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# Stochastic Cost Modeling for Second-hand Products' Optimum Warranty Period and Upgrade Level

Maryam Dehghanbaghi<sup>1,\*</sup>, Azadeh Dabbaghi<sup>2</sup><sup>1</sup> Department of Industrial Engineering, Robat karim Branch, Islamic Azad University, Tehran Iran; dehghanbaghi@yahoo.com;<sup>2</sup> Research institute of petroleum Industry, Tehran, Iran; azadehdabbaghi@yahoo.com.

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## Abstract

After the second-hand products return to the second-hand markets, either goods, parts, or materials are reused or disposed. For optimum sales, these products are under the process of determining the optimal warranty period and warranty policy, so that both the seller's profit and the consumer's profit are met. Therefore, estimating the warranty costs for future claims on second-hand products is very necessary. In this paper, a hybrid stochastic model is presented to improve the reliability of second-hand products under the free repair replacement warranty policy to determine the level of upgrade, with the aim of reducing warranty costs. Using the upgrade actions, the model, based on three approaches, minimal-perfect repair, virtual age, and improvement factor, is developed for estimating the warranty costs of second-hand products. The contribution of this research is the application of three upgrade approaches in the warranty cost model simultaneously to estimate the warranty costs and optimum upgrade level more realistic. Finally, under different product lifetime, a numerical example and sensitivity analysis are provided. Evaluation is presented in four lifetimes for second-hand products ranging from one to four years for which the optimum upgrade level and the warranty period are determined. The results show that the higher the level of second-hand product upgrades to a certain extent, the higher the savings, but the more upgrades are not cost-effective for second-hand products.

**Keywords:** Warranty, Second-hand product, Warranty costs, Stochastic modeling, Upgrade.



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## 1 | Introduction

During the past 10 years, the market share of the second-hand products has rapidly increased due to the fact that the highly developed technologies have resulted in a longer product life and a higher price of the new product. Warranty for Second-hand product is now provided by dealers in home appliances, personal computers, automobiles, etc. The warranty offers useful product information and leads to customer confidence in the quality of the product and serves as a



Corresponding Author: dehghanbaghi@yahoo.com



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promotional tool for sellers or dealers to provide different types of warranty policies for their used products. Warranty policy is a statement of compensation offered by the dealer in the event of failure. A better warranty policy usually provides a higher product quality and an assurance to buyers. Warranty provides assurance to customers and signals product reliability. In fact, the laws, regulations and standards for second-hand items aim to guarantee that they will operate as expected at least for a specific warranty period [1]. Hence, in order to promote sales as well as to improve the reliability of the second-hand product, dealers usually perform various maintenance actions not only at the pre-sale point but also during warranty period [2]. According to the definitions provided, the warranty improves the quality of the product (from the customer's point of view) and guarantees repair, replacement or refund in case of the faulty product.

Due to some problems may occur for these products, such as uncertain durability and performance, dealers tend to carryout upgrade actions on the used products to restore the failed products during the warranty period. The main goal of the dealers is to minimize warranty costs. To reach this goal, numerous methods and approaches have been developed to obtain optimal upgrade level and warranty period. On the one hand, the warranty period is determined by indicators such as the product usage period, product lifetime, and reliability and maintenance history. On the other hand, the quality of returned products plays a significant role in product warranty decisions. Therefore, the warranty period may differ for each product categories. It is also true in general that users of second-hand products have more reason to be concerned about potential product failures than do purchasers of new products and may thus want longer warranty periods. From the dealer's perspective, however, the longer the warranty period, the higher risk of a product failure and thus the higher the cost of the warranty [3]. Thus, the determination of optimal warranty period can play an active role in the costs incurred.

The main objective of this paper is to investigate a stochastic cost model for second-hand products under given warranty costs and failure rate structures. The presented model incorporates different upgrade actions (explained in section 3.1) and results in the derivation of the optimal expected upgrade level to minimize the dealer's expected costs per second-hand product during the different warranty periods. The model makes a useful contribution to the reliability improvement literature as it captures different upgrade actions occurred in reality. In our study, the product lifetime distribution is assumed to be the Weibull distribution for which an appropriate choice of the parameter values results in a flexible model for the failure rate distribution.

