ONSET OF ELECTROCONVECTION IN A COMPACTLY PACKED DIELECTRIC LIQUID-PERMEABLE LAYER WITH A MODULATED ELECTRIC FIELD

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(Received 20 September 2022; in final form 14 November 2022; accepted 26 December 2022; published online 04 March 2023)

The effect of time-periodic electric field modulation on electroconvection in a compactly packed dielectric liquid-permeable layer is investigated using the small perturbation method coupled with the regular perturbation method. The dielectric constant is assumed to be a linear function of temperature. For small amplitude electric field modulation, the critical correction Rayleigh number is determinedusing the regular perturbation method. The critical Rayleigh number is obtained in terms of the electrical Rayleigh number, Vadasz number, normalized porosity, and the modulation frequency to determine the stability of the system. It is found that electric field modulation at low frequencies can create subcritical convective motion. The impact of Vadasz number is shown to be akin to that of the dielectrophoretic force. The stabilizing influence of normalized porosity is more pronounced when the frequency of electric field modulation is modest and large. The study reveals that time-varying electric fields and a densely packed porous layer may have implications for the control of electroconvection in heat transfer applications involving dielectric fluids as working media.

Key words: Darcy model, dielectric fluid, electric field, porous medium, porosity and modulation.

DOI: https://doi.org/10.30970/jps.27.1401

I. INTRODUCTION

The simultaneous movement of mass and electric charge is important in many areas of science and technology, and applications range from improving heat and mass transfer in nuclear reactors to inkjet printing and coalescence [1, 2]. Several studies have been conducted to investigate how the electric field affects natural convection, since there are many practical problems involving dielectric fluids which have poor conductivity and whose motion is governed mainly by electric forces rather than magnetic forces. Considering the dielectric constant and electrical conductivity as linear functions of temperature, Roberts [3] conducted the first individual study of electrohydrodynamic convection. A theoretical framework has been developed for studying thermocapillary and/or buoyancy instabilities in earth laboratories for electrically conducting fluid layers under an *ac* electric field by Takashima and Aldridge [4], Martin and Richardson [5], Maekawa *et al.* [6], and Douiebe *et al.* [7]. According to Smorodin [8], an alternating electric field of arbitrary frequency affects the stability of convection of a dielectric fluid in a vertical layer. With the help of the Floquet theory, the stability thresholds are determined linearly. During the study of electroconvective instability in a dielectric fluid, Maruthamanikandan [9] investigated the effect of internal heat generation, surface tension, radiation, and viscoelasticity. While some research has been done on anisotropic media, most is conducted on isotropic media. Again, Maruthamanikandan and Smita [10] investigated how the second sound influences the onset of Rayleigh–Bénard instability in a dielectric fluid subjected to alternating vertical electric fields and vertical temperature gradients simultaneously. The problem of convection in a thermally radiating dielectric fluid saturating a porous medium was considered by Smita and Maruthamanikandan [11]. In order to convert radiative heat flux into thermal heat flux, the Milne–Eddington approximation is used. It is made clear that as the radiation parameter increases, the fundamental temperature profile increases exponentially, delaying the onset of electroconvection.

The Darcy–Bénard convection occurs when buoyancy causes heat to flow uniformly through a porous layer heated from below. As thermal convection occurs naturally in porous fluid-saturated media, extensive studies have been conducted on its effects on a variety of scientific, engineering, and technological fields, including, but not limited to, medicine and aerospace engineering [12, 13]. There are many applications of this configuration, such as in biomedical engineering, drying processes, thermal insulation, radioactive waste disposal, transpiration cooling, geophysics, transport of contaminants in groundwater, ceramic processing, and solid matrix heat exchangers. It is well documented in the literature that various developments in this field have occurred; see for instance Bear [14], Rees [15, 16], Bejan *et al.* [17], Nield and Bejan [18], Vafai [19], Nield and Simmons [20]. When magnetic fluids and couple stresses are involved, Saravanan and Yamaguchi [21, 22] studied the same problem.

