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LETTER

An explicit and absolutely stable FDTD method for electromagnetic analysis

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Abstract In this letter, an explicit and absolutely stable finite-difference time-domain (FDTD) algorithm is designed for electromagnetic analysis. The algorithm works through a structure composed of several vectors and matrices acting on these vectors. Excitation source is linearly approximated in a time interval, fields in the computation domain are expressed by a vector, and these matrices are derived based on the FDTD method. The proposed algorithm solves electromagnetic problems in an explicit way, its time step size is beyond the Courant-Friedrich-Levy (CFL) stability condition and the computation efficiency of the proposed method is also higher than the conventional FDTD method. Two numerical examples are tested and validate that the proposed algorithm can solve electromagnetic problems correctly and also improves calculation efficiency.

Keywords: explicit, absolutely stable, FDTD algorithm, CFL
Classification: Electromagnetic theory

1. Introduction

The finite-difference time-domain (FDTD) method is a popular algorithm for solving electromagnetic problems [1, 2]. The explicit algorithm is free of solving matrix equations, while it is limited by the Courant-Friedrich-Levy (CFL) stability condition and inefficient for numerical problem with fine structures in that high temporal resolution means heavy burden of operation time. In recent years, various methods have been proposed such as the hybrid implicit-explicit (HIE) FDTD method [3, 4], magnetically-mixed Newmark-Leapfrog (MNL) FDTD method [5, 6, 7, 8], weakly conditionally stable (WCS) FDTD method [9, 10], FDTD method with filtering scheme [11, 12], Crank-Nicolson (CN) FDTD method [13, 14, 15], alternating-direction-implicit (ADI) FDTD method [16, 17, 18, 19], locally-one-dimensional (LOD) FDTD method [20, 21, 22, 23] and the Weighted-Laguerre-Polynomial (WLP) FDTD method [24, 25, 26, 27]. Among the methods above, the method in [11, 12] is an explicit and unconditionally stable FDTD method by filtering part of high frequency components to extend the time step size. At each time step, Fourier transform is executed for the whole fields in the calculation domain. Recently, authors in [28, 29, 30] also developed an explicit and unconditionally stable method. The method firstly selects a work time step and expresses the FDTD solution into a matrix form. Then the method executes eigenvalue decomposition for the matrix, groups the eigenvectors into stable modes and unstable modes based on Nyquist theorem, and expands fields in a subspace composed of the stable modes and finally solves fields in the new stable subspace explicitly. As far as we can see, except these two method, other methods [3, 5, 9, 13, 16, 20, 24] all take implicit difference schemes and need to solve matrix equations. On the whole, a method that works as an explicit and absolutely stable method still seems to be rare.

It can be seen that the existing explicit and absolutely stable methods in [11, 12, 28, 29, 30] are moderately complex and relatively difficult to implement [31]. As a result, an explicit and absolutely stable method with relatively simple implementation may be of value. In the present letter, an explicit and absolutely stable FDTD method with simple implementation is proposed. The proposed method works through a structure composed of vectors and matrices that act on these vectors and are derived from the FDTD method without complicated matrix operations. The proposed method can select time step freely, improves computation efficiency and is also accurate when time step chooses large values.

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2. Formulation

In simple, lossless and isotropic media, the general governing equations can be written as

$$\frac{\partial E}{\partial t} = \frac{1}{\varepsilon}\nabla \times H - \frac{1}{\varepsilon}J \tag{1}$$

$$\frac{\partial H}{\partial t} = -\frac{1}{\varepsilon}\nabla \times E \tag{2}$$

The conventional FDTD solution is

$$H^{n+1} = H^n - \frac{\Delta t}{\mu}\nabla \times E^{n+\frac{1}{2}} \tag{3}$$

$$E^{n+\frac{3}{2}} = E^{n+\frac{1}{2}} + \frac{\Delta t}{\varepsilon}\nabla \times H^{n+1} - \frac{\Delta t}{\varepsilon}J^{n+1} \tag{4}$$