The remainder of the paper is organized as follows: in Section 2, we present related literature review of warranty for second hand products and a brief overview of upgrade actions so as to set the background for the main contribution of the paper. In Section 3, first, we discuss upgrade actions then we formulate a new upgrade-based costing stochastic model. In Section 4, we numerically analyzed the proposed model using a real case study. Finally, the concluding remarks are given in Section 5.

## 2 | Literature Review

Although the number of papers in the fields of new products' warranty, reliability and maintenance ([4]-[6]) is considerable, the number of similar surveys on second-hand products is limited. This brief review of the literature mainly emphasizes and discusses the warranty policies, upgrade actions and the cost modeling of second-hand products.

In 1999, Murthy and Chattopadhyay [7] present a classification on the second-hand products warranty. They calculate the average warranty costs using product lifetime and various warranty policies. Chattopadhyay and Murthy [8] introduce new warranty policies for second-hand products and develop statistical models for cost analysis. In the presented policy, both the buyer and the seller participate in the warranty costs, and if a product is damaged during the warranty period, each buyer and seller will be responsible for a share of it. Chattopadhyay and Murthy [9] set the optimal upgrade level for second-hand products under the warranty policy. Shafiee et al. [10] present a sustainable improvement approach for the second-hand items under the free repair/replacement policy to determine the optimal upgrade level in

which uncertainties are reduced and the remanufacturing process of second-hand items is also improved. Yu and Peng [11] analyze the impact of remanufactured product quality, including initial product design, component mismatch, and reassembly errors, on the warranty costs of these products by modelling reliability and warranty costs under free rectification lifetime warranty policy. Saidi-Mehrabad et al. [12] use two strategic approaches to decide on reliability improvement strategies for second-hand products sold under different warranty policies. These approaches are achieved when the reduction in warranty costs outweighs the additional costs of upgrade actions.

A review of warranty cost models is conducted by Aksezer [13] with an emphasis on the analysis of failures of second-hand vehicles considering their age, function and maintenance records. Chukova and Shafiee [14] develop a model of one-dimensional warranty cost estimates for second-hand products, in which warranty policies are examined and compared. Yazdian et al. [15] present a combined mathematical-stochastic model to determine the optimal price and warranty period for the remanufacturing of second-hand products. This approach improves the reliability of the second-hand products to reach higher quality levels. Park et al. [3] propose a cost analysis for second-hand products to determine an optimal warranty period. The authors introduce the concept of full refund rather than replacement in order to deal with the situation when a failed product cannot be repaired within the warranty period. Lu and Shang [16] propose a warranty mechanism for the second-hand electronic market.

Other studies on the second-hand products warranty are related to two-dimensional warranty approaches. There are three main approaches for cost modeling under a two-dimensional warranty; "Two-variable distribution function" approach, "linear combination" approach and the "performance rate as a random variable" approach.

Dealers in second-hand products typically upgrade the product prior to sale to minimize the warranty costs, which would be incurred during the warranty period, as well as to enhance the product value by improving its performance and reliability [17]. Upgrade actions on second-hand products are with the aim of restoring the products to working condition, reducing its failure rate during the warranty period, increasing the remaining life of the product, etc. [18]. Therefore, dealers of second-hand products often have the option to upgrade an item to improve its reliability. It is notable that upgrade is worthwhile only if the cost of upgrade is less than the expected reduction in the warranty cost. Detecting defective parts, they are divided into two categories, repairable and non-repairable. Repairable parts are repaired and upgraded. If these actions are not cost-effective, they are recycled or destroyed. If the repair is done, the reliability returns to pre-failure condition or increases. Thus, the failure rate decreases during the warranty period. The following three upgrade action strategies are considered in this study:

- *A minimal repair makes a minor improvement, and the product operational state is "as bad as old."*
- *An imperfect repair restores the item to an operational state that is between "as good as new" and as bad as old.*
- *A perfect (complete) repair makes the item's operational state as good as new [19].*

The higher the quality of supplied second-hand products, the stronger the application of each strategy, from the first to the third, will be. On the other hand, the seller bears more cost. Increasing the reliability of the second-hand products is related to the products entered the second-hand market with a specific warranty period and upgrade actions.