According to Bhatta *et al.* [23], a horizontal mushy layer saturating a porous medium exhibits steady magnetoconvection. When the fluid is dielectric, the electric force drives the motion more than the magnetic force. Swamy [24] performed linear and nonlinear stability analysis for a porous layer saturated with a dielectric fluid in the presence of a vertical *ac* electric field and time-periodic vertical oscillations. A fluid semiconductor layer that has an open unstable surface was investigated by Smorodin *et al.* [25] for its effect on the instability of a first quasi-equilibrium fluid caused by intermittent temperature fluctuations. The number of Rayleigh numbers and the frequency of the electric field play an important role in destabilizing and stabilizing the ground state, according to Smorodin [26]. It is only the positive response to the electric field that is considered in the horizontal layer. In the fluctuation period, the dielectrophoretic force does not change its direction because it does not depend on the direction of the electric field [27–29]. Rudresha *et al.* [30, 31] investigated thermo-electroconvection in a dielectric fluid subjected to time-periodic electric field modulation. It is shown that the onset of electroconvection can be accelerated or delayed by proper adjustment of the mechanisms of electric field modulation, electric force and couple stresses.

This study aims at analyzing the combined effect of small amplitude electric field modulation on the onset of electroconvection in a horizontal densely packed porous fluid layer with a wide range of values of the frequency of modulation besides the Vadasz number, the Rayleigh number, the electric Rayleigh number, and normalized porosity. The Darcy model is taken into account because the fluid layer under consideration is treated as a permeable one. The outcomes of this work are expected to be helpful for crystal development under microgravity conditions.

II. MATHEMATICAL FORMULATION

The electric force per unit volume acting on the fluid is represented as follows:

$$\mathbf{f_e} = \rho_e\,\mathbf{E} - \frac{1}{2}\ (\mathbf{E}\cdot\mathbf{E})\ \nabla\varepsilon + \nabla\cdot\left(\frac{1}{2}\rho\frac{\partial\varepsilon}{\partial\rho}\ (\mathbf{E}\cdot\mathbf{E})\right). \quad (1)$$

The Coulomb force owing to a free charge is the first term on the right. It is the most powerful term of the EHD force and generally takes precedence when *dc* electric fields are present. The second term is determined by the gradient of ε and takes precedence when an *ac* electric field acts on the dielectric fluid. The application of a *dc* electric field causes the dielectric fluid to accumulate free charges. The free charge has no time to accumulate when an *ac* electric field is supplied at a frequency substantially greater than the reciprocal of the electric relaxation period. Furthermore, the electrical relaxation durations of most dielectric fluids appear to be long enough to exclude free charge accumulation at typical power frequencies. At the same time, the dielectric loss at these frequencies is so little that it has little effect on the temperature field [4]. Under these conditions, only the force caused by the non-uniformity of the dielectric constant is taken into account. The last term in Eq. (1), known as electrostriction force, may be summed up with the pressure term and has no effect on an incompressible dielectric fluid.

We investigate a densely packed porous layer of a dielectric fluid that spreads between infinite horizontal surfaces $z = 0$ and $z = d$ under the influence of a vertically acting electric field and a varying electric potential with time t.

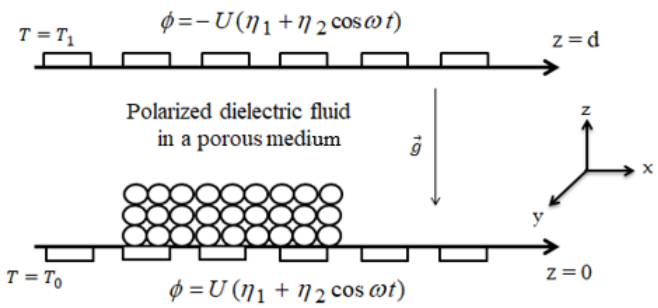

Fig. 1. Physical configuration

On the boundaries of the horizontal layer, $\phi = \pm U\,(\eta_1 + \eta_2\,\cos\omega\,t)$ (see Fig. 1) is achieved, where U is the magnitude of the modulation of the electric potential, ω is the frequency of modulation and, η_1 and η_2 are the relative amplitudes of the components of constant and alternating potential difference. We assume that the dielectric fluid constant ε is a linear function of temperature, the fluid is incompressible, and the porous medium is densely packed. The governing equations for the problem of convection in a densely packed porous layer saturated with an incompressible dielectric Boussinesq fluid are as follows [3, 4, 30]

$$\nabla\cdot\mathbf{q} = 0, \quad (2)$$

$$\rho_0\frac{1}{\delta}\left[\frac{\partial\,\mathbf{q}}{\partial\,t} + \frac{1}{\delta}\,(\mathbf{q}\cdot\nabla)\,\mathbf{q}\right] = -\nabla p + \rho\mathbf{g} - \frac{\mu}{K}\mathbf{q} - \frac{1}{2}\,(\mathbf{E}\cdot\mathbf{E})\,\nabla\varepsilon, \quad (3)$$

