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ERRATUM: SIMPSON TYPE QUANTUM INTEGRAL INEQUALITIES FOR CONVEX FUNCTIONS

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Abstract. We have shown that the results of [4] were wrong. Additionally, correct results concerning the Simpson type quantum integral inequalities are proved.

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1. INTRODUCTION

In 2018 Tunç et al. [4] obtained Simpson's type quantum integral inequalities. Unfortunately, there are many mistakes in the proofs. Many q-integrals are calculated incorrectly. Besides, the results of lemma and theorems are also wrong. In this paper, we show the errors in the [4].

2. PRELIMINARIES AND DEFINITIONS OF q-CALCULUS

Throughout this paper, let $a < b$ and $0 < q < 1$ be a constant. The following definitions and theorems for q- derivative and q- integral of a function f on $[a,b]$ are given in [2, 3].

Definition 1. For a continuous function $f : [a,b] \to \mathbb{R}$ then q- derivative of f at $x \in [a,b]$ is characterized by the expression

$$ {}_aD_q f(x) = \frac{f(x) - f(qx + (1-q)a)}{(1-q)(x-a)},\ x \neq a. \tag{2.1}$$

Since $f : [a,b] \to \mathbb{R}$ is a continuous function, thus we have ${}_aD_q f(a) = \lim_{x\to a} {}_aD_q f(x)$. The function f is said to be q- differentiable on $[a,b]$ if ${}_aD_q f(t)$ exists for all $x \in [a,b]$. If $a = 0$ in (2.1), then ${}_0D_q f(x) = D_q f(x)$, where $D_q f(x)$ is familiar q-derivative of f at $x \in [a,b]$ defined by the expression (see [1])

$$ D_q f(x) = \frac{f(x) - f(qx)}{(1-q)x}, \qquad x \neq 0. \tag{2.2}$$

Definition 2. Let $f:[a,b]\to\mathbb{R}$ be a continuous function. Then the q-definite integral on $[a,b]$ is delineated as

$$\int_a^x f(t)_a\, d_qt = (1-q)(x-a)\sum_{n=0}^{\infty} q^n f(q^n x + (1-q^n)a) \tag{2.3}$$

for $x\in[a,b]$.

If $a=0$ in (2.3), then $\int_0^x f(t)_0\, d_qt = \int_0^x f(t)\, d_qt$, where $\int_0^x f(t)\, d_qt$ is familiar q-definite integral on $[0,x]$ defined by the expression (see [1])

$$\int_0^x f(t)_0\, d_qt = \int_0^x f(t)\, d_qt = (1-q)\,x\sum_{n=0}^{\infty} q^n f(q^n x). \tag{2.4}$$

If $c\in(a,x)$, then the q- definite integral on $[c,x]$ is expressed as

$$\int_c^x f(t)_a\, d_qt = \int_a^x f(t)_a\, d_qt - \int_a^c f(t)_a\, d_qt. \tag{2.5}$$

$[n]_q$ notation

$$[n]_q = \frac{q^n-1}{q-1}$$

Lemma 1. *[3] For $\alpha\in\mathbb{R}\setminus\{-1\}$, the following formula holds:*

$$\int_a^x (t-a)_a^{\alpha}\, d_qt = \frac{(x-a)^{\alpha+1}}{[\alpha+1]_q}. \tag{2.6}$$

3. Erratum: Simpson type quantum integral inequalities for convex functions

Here, we will show the errors we mentioned above. For example, in Lemma 4 the followin equality is not correct:

$$\begin{aligned}\int_0^{\frac{1}{2}} (1-t)\left|qt-\frac{1}{6}\right|_0 d_qt &= \int_0^{\frac{1}{2}} \left|qt-\frac{1}{6}\right|_0 d_qt - \int_0^{\frac{1}{2}} t\left|qt-\frac{1}{6}\right|_0 d_qt \\ &= \int_0^{\frac{1}{6q}} \left(qt-\frac{1}{6}\right)_0 d_qt + \int_{\frac{1}{6q}}^{\frac{1}{2}} \left(\frac{1}{6}-qt\right)_0 d_qt\end{aligned}$$

$$-\left(\int_0^{\frac{1}{6q}} t\left(qt-\frac{1}{6}\right)_0 d_qt+\int_{\frac{1}{6q}}^{\frac{1}{2}} t\left(\frac{1}{6}-qt\right)_0 d_qt\right).$$