In Eq. (3) and Eq. (4), the spatial derivatives are approximated by central finite difference. When the fields in Eq. (3) and Eq. (4) are numerically expressed, H^{n+1} in Eq. (4) are replaced with those in Eq. (3) and a different update equation linking fields at past time step with fields at current time step directly is acquired. For simplicity, only E_x and H_y in this form are presented.

$$E_x^{n+3/2}(i+1/2,j,k) = E_x^{n+1/2}(i+1/2,j,k) + \frac{\Delta t}{\varepsilon}\left(\frac{H_z^n(i+1/2,j+1/2,k) - H_z^n(i+1/2,j-1/2,k)}{\Delta y} - \frac{H_y^n(i+1/2,j,k+1/2) - H_y^n(i+1/2,j,k-1/2)}{\Delta z}\right)$$

$$+\frac{\Delta t}{\mu}\frac{\Delta t}{\varepsilon}\left(\frac{\begin{array}{l}E_x^{n+1/2}(i+1/2,j+1,k)-2E_x^{n+1/2}(i+1/2,j,k)\\+E_x^{n+1/2}(i+1/2,j-1,k)\end{array}}{\Delta y^2}\right)$$

$$+\frac{\Delta t}{\mu}\frac{\Delta t}{\varepsilon}\left(\frac{\begin{array}{l}E_x^{n+1/2}(i+1/2,j,k+1)-2E_x^{n+1/2}(i+1/2,j,k)\\+E_x^{n+1/2}(i+1/2,j,k-1)\end{array}}{\Delta z^2}\right)$$

$$-\frac{\Delta t}{\mu}\frac{\Delta t}{\varepsilon}\frac{1}{\Delta x}\left(\begin{array}{l}\frac{E_y^{n+1/2}(i+1,j+1/2,k)-E_y^{n+1/2}(i+1,j-1/2,k)}{\Delta y}\\-\frac{E_y^{n+1/2}(i,j+1/2,k)-E_y^{n+1/2}(i,j-1/2,k)}{\Delta y}\end{array}\right)$$

$$-\frac{\Delta t}{\mu}\frac{\Delta t}{\varepsilon}\frac{1}{\Delta x}\left(\begin{array}{l}\frac{E_z^{n+1/2}(i+1,j,k+1/2)-E_z^{n+1/2}(i+1,j,k-1/2)}{\Delta z}\\-\frac{E_z^{n+1/2}(i,j,k+1/2)-E_z^{n+1/2}(i,j,k-1/2)}{\Delta z}\end{array}\right)$$

$$-\frac{\Delta t}{\varepsilon}J_x^{n+1} \tag{5}$$

$$H_y^{n+1}(i+1/2,j,k+1/2)=H_y^n(i+1/2,j,k+1/2)$$
$$-\frac{\Delta t}{\mu}\left(\begin{array}{l}\frac{E_x^{n+1/2}(i+1/2,j,k+1)-E_x^{n+1/2}(i+1/2,j,k)}{\Delta z}\\-\frac{E_z^{n+1/2}(i+1,j,k+1/2)-E_z^{n+1/2}(i,j,k+1/2)}{\Delta x}\end{array}\right) \tag{6}$$

Eq. (5), Eq. (6), Eq. (3) and Eq. (4) show a clear and direct relationship between fields at current time step and fields at previous time step. As a result, the solution of the FDTD method can be arranged into a matrix form as

$$F^{n+1}=aF^n+S^{n+1} \tag{7}$$

In Eq. (7) F^{n+1} represents vector $\left(E^{n+\frac{3}{2}} \quad H^{n+1}\right)^T$ that covers all electric and magnetic fields at current time step, F^n represents these fields at the previous time step, a represents the matrix derived from the coefficients of numerical spatial derivatives and media parameters in Eq. (3) and Eq. (4). S^{n+1} presents vector $\left(\frac{\Delta t}{\varepsilon}J^{n+1} \quad 0\right)^T$ covering current terms at different points.