$$A\,\frac{\partial\,T}{\partial\,t} + (\mathbf{q}\cdot\nabla)\,T = \kappa\,\nabla^2\,T, \quad (4)$$

where $\mathbf{E}$ is the electric field, T is the temperature, $\mathbf{g}$ is the gravitational acceleration ε is the dielectric constant, $p^* = p - \frac{1}{2}\rho\frac{\partial\,\varepsilon}{\partial\,\rho}\,(\mathbf{E}\cdot\mathbf{E})$ and $A = \frac{(\rho\,c)_m}{(\rho\,c_p)_f}$ is the specific heat ratio, ρ_0 is the density at reference temperature, ρ is the density of the fluid, δ is the porosity of the porous medium, μ is the fluid viscosity, K is the porous permeability, κ is the thermal diffusivity. Assuming that the free charge density is negligibly small, the relevant Maxwell equations are

$$\nabla\cdot[\varepsilon\,\mathbf{E}] = 0, \quad (5)$$

$$\nabla \times \mathbf{E} = 0 \Rightarrow \mathbf{E} = -\nabla \phi, \tag{6}$$

where ϕ is the electric potential and, ρ and ε are assumed to be a linear function of temperature of the form

$$\rho = \rho_0 \left[1 - \alpha \left(T - T_0\right)\right], \tag{7}$$

$$\varepsilon = \varepsilon_0 \left[1 - e \; (T - T_0)\right], \tag{8}$$

where $e\,(> 0)$ denotes the thermal expansion coefficient of the dielectric constant and α is the thermal expansion coefficient of the temperature. For example, for 10 cs silicone oil, $e = 2.86 \times 10^{-1}\,\mathrm{K}^{-3}$ and $\varepsilon = 2.6 \times 10^{-11}\,\mathrm{F} \cdot \mathrm{m}^{-1}$ [6].

III. BASIC STATE

The ground state is at rest and is given by $\frac{\partial}{\partial t} = 0$; $\mathbf{q} = \mathbf{q}_b\,(z) = 0$; $T = T_b\,(z)\,;\, p = p_b\,(z)\,;\, \rho = \rho_b\,(z)\,;\, \varepsilon = \varepsilon_b\,(z)\,;\, \phi = \phi_b\,(z)\,;\, \mathbf{E} = \mathbf{E}_b = [0, 0, E_b\,(z)]$, where the suffix represents the basic state. Using these in equations (2) though (8), we obtain

$$\mathbf{0} = -\nabla p_b + \rho_b\,\mathbf{g} - \frac{1}{2} E_b^2\,\nabla \varepsilon_b, \tag{9}$$

$$T_b = T_0 - \beta\, z, \tag{10}$$

$$\rho_b = \rho_0\;[1 + \alpha\beta\, z]\,, \tag{11}$$

$$\varepsilon_b = \varepsilon_0\;[1 + e\beta\, z] \tag{12}$$

with $\mathbf{E}_b = -\nabla \phi_b$ and the solution of Eq. (10) using the boundary conditions $T_b = T_0$ at $z = 0$ and $T_b = T_1$ at $z = 1$, we obtain

$$\phi_b = \frac{-E_0}{e\beta} \log\left(1 + e\beta\, z\right) + U\left(\eta_1 + \eta_2 \cos \omega\, t\right) \tag{13}$$

and

$$E_b = \frac{2\,U\left(\eta_1 + \eta_2 \cos \omega\, t\right)}{d} \left(1 - e\beta\, z\right), \tag{14}$$

where $E_0 = \frac{2\,U(\eta_1 + \eta_2 \cos \omega\, t) e\beta}{\log(1 + e\beta\, d)}$ and $\beta = \frac{T_0 - T_1}{d}$.

