain $\frac{\partial^2 P_M(w,x)}{\partial w^2} < 0$, $\frac{\partial^2 P_M(w,x)}{\partial x^2} < 0$ and $\left(\frac{\partial^2 P_M(w,x)}{\partial w^2}\right)\left(\frac{\partial^2 P_M(w,x)}{\partial x^2}\right) - \left(\frac{\partial^2 P_M(w,x)}{\partial x\partial w}\right)^2 > 0$, by using numerical data of Example 4.2 (see Fig. 5).

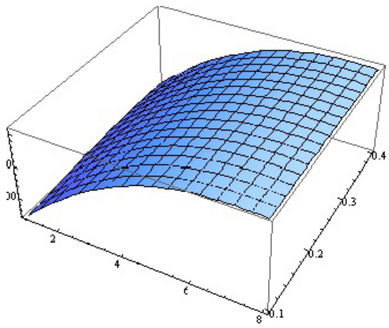

Figure 5: Graphical representation of P_M with respect to w and x

3. NUMERICAL EXAMPLES AND SENSITIVITY ANALYSIS

In this section, we present numerical examples to illustrate the proposed model. Also, we have studied the changes in output parameters with respect to changes in input parameters. Then, we made suggestions to the inventory manager based on sensitivity analysis.

Example 4.1 *For Scheme without cohabitation:*

For an illustration of the proposed model, we consider an inventory situation with randomly selected data set values of cost and basic parameters for a closed-loop supply chain with their usual units as $a = 20, b = 2, C_M = 4, C_R = 2, t = 1, h_M = 1, h_R = 0.5, h_{3P} = 0.001, O_M = 2, O_R = 2, \delta = C_M - C_R = 2, C = 7.$ With the help of the proposed model Eq. (3.11), we get the optimal wholesale price $w^* = 7.20$, and putting the optimal value of $w^* = 7.20$, in Eq. (3.2) and Eq. (3.4), we get the optimal time horizon $N^* = 3.7$, , optimal retail price $p^* = 9.07$, optimal return rate $r^* = 0.497$, respectively. For maximization of profits with respect to the decision variables, the necessary condition is satisfied. After simplification, we obtain the optimal manufacturer's profit $P_M^* = \$10.76$, optimal retailer's profit $P_R^*(p, N) = \$4.49$, and optimal collector's profit $P_{3P}^*(p, N) = \$1.72$.

Example 4.2 *For Financial Cohabitation Scheme:*
To illustrate the preceding theory, let us consider a closed-loop supply chain with the following data in appropriate units as mentioned in the notations section. We set values of cost and basic parameters as $a = 20, b = 2, C_M = 4, C_R = 2, t = 1, h_M = 1, h_R = 0.5, h_{3P} = 0.001, O_M = 2, O_R = 2, \delta = C_M - C_R = 2, C = 7.$
This is heterogeneous data only for illustration of the proposed model.
The computation is done through MS-excel and Mathematica Software and obtains the following optimal values: $w^* = 7.13, x^* = 0.33, p^* = 9.04, N^* = 3.83, r^* = 0.78, P_M^*(w) = 11.22, P_R^*(p, N) = 5.00,$ and $P_{3P}^*(r, x) = 2.84.$
The following Similarities and differences can be found between the scheme without cohabitation and the Financial Cohabitation (FC) scheme:

- The return rate of used products is more with the FC scheme as the third party is fully involved in collection activity due to financial support.
- The profit of each member of the supply chain is more with the FC scheme than without the FC scheme.
- Profit functions are concave with respect to their decision variables.
- Without the FC scheme, the profit of a third party is a function of one variable r, whereas, in the FC scheme, it is a function of two variables r and x.
- The profit of the third party with the FC scheme is more whereas the profit of the third party without the FC scheme is less.

Fig. 6 depicts the impact of the coefficient b on the profits of the manufacturer, retailer, and third party, which indicates that profits decrease in b. As illustrated in the figure, the profit of the manufacturer P_M is concave, and the profit functions of the retailer P_R, and collector P_C linearly decrease due to the increasing price-sensitive coefficient b. The optimal return rate over the transfer price is illustrated in Fig. 7. It shows that r will increase as the t increases.