From Eq. (7) we can get an equation

$$F^{n+k}=a^kF^n+\sum_{i=1}^{k}a^{k-i}S^{n+i} \tag{8}$$

In the time interval from the $(n+1)th$ time step to the $(n+k)th$ time step, the current is approximated linearly and is expressed as

$$S^{n+i}=\frac{k-i}{k-1}S^{n+1}+\frac{i-1}{k-1}S^{n+k} \tag{9}$$

Then replacing the current at different time step in Eq. (8) with its new representation in Eq. (9), we get

$$F^{n+k}=a^kF^n$$

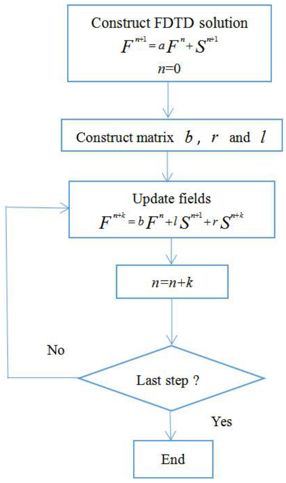


Fig. 1 Flowchart of the proposed method.

$$+\sum_{i=1}^{k}a^{k-i}\left(S^{n+1}+\frac{i-1}{k-1}S^{n+k}\right) \tag{10}$$

Merging similar terms, Eq. (10) can be written as

$$F^{n+k}=a^kF^n+lS^{n+1}+rS^{n+k} \tag{11}$$

The l and r in Eq. (11) are matrices by merging similar terms in Eq. (10).

The proposed method now can be written as

$$F^{n+k}=bF^n+lS^{n+1}+rS^{n+k} \tag{12}$$

In Eq. (12) matrix b equals matrix a^k.

A flowchart of the proposed explicit and absolutely stable FDTD method is shown in Fig. 1.

Wave equation is also a popular governing equation and is used in this letter. It is expressed as

$$\frac{\partial^2 E}{\partial t^2}=\frac{1}{\mu\varepsilon}\nabla^2 E-\frac{1}{\varepsilon}\frac{\partial J}{\partial t} \tag{13}$$

The FDTD method in Eq. (13) is

$$\begin{aligned}E^{n+1}&=2E^n-E^{n-1}\\&+\frac{\Delta t^2}{\mu\varepsilon}\nabla^2E^n-\frac{\Delta t}{\varepsilon}\left(J^{n+\frac{1}{2}}-J^{n-\frac{1}{2}}\right)\end{aligned} \tag{14}$$

Again the central finite difference scheme is applied to spatial derivatives in Eq. (14), and an equivalent description of Eq. (14) can be written as

$$e^{n+1}=2e^n-e^{n-1}+Ce^n+j^{n+1} \tag{15}$$

where vector e^{n+1}, e^n and e^{n-1} represents all electric fields in computation domain at the $(n+1)th$, nth and $(n-1)th$ time step, matrix C is derived from the coefficients of numerical spatial derivatives and media parameters in Eq. (14), and vector j^{n+1} stands for the excitation current $-\frac{\Delta t}{\varepsilon}\left(J^{n+\frac{1}{2}}-J^{n-\frac{1}{2}}\right)$. Then a different expression of Eq. (18) is

$$\begin{pmatrix}e^{n+1}\\e^n\end{pmatrix}=\begin{pmatrix}2+C & -1\\1 & 0\end{pmatrix}\begin{pmatrix}e^n\\e^{n-1}\end{pmatrix}+\begin{pmatrix}-j^{n+1}\\0\end{pmatrix} \tag{16}$$

The Eq. (16) can be expressed as a more meaning expression of

$$u^{n+1} = Tu^n + w^{n+1} \tag{17}$$

It is evident that the form of Eq. (17) is just the same as that of Eq. (7), so all the rest procedures can be completed in a similar way. In Eq. (17), $u^{n+1} = \left(e^{n+1} \quad e^n\right)^T$, $T = \left(2+C \quad -1; 1 \quad 0\right)$, $u^n = \left(e^n \quad e^{n-1}\right)^T$ and $w^{n+1} = \Delta t\left(-j^{n+1/2} + j^{n-1/2} \quad 0\right)^T \Big/ \varepsilon$.

