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An inexact multiblock alternating direction method for grasping-force optimization of multifingered robotic hands

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Abstract

In this paper, we present an inexact multiblock alternating direction method for the point-contact friction model of the force-optimization problem (FOP). The friction-cone constraints of the FOP are reformulated as the Cartesian product of circular cones. We focus on the convex quadratic circular-cone programming model of the FOP, which is an exact cone-programming model. Coupled with the separable convex quadratic objective function, we recast the circular-cone-programming model as a multiblock separable cone model. A parallel inexact multiblock alternating direction method is used to solve the FOP. We prove the global convergence of the proposed method. Simulation results of the three-fingered FOP are reported, which verified the efficiency of the proposed method.

Keywords: Circular-cone programming; Second-order cone programming; Alternating direction method; SeDuMi software

1 Introduction

Multifingered robotic hands have been extensively studied for their superior performance in dextrous manipulation. A fundamental issue on the optimal control of multifingered robot hands is the grasping-force-optimization problem. For the grasping-force-optimization problem, we seek the minimum contact forces from every finger applied to balance any external wrench [1, 2].

Previous studies of the force-optimization problems are mainly based on the cone-programming models. There are two types of optimization methods for the force-optimization problems, one is the approximation method, the other is the exact method. The friction cone constraints of the force-optimization problem are nonlinear, which significantly complicates the solution of the force-optimization problem [3]. The early method is the linear programming method, which is based on the linearization of the friction cone [4, 5]. However, linearization methods violate the nonlinear constraint, hence the linear programming methods are approximation methods. The semidefinite programming model and the second-order cone programming are types of exact cone models for the force-optimization problems. In the papers [6] and [7], the semidefinite programming models are proposed for the force-optimization problems, which are solved by the

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interior-point algorithms. In the paper [8], the second-order cone-programming model is proposed for the optimal grasping forces. Furthermore, the interior-point method is used to solve the exact second-order cone-programming model. In the paper [9], a kernel-based interior-point method is proposed for the convex quadratic programming over circular cones, which is another exact model for the force-grasping-optimization problems. In the paper [10], two projection and contraction methods are presented for solving the grasping-force-optimization problem, in which the grasping-force-optimization problem is transformed as a convex quadratic circular-cone programming (CQCCP) model like that in [9]. In the paper [11], the grasping-force synthesis problem, while considering the physical constraints and hand capabilities, can be formulated as a second-order cone programming, which is solved by the software CVX [12]. In CVX software, it solved the second-order cone programming by SeDuMi (the compared method). In the paper [13], the authors optimize grasp stability by the principle of the maximum internal tangent sphere and conduct a stable grasp planning based on minimum force. The nonlinear programming model is solved by Newton's method. In the paper [14], the authors proposed a control barrier function, which is a useful method of ensuring constraint satisfaction for a wide class of controllers. The algorithm is implemented by the hardware.

The aim of this paper is to develop an efficient method to solve the grasping-force minimization of a multifingered robotic hand. We first reformulate the CQCCP model of the grasping-force optimization as a multiblock separable cone programming. Then, we propose a parallel inexact alternating direction method for the equivalent separable CQCCP problem. Furthermore, we analyze the global convergence property of the proposed method. By the three-fingered grasping-force optimization example, the numerical experimental results are given to illustrate the performance of the method.

2 The CQCCP model for the force-optimization problem

In this section, we consider the point-contact friction model for the force-optimization problem. The multifingered robot hands grasp the rigid object at M contact points $p_i \in \mathbb{R}^3$, for $i = 1, \dots, M$. The force applied at a contact point p_i is denoted by $f_i = (f_x^i, f_y^i, f_z^i)^T \in \mathbb{R}^3$, in which $(f_x^i, f_y^i)^T$ is the tangential force, and f_z^i is the normal force.

The friction cone constraint of the force-optimization problem for the contact forces f_i is [8]

$$\|(f_x^i, f_y^i)^T\| \leq \mu_i f_z^i, \quad (1)$$

where μ_i is the friction coefficient at the i th finger, and $\|\cdot\|$ denotes the Euclidean norm.

Let $f = (f_1, \dots, f_M) \in \mathbb{R}^{3M}$. To balance the external wrench $\omega_{ext} \in \mathbb{R}^6$, the linear equation constraint of the force-optimization problem is

$$Af + \omega_{ext} = 0, \quad (2)$$

where $A \in \mathbb{R}^{6 \times 3M}$ is the grasping transformation matrix, which is full row rank. Thus, the point-contact friction model for the force-optimization problem is

$$\begin{aligned} & \min \frac{1}{2} f^T f \\ & \text{s.t. } Af + \omega_{ext} = 0, \\ & \|(f_x^i, f_y^i)^T\| \leq \mu_i f_z^i, \quad i = 1, \dots, M. \end{aligned} \quad (3)$$

Usually, problem (3) is dealt with as a convex quadratic second-order cone-programming problem by adding M equations and M variables.

$$\begin{aligned} \min \quad & \frac{1}{2} f^T f \\ \text{s.t.} \quad & Af + \omega_{ext} = 0, \\ & x_i = \mu_i f_z^i, \quad i = 1, \dots, M, \\ & \|(f_x^i, f_y^i)^T\| \leq x_i, \quad i = 1, \dots, M. \end{aligned} \quad (4)$$

In addition, problem (3) is also written as a convex quadratic circular-cone program (CQCCP)

$$\begin{aligned} \min \quad & \frac{1}{2} f^T f \\ \text{s.t.} \quad & Af + \omega^{ext} = 0, \\ & f \in L_\theta, \end{aligned} \quad (5)$$

where, $L_\theta = L_{\theta_1} \times L_{\theta_2} \times \dots \times L_{\theta_M}$, and L_{θ_i} is the circular cone [10]

$$L_{\theta_i} := \{f_i = (f_x^i, f_y^i, f_z^i)^T : \|(f_x^i, f_y^i)^T\| \leq \tan \theta_i f_z^i\}$$

with the rotation angle $\theta_i = \tan^{-1} \mu_i$.