IV. LINEAR STABILITY ANALYSIS

To study the stability of the basic state, we superimpose infinitesimally small perturbations on the basic state in the form $\mathbf{q} = \mathbf{q}' = (u', v', w')\,;\, p = p_b + p', T = T_b + T', \varepsilon = \varepsilon_b + \varepsilon', \phi = \phi_b + \phi', \mathbf{E} = \mathbf{E}_b + \mathbf{E}'$. Substituting these into equations (2) through (8), linearizing and eliminating the pressure term, we obtain

$$\nabla \cdot\; \mathbf{q}' = 0, \tag{15}$$

$$\mathbf{E}' = -\nabla \phi', \tag{16}$$

$$\rho' = -\alpha\, \rho_0\, T', \tag{17}$$

$$\varepsilon' = -e\, \varepsilon_0\, T'. \tag{18}$$

The perturbed equations of the study at hand are the following

$$\begin{aligned} \frac{\rho_0}{\delta} \frac{\partial}{\partial t} \left(\nabla^2\, w'\right) \;&=\; \alpha\, \rho_0\, g\, \nabla_1^2 - \frac{\mu}{K} \nabla^2\, w' + \frac{A_1}{d} \frac{\partial}{\partial z} \left(\nabla^2\, \phi'\right) \\ &+\; \frac{2 A_1 e}{d^2} \nabla_1^2\, T', \end{aligned} \tag{19}$$

$$\nabla^2 \phi' = \frac{-2\,U\left(\eta_1 + \eta_2 \cos \omega\, t\right)\; e}{d} \frac{\partial T'}{\partial\, z}, \tag{20}$$

$$A \frac{\partial T'}{\partial\, t} - \beta\, w' = \kappa\, \nabla^2 T', \tag{21}$$

where $A_1 = 2\,U\,(\eta_1 + \eta_2 \cos \omega\, t)\, e\beta\, \varepsilon_0$, $\nabla_1^{\,2} = \;\partial^2/\partial\, x^2 + \partial^2/\partial\, y^2$, $f = \cos \omega\, t$, $\nabla^{\,2} = \partial^2/\partial\, x^2 + \;\partial^2/\partial\, y^2 + \;\partial^2/\partial\, z^2$. Equations (19) through (21) are rendered dimensionless through the following transformations $(x^*, y^*, z^*) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right)$; $\phi^* = \frac{\phi'}{2U(\eta_1 + \eta_2 f)\, e\, \Delta T}$; $T^* = \frac{T'}{\Delta T}$; $t^* = \frac{\kappa\, t}{A\, d^2}$; $w^* = \frac{w' d}{\kappa}$ to obtain the following dimensionless equations (after omitting the asterisks for simplicity)

$$\begin{aligned} \left(\frac{1}{Va} \frac{\partial}{\partial t} + 1\right) \nabla^2 w \;&=\; \left[R + R_e (1 + \eta_3 f)^2\right] \nabla_1^2 T \\ &+\; R_e (1 + \eta_3 f)^2 \frac{\partial}{\partial z} \nabla_1^2 \phi, \end{aligned} \tag{22}$$

$$\left(\frac{1}{\chi} \frac{\partial}{\partial t} - \nabla^2\right) T = w, \tag{23}$$

$$\nabla^2 \phi = -\frac{\partial T}{\partial\, z}, \tag{24}$$

where $R = \frac{\alpha \rho_0 g K d \Delta T}{\mu\, \kappa}$ is the Darcy-Rayleigh number, $Va = \frac{\delta\, \gamma d^2}{K\, \kappa}$ is the Vadasz number, $R_e = \frac{4 e^2 U^2 \beta^2 \varepsilon_0 d \eta_1^2 K}{\mu\, \kappa}$ is the electrical Rayleigh number, $\chi = \frac{\kappa}{A d^2}$ is the normalized porosity, $\eta_3 = \frac{\eta_2}{\eta_1}$ is the ratio of amplitudes. The appropriate boundary conditions are

$$w = T = D\phi = 0 \quad at \quad z = 0, 1. \tag{25}$$

Combining equations (22) through (24) yields

$$\begin{aligned} &\left(\frac{1}{Va} \frac{\partial}{\partial\, t} + 1\right) \left(\frac{1}{\chi} \frac{\partial}{\partial\, t} - \nabla^2\right) \nabla^4\, w \\ &= \left[R\, \nabla^2 + R_e \nabla_1^2 (1 + \eta_3 \cos \omega\, t)^2\right] \nabla_1^2\, w. \end{aligned} \tag{26}$$

Equation (26) must be solved under the dimensionless homogeneous boundary conditions [31, 32]

$$w = \frac{\partial^2\, w}{\partial\, z^2} = \frac{\partial^4\, w}{\partial\, z^4} = 0 \text{ at } z = 0, 1. \tag{27}$$

We employ the regular perturbation method to derive an expression for the correction critical Rayleigh number.