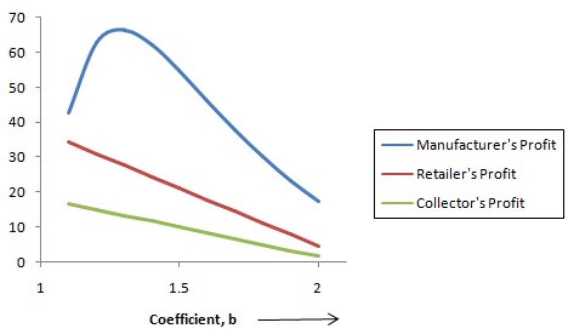

Figure 6: Effect of b on profits of the Manufacturer, Retailer, and Collector

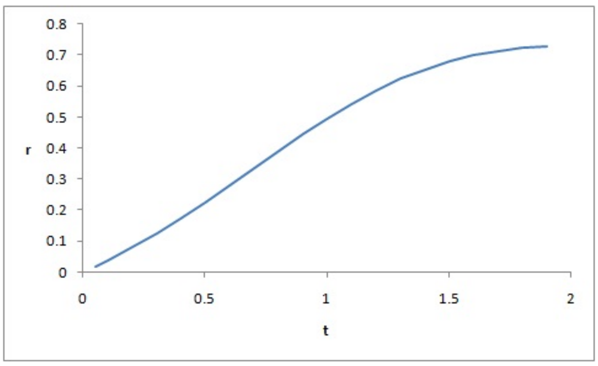

Figure 7: Effect of the transfer price on the optimal return rate

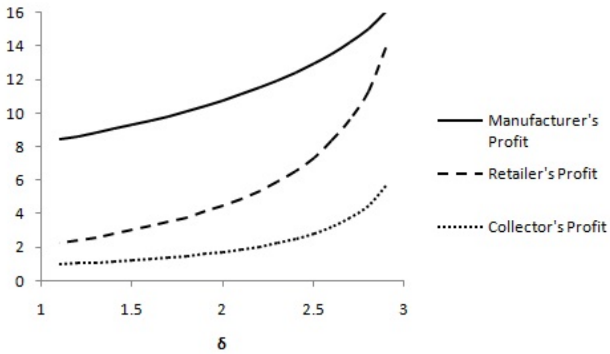

Figure 8: Relation between unit cost-savings δ and profits

Proposition 4.1
The optimal finite time horizon N^* is decreasing in wholesale price *w*.
Proof.
From Eq. (3.2), $N^* = \dfrac{2(\text{a}-\text{bw})}{3bh_R}$, then the first-order condition $\dfrac{\partial N}{\partial w} < 0$, and therefore N^* is decreasing in p for any given demand and holding cost. As the wholesale price increases, the demand for the product automatically decreases due to the high retail price offered by the retailer, so each member in the supply chain requires less time to fulfill their task and complete market demand.

Proposition 4.2
With $r > 0$, the optimal return rate r^* is decreasing with the wholesale price w and the coefficient to retail price b; r^* is increasing with the transfer price t.
Proof.
The fact that decreases with b and w; r^* increases with t, comes directly from observing Eq. (3.4), that is, $r^* = \dfrac{t(a-bw)^2}{9bCh_R}$. Clearly $\dfrac{\partial r}{\partial b} < 0$, $\dfrac{\partial r}{\partial w} < 0$, and $\dfrac{\partial r}{\partial t} < 0$. So, the optimal return rate is increasing in the transfer price (from Fig. 7). The motive is value creation through the recovery of returned products.

We see from Fig. 8 that the manufacturer's profit, retailer's profit, and third party's profit are all increasing in the unit cost-savings δ. Furthermore, when δ tend towards transfer rate is r, the profit of each member has decreased abruptly. The reason behind this is that the profit functions contain the term $(\delta - t)$. Fig. 9, shows the effect of r on the profit of the third party both without the FC scheme and with the FC scheme. The profit of the third party for the FC scheme is more than the profit of the third party without the FC scheme due to financial assistance obtained from the manufacturer.

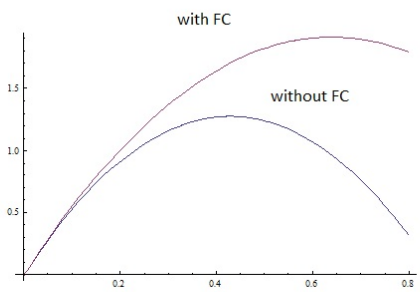

Figure 9: Graphical representation of Profits of the Third-party with respect to r

5. CONCLUSION

Companies have recognized that besides maximizing the profit of big parties, third parties (or small parties), satisfaction, and getting sufficient profit to play an important role in getting a successful position in a competitive market. In this study, we have proposed a closed-loop supply chain model with a manufacturer (remanufacturer), retailer, and third party, for remanufacturing used products within a finite time horizon. The remanufacturing of used products cuts down the consumption of raw materials and reduces environmental pollution. We developed two models: The Scheme without cohabitation and Financial Cohabitation (FC) scheme. To amend the recovery efficiency, we modeled the second scheme FC. The procedure assumes a general demand function dependent on retail price. The results suggest that it is advantageous to adopt financial cohabitation policy, as it will promote coordination among members of the supply chain and strengthen financially.

This paper studies the manufacturer, the retailer, and the third-party profits, where the third party bears the used products collecting responsibility. The purpose of this study is to determine the optimal retail price, time horizon, return rate, and wholesale price for maximizing the profit functions of a retailer, third party, and manufacturer. We have given the analytic solution to the problem. A numerical example is presented to demonstrate the developed model, and decision variables are solved by using Mathematica software. Compressive sensitivity studies were also done to explore the effect of parameters and similarities/differences between the two models. After that, we conduct a detailed analysis of the optimality of these profits. Finally, we have shown that profit functions are concave with respect to decision variables. Motivated by examples from the industry, we look at a market where a remanufactured product is valued less than a new product and is targeted to the lower end of the market. Our analysis yields the following insights. The collector delegates more effort to collecting used products under more transfer price, which increase the return rate of used products. In brief, increasing the price-sensitive parameter b is unfavorable for all members of the supply chain. The result shows that the finite time horizon N decreases with respect to p for any given demand and holding cost. We found that the profit functions are increasing in the unit cost-saving δ and as the δ tends towards the transfer price t, the profit decrease abruptly.

There are many opportunities for future research. The proposed model can be extended by considering the competitive retailing environment. Demand is assumed to be time-dependent or stochastic. The model may be further extended by considering backlogging. The manufacturer may assist the third party physically, to increase the profit of the third party.

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