Problem (5) is a convex quadratic circular-cone programming problem with a linear equation of $6 \times 3M$ dimension, and M circular-cone constraints. Problem (4) is a linear second-order cone-programming problem with a linear equation of dimension $(6 + M) \times 4M$, and M second-order cone constraints. Hence, the scale of problem (5) is smaller than that of problem (4).

The projection over the circular L_{θ_i} is given in the paper [15]. Hence, the projection $P_{L_\theta}(f)$ on cone L_θ is

$$P_{L_\theta}(f) = [P_{L_{\theta_1}}(f_1), \dots, P_{L_{\theta_M}}(f_M)] \in \mathbb{R}^3 \times \dots \times \mathbb{R}^3. \quad (6)$$

3 An inexact multiblock alternating direction method for circular-cone programming problems

In this section, we rewrite the convex quadratic circular-cone programming as a multi-block separable circular-cone programming problem based on the circular-cone block variables

$$\begin{aligned} \min \quad & \frac{1}{2} \sum_{i=1}^M f_i^T f_i \\ \text{s.t.} \quad & \sum_{i=1}^M A_i f_i = -\omega_{ext}, \\ & f_i \in L_{\theta_i}, \quad i = 1, 2, \dots, M, \end{aligned} \quad (7)$$

where $A = (A_1, A_2, \dots, A_M)$ and $A_i \in \mathbb{R}^{6 \times 3}$.

Since the objective function of (7) are sums of terms on individual f_i (they are separable), problem (7) can be decomposed into M smaller subproblems, which is solved in a parallel and distributed manner. The circular-cone programming (7) can be solved by the multiblock alternating direction method. When $M \geq 3$, the direct extension of the multiblock alternating direction method is not necessarily convergent [16]. In the papers [17–19], the multiblock problem with $M \geq 3$ is converted into an equivalent two-block problem via variable splitting. Here, we extend the idea, and give an inexact multiblock alternating direction method for the convex quadratic circular-cone programming problem (7). The inexact multiblock alternating direction method applied the simple projection method to solve the subproblem. In each subproblem, the multiblock variables are computed in a parallel and distributed manner.

The multiblock separable circular-cone programming problem is converted into an equivalent two-block problem via variable splitting:

$$\begin{aligned} \min \quad & \frac{1}{2} \sum_{i=1}^M f_i^T f_i \\ \text{s.t.} \quad & z_i = A_i f_i, \\ & f_i \in L_{\theta_i}, \quad z \in \Omega, i = 1, 2, \dots, M, \end{aligned} \quad (8)$$

where

$$\Omega = \left\{ z = (z_1, z_2, \dots, z_M) \mid \sum_{i=1}^M z_i = -\omega_{ext} \right\}. \quad (9)$$

The Lagrangian dual for the multiblock separable circular-cone programming problem (8) is

$$\max_{\lambda} \min_{f \in L_{\theta}, z \in \Omega} L(f, z, \lambda) = \frac{1}{2} \sum_{i=1}^M f_i^T f_i + \sum_{i=1}^M \lambda_i^T (z_i - A_i f_i), \quad (10)$$

where $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_M)$ and $\lambda_i \in \mathbb{R}^6$.

Under Slater's condition, we have that f^* is an optimal solution of (7) if and only if there exists $(f^*, z^*, \lambda^*) \in L_{\theta} \times \Omega \times \mathbb{R}^{6M}$, which satisfies the Karush–Kuhn–Tucker (KKT) conditions in variational inequality form

$$\begin{cases} \langle f_i - f_i^*, f_i^* - A_i^T \lambda_i^* \rangle \geq 0, & f_i \in L_{\theta_i}, i = 1, 2, \dots, M, \\ \langle z_i - z_i^*, \lambda_i^* \rangle \geq 0, & \forall z_i \in \Omega, i = 1, 2, \dots, M, \\ A_i f_i^* = z_i, & i = 1, 2, \dots, M. \end{cases} \quad (11)$$

The augmented Lagrangian function for the separate structure convex quadratic cone-programming problem is

$$L_{\beta}(f, z, \lambda) = \sum_{i=1}^M \frac{1}{2} f_i^T f_i + \lambda_i^T (z_i - A_i f_i) + \frac{1}{2\beta} \|z_i - A_i f_i\|^2, \quad f_i \in L_{\theta_i}, z_i \in \Omega, \quad (12)$$

where $\beta > 0$.