From Eq. (17) we can get

$$u^{n+k} = T^k u^n + \sum_{i=1}^{k} T^{k-i} w^{n+i} \tag{18}$$

In the time interval from the $(n+1)th$ time step to the $(n+k)th$ time step, the current term is also expressed as

$$w^{n+i} = \frac{k-i}{k-1} w^{n+1} + \frac{i-1}{k-1} w^{n+k} \tag{19}$$

Replacing the current term at different time step in Eq. (18) with its new representation in Eq. (19) and merging similar terms, the proposed method using wave equation as governing equation can be acquired and it is put as the form of

$$u^{n+k} = T^k u^n + Lw^{n+1} + Rw^{n+k} \tag{20}$$

We can also write it as a concise form of

$$u^{n+k} = Bu^n + Lw^{n+1} + Rw^{n+k} \tag{21}$$

In Eq. (21) matrix B is equal to matrix T^k. L and R are matrices by merging similar terms in Eq. (18) after inserting Eq. (19) into Eq. (18). The time step size of the proposed algorithm is k (arbitrary value) times that of the FDTD method, as fields at nth time step are transferred to $(n+k)th$ time step in one iteration.

It can be seen from the process of constructing the proposed method that the required operations are simple multiplications among matrices and vectors and the the process is also very direct and simple to finish.

3. Numerical stability analysis

Firstly the von Neumann method is used to analyse the stability condition. Applying plane wave

$$E^t(x,y,z) = E^t \exp\left(j\left(k_x x + k_y y + k_z z\right)\right) \tag{22}$$

$$H^t(x,y,z) = H^t \exp\left(j\left(k_x x + k_y y + k_z z\right)\right) \tag{23}$$

is revised as to the FDTD solution of Eq. (3) and

$$E^{n+\frac{3}{2}} = E^{n+\frac{1}{2}} + \frac{\Delta t}{\varepsilon} \nabla \times H^{n+1} \tag{24}$$

and inserting Eq. (3) into Eq. (24), a matrix equation describing Eq. (3) and Eq. (24) is

$$U^{n+1} = MU^n \tag{25}$$

$U^{n+1} = \left(E^{n+3/2} \quad H^{n+1}\right)^T$ and $U^{n+1} = \left(E^{n+1/2} \quad H^n\right)^T$ in Eq. (25) are vectors both with six field components. Matrix$M$is derived from the coefficients of numerical spatial derivative and media parameters in Eq. (3) and Eq. (24), and can be understood by referring to Eq. (5) and Eq. (6). In this situation, numerical spatial derivative in an arbitrary direction is

$$D_\xi = 2j\sin(k_\zeta \Delta x/2)/\Delta\zeta \tag{26}$$

where $\zeta = x, y, z$. To maintain the stability, the amplitudes of eigenvalues of matrix$M$should be not bigger than unity.

The eigenvalues of matrix M are

$$\lambda_1 = \lambda_2 = 1 \tag{27}$$

$$\lambda_3 = \lambda_4 = 1 + \frac{g}{2} + \frac{\sqrt{g(g+4)}}{2} \tag{28}$$

$$\lambda_5 = \lambda_6 = 1 + \frac{g}{2} - \frac{\sqrt{g(g+4)}}{2} \tag{29}$$

where $g = \frac{\Delta t}{\mu}\frac{\Delta t}{\varepsilon}\left(D_x^2 + D_y^2 + D_z^2\right)$.

When $g + 4 > 0$, the amplitudes of eigenvalues of matrix M in Eq. (25) will be not bigger than unity. That is to say

$$|\lambda(M)| = 1 \tag{30}$$

when

$$c\Delta t < 1/\sqrt{1/\Delta x^2 + 1/\Delta y^2 + 1/\Delta z^2} \tag{31}$$

Eq. (31) is the CFL stability condition.

The proposed method in frequency domain can be written as

$$U^{n+k} = M^k U^n \tag{32}$$

For clarity, we write Eq. (32) into a different expression as

$$U^{n+k} = mU^n \tag{33}$$

where matrix m equals to M^k. Time step in this method is ΔT.

