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Research article

Precise large deviations of aggregate claims in a nonstandard risk model with arbitrary dependence between claim sizes and waiting times

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Abstract: Recently, Chen et al. [3] investigated the precise large deviations of aggregate claims in a renewal risk model with arbitrary dependence between claim sizes and their waiting times. In this paper, we extend their results to a nonstandard risk model in which various dependence structures are imposed on the modeling components, and obtain the asymptotic lower and upper bounds of precise large deviations for aggregate claims, which hold uniformly for all x in a t-interval.

Keywords: asymptotics; precise large deviations; dependence; consistent variation; aggregate claims
Mathematics Subject Classification: Primary 60F10; Secondary 62P05, 91B30

1. Introduction

It is well known that the precise large deviations with applications in finance and insurance have attracted much attention from many researchers. See, e.g., Cline and Hsing [5], Klüppelberg and Mikosch [12], Mikosch and Nagaev [19], Tang et al. [24], Ng et al. [20], Konstantinides and Loukissas [11], Loukissas [17], Lu et al. [18] and Jiang et al. [9], just to name a few in this study. The above-referenced results hold under the independence assumption, which appears far too unrealistic in practice and then considerably limits the usefulness of the obtained results. More contributions in risk theory have imposed various dependence structures in investigations of precise large deviations and other risk-related topics. The readers are referred to Kass and Tang [10], Tang [23], Wang and Wang [27], Liu [14], Chen et al. [4], Wang and Cheng [28], Yang and Wang [29], He et al. [8], Wang and Chen [26] and many others. Especially in risk models, the dependence structures between the claim size and its waiting time were considered by Li et al. [13], Chen and Yuen [2], Shen et al. [22], Liu et al. [16], Gao et al. [7] and references therein.

Following the earlier works of this study, we first introduce the renewal risk model with the assumptions as below.

Assumption H_1. The claim sizes $\{X_i, i \geq 1\}$ form a sequence of nonnegative, independent and

identically distributed (i.i.d) random variables (r.v.s) with the common distribution F.
Assumption H_2**.** The claim inter-arrival times $\{\theta_i, i \geq 1\}$ form another sequence of nonnegative and i.i.d. r.v.s with the common distribution G and finite mean λ^{-1}. Denote the arrival times of claims by $\tau_n = \sum_{i=1}^{n} \theta_i$, $n \geq 1$, which constitute the renewal counting process

$$N(t) = \sup\{n \geq 1 : \tau_n \leq t\}, \quad t \geq 0,$$

with a finite renewal function $\lambda(t) = EN(t)$ such that $\lambda(t) \to \infty$ and $\lambda(t)/\lambda t \to 1$ as $t \to \infty$.
Assumption H_3**.** Assume that $\{X_i, i \geq 1\}$ and $\{\theta_i, i \geq 1\}$ are mutually independent.

In this way, the aggregate amount of claims accumulated up to time $t \geq 0$ is a random sum of the form

$$S(t) = \sum_{i=1}^{N(t)} X_i, \tag{1.1}$$

where, and henceforth, a summation over an empty index set is 0.

A recent trend in risk theory is to introduce various dependence structures to risk models. So, now we present some dependence structures, among which the widely upper orthant dependence structure was proposed by Wang et al. [25]. Say that r.v.s $\{\xi_i, i \geq 1\}$ are widely upper orthant dependent (WUOD), if there exists a sequence of finite and positive numbers $\{g_U(n), n \geq 1\}$ such that, for each $n \geq 1$ and for all $x_i \in (-\infty, \infty)$, $1 \leq i \leq n$,

$$P\left(\bigcap_{i=1}^{n} \left\{\xi_i > x_i\right\}\right) \leq g_U(n) \prod_{i=1}^{n} P(\xi_i > x_i).$$

Adopting the term of Liu et al. [15], say that $\{\xi_i, i \geq 1\}$ are upper-tail asymptotically independent (UTAI), if $P(\xi_i > x) > 0$ for all $x \in (-\infty, \infty)$, $i \geq 1$ and

$$\lim_{\min\{x_i, x_j\} \to \infty} P\left(\xi_i > x_i | \xi_j > x_j\right) = 0 \quad \text{for all } 1 \leq i \neq j < \infty.$$