Algorithm 1 The Exact Multiblock Alternating Direction Method

Given $f^0 \in \mathbb{R}^n$, $z^0 \in \mathbb{R}^{6M}$, $\lambda^0 \in \mathbb{R}^{6M}$, and $\beta > 0$. For $k = 0, 1, 2, \dots$, then

Step 1. Compute f_i^{k+1}

$$\left\langle f_i - f_i^{k+1}, f_i^{k+1} - A_i^T \lambda_i^k + \frac{1}{\beta} A_i^T (A_i f_i^{k+1} - z_i^k) \right\rangle \geq 0, \quad \forall f_i \in L_{\theta_i}. \quad (13)$$

Step 2. Compute z_i^{k+1}

$$\left\langle z_i - z_i^{k+1}, \lambda_i^k - \frac{1}{\beta} (A_i f_i^{k+1} - z_i^{k+1}) \right\rangle \geq 0, \quad \forall z_i \in \Omega. \quad (14)$$

Step 3. Update the Lagrange multiplier

$$\lambda_i^{k+1} = \lambda_i^k - \frac{1}{\beta} (A_i f_i^{k+1} - z_i^{k+1}). \quad (15)$$

The variational inequality form of the exact multiblock alternating direction method for problem (7) is as follows.

In Steps 1 and 2 of Algorithm 1, we should solve variational inequalities (13) and (14). We need to solve a $6M \times 6M$ linear equation to obtain f_i^{k+1} due to the existence of the term $A_i^T A_i f_i^{k+1}$ in (13). In the following analysis, we will convert them to simple projection operations.

Lemma 1 Let Θ be a closed convex set in a Hilbert space and let $P_\Theta(x)$ be the projection of x onto Θ , then [20]

$$\langle z - y, y - x \rangle \geq 0, \quad \forall z \in \Theta \iff y = P_\Theta(x). \quad (16)$$

Substituting $f = f_i^{k+1} - \alpha_i (f_i^{k+1} - A_i^T \lambda_i^k + \frac{1}{\beta} A_i^T (A_i f_i^{k+1} - z_i^k))$ and $y = f_i^{k+1}$ into (16), solving (13) is equivalent to computing the nonlinear equation

$$f_i^{k+1} = P_{L_{\theta_i}} \left[f_i^{k+1} - \alpha_i \left(f_i^{k+1} - A_i^T \lambda_i^k + \frac{1}{\beta} A_i^T (A_i f_i^{k+1} - z_i^k) \right) \right], \quad (17)$$

where α_i can be any positive number.

Substituting $x = z_i^{k+1} - \mu (\lambda_i^k - \frac{1}{\beta} (A_i f_i^{k+1} - z_i^{k+1}))$ and $y = z_i^{k+1}$ into (16), we see that (14) is equivalent to the following nonlinear equation

$$z_i^{k+1} = P_\Omega \left[z_i^{k+1} - \mu \left(\lambda_i^k - \frac{1}{\beta} (A_i f_i^{k+1} - z_i^{k+1}) \right) \right], \quad (18)$$

where μ can be any positive number.

It is not easy to compute f_i^{k+1} directly due to the existence of the term $A_i^T A_i f_i^{k+1}$ in (17). We therefore use a similar approximate approach as that in the paper [21]. Let

$$R_i(f_i^k, f_i^{k+1}) = A_i^T A_i f_i^{k+1} - A_i^T A_i f_i^k - \gamma_i (f_i^{k+1} - f_i^k), \quad i = 1, \dots, M$$

be the residual between $A_i^T A_i f_i^{k+1}$ and their linearization at f_i^k , where $\gamma_i > \lambda_{\max}(A_i^T A_i)$, $\lambda_{\max}(A_i^T A_i)$ is the largest eigenvalue of $A_i^T A_i$. We can use the power method to compute the largest eigenvalue of $A_i^T A_i$ [22].

Instead of computing (17), we compute

$$\begin{aligned} f_i^{k+1} &= P_{L_{\theta_i}} \left[f_i^{k+1} - \alpha_i \left(f_i^{k+1} - A_i^T \lambda_i^k + \frac{1}{\beta} A_i^T (A_i f_i^{k+1} - z_i^k) - \frac{1}{\beta} R_i(f_i^k, f_i^{k+1}) \right) \right] \\ &= P_{L_{\theta_i}} \left[f_i^{k+1} - \alpha_i \left(\frac{\gamma_i + \beta}{\beta} f_i^{k+1} - A_i^T \lambda_i^k - \frac{1}{\beta} A_i^T z_i^k + \frac{1}{\beta} (A_i^T A_i f_i^k - \gamma_i f_i^k) \right) \right]. \end{aligned} \quad (19)$$

Setting $\alpha_i = \frac{\beta}{\gamma_i + \beta}$ in (19), we have

$$f_i^{k+1} = P_{L_{\theta_i}} \left[\alpha_i (A_i^T \lambda_i^k + \frac{1}{\beta} A_i^T z_i^k - \frac{1}{\beta} (A_i^T A_i f_i^k - \gamma_i f_i^k)) \right]. \quad (20)$$

Letting $\mu = \beta$ in (18), we have

$$z_i^{k+1} = P_{\Omega} [A_i f_i^{k+1} - \beta \lambda_i^k]. \quad (21)$$

The projection of z on Ω can be computed easily as follows [17, 19]:

$$z_i^{k+1} = A_i f_i^{k+1} - \beta \lambda_i^k - \frac{1}{M} \left[\sum_{i=1}^M (A_i f_i^{k+1} - \beta \lambda_i^k) + \omega_{ext} \right]. \quad (22)$$

In summary, the inexact multiblock alternating direction method is given as follows.