Clearly, as $m = M^k$, the values of

$$|\lambda\,(m)| = 1 \tag{34}$$

when

$$\Delta T = k\Delta t \tag{35}$$

We also discuss the stability of the proposed method in time domain. Eq. (7) can be written as

$$F^{n+1} = aF^n \tag{36}$$

It is just the expression of the FDTD method in a matrix form. According to [32], to keep the stability of the FDTD method expressed in Eq. (36), the time step has to satisfy the CFL stability condition. That is to say, the amplitudes of eigenvalues of matrix a

$$|\lambda(a)| \le 1 \tag{37}$$

is revised as when Eq. (31) is satisfied.

The proposed method in this letter is

$$F^{n+k} = bF^n \tag{38}$$

In Eq. (38), the time step is k times that of Δt, and matrix b equals matrix a^k, so eigenvalues of b are k times that of a accordingly. Evidently, the eigenvalues of matrix b

$$\lambda(b) \le 1 \tag{39}$$

is revised as when Eq. (35) is satisfied.

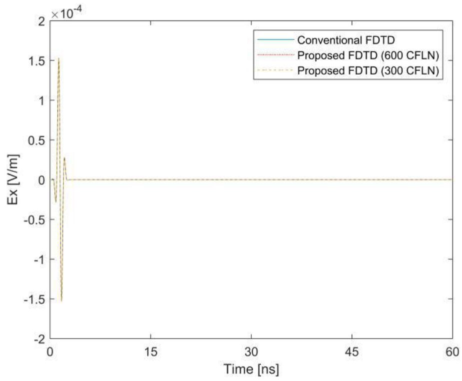


Fig. 2 Transient E_x at r point.

It can be seen from Eq. (34) and Eq. (39) that magnitudes of eigenvalues of system matrix are always not bigger than unity whatever value k selects. As a result, the proposed method overcomes the CFL stability condition in such a way and becomes an absolutely stable FDTD method. So time step ΔT that is k times of Δt in the proposed method can choose arbitrary value.

The numerical dispersion relationship in the proposed method is the same as that of the FDTD method, which can be acquired by using Eq. (25) in Eq. (32) repeatedly and is shown in

$$-4\sin(\omega\Delta t/2)^2/\Delta t^2 = (vD_x)^2 + \left(vD_y\right)^2 + (vD_z)^2 \quad (40)$$

The case that selects wave equation as governing equation can be interpreted in a similar way.

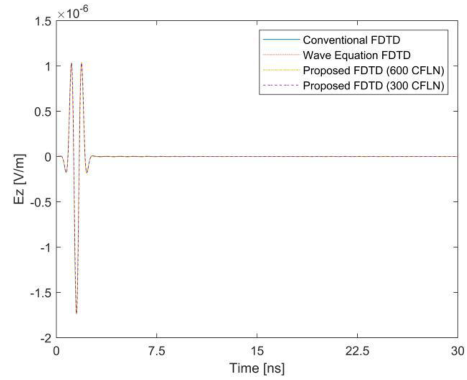
4. Numerical validation

Two numerical examples are solved to validate the proposed algorithm.

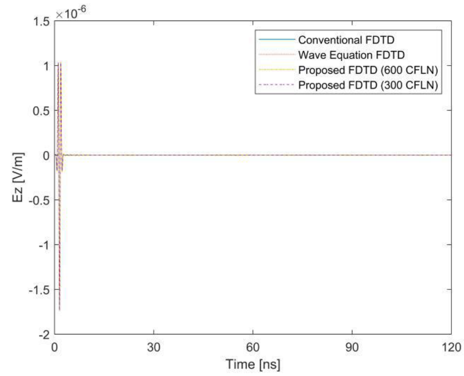
In the first example, the radiation of an infinite surface current in free space is simulated. The space has 200 cells with the size of 1 μm, the time step size in FDTD method is 2 fs and 60 ns are simulated. In the proposed method $k = 200$, 300 and 600 are used so the time step sizes are 0.4 ps, 0.6 ps and 1.2 ps respectively. We define CFLN as $\frac{\Delta t_{ProposedFDTD}}{\Delta t_{FDTD}}$, and it is obviously that CFLN is equal to k used in this letter. The region is excited by a sinusoidally modulated Gaussian current at $z = 10$ which takes its profile from [24] but is along the x direction. This is a 1D problem and only one electric field component and one magnetic field component exist. And the region is truncated by Mur's first-order absorbing boundary condition.