Due to the limited number of articles in the field of upgrade actions on used products, in this section, a review of the research background related to improving the reliability of second-hand products until 2019 is done. An example of basic research in which the proposed model has been used in many reliability studies is the paper by Hussain and Murthy [20]. They proposed a stochastic model based on the upgrade coefficient in improving reliability of the second-hand products. One of the most important articles increasing the reliability of second-hand products offered with warranty policies is presented by Chattopadhyay and Murthy [9]. Given that the supply of second-hand products under different warranty

policies leads to additional costs for sellers, they tend to take steps to improve and enhance the reliability of these products before offering so that they can bear lower costs during the warranty period. Shafiee et al. [21] and [22] develop two costing models for warranty with upgrade actions. The first model is based on the combination of virtual age and minimal repair approaches, and the second model is based on a change in the improvement coefficient approach and based on the upgrade level. By the two proposed models, the sum of warranty costs and the optimal upgrade level are determined. They also proposed other models in the form of joint determination of warranty period, upgrade level and price of second-hand products under different warranty policies [23] and [24]. Soltani Neshan and Asgharizadeh [25] present a model for estimating warranty costs after upgrade actions using a combination of the virtual age approach and minimal repairs along with pricing for the second-hand biochemical equipment. The result of the model is a reduction in failure costs during the warranty period under the above approaches. A cost modeling is also developed by Darghouth et al. [26] to determine the optimal upgrade level for second-hand products under the free repair replacement policy with periodic maintenance and repairs during the warranty period. A pre-sale upgrade model is developed for repairable used products sold with two-dimensional warranty policies by Su and Wang [27]. Two kinds of two-dimensional warranty regions, including rectangular and L-shaped, are considered. The optimal upgrade policy is determined to minimize the dealer's total expected servicing cost. Yazdian et al. [15] present a model for simultaneously determining the optimal price, warranty and the upgrade level of second-hand products by the virtual age approach while taking into account price and warranty based on demand. Wang et al. [28] proposed a model for determining the optimal upgrade level in second-hand production equipment with and without maintenance programs. Zhang et al. [29] presented a stochastic model for minimizing short-term warranty costs for two-component systems. Wang et al. [30] developed an upgrade model for sophisticated second-hand products under the of free repair-replacement policy.

From the warranty literature reviewed, second-hand products' warranty is offered under two approaches: warranty after upgrades and warranty without upgrades. In this paper, a hybrid stochastic model is presented to improve the reliability of second-hand products under the free repair replacement warranty policy, which aims to determine the optimum level of upgrade, as a decision variable, with the aim of reducing warranty costs. Where the modelling proposed by Shafiee et al. [31] is developed, and a new approach to estimating warranty costs under the upgrade approach is presented. *Table 1* compares the model developed in this research and the model presented by Shafiee et al. [31]. The contribution of the presented model is that we integrated the three upgrade actions including "virtual age", "improvement coefficient" and "minimal perfect repair" actions. The proposed integration helps modeling almost all real situations occurred during the second-hand products' upgrade.

**Table 1. Comparing the proposed model and the existing approach.**

Authors	Virtual age	Improvement coefficient	Minimal- perfect repair
Shafie et al. [23]	☒	☐	☒
The presented model	☒	☒	☒

### 3 | Modeling Warranty Costs under the Upgrade Approach

This section first analyzes and describes upgrade approaches. Then, a new upgrade-based costing stochastic model is also developed as a contribution to the reliability improvement literature. The warranty costs are determined based on the upgrade approaches in the developed model. All the steps that the seller can take to sell the second-hand product in the best possible way are presented. The model considers both the seller and the buyer's profits simultaneously. This study supports dealers' decision making on how much to invest in reliability improvement programs.

There exist different approaches for modelling reliability improvement programs in the literature. But in general, the three main approaches, "minimal-perfect repair", "improvement factor" and "virtual age" approaches are considered:

- **Minimal- perfect repair Approach:** In this approach, a second-hand product is returned to the ‘good-as-new’ state and fully repaired with probability  $p$ . As well, a used product is minimally repaired and restored to its functioning state, with probability  $1-p$ . This approach is proposed by Brown and Proschan [32] developed in later years.
- **Improvement factor Approach:** In this approach, improvement coefficient indicates the effect of upgrade actions on the second-hand product in reducing the failure rate during the warranty period. The coefficient is considered between 0 and 1. For which, 0 indicates the ineffectiveness of the action and 1 shows the complete refurbishing of the second-hand product.
- **Virtual Age Approach:** Each improvement action on a second-hand product leads to its rejuvenation depending on the upgrade level. It is the basis of the virtual age approach proposed by Kijima [33], in which the modelling is based on the virtual age of the product after the upgrade action.