He et al. [8] initiated a dependence structure among r.v.s $\{\xi_i, i \geq 1\}$ such that, for any $\gamma > 0$,

$$\lim_{n\to\infty} \sup_{x \geq \gamma n} \sup_{1 \leq i < j \leq n} xP(\xi_i > x | \xi_j > x) = 0. \tag{1.2}$$

Remark that the UTAI structure can properly cover the WUOD structure; see Example 3.1 of Liu et al. [15]. And, the dependence structure defined by the relation (1.2) is a special case of the UTAI structure. In fact, He et al. [8] showed that, if $\{X_i, i \geq 1\}$ are WUOD, then the relation (1.2) follows from

$$\lim_{x\to\infty} x\overline{F}(x) = 0,$$

which is slightly weaker than that the r.v. has a finite mean. In doing so, the dependence structure defined by (1.2) at least properly covers the WUOD r.v.s with finite means. See an example of Liu et al. [16], indicating that there exist WUOD r.v.s such that (1.2) holds, while two examples given by He et al. [8] show that there exist UTAI r.v.s satisfying (1.2), that are not WUOD.

Practitioners in non-life insurance are interested in distributions with a heavy tail, which is formally adopted to model the large claims caused by extremal events. Finally, in the section, we give some heavy-tailed distribution classes. For a proper distribution V, we denote its tail by $\overline{V}(x) = 1 - V(x)$ and its upper Matuszewska index by

$$J_V^+ = -\lim_{y\to\infty} \frac{\log \overline{V}_*(y)}{\log y} \quad \text{with} \quad \overline{V}_*(y) = \liminf_{x\to\infty} \frac{\overline{V}(xy)}{\overline{V}(x)}, \quad y > 1.$$

Say that a distribution V on $[0, \infty)$ belongs to the long-tailed class, denoted by $V \in \mathscr{L}$, if, for any $y > 0$,

$$\lim_{x\to\infty} \frac{\overline{V}(x+y)}{\overline{V}(x)} = 1;$$

it belongs to the dominated variation class, denoted by $V \in \mathscr{D}$, if, for any $0 < y < 1$,

$$\overline{V}^*(y) < \infty,$$

where $\overline{V}^*(y) = \limsup_{x\to\infty} \overline{V}(xy)/\overline{V}(x)$; it belongs to the consistent variation class, denoted by $V \in \mathscr{C}$, if

$$\lim_{y\searrow 1} \overline{V}_*(y) = 1, \quad \text{or, equivalently,} \quad \lim_{y\nearrow 1} \overline{V}^*(y) = 1.$$

More generally, we say that a distribution V on $(-\infty, \infty)$ belongs to a distribution class if $V(x)\mathbf{1}_{\{x\geq 0\}}$ belongs to the class, where $\mathbf{1}_A$ denotes the indicator function of a set A. Note that the class $\mathscr{C}$ is an important subclass of $\mathscr{L} \cap \mathscr{D}$. For more details on heavy-tailed distributions and their applications, we refer to Bingham et al. [1] and Embrechts et al. [6].

Based on the above-mentioned preparatory work, in the present paper, we make an effort to relax the independence assumptions imposed in the renewal risk model, and further study the precise deviations of the aggregate amount of claims described by (1.1) with various dependence structures.

The rest of this paper is organized as follows: Section 2 states the main results, Section 3 presents some lemmas and Section 4 proves the main results.

2. Motivation and main results

All limit relationships hereafter are taken as $t \to \infty$ unless mentioned otherwise. For two positive functions $a(\cdot)$ and $b(\cdot)$, we write $a(t) \lesssim b(t)$ if $\limsup a(t)/b(t) \leq 1$, write $a(t) \gtrsim b(t)$ if $\liminf a(t)/b(t) \geq 1$, write $a(t) \sim b(t)$ if both and write $a(t) = o(1)b(t)$ if $\lim a(t)/b(t) = 0$.