Remark 3.1 Compared with the exact alternating direction method, the proposed method only needs computation of the proximal points by solving the regularization subproblems

Algorithm 2 The Inexact Multiblock Alternating Direction Method

Given $f^0 \in \mathbb{R}^n$, $z^0 \in \mathbb{R}^{6M}$, $\lambda^0 \in \mathbb{R}^{6M}$, and $\beta > 0$. For $k = 0, 1, 2, \dots$, then

Step 1. Compute f_i^{k+1}

$$f_i^{k+1} = P_{L_{\theta_i}} \left[f_i^{k+1} - \alpha_i \left(\frac{\gamma_i + \beta}{\beta} f_i^{k+1} - A_i^T \left(\lambda_i^k - \frac{1}{\beta} z_i^k \right) + \frac{1}{\beta} (A_i^T A_i f_i^k - \gamma_i f_i^k) \right) \right]. \quad (23)$$

Step 2. Compute z_i^{k+1}

$$z_i^{k+1} = P_{\Omega} [A_i f_i^{k+1} - \beta \lambda_i^k]. \quad (25)$$

Step 3. Update the Lagrange multiplier by

$$\lambda_i^{k+1} = \lambda_i^k - \frac{1}{\beta} (A_i f_i^{k+1} - z_i^{k+1}), \quad i = 1, 2, \dots, M.$$

(19). The proximal points can be obtained by the simple projection onto the cones L_θ and Ω .

Remark 3.2 In Algorithm 2, we compute the f -subproblem and z -subproblem alternately. For the inner of f -subproblem and z -subproblem, we solve the M individual f_i -subproblems and z_i -subproblems in parallel.

4 The convergence result

In this section, we give the convergence analysis of the inexact multiblock alternating direction method for the grasping-force-optimization problem. The convergence analysis method can be extended from the convergence results of the alternating direction methods for convex quadratically constrained quadratic semidefinite programs in the paper [21].

Lemma 2 *The sequence $\{f_i^k, z_i^k, \lambda_i^k\}$ generated by the inexact multiblock alternating direction method satisfies*

$$\begin{aligned} & \left\langle f_i^{k+1} - f_i^*, \frac{1}{\beta} R_i(f_i^k, f_i^{k+1}) \right\rangle + \frac{1}{\beta} \langle z_i^{k+1} - z_i^*, z_i^k - z_i^{k+1} \rangle \\ & + \beta \langle \lambda_i^{k+1} - \lambda_i^*, \lambda_i^k - \lambda_i^{k+1} \rangle \geq 0, \end{aligned} \quad (25)$$

where $\{f_i^*, z_i^*, \lambda_i^*\}$ is a KKT point of system (11).

Proof Substituting $z_i = z_i^{k+1}$ into the second inequality in system (11), we have

$$\langle z_i^{k+1} - z_i^*, \lambda_i^* \rangle \geq 0. \quad (26)$$

Substituting $z_i = z_i^*$ into (11), and coupled with (15), we have

$$\langle z_i^* - z_i^{k+1}, \lambda_i^{k+1} \rangle \geq 0. \quad (27)$$

Adding (26) and (27) together, we have

$$\langle z_i^{k+1} - z_i^*, \lambda_i^* - \lambda_i^{k+1} \rangle \geq 0. \quad (28)$$

In addition, coupled with (14) and (15), we have

$$\langle z_i^k - z_i^{k+1}, \lambda_i^{k+1} \rangle \geq 0, \langle z_i^{k+1} - z_i^k, \lambda_i^k \rangle \geq 0.$$

Adding the two inequalities above, we have

$$\langle z_i^{k+1} - z_i^k, \lambda_i^k - \lambda_i^{k+1} \rangle \geq 0. \quad (29)$$

Note that (23) can be written equivalently as

$$\left\langle f_i - f_i^{k+1}, f_i^{k+1} - A_i^T \lambda_i^k + \frac{1}{\beta} A_i^T (A_i f_i^{k+1} - z_i^k) - \frac{1}{\beta} R_i(f_i^k, f_i^{k+1}) \right\rangle \geq 0,$$

$$\forall f_i \in L_{\theta_i}.$$

Substituting $f_i = f_i^*$ into (23), we have

$$\left\langle f_i^* - f_i^{k+1}, f_i^{k+1} - A_i^T \lambda_i^{k+1} + \frac{1}{\beta} A_i^T (z_i^{k+1} - z_i^k) - \frac{1}{\beta} R_i(f_i^k, f_i^{k+1}) \right\rangle \geq 0. \quad (30)$$

Substituting $f_i = f_i^{k+1}$ into the first inequality in system (11), we have

$$\langle f_i^{k+1} - f_i^*, f_i^* - A_i^T \lambda_i^* \rangle \geq 0. \quad (31)$$

Adding (30) and (31) together, we have

$$\begin{aligned} & \langle f_i^{k+1} - f_i^*, A_i^T (\lambda_i^{k+1} - \lambda_i^*) \rangle + \left\langle f_i^{k+1} - f_i^*, \frac{1}{\beta} A_i^T (z_i^k - z_i^{k+1}) \right\rangle \\ & + \left\langle f_i^{k+1} - f_i^*, \frac{1}{\beta} R_i(f_i^k, f_i^{k+1}) \right\rangle \geq \langle f_i^{k+1} - f_i^*, f_i^{k+1} - f_i^* \rangle \geq 0. \end{aligned} \quad (32)$$