V. PERTURBATION PROCEDURE WITH SMALL AMPLITUDE APPROXIMATION

We search for the fundamental temperature profile which deviates from the linear profile by measures of order η_3. We, therefore, assume that the solution of equation (26) has the form

$$(w, R) = (w_0, R_0) + \eta_3 (w_1, R_1) + \eta_3^2 (w_2, R_2) + \ldots \quad (28)$$

Substituting equation (28) into (23) and equating the coefficients of like powers of η_3, we obtain the following system of equations

$$L\, w_0 = 0, \quad (29)$$

$$L w_1 = R_1 \nabla^2 \nabla_1^2 w_0 + 2 R_e f \nabla_1^2 w_0, \quad (30)$$

$$\begin{aligned} L w_2 &= R_1 \nabla^2 \nabla_1^2 w_1 + R_2 \nabla^2 \nabla_1^2 w_0 \\ &+ 2 R_e f \nabla_1^4 w_1 + R_e f^2 \nabla_1^4 w_0, \end{aligned} \quad (31)$$

where

$$L = \left(\frac{1}{Va}\frac{\partial}{\partial t} + 1\right)\left(\frac{1}{\chi}\frac{\partial}{\partial t} - \nabla^2\right)\nabla^4 - R_0 \nabla_1^2 \nabla^2 - R_e \nabla_1^4. \quad (32)$$

It is necessary to determine w_0, w_1, and w_2 using the boundary conditions in equation (27).

When studying convection in a horizontal, dielectric, fluid-saturated, tightly packed porous layer exposed to a uniform electric field, equation (29) is utilized, and it should be only minimally stable. The marginally stable solution for that problem is

$$w_0 = \sin \pi z. \quad (33)$$

The corresponding eigenvalues are given by

$$R_0 = \frac{\left(\alpha^2 + \pi^2\right)^3 - R_e \alpha^4}{\alpha^2 \left(\alpha^2 + \pi^2\right)}. \quad (34)$$

Equation (34) expresses the thermal Rayleigh number as a function of wavenumber α and the electric Rayleigh number R_e. It is identical to that obtained by Nagouda and Maruthamanikandan [33] in the case of the non-classical heat conduction effect being absent.

In the absence of electric force (i.e., when $R_e = 0$), Eq. (34) reduces to

$$R_0 = \frac{\left(\alpha^2 + \pi^2\right)^2}{\alpha^2}$$

which is exactly the same relation as available in the literature [19, 20].

In Fig. 2, the thermal Rayleigh number R_0 is plotted against wave number α for different values of the electric Rayleigh number R_e. The destabilizing effect of the dielectrophoretic force is evident from Fig. 2.

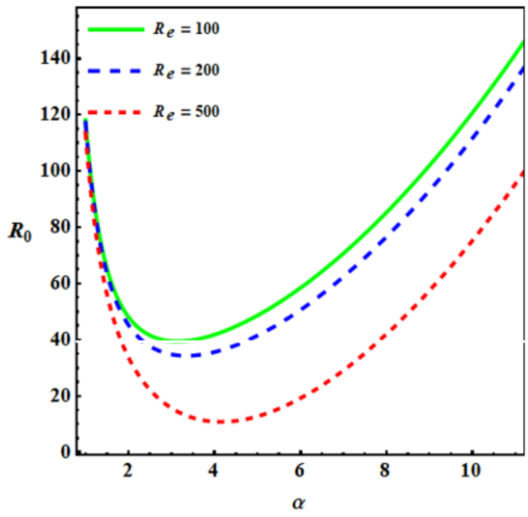

Fig. 2. Variation of the thermal Rayleigh number R_0 with the wavenumber α for different values of the electric Rayleigh number R_e

We take the solution of w in the form $w(x, y, z, t) = w(z, t)\, e^{i(\alpha_x x + \alpha_y y)}$ and thus obtain $\nabla_1^2 w = -\alpha^2 w$. Equation (32) now becomes

$$L\, e^{-i\omega t} \sin n\pi z = L(\omega, n)\, e^{-i\omega t} \sin n\pi z, \quad (35)$$

where

$$\begin{aligned} L(\omega, n) &= -\frac{1}{\chi}\frac{\omega^2}{Va}\left(n^2\pi^2 + \alpha^2\right)^2 + \left(n^2\pi^2 + \alpha^2\right)^3 \\ &- R_0 \alpha^2 \left(n^2\pi^2 + \alpha^2\right) - R_e \alpha^4 \\ &- i\omega \left\{\frac{1}{Va}\left(n^2\pi^2 + \alpha^2\right)^3 + \frac{1}{\chi}\left(n^2\pi^2 + \alpha^2\right)^2\right\}. \end{aligned} \quad (36)$$

The above equation is inhomogeneous and its solution poses a problem because of the presence of a resonance term. The mathematical properties and solvability conditions of the differential equations with time periodic coefficients have been extensively discussed by Yakubovich and Starzhinskii [34]. If this equation is to have a solution, the right-hand side must be orthogonal to the null space of the operator L. This requires that the time-independent part of the right-hand side should be orthogonal to its steady state solution w_0. It follows that all the odd coefficients $R_1, R_3, R_5, \ldots$ in equation (28) are zero because a change of the sign of η_3 shifts the time origin by half period but does not change the physical problem.