The second-hand product’s age is reduced to a virtual value of  $v_t$  at time  $t$  after the upgrade at  $a$ , usually expressed as  $ka$  in Eq. (1).  $K$  is the ‘virtual age parameter’ ( $0 \leq k \leq 1$ ). There are only a few references in the literature that deal with the estimation of parameter  $k$ .

$$v_t = ka + (t-a) \quad t \geq a. \quad (1)$$

### 3.1 | Warranty Cost Modeling for Second-hand Product with Upgrade Action

Based on the approaches presented for the second-hand products upgrade, in this section, a hybrid stochastic model of existing approaches has been developed to find the cost and determine the optimal warranty period and the upgrade level. The proposed model is designed to facilitate the use of each approach separately, combining the three approaches and applying the characteristics of each. The model is based on the models presented by Shafiee et al. [31] described in the literature. They presented two models. One is based on the combination of two approaches of virtual age and minimal-perfect repair and the other is based on the improvement coefficient approach. During the upgrade action, a second-hand product can be in the range of minimal to perfect repair to improve the reliability, which is not 100% effective. Also, it may reduce its age after the upgrade action. Therefore, in the real world, a combination of three approaches to estimating warranty costs seems reasonable and practical. The model presented in this section is obtained by combining these three approaches in the form of a new model described below:

Let  $T$  be a random variable describing the failure time of the product with cumulative distribution function  $F(t)$  and the density function  $f(t) = dF(t)/dt$ . The Rate of Occurrence of Failures (ROCOF) is the Non- Homogeneous Poisson Process (NHPP) based on Eq. (2).

$$\Lambda_T(t) = \frac{f_T(t)}{1 - F_T(t)}. \quad (2)$$

On the basis of minimal-perfect approach, when the product is under repair, the failure rate turns to the failure rate before repair,  $\Lambda_T(t)$ . In complete repair the failure rate decreases to  $\Lambda_T(t-a)$ . It is assumed that failures occur based on point process.

Based on ‘Minimal-perfect repair approach’, we assume that, for given  $p$  the used product undergoes a ‘complete repair’ with probability  $p$  and a ‘minimal repair’ with a probability  $(1-p)$ . Thus, the rate of occurrence of failures of a second-hand products with age “ $a$ ” after an upgrade action of level  $u$  is given by Eq. (3).



On the other hand, according to Hussain and Murthy [20], the improvement coefficient, which is a function of the second-hand products' upgrade, is defined as *Eq. (4)*.  $p'$  is the effectiveness of the upgrade action value. If  $p'$  is equal to 1, the upgrade action is quite effective.

$$\Lambda_T(t, u) = p \times \Lambda_T(t - a) + (1 - p) \times \Lambda_T(t) \quad t > a. \quad (3)$$

$$\delta_u = (1 - e^{-\theta u}) \times p' \quad p' \in \{0, 1\}. \quad (4)$$

This parameter is considered due to the uncertainty in the effectiveness of improvement upgrade actions and to create conformity to the real world. Therefore, the parameters  $p$  and  $p'$  can be considered as random variables. According to the related literature, one of the most suitable random distributions for these two parameters is the Beta distribution ([34] and [35]). Because this function accepts finite values and can model the results realistically and experimentally. In addition, the number of parameters of this distribution for estimation is limited, which is defined as *Eq. (5)* [24]. Its mathematical expectation is equal to *Eq. (6)*.

$$\beta_p(p) = \frac{1}{\beta(\alpha_1, \alpha_2)} p^{\alpha_1-1} (1-p)^{\alpha_2-1}, \delta_u = (1 - e^{-\theta u}) \times p', \quad (5)$$

$$\beta_p(p) = \frac{1}{\beta(\alpha_1, \alpha_2)} p^{\alpha_1-1} (1-p)^{\alpha_2-1}, \delta_u = (1 - e^{-\theta u}) \times p'. \quad (6)$$

$$E(p) = \frac{\alpha_1}{\alpha_1 + \alpha_2}, \quad E(p') = \frac{\alpha'_1}{\alpha'_1 + \alpha'_2}. \quad (6)$$