As was stated in Section 1, an increasing number of researchers have paid attentions to the precise large deviations and their applications. More recently, Chen et al. [3] considered the renewal risk model with arbitrary dependence between the claim size X and its waiting time θ, and obtained a precise large deviation formula of the aggregate amount of claims, as follows:

Theorem A. *Consider the aggregate claims described by (1.1) in the renewal risk model with Assumptions H_1 and H_2. If $F \in \mathscr{C}$ and $\overline{G}(x) = o(1)\overline{F}(x)$ as $x \to \infty$, then, for any $0 < \gamma < \Gamma < \infty$, it holds uniformly for all $x \in [\gamma t, \Gamma t]$ that*

$$P(S(t) - \mu\lambda t > x) \sim \lambda t \overline{F}(x),$$

namely,

$$\lim_{t\to\infty}\sup_{x\in[\gamma t,\Gamma t]}\left|\frac{P(S(t)-\mu\lambda t>x)}{\lambda t\overline{F}(x)}-1\right|=0.$$

It is worth noting that, because of the arbitrary dependence between X and θ, we can get rid of the independence assumption in Assumption H_3 and some specific dependence structures, such as the size dependence originated by Chen and Yuen [2]. Motivated by Chen et al. [3], we will extend their result to a nonstandard renewal risk model with various dependence structures satisfying the following assumptions.

Assumption H_1^*. The claim sizes $\{X_i, i \geq 1\}$ are nonnegative r.v.s with the common distribution F, and they satisfy the dependence structure defined by (1.2).

Assumption H_1^{**}. The claim sizes $\{X_i, i \geq 1\}$ are nonnegative and WUOD r.v.s with the common distribution F, and they satisfy

$$\lim_{n\to\infty} n^{-1}\log g_U(n) = 0. \tag{2.1}$$

Assumption H_2^*. The claim inter-arrival times $\{\theta_i, i \geq 1\}$ are nonnegative r.v.s with the common distribution G and finite mean λ^{-1}.

Assumption H_3^*. For any $i \geq 1$, the claim size X_i and its waiting time θ_i are arbitrarily dependent.

Obviously, Assumptions H_1^* and H_1^{**} impose two specific dependence structures on the claim sizes to weaken the independence assumption in Assumption H_1; Assumption H_2^* indicates that no assumption is made on the dependence structure of claim inter-arrival times, namely, that the claim inter-arrival times are allowed to be arbitrarily dependent in order to drop the independence assumption in Assumption H_2; Assumption H_3^* implies that, for any fixed $i \geq 1$, neither independence, nor a specific dependence structure, is taken between X_i and θ_i, which means that we can avoid the independence assumption and a specific dependence structure between both. As for the relation (2.1) in Assumption H_1^{**}, see an example given by Liu et al. [16], which illustrates that there exists a sequence of WUOD r.v.s such that the relation (2.1) holds.

The main results of this paper are given below, where the first theorem provides an asymptotic lower bound of the precise large deviations of aggregate claims in a nonstandard renewal risk model.

Theorem 2.1. *Consider the aggregate claims described by (1.1) in the nonstandard renewal risk model with Assumptions H_1^*– H_3^*. If $F \in \mathscr{C}$ and $\overline{G}(x) = o(1)\overline{F}(x)$ as $x \to \infty$, then, for any $0 < \gamma < \Gamma < \infty$, it holds uniformly for all $x \in [\gamma t, \Gamma t]$ that*

$$P(S(t) > x) \gtrsim \lambda t\overline{F}(x), \tag{2.2}$$

namely,

$$\liminf_{t\to\infty}\inf_{x\in[\gamma t,\Gamma t]}\frac{P(S(t)>x)}{\lambda t\overline{F}(x)} \geq 1.$$

The second theorem gives an asymptotic upper bound of the precise large deviations of aggregate claims with some conditions that are slightly stronger than those in the first theorem.