The running time of the FDTD method is 93.33 s, but the proposed algorithm takes 21.81 s, 44.05 s and 69.50 s when $k = 600$, 300 and 200, which means the proposed method realizes a noticeable improvement in calculation efficiency. E_x at point $r(50)$ in two methods are recorded and results from the proposed method when $k = 600$ and 300 are shown in Fig. 2. It can be seen that the two solutions are in good agreement. In this case, the memory for the simulations run by the FDTD method and the proposed method is 0.38 M and 6.06 M, respectively.

We define an error function as



(a)



(b)

Fig. 3 Transient E_z at p point (a) from 0 to 30 ns and (b) from 0 to 120 ns.

$$error = \sqrt{\sum_{i=1}^{T}\left(E_x^i - E_{x,\mathrm{Ref}}^i\right)^2 \Big/ \sum_{i=1}^{T}\left(E_{x,\mathrm{Ref}}^i\right)^2} \quad (41)$$

Setting solution of the FDTD method as reference, we make a comparison among the reference and solutions solved by the proposed method when k changes. In detail, E_x at observing point are recorded and are used to measure the accuracy of the proposed method. When $k = 200$, 300, and 600, the error is 0.0026, 0.0040 and 4.0765×10^{-6}. T is 10000 which is the iteration number of simulation run by the proposed method when $k = 600$. It can be seen that the error is relatively small when k gets different value.

In the second example, radiation of a infinite line current at $(10, 5)$ along the z direction in a 2D metal cavity is simulated. In this TM case, the scalar wave equation of E_z is selected as governing equation for the proposed method. There are both 20 cells along the x and y direction with the cell sizes of 10 μm and 0.01 m respectively. The FDTD method and wave equation FDTD method both choose 20 fs as time step and 120 ns are simulated. The proposed algorithm uses time steps 600, 300 and 200 times that of the 20 fs, that is to say, 12 ps, 6 ps and 4 ps. E_z at $p(5, 5)$ in three methods are recorded and results from the proposed method when $k = 600$ and 300 are shown in Fig. 3. It can be seen from Fig. 3 that they agree with each other very well. The FDTD method takes 93.96 s, the wave equation FDTD method takes 66.18 s, while the proposed method costs 36.92 s, 42.42 s and 54.62 s when $k = 600$, 300 and 200, which shows a higher computation efficiency. In this case, the memory cost for the conventional FDTD method and wave equation FDTD method is 0.086 M and 0.086M, and the memory cost of the proposed method is 17.14 M.

Setting solution of E_z from the FDTD method as reference and using the same error function form in Eq. (40), we make a comparison to measure the accuracy of the proposed method. When $k = 200$, 300 and 600, the error is 0.0334, 0.0458 and 0.0171. In this case, T is 10000 which is the iteration number of simulation run by the proposed method when $k = 600$. And the error between the solution calculated by the wave equation FDTD method and that by the proposed method is 0.0333, 0.0457 and 0.0172 when $k = 200$, 300 and 600, respectively. It can be seen that two groups of errors are almost the same and stay in a stable and low level when k changes.

All simulations are operated on Intel(R) core (TM) i3-3220 CPU @3.30 GHZ with memory of 4G and Matlab 2018b is used for programming.

5. Conclusion

In this letter, an explicit and absolutely stable FDTD method is proposed. The algorithm is derived from the FDTD method and is an explicit method as there is no need of solving matrix equations, while it is also an absolutely stable method as the time step in it can choose an arbitrary value. Numerical results validate that the solutions solved by the proposed method are in good agreement with those calculated by the FDTD method and the computation efficiency of the proposed algorithm is also higher than the FDTD method.

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