We now solve equation (30) by inverting the operator L term by term and obtain the expression for w_1 in the form

$$w_1 = 2 R_e \alpha^4 \,\mathrm{Re}\left[\sum_{n=1}^{\infty} \frac{1}{L(\omega, n)}\, e^{-i\omega t} \sin n\pi z\right]. \quad (37)$$

It is not essential to solve equation (31) for w_2 since we are only concerned with identifying the value of R_2, the non-zero correction of R. The continuity of the right-hand side of equation (31) must be orthogonal to $\sin\pi z$ in order for it to be solvable. It follows that

$$R_2 = \frac{-2R_e\,\alpha^2}{(\alpha^2+\pi^2)}\left[2\int\limits_0^1 \overline{f\,w_1}\sin\pi z\,dz\right], \qquad (38)$$

where the overbar indicates the time average. Using equation (30) and (37) in equation (38) yields

$$R_2 = \frac{2R_e^2\,\alpha^6}{(\alpha^2+\pi^2)}\left[\sum_{n=1}^{\infty}\frac{C_n}{D_n}\right], \qquad (39)$$

where

$$C_n = \frac{1}{\chi}\frac{\omega^2}{Va}\left(n^2\pi^2+\alpha^2\right)^2 - \left(n^2\pi^2+\alpha^2\right)^3 + R_0\,\alpha^2\left(n^2\pi^2+\alpha^2\right) + R_e\,\alpha^4$$

and

$$D_n = \left[\frac{1}{\chi}\frac{\omega^2}{Va}\left(n^2\pi^2+\alpha^2\right)^2 - \left(n^2\pi^2+\alpha^2\right)^3 + R_0\,\alpha^2\left(n^2\pi^2+\alpha^2\right) + R_e\,\alpha^4\right]^2$$
$$+\,\omega^2\left\{\frac{1}{Va}\left(n^2\pi^2+\alpha^2\right)^3 + \frac{1}{\chi}\left(n^2\pi^2+\alpha^2\right)^2\right\}^2.$$

VI. RESULTS AND DISCUSSION

Analytical research involving the linear stability theory is done on the simultaneous impact of the time-periodically varying electric field and a porous medium on the initiation of thermal convection in a horizontal dielectric fluid layer. The critical Rayleigh number R_{2c} and the wave number are determined using the regular perturbation approach, which is based on a limited amplitude of modulation. The expression for the critical correction Rayleigh number R_{2c} is computed as a function of the frequency of modulation, the Rayleigh number, the Vadasz number, the electrical Rayleigh number, and the normalized porosity, and the impact of these parameters on the stability of the system is discussed.

The analysis presented in this work is based on the assumptions that the amplitude of the electric field modulation is very small compared to the mean electric field and that the convective currents are weak so that nonlinear effects may be neglected. The violation of these assumptions would alter the results significantly only when the modulating frequency ω is low. This is due to the perturbation method's requirement that the amplitude of $\eta_3 w_1$ should not be more than that of w_0, which results in the condition $\omega > \eta_3$. Thus, the value of the frequency of the modulation determines the applicability of the results achieved here. The boundaries of the fluid are affected by the modulation of the electric field when ω is sufficiently small (i. e., when the period of the modulation is large). High frequencies, on the other hand, correspond to a renormalization of the static modulation of the electric field, which means that for large values of ω, the influence of the modulation is restricted to a thin border layer close to the boundary. As a result, the buoyancy force takes an average value outside of this layer thickness, resulting in the non-modulated case's equilibrium state.

Since the modulation amplitude is an externally controllable variable, it is possible to prevent finite amplitude instabilities by preventing it from growing too large. Although it cannot be controlled, the nonlinear interactions are used to determine the magnitude of the convection currents. In order to maintain the notion that the nonlinear terms are tiny, it is significantly more crucial that the flow fields under discussion remain of modest amplitude at some point in a modulation cycle.