The random variable  $p$  is related to the value of  $u$ . It is assumed that based on the nonlinear *Eq. (7)* and *Eq. (8)* that Hussain [36] presented,  $a_1, b_1, a_2, b_2$ , and also  $a'_1, b'_1, a'_2, b'_2$  can be calculated through logarithmic regression.

$$\alpha_1 = a_1 u^{a_2}, \alpha'_1 = a'_1 u^{a'_2}. \quad (7)$$

$$\alpha_2 = b_1 u^{b_2}, \alpha'_2 = b'_1 u^{b'_2}. \quad (8)$$

For modeling warranty costs using the combined approach, it is possible to calculate the average warranty costs of a second-hand product with the age of " $a$ " to compare the costs in the case of upgrade and without upgrade under the free replacement warranty policy. The savings from upgrade actions are also obtained from the difference in costs with and without the upgrade actions.

### 3.1.1 | Costs of upgrade actions for second hand product

If the recovery of second-hand products leads to their upgrade, an investment, equal to the upgrade cost, is made through the seller or the manufacturer. The costs are usually in the form of initial costs for start-up, upgrade costs, equipment procurement costs, overhead costs, parts replacement and etc.

Assume that  $C_u(a)$  is the amount of capital spent by the seller to improve the quality and performance of the second-hand product with the age of " $a$ " and upgrade level of  $u$ . Using the model of Pongpech and Murthy [19], the cost of upgrade action, which is a non-linear function of product age and upgrade strategy, is formulated as *Eq. (9)*.

$$C_u(a) = \frac{\alpha \times u}{1 - \exp[-\phi(a - u)]}. \quad (9)$$

In this model,  $\phi$  and  $a$  are two positive parameters estimated using the past history of the product. If  $u = 0$ , no upgrade action is made on the second-hand product, and the cost is 0, and if  $u = a$ , the product

undergoes complete repair and the cost is maximum. Therefore, the higher the upgrade level is, the higher the upgrade costs will be.

### 3.1.2 | Modeling warranty costs without upgrade action

$E[N_w(a)]$  indicates the expected number of claims for a second-hand product with age of “a” over the warranty period w based on Eq. (10).

$$E(N_w(a)) = \int_a^{a+w} \Lambda_T(t) dt. \quad (10)$$

Therefore, the seller’s expected warranty cost over the warranty period is given by Eq. (11).

$$E(C_w(a)) = \bar{C} \int_a^{a+w} \Lambda_T(t) dt. \quad (11)$$

### 3.1.3 | Modeling warranty costs with upgrade action

Suppose  $E[N_w(a, u)]$  represents the number of claims that occurred for a second-hand product with age “a” over the warranty period w with upgrade value of u (age reduction following the upgrade action). Based on Eq. (12), the expected warranty over the warranty period w is equal to Eq. (13).

$$E(N_w(a, u)) = \int_a^{a+w} \Lambda_T(t, u) dt. \quad (12)$$

$$\begin{aligned} E(C_w(a, u)) = & \bar{C} \int_a^{a+w} \int_0^1 \int_0^1 p \times \Lambda_T(t) \times \delta_u \times B(p) \times B(p') dp dp' dt \\ & + \\ & \bar{C} \int_a^{a+w} \int_0^1 \int_0^1 (1-p) \times \Lambda_T(t) \times \delta_u \times B(p) \times B(p') dp dp' dt. \end{aligned} \quad (13)$$

Where  $\bar{C}$  is the average warranty cost over the warranty period. Based on the modeling by Hussain [36], the improvement coefficient ( $\delta_u$ ) is modeled as Eq. (14) with minimal changes in the failure parameter.

$$\delta_u = (1 - e^{-qu}) \times p'. \quad (14)$$

The improvement factor is function of u (upgrade value) in which q is a positive parameter, and the p' is between 0 and 1, indicating the effectiveness of the improvement action. If  $p = 1$ , the upgrade action has been quite effective.