Here, for $q\in(0,1)$, $\frac{1}{6q}\not\leq\frac{1}{2}$. For instance, $q=\frac{1}{6}\to 1\not\leq\frac{1}{2}$. So, the proof of Lemma 4 is not correct. Lemma 5 also have the same errors. On the other hand, since Lemma 4 and Lemma 5 are used in proof of Theorem 1, there are errors in this theorem. Moreover, Theorem 2 and 3 have the same mistakes. For instance, because of (2.6), the following equalities are also not true:

$$\int_0^{\frac{1}{2}}\left|qt-\frac{1}{6}\right|_0^p d_qt=\frac{\left(1+(3q-1)^{p+1}\right)(1-q)}{6^{p+1}q\left(1-q^{p+1}\right)},$$

$$\int_{\frac{1}{2}}^{1}\left|qt-\frac{5}{6}\right|_0^p d_qt=\frac{\left[(5-3q)^{p+1}+(6q-5)^{p+1}\right](1-q)}{6^{p+1}q\left(1-q^{p+1}\right)}.$$

The integral boundaries that cause all these errors are chosen independently of q.

Now, let show the following Theorem 1 in [4] is not correct. For this, we give an example.

Theorem 1. *Suppose that $f:[a,b]\to\mathbb{R}$ is a q-differentiable function on (a,b) and $0<q<1$. If $|{}_aD_qf|$ is convex and integrable function on $[a,b]$, then we possess the inequality*

$$\frac{1}{6}\left|f(a)+4f\left(\frac{a+b}{2}\right)+f(b)-\frac{1}{(b-a)}\int_a^b f(t)_a d_qt\right| \tag{3.1}$$
$$\leq\frac{(b-a)}{12}\left\{\frac{2q^2+2q+1}{q^3+2q^2+2q+1}\left|{}_aD_qf(b)\right|+\frac{1}{3}\frac{6q^3+4q^2+4q+1}{q^3+2q^2+2q+1}\left|{}_aD_qf(a)\right|\right\}.$$

Example 1. Let choose $f(t)=1-t$ on $[0,1]$ and $f(t)$ satisfies the conditions of Theorem 1. On the other hand, $|{}_aD_qf|=|{}_aD_q(1-t)|=1$ is convex and integrable on $[0,1]$. Then we have

$$\frac{1}{6}\left|f(a)+4f\left(\frac{a+b}{2}\right)+f(b)-\frac{1}{(b-a)}\int_a^b f(t)_a d_qt\right| \tag{3.2}$$
$$=\frac{1}{6}\left|1+2+0-\int_0^1(1-t)_0 d_qt\right|$$

$$= \frac{1}{6}\left|3-\left(t-\frac{t^2}{1+q}\right)\Big|_0^1\right| = \frac{3+2q}{6(1+q)}.$$

Also,

$$\begin{aligned}
&\frac{(b-a)}{12}\left\{\frac{2q^2+2q+1}{q^3+2q^2+2q+1}\left|{}_aD_qf(b)\right| + \frac{1}{3}\frac{6q^3+4q^2+4q+1}{q^3+2q^2+2q+1}\left|{}_aD_qf(a)\right|\right\}\\
&\quad = \frac{1}{12}\left\{\frac{2q^2+2q+1}{q^3+2q^2+2q+1} + \frac{1}{3}\frac{6q^3+4q^2+4q+1}{q^3+2q^2+2q+1}\right\}\\
&\quad = \frac{1}{36}\frac{6q^2+6q+3+6q^3+4q^2+4q+1}{q^3+2q^2+2q+1}\\
&\quad = \frac{1}{36}\frac{6q^3+10q^2+10q+4}{q^3+2q^2+2q+1}\\
&\quad = \frac{1}{18}\frac{3q^3+5q^2+5q+2}{q^3+2q^2+2q+1}. \qquad (3.3)
\end{aligned}$$

As we seen, from (3.2) and (3.3) and for $q \in (0,1)$ we write

$$\frac{3+2q}{6(1+q)} \nleq \frac{1}{18}\frac{3q^3+5q^2+5q+2}{q^3+2q^2+2q+1}.$$

For instance, choosing $q = \frac{1}{2}$ we have

$$\frac{4}{9} \nleq \frac{7}{54}.$$

Therefore, Inequality (3.1) is not correct.

Similarly, other theorems can be shown to be false.

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