From the first part of the left side of (32) and the third equation in system (11), we have

$$\begin{aligned} & \langle f_i^{k+1} - f_i^*, A_i^T (\lambda_i^{k+1} - \lambda_i^*) \rangle \\ & = \langle A_i f_i^{k+1} - A_i f_i^*, \lambda_i^{k+1} - \lambda_i^* \rangle \\ & = \beta \langle \lambda_i^{k+1} - \lambda_i^*, \lambda_i^k - \lambda_i^{k+1} \rangle - \langle z_i^{k+1} - z_i^*, \lambda_i^* - \lambda_i^{k+1} \rangle. \end{aligned} \quad (33)$$

From the second part of the left side of (32), we have

$$\begin{aligned} & \left\langle f_i^{k+1} - f_i^*, \frac{1}{\beta} A_i^T (z_i^k - z_i^{k+1}) \right\rangle \\ & = \frac{1}{\beta} \langle A_i (f_i^{k+1} - f_i^*), z_i^k - z_i^{k+1} \rangle \\ & = \frac{1}{\beta} \langle z_i^{k+1} - z_i^*, z_i^k - z_i^{k+1} \rangle - \langle z_i^{k+1} - z_i^k, \lambda_i^k - \lambda_i^{k+1} \rangle. \end{aligned} \quad (34)$$

It follows from (28), (29), and (32)–(34) that

$$\begin{aligned} & \left\langle f_i^{k+1} - f_i^*, \frac{1}{\beta} R_i(f_i^k, f_i^{k+1}) \right\rangle + \frac{1}{\beta} \langle z_i^{k+1} - z_i^*, z_i^k - z_i^{k+1} \rangle \\ & + \beta \langle \lambda_i^{k+1} - \lambda_i^*, \lambda_i^k - \lambda_i^{k+1} \rangle \geq 0. \end{aligned} \quad \square$$

Now, we give the convergent conclusion.

Theorem 1 *The sequence $\{f_i^k, z_i^k, \lambda_i^k\}$ generated by the inexact multiblock alternating direction method converges to a solution $\{f_i^*, z_i^*, \lambda_i^*\}$ of problem (11).*

Proof We denote

$$w_i = \begin{bmatrix} f_i \\ z_i \\ \lambda_i \end{bmatrix}, \quad G_i = \begin{bmatrix} \frac{1}{\beta} (\gamma_i I_3 - A_i^T A_i) & 0 & 0 \\ 0 & \frac{1}{\beta} I_3 & 0 \\ 0 & 0 & \beta I_3 \end{bmatrix},$$

where I_3 denotes the 3-dimensional unit matrix. Obviously, G_i is positive-definite. Here, we define the G_i -inner product of w_i and \bar{w}_i as

$$\langle w_i, \bar{w}_i \rangle_{G_i} = \left\langle x_i, \frac{1}{\beta} (\gamma_i I_3 - A_i^T A_i) \bar{x}_i \right\rangle + \frac{1}{\beta} \langle z_i, \bar{z}_i \rangle + \beta \langle \lambda_i, \bar{\lambda}_i \rangle,$$

and the associated G_i -norm as

$$\|w_i\|_{G_i} = \left\{ \|x_i\|_{\frac{1}{\beta}(\gamma_i I_3 - A_i^T A_i)}^2 + \frac{1}{\beta} \|z_i\|_2^2 + \beta \|\lambda_i\|_2^2 \right\}^{\frac{1}{2}}.$$

Observe that, solving the optimal condition (11) of problem (7) is equivalent to finding a zero point of the residual function

$$e(w) = \left\| \begin{array}{c} f_i - P_{L_{\theta_i}}(f_i - \alpha_i(f_i - A_i^T \lambda_i)) \\ z_i - P_{\Omega}(z_i - \mu \lambda_i) \\ A_i f_i - z_i \end{array} \right\|_2. \quad (35)$$

From (13) and (17), we have that

$$f_i^{k+1} = P_{L_{\theta_i}} \left[f_i^{k+1} - \alpha_i \left(f_i^{k+1} - A_i^T \lambda_i^{k+1} + \frac{1}{\beta} A_i^T (z_i^{k+1} - z_i^k) - \frac{1}{\beta} R_i(f_i^k, f_i^{k+1}) \right) \right]. \quad (36)$$

From (15) and (18), we have

$$z_i^{k+1} = P_{\Omega} [z_i^{k+1} - \mu \lambda_i^{k+1}]. \quad (37)$$

Based on (35)–(37), and the nonexpansion property of the projection operator, we have

$$\begin{aligned} \|e(w_i^{k+1})\|_2 &\leq \left\| \begin{array}{c} \frac{\alpha_i}{\beta} A_i^T (z_i^k - z_i^{k+1}) + \frac{\alpha_i}{\beta} R_i(f_i^k, f_i^{k+1}) \\ 0 \\ \beta (\lambda_i^k - \lambda_i^{k+1}) \end{array} \right\|_2 \\ &\leq \left\| \begin{array}{c} \frac{\alpha_i}{\beta} R_i(f_i^k, f_i^{k+1}) \\ 0 \\ \beta (\lambda_i^k - \lambda_i^{k+1}) \end{array} \right\|_2 + \left\| \begin{array}{c} \frac{\alpha_i}{\beta} (z_i^k - z_i^{k+1}) \\ 0 \\ 0 \end{array} \right\|_2 \\ &\leq \delta \|w_i^k - w_i^{k+1}\|_{G_i}, \end{aligned} \quad (38)$$

where δ is a positive constant depending on parameters α_i , β , γ_i , and the largest eigenvalue of $A_i^T A_i$, for example, setting

$$\delta = \sqrt{\max \left\{ \beta, \frac{\alpha_i^2}{\beta}, \alpha_i^2 \lambda_{\max} \left(\frac{1}{\beta} (\gamma_i I_3 - A_i^T A_i) \right) \right\}}. \quad (39)$$