By placing Eq. (14) and Eq. (6) in Eq. (13) we have Eq. (15) and by placing Eq. (7) and Eq. (8) in Eq. (15), the expected warranty costs over the warranty period based on the combination of three approaches is given by Eq. (16).

$$\begin{aligned} E(C_w(a, u)) = & \bar{C} \times E(p) \times E(p') \int_a^{a+w} \Lambda_T(t-a) \times (1 - e^{-qu}) dt \\ & + \\ & \bar{C} \times E(1-p) \times E(p') \int_a^{a+w} \Lambda_T(t) \times (1 - e^{-qu}) dt. \end{aligned} \quad (15)$$

$$E(C_w(a, u)) = \frac{(1 - e^{-au})}{a_1 u^{a_2} + b_1 u^{b_2}} \times \frac{a_1' u^{a_2'} + b_1' u^{b_2'}}{a_1' u^{a_2'} + b_1' u^{b_2'}} \times [a_1 u^{a_2} \times \int_a^{a+w} \Lambda_T(t-a) dt + b_1 u^{b_2} \int_a^{a+w} \Lambda_T(t) dt]. \quad (16)$$

### 3.2 | Average Savings in Warranty Costs with and without Upgrade Action

It has already been mentioned that upgrade actions reduce the seller's costs over the warranty period. This leads to cost savings for the seller. The amount of this savings is equal to the difference between the warranty costs with the upgrade program and without the upgrade program (*Eq. (17)*).

$$E(C_w(a, u, w)) = E(C_w(a, u)) + C_u(a). \quad (17)$$

There are two conditions for analyzing upgrade costs:

- 1) Cost-effectiveness of upgrade action

Upgrade actions on second-hand products cost-effective product will be cost-effective if the average amount of savings resulting from upgrade programs exceeds the upgrade costs (*Eq. (18)*).

$$E(CS(a, u)) = E(C_w(a)) - E(C_w(a, u)). \quad (18)$$

This means that the seller's investment in upgrading a second-hand product should not exceed the savings.

- 2) Determining the minimum cost and the level of optimal upgrade.

To determine the best upgrade level, for the known warranty periods, it is necessary to determine the total cost. According to *Eq. (19)* it is equal to the sum of the warranty and the upgrading costs of the second-hand products.

$$E[CS_u(a)] > C_u(a). \quad (19)$$

By minimizing the above equation, if the upgrade action is cost-effective, the optimal upgrade level can be achieved in different warranty periods. Because, the higher the level of upgrade, the higher the initial investment on the one hand, and the reduction of warranty costs on the other hand. Therefore, the necessary balance must be settled between the two.

## 4 | Numerical and Sensitivity Analysis

The case study intended for this research is the air conditioner. The air conditioning system consists of 17 main components, including thermostat, motor, filter, compressor, coil, and several by-products that do not play a critical role in the product's operation. The main parts are the main ones because they suffer the most damage. In case of failure over the warranty period, "free repair replacement" policy, the system undergoes repairing the parts or replacing them.

In order to determine the statistical distribution of failures of second-hand products sold during the warranty period, the relevant information from the representatives of this product with a known brand located in Italy has been collected for analysis. The collected data regards customer claims about product



failure during the warranty period. Of the 4212 upgraded products distributed over 18 months, 1307 products were accompanied by failure claims. Thus, about 30 percent have been damaged during the warranty period. The type of warranty is “free repair – replacement”. Number of products damaged again after repair are excluded from this study due to lack of information.

Among the registered claims, the engine and compressor parts have the highest number of failures. Accordingly, in order to determine the failure distribution function of these parts within the warranty period, EASY-FIT software is used to find the appropriate distribution function. Different distributions (more than 50 types) were tested on the data. Finally, among the existing distributions, based on the warranty literature and due to the increasing-decreasing trend of engine failure rate, Weibull distribution is considered as the most appropriate distribution function for the main components failures. The results of the k-s test at different levels of reliability indicate a good fit for this distribution on failure data.

In the present study, an estimation of the Weibull distribution parameters for the engine obtained as  $\lambda = 0.56$  and  $\beta = 2.8$  using MATLAB software.