Theorem 2.2. *Under the same conditions of Theorem 2.1 except Assumption H_1^*, which is replaced by H_1^{**}, it still holds uniformly for all $x \in [\gamma t, \Gamma t]$ that*

$$P(S(t) - \mu\lambda t > x) \lesssim \lambda t\overline{F}(x), \tag{2.3}$$

namely,

$$\limsup_{t\to\infty} \sup_{x\in[\gamma t,\Gamma t]} \frac{P(S(t) - \mu\lambda t > x)}{\lambda t\overline{F}(x)} \leq 1.$$

3. Some lemmas

To prove the main results of this paper, we now present some lemmas, among which the first one is due to Lemma 2.5 of Liu et al. [16].

Lemma 3.1. *Let $\{\xi_i, i \geq 1\}$ be a sequence of nonnegative and WUOD r.v.s with the common distribution $V \in \mathscr{C}$ and mean μ. If the relation (2.1) holds, then, for any $\gamma > 0$, it holds uniformly for all $x \geq \gamma n$ that, as $n \to \infty$,*

$$P\left(\sum_{i=1}^{n} \xi_i - n\mu > x\right) \lesssim n\overline{V}(x),$$

namely,

$$\limsup_{n\to\infty} \sup_{x\geq\gamma n} \frac{P\left(\sum_{i=1}^{n} \xi_i - n\mu > x\right)}{n\overline{V}(x)} \leq 1.$$

In the following lemma, we extend Lemma 3.4 of Chen et al. [3] to the case in which no assumption is made on the dependence structure of underlying r.v.s.

Lemma 3.2. *Let $\{\xi_i, i \geq 1\}$ be a sequence of real-valued r.v.s with the generic r.v. ξ and mean 0. If $P(\xi > x) = o(1)\overline{V}(x)$ for some $V \in \mathscr{C}$, then, for any $\gamma > 0$, it holds uniformly for all $x \geq \gamma n$ that, as $n \to \infty$,*

$$P\left(\sum_{i=1}^{n} \xi_i > x\right) = o(1)n\overline{V}(x),$$

namely,

$$\lim_{n\to\infty} \sup_{x\geq\gamma n} \frac{P\left(\sum_{i=1}^{n} \xi_i > x\right)}{n\overline{V}(x)} = 0.$$

Proof. Following the ideas in the proof of Lemma 3.4 of [3], we prove the lemma. Set $\xi^+ = \max\{0, \xi\}$. Clearly,

$$P(\xi^+ > x) \sim P(\xi > x),$$

which, along with $P(\xi > x) = o(1)\overline{V}(x)$, implies that, for an arbitrarily fixed $\varepsilon > 0$, there exists a sufficiently large x such that

$$P(\xi^+ > x) \leq \varepsilon\overline{V}(x).$$

Introduce a r.v. ξ^* with its tail distribution as

$$\overline{V^*}(x) := \max\{P(\xi^+ > x), \varepsilon\overline{V}(x)\}.$$

It is easy to see that

$$\overline{V^*}(x) \sim \varepsilon\overline{V}(x) \quad \text{and} \quad \xi^+ \leq_{st} \xi^*,$$

where $\xi^+ \leq_{st} \xi^*$ means that, for all increasing functions $g : (-\infty, \infty) \mapsto (-\infty, \infty)$, $Eg(\xi^+) \leq Eg(\xi^*)$, provided the expectations $Eg(\xi^+)$ and $Eg(\xi^*)$ exist and are finite. See Definition 3.2.1 of [21].

Now we construct a sequence of WUOD r.v.s $\{\xi_i^*, i \geq 1\}$ with the generic r.v. ξ^* such that (2.1) holds. In fact, see an example given by [16], which illustrates that there exists a sequence of WUOD r.v.s satisfying (2.1), and then ensures that the construction is well-grounded. Consider that ξ^* stochastically decreases to ξ as $\varepsilon \downarrow 0$; one knows that, for any $\gamma > 0$, there exists a sufficiently small $\varepsilon > 0$ such that

$$0 = E\xi \leq E\xi^* \leq \frac{\gamma}{2},$$

and then, for all $x \geq \gamma n$,

$$x - nE\xi^* \geq \frac{\gamma n}{2}.$$

Hence, by Lemma 3.1, it holds uniformly for all $x \geq \gamma n$ that, as $n \to \infty$,