From Lemma 2, we can write (25) as

$$\langle w_i^{k+1} - w_i^*, w_i^k - w_i^{k+1} \rangle_{G_i} \geq 0,$$

which implies that

$$\langle w_i^k - w_i^*, w_i^k - w_i^{k+1} \rangle_{G_i} \geq \|w_i^k - w_i^{k+1}\|_{G_i}.$$

Thus,

$$\begin{aligned} & \|w_i^{k+1} - w_i^*\|_{G_i}^2 \\ &= \|(w_i^k - w_i^*) - (w_i^k - w_i^{k+1})\|_{G_i}^2 \\ &= \|w_i^k - w_i^*\|_{G_i}^2 - 2\langle w_i^k - w_i^*, w_i^k - w_i^{k+1} \rangle_{G_i} + \|w_i^k - w_i^{k+1}\|_{G_i}^2 \\ &\leq \|w_i^k - w_i^*\|_{G_i}^2 - \|w_i^k - w_i^{k+1}\|_{G_i}^2 \\ &\leq \|w_i^k - w_i^*\|_{G_i}^2 - \frac{1}{\delta^2} \|e(w_i^{k+1})\|_2^2. \end{aligned} \quad (40)$$

From the above inequality, we have

$$\|w_i^{k+1} - w_i^*\|_{G_i}^2 \leq \|w_i^k - w_i^*\|_{G_i}^2, \quad k = 1, 2, \dots \quad (41)$$

That is, the sequence $\{w_i^k\}$ is bounded. Thus, there exists at least one cluster point of $\{w_i^k\}$.

It also follows from (40) that

$$\sum_{k=0}^{\infty} \frac{1}{\delta^2} \|e(w_i^{k+1})\|_2^2 < +\infty,$$

and thus

$$\lim_{k \rightarrow \infty} \|e(w_i^{k+1})\|_2 = 0.$$

Let \bar{w}_i be a cluster point of $\{w_i^k\}$ and the subsequence $\{w_i^{k_j}\}$ converges to \bar{w}_i . We have

$$\|e(\bar{w}_i)\|_2 = \lim_{j \rightarrow \infty} \|e(w_i^{k_j})\|_2 = 0,$$

so \bar{w}_i satisfies system (11). Setting $w_i^* = \bar{w}_i$, we have

$$\|w_i^{k+1} - \bar{w}_i\|_{G_i} \leq \|w_i^k - \bar{w}_i\|_{G_i},$$

the sequence $\{w_i^k\}$ satisfies

$$\lim_{k \rightarrow \infty} w_i^k = \bar{w}_i. \quad \square$$

Theorem 2 *The sequence $\{f^k\}$ generated by the inexact multiblock alternating direction method converges to a solution point f^* of problem (3).*

Proof It is easily proved by using Theorem 1. □

5 Simulation experiments

In this section, we describe simulation experiments to verify the efficiency of the proposed method. All the algorithms are run on a personal computer with a 1.80 GHz Intel Core processor and 2.0 GB Ram.

Example 5.1 We consider a three-fingered grasping-force-optimization example, which is from Example 5.3 in [23]. A polyhedral is grasped by a three-fingered robot, and the positions of the grasping points are $p_1 = (0, 1, 0)^T$, $p_2 = (1, 0.5, 0)^T$, and $p_3 = (0, -1, 0)^T$. The mass of the polyhedral is $M = 0.1$ kg, and the friction coefficient of each finger is $\mu_i = 0.6$. The robot hand moves along a vertical circular trajectory of radius $r = 0.2$ with a constant velocity $v = 0.4\pi$ m/s. The grasping transformation matrix is

$$A = \begin{pmatrix} 0 & 0 & 1 & -1 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & -0.5 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0.5 & 0 & -1 & 0 & 1 & 0 \end{pmatrix}.$$

A time-varying external wrench applied to the center of mass of the object is

$$\omega_{ext} = (0, f_c \sin \theta(t), f_c \cos \theta(t) - Mg, 0, 0, 0)^T,$$

where $g = 9.8$ m/s², $f_c = Mv^2/r$ is the centripetal force, $t \in [0, 1]$, and $\theta(t) = vt/r \in [0, 2\pi]$.