#### 4.1 | Stochastic Model: Determining Minimum Cost and Optimal Upgrade Level

The seller sells his products under a "free repair replacement" policy and offers his second-hand products after repair with a one-year warranty period. Due to the high costs of the warranty period, the seller tends to better control and optimize the costs of the warranty period by finding the optimal upgrade level. In this study, the costs are considered as the sum of the warranty period costs and the upgrade investment costs. According to the selected Weibull distribution as the failure time distribution and by placing its failure rate function, *Eq. (19)* becomes *Eq. (20)*, and the total cost function is obtained based on *Eq. (21)*.

$$E(C_w(a, u)) = \frac{(1 - e^{-\varrho u})}{a_1 u^{a_2} + b_1 u^{b_2}} \times \frac{a_1' u^{a_2'}}{a_1' u^{a_2'} + b_1' u^{b_2'}} \quad (20)$$

$$\times [a_1 u^{a_2} \times \lambda^\beta + ((a + w)^\beta - a^\beta) \times b_1 u^{b_2}].$$

$$E(C_w(a, u)) = \frac{\delta \times u}{1 - e^{-\theta(a-u)}} + \frac{(1 - e^{-\varrho u})}{a_1 u^{a_2} + b_1 u^{b_2}} \times \frac{a_1' u^{a_2'}}{a_1' u^{a_2'} + b_1' u^{b_2'}} \quad (21)$$

$$\times [a_1 u^{a_2} \times \lambda^\beta + ((a + w)^\beta - a^\beta) \times b_1 u^{b_2}].$$

To determine the optimal upgrade level, the above equation is minimized. Due to the complexity of this function, and the impossibility of using derivation methods, the optimization code of this function has been created using a network search algorithm. Defining the variable in the specified interval, the algorithm searches in this interval to find the optimal value. Accordingly, the proposed algorithm searches the function at the midpoints of the intervals to find the points with the lowest costs.

Also, in order to find the parameters in the upgrade cost function ( $\delta$  and  $\theta$ ) - *Eq. (9)*, beta distribution parameters ( $a_1, \alpha_2, \beta_1, \beta_2$ ) - *Eq. (16)* and also the parameters of improvement coefficient ( $\varrho$ ) - *Eq. (7)* and *Eq. (8)* logarithmic regression method is utilized using the existing records of the used products.

The information obtained is as follows:

$\beta=2.8$ ,  $\lambda=0.56$ ,  $\delta=5.01$ ,  $\theta=0.31$ ,  $\rho=0.8$ ,  $a1=2.13$ ,  $a2=1$ ,  $b1=3.04$ ,  $b2=1$ ,  $a1'=2.13$ ,  $a2'=1$ ,  $b1'=3.04$ ,  $b2'=1$ .

The results of warranty cost optimization and the optimal value of  $u$  ( $u^*$ ) for a range of values of  $w$  and  $a$  are given in Fig. 2. Analyses are presented in four different ages, from 1 to 4 years. In second-hand products at age 1, it is not cost-effective to upgrade the products. Thus, they have no feasible solution. For other periods, for example, a product at age 4 and a warranty period 1.5 years, the optimal upgrade level is 1.44 years. This means that the second-hand product at its current age needs to be rejuvenated 1.44 years to meet the lowest cost. That is, the age of the product should be reduced from four years and rejuvenated accordingly. After all, the product is offered to the market with a 1.5 years free repair replacement warranty.

Warranty costs will increase if the product age and the warranty period increase. For example, in a given age of 2 years and a 1 year warranty period, the upgrade rate is 0.43 years, and for a three years old product with the same warranty period, the upgrade level is 0.76 years. On the other hand, for a given product with a 1.5 years warranty, the upgrade value is 0.60, and the same product with a warranty period of 2 years has the upgrade value of 0.73. Besides, the higher the upgrade level as the age of the product increases, the higher the savings. Also, all the saving amounts are higher than the investment amounts. Fig. 1 and Fig. 2 show the expected savings and the upgrade level incurred versus the improvement level for a second-hand product sold with the past age from 2 to 4 years with a warranty period of 3 years.

Although, the higher the level of upgrade, the greater the amount of savings. The more upgrades than a certain extent does not lead to more savings. Therefore, selling second-hand products after upgrade actions is more cost-effective than selling them without upgrading.