For a densely packed porous dielectric fluid layer, Figs. 3 through 5 show the frequency dependence of the critical correction Rayleigh number R_{2c}. These figures demonstrate that across a narrow range of values ω, R_{2c} is negative, showing that the system is destabilised by the application of electric field modulation, with electroconvection occurring at an earlier point when compared with the unmodulated system. However, the critical correction Rayleigh number is positive for modest and large values of the frequency of modulation indicating that the electric field modulation has a stabilizing impact on the system with convection occurring at a later point in comparison with the unmodulated system. In addition, we discover that the critical Rayleigh number magnitude rises with rising ω, reaches a peak value at some frequency $\omega = \omega^*$, and then falls with rising ω. The magnitude of the electric force determines the frequency at which the critical Rayleigh number peaks. Figure 3 demonstrates the effect of the electrical Rayleigh number on the correction Rayleigh number with fixed values of the Vadasz number and normalized porosity. The fixed values are taken to be $Va = 50$ and $\chi = 0.5$.

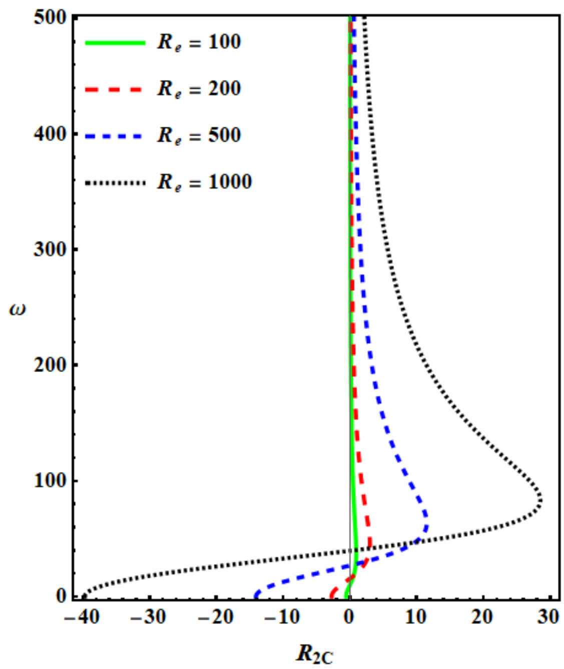

Fig. 3. Variation of R_{2c} with ω for different values of R_e for the Darcy porous layer

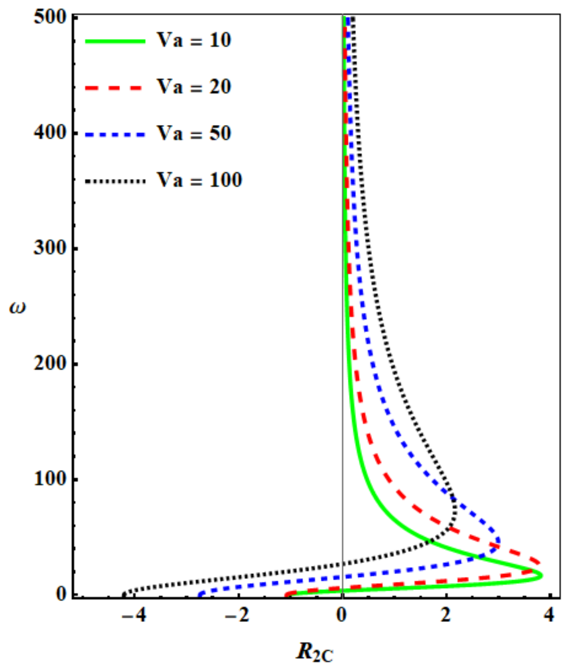

Fig. 4. Variation of R_{2c} with ω for different values of Va for the Darcy porous layer

It is observed that the value of the critical correction Rayleigh number increases negatively with the electrical Rayleigh number at low frequencies, but positively with the electrical Rayleigh number at moderate and high frequencies, indicating that the effect of the electrical Rayleigh number is to destabilize the system at low frequencies while stabilizing the system at moderate and high values of frequency of the electric field modulation.

The impact of the Vadasz number Va on the stability system is seen in Fig. 4. The size of the critical correction Rayleigh number R_{2c} intensifies adversely with cumulative Va when the value of ω is large. The tendency does, however, sharply reverse itself. As a result, when the frequency is low, an increase in the Vadasz number destabilizes the system and when the frequency is high, it enhances the stability of the system. It is noteworthy that the critical correction Rayleigh number R_{2c} begins to positively decline over a period of Va for adequately large ω values, showing that the electric field modulation has the stabilizing effect for this range of frequencies.