In the simulation experiments, we compare the performances of the proposed inexact multiblock alternating direction method with the primal-dual interior-point method for grasping-force optimization. As is known, the primal-dual interior-point method has been proved to be one of the most efficient methods for SOCP. Here, the Matlab codes for primal-dual interior-point method are designed by the SeDuMi software package [24]. However, SeDuMi cannot solve the grasping-force-optimization problem (5) directly, hence we solve the equivalent linear second-order cone-programming problem [25]

$$\begin{aligned} & \min t \\ & \text{s.t. } Af + \omega_{ext} = 0, \\ & \quad \sqrt{(t-1)^2 + 2\|f\|^2} \leq t+1, \\ & \quad \|(f_x^i, f_y^i)^T\| \leq \mu_i f_z^i, \quad i = 1, \dots, M. \end{aligned} \tag{42}$$

The stopping criteria for SeDuMi is $\text{pars.eps} = 10^{-4}$, which indicates that the duality gap of the problem (42) is less than 10^{-4} .

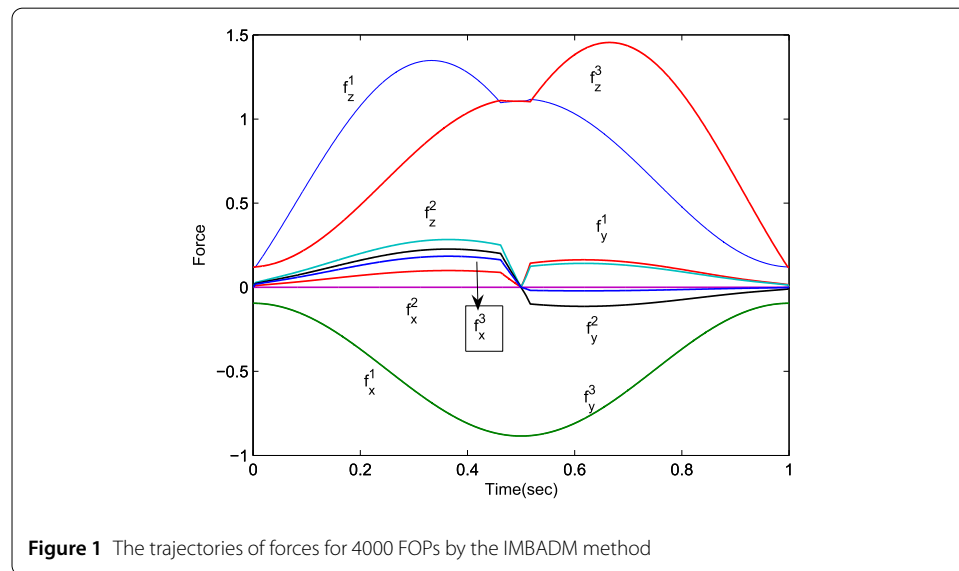
Let $g(f) = \frac{1}{2}f^T f$. The stopping criteria of our proposed algorithm is

$$\max \{ \|f^k - f^{k-1}\|, \|z^k - z^{k-1}\|, \|\lambda^k - \lambda^{k-1}\|, \|g(f^k) - g(f^{k-1})\| \} \leq \epsilon.$$

Here, we set $\beta_i = 0.2$, $\gamma_i = \lambda_{\max}(A_i^T A_i) + 0.000001$, and $\epsilon = 10^{-4}$.

Table 1 The test results for the multiple force-optimization problems of Example 5.1

Methods	4000		2000	
	Iter.	Time	Iter.	Time
IMBADM	88	0.072	88	0.072
SeDuMi	6.47	0.248	6.47	0.248



We give the test results for the 4001 force-optimization problems with $t = 0 : 1/4000 : 1$, and the 2001 force-optimization problems with $t = 0 : 1/2000 : 1$ in Table 1. The value of ω_{ext} will be recalculated for each of the 4000 and 2000 force-optimization problems since ω_{ext} is a time-varying external wrench. The inexact multiblock alternating direction method uses zeros as the initial point for each of the grasping-force-optimization problems. In Table 1, “Time” represents the average CPU time (in seconds), and “Iter.” represents the average number of iterations. In addition, “IMBADM” represent the inexact multiblock alternating direction method, respectively.

The results in Table 1 show that the inexact multiblock alternating direction method has more iteration steps than SeDuMi, but costs less CPU time than SeDuMi.

The optimal forces for the 4000 force-optimization problems solved by the inexact multiblock alternating direction method are shown in Fig. 1. In addition, the inexact multiblock alternating direction method is compared with SeDuMi, and the comparative results of optimal force f_z^1 , f_x^1 , and f_y^1 are shown in Fig. 2.

The results in Fig. 2 show that the optimal forces solved by the two methods are similar.

Example 5.2 The second test example is the min-max force-optimization problem from [8]. In the model, the size of the set of contact forces is measured by the maximum magnitude of the M contact forces:

$$F = \max\{\|f^i\|, i = 1, 2, \dots, M\}.$$

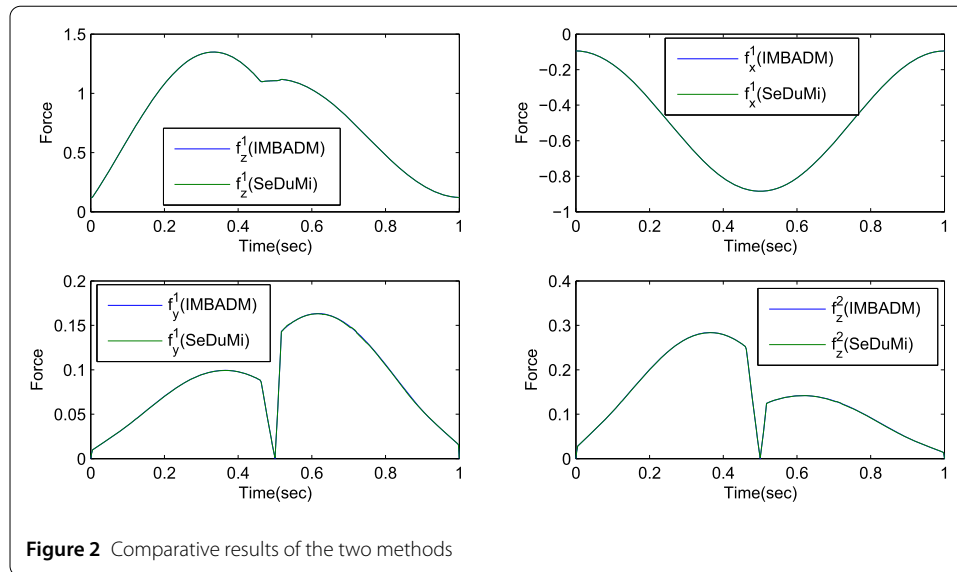


Table 2 The test results for the min-max force-optimization problems of Example 5.2

Methods	4000		2000	
	Iter.	Time	Iter.	Time
IMBADM	223	0.223	223	0.223
SeDuMi	6.47	0.248	6.47	0.248

The new model minimizes the force F , while satisfying the constraints (1) and (2), which is modeled as [8]

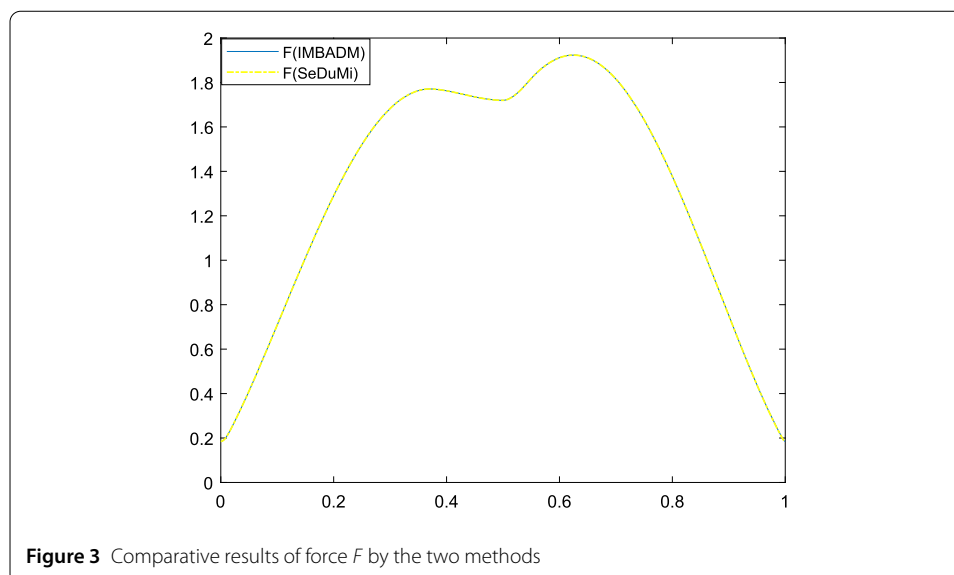
$$\begin{aligned}
 &\min F \\
 &\text{s.t. } Af + \omega^{ext} = 0, \\
 &\quad \|(f_x^i, f_y^i)^T\| \leq \tan\theta_j f_z^i, \quad i = 1, 2, \dots, M, \\
 &\quad \|(f_x^i, f_y^i, f_z^i)^T\| \leq F, \quad i = 1, 2, \dots, M.
 \end{aligned} \tag{43}$$

The inexact multiblock alternating direction method and the primal-dual interior-point method are used to solve Example 5.2. For the 4000 and 2000 min-max force-optimization problems, the test results are shown in Table 2 and Fig. 3.

From Table 2, we see the IMBADM costs less CPU time than that of the interior-point method. However, the number of iteration steps of SeDuMi is less than that of the IMBADM.

In addition, we compare the optimal forces for the 4000 force-optimization problems solved by two methods, and the comparative results of optimal forces F is shown in Fig. 3. The result in Fig. 3 shows that the optimal forces solved by the two methods have similar values to the same accuracy.

Figures 1–3 and Tables 1 and 2 demonstrate that our method is efficient for the two grasping-force-optimization problems.



6 Conclusions

The two models of the grasping-force-optimization problems are exact, which rewrite the friction-cone constraints as the Cartesian product of circular cones. We propose the parallel inexact alternating direction method to solve the multiblock two problems. In the proposed method, we compute the f -subproblem and z -subproblem alternately. For the inner of the f -subproblem and z -subproblem, we solve the M individual f_i -subproblems and z_i -subproblems in parallel. Furthermore, we prove the global convergence of the proposed method. By the numerical examples of the grasping-force-optimization problems, we conclude that the proposed method is efficient.

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Availability of data and materials

The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Competing interests

The authors declare that they have no competing interests.

Author contribution

YZ designed the inexact multiblock alternating direction method for the grasping-force optimization of multifingered robotic hands and carried out the convergence analysis. XM performed the experiments of the grasping-force optimization of multifingered robotic hands, and was a major contributor in writing the manuscript. All authors read and approved the final manuscript.

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