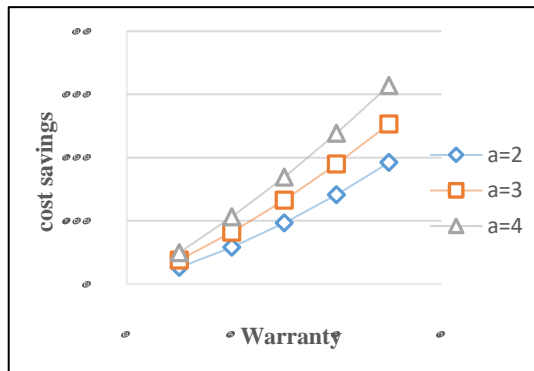


Fig. 1. Average savings of upgrade action.

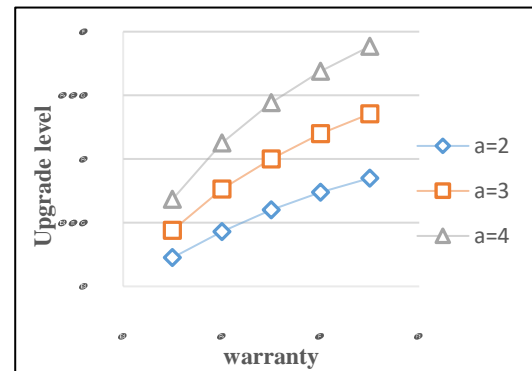


Fig. 2. Optimum upgrade levels in different ages.

#### 4.1.1 | Sensitivity analysis

In this section, we illustrate our results and investigate the effects of relevant parameters on the optimal upgrade level and warranty cost saving numerically. For the purpose of the numerical evaluation, each time, we fix repair costs in different values and calculate the results by several combinations of the repair costs together with different warranty periods and product ages measured in years. Fig. 1 shows the average investment and warranty costs and savings resulting from upgrades for different warranty periods of 6 months to 3 years. The results indicate that upgrade action is not cost-effective within the age of less than one year. In addition, the average warranty costs increase with the length of the warranty period. The sensitivity of the total cost model with the average change in repair costs for \$ 120, \$ 140 and \$ 160 for second-hand products with the age of 1 to 4 years and warranty periods of 6 months to 2 years, is presented in Fig. 2. The results show that although increasing the warranty period and upgrade actions cost the seller more and consequently a higher price for the buyer, but in the long run, these actions will reduce the warranty costs after the end of the warranty period. Therefore, it is better to take the upgrade actions on the products whose performance is more important than their appearance for the customer. In fact, the

higher the repair costs, the higher the upgrade under the same conditions. It is due to the use of higher quality parts when performing a second-hand product improvement operation. The level of second-hand product upgrade depends on factors such as product age, the severity of the failure, second-hand product market, and related costs. The higher the repair costs, the higher the quality of the parts used, or the higher the cost of the repair. Therefore, upgrade actions are more cost-effective for high-value products. In addition, the upgrade actions are more cost effective when the second-hand products have higher past age. Therefore, it is suggested to focus on upgrading second-hand products with longer past age and high value. Also, it is better to take upgrade actions for products that have more possibility to increase the warranty period.

## 5 | Conclusion

Despite the competitive environment for second-hand products markets, reusing these products has become an attractive investment market. In order to sell better, these products are subjected to an optimal upgrade level in order to provide the seller's profit on the one hand and the consumer's profit on the other hand. In this paper, a new hybrid stochastic model is developed to model the estimation of the minimum warranty costs of second-hand products under the "free repair replacement" warranty policy to find the optimal upgrade level. The structure of the model is based on combining three upgrade approaches, including "improvement factor approach", "minimal-perfect repair" and "virtual age". Under given cost structures, the optimal upgrade action strategy is identified, leading to minimization of the sellers' expected warranty costs. A case study is conducted to evaluate the validity of the model. The results show that warranty length, the past life of second-hand products, and the upgrade level have all significant effects on the warranty costs and sellers' expected profit considering all occurrences in the real world. The results showed that although increasing the warranty period and the upgrade level imposes higher costs on the seller and consequently a higher price, but in the long run, these improvements actions lead to a reduction in warranty costs after the end of the warranty period. In fact, the higher the repair costs, the higher the upgrade under the same conditions will be. Therefore, given the upgrade costs, upgrade actions are more cost-effective for high-value products. Furthermore, the longer the life of the second-hand product, the more cost-effective the upgrade actions, and the higher the savings.

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