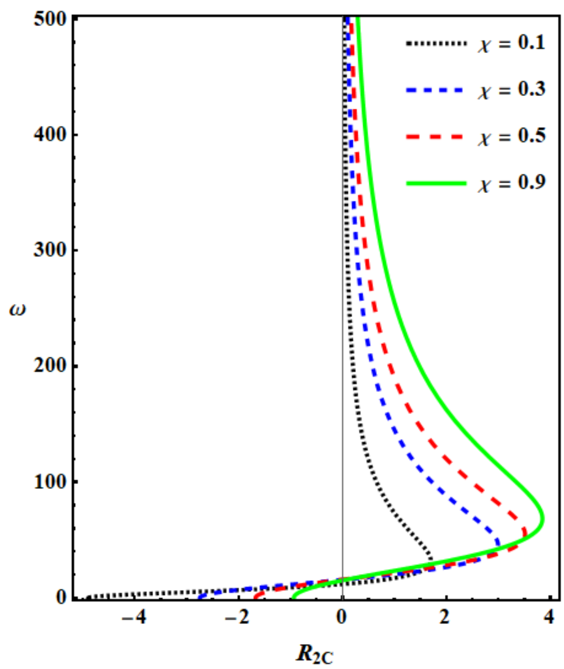

Fig. 5. Variation of R_{2c} with ω for different values of χ for the Darcy porous layer

Figure 5 illustrates the effect of normalized porosity χ on the stability of the system in the presence of electric field modulation. we discover that the impact of χ on the stability of the system is less significant for small values of ω. However, normalized porosity χ tends to stabilize the dielectric fluid layer when ω is moderate and large. As a result, electroconvection can be postponed with increasing values of χ provided frequency of the electric field modulation is not small enough.

VII. CONCLUSIONS

The effect of electric field modulation on the onset of convection in a horizontal dielectric fluid layer and a fluid-densely packed porous layer is examined using the perturbation method. The following conclusions are drawn:

1. Subcritical instability is noticeable due to the electric field modulation for low frequency of the modulation.
2. Dielectrophoretic force tends to destabilize the system for low frequency of the modulating electric field and the opposite is true for moderate and large values of the frequency.

3. The impact of the Vadasz number on the stability of the system is akin to that of the dielectrophoretic force.

4. The system is stabilized only slightly due to the normalized porosity for low frequency of the modulation.

5. Electroconvection can be delayed by the normalized porosity for moderate and large values of the frequency of the modulation.

In summary, electric field modulation in a horizontal dielectric fluid layer saturating a Darcy porous medium induces or delays electroconvection in a porous medium. As a result, the mechanism of electric field modulation could be employed to control convection in compactly packed porous media saturated with dielectric fluids.

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ВИНИКНЕННЯ ЕЛЕКТРОКОНВЕКЦІЇ В КОМПАКТНО УПАКОВАНОМУ ДІЕЛЕКТРИЧНОМУ ПРОНИКНОМУ ДЛЯ РІДИНИ ШАРІ З МОДУЛЬОВАНИМ ЕЛЕКТРИЧНИМ ПОЛЕМ

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Методом малих збурень у поєднанні з методом реґулярних збурень досліджено вплив періодичної в часі модуляції електричного поля на електроконвекцію в компактно упакованому діелектричному проникному для рідини шарі. Вважається, що діелектрична проникність є лінійною функцією температури. Для модуляції електричного поля малої амплітуди критичне поправкове число Релея визначають за допомогою методу реґулярних збурень. Критичне число Релея отримують через електричне число Релея, число Вадаса, нормовану пористість і частоту модуляції для визначення стабільності системи. Виявлено, що модуляція електричного поля на низьких частотах може створювати субкритичний конвективний рух. Показано, що вплив числа Вадаса схожий на вплив діелектрофоретичної сили. Стабілізуювальний вплив нормалізованої пористості більш виражений, коли частота модуляції електричного поля помірна і велика. Дослідження показує, що електричні поля, що змінюються в часі, і щільно упакований пористий шар можуть мати наслідки для контролю електроконвекції в програмах теплопередачі, що включають діелектричні рідини як робочі середовища.

Ключові слова: модель Дарсі, діелектрична рідина, електричне поле, пористе середовище, пористість і модуляція.