$$\begin{aligned} P\left(\sum_{i=1}^n \xi_i > x\right) &\leq P\left(\sum_{i=1}^n \xi_i^+ > x\right) \\ &\leq P\left(\sum_{i=1}^n (\xi_i^* - E\xi^*) > x - nE\xi^*\right) \\ &\lesssim n\overline{V^*}(x - nE\xi^*) \\ &\leq n\overline{V^*}\left(x\left(1 - \frac{E\xi^*}{\gamma}\right)\right), \end{aligned}$$

which yields that

$$\begin{aligned} \limsup_{n\to\infty} \sup_{x\geq\gamma n} \frac{P\left(\sum_{i=1}^n \xi_i > x\right)}{n\overline{V}(x)} &\leq \limsup_{x\to\infty} \frac{\overline{V^*}\left(x\left(1 - \frac{E\xi^*}{\gamma}\right)\right)}{\overline{V}(x)} \\ &= \varepsilon \limsup_{x\to\infty} \frac{\overline{V}\left(x\left(1 - \frac{E\xi^*}{\gamma}\right)\right)}{\overline{V}(x)}. \end{aligned} \tag{3.1}$$

Thus, by $V \in \mathscr{C} \subset \mathscr{D}$, the last lim sup in (3.1) is finite, and then the right-hand side of (3.1) tends to 0 as $\varepsilon \downarrow 0$. This completes the proof of Lemma 3.2. □

By the symmetric derivation of Lemma 3.2, we put forward a lemma below which is the extended version of Corollary 3.1 of [3].

Lemma 3.3. *Let the conditions of $\{\xi_i, i \geq 1\}$ in Lemma 3.2 be true and $P(|\xi| > x) = o(1)\overline{V}(x)$ for some $V \in \mathscr{C}$; then, for any $\gamma > 0$, it holds uniformly for all $x \geq \gamma n$ that, as $n \to \infty$,*

$$P\left(\left|\sum_{i=1}^n \xi_i\right| > x\right) = o(1)n\overline{V}(x).$$

4. Proof of main results

Proof of Theorem 2.1. For any, but small, $0 < \delta < 1$, we have

$$\begin{aligned} P(S(t) > x) &\geq P\left(\sum_{i=1}^{(1-\delta)\lambda t} X_i > x,\ N(t) \geq (1-\delta)\lambda t\right) \\ &\geq P\left(\sum_{i=1}^{(1-\delta)\lambda t} X_i > x\right) - P(N(t) < (1-\delta)\lambda t) \\ &:= I_1(x,t) - I_2(t). \end{aligned} \tag{4.1}$$

For $I_1(x,t)$, we obtain by (1.2) that, uniformly for all $x \geq \gamma t$,

$$\begin{aligned} I_1(x,t) &\geq \sum_{i=1}^{(1-\delta)\lambda t} P(X_i > x) - \sum_{1\leq i<j\leq(1-\delta)\lambda t} P\left(X_i > x,\ X_j > x\right) \\ &\geq (1-\delta)\lambda t\overline{F}(x) - x^{-1} \sum_{1\leq i<j\leq(1-\delta)\lambda t} xP(X_i > x|X_j > x)P(X_j > x) \\ &= (1-\delta)\lambda t\overline{F}(x) - o(1)\frac{\lambda t}{x}(1+\delta)^2\lambda t\overline{F}(x), \end{aligned}$$

which, along with the arbitrariness of $0 < \delta < 1$, implies that, uniformly for all $x \geq \gamma t$,

$$I_1(x,t) \gtrsim \lambda t\overline{F}(x). \tag{4.2}$$

For $I_2(t)$, by Lemma 3.2 and $F \in \mathscr{C} \subset \mathscr{D}$, it holds uniformly for all $x \leq \Gamma t$ that

$$\begin{aligned} I_2(t) &\leq P\left(\sum_{i=1}^{(1-\delta)\lambda t} \theta_i > t\right) \\ &= P\left(\sum_{i=1}^{(1-\delta)\lambda t} \left(\theta_i - \frac{1}{\lambda}\right) > \delta t\right) \\ &= o(1)\lambda t\overline{F}(\delta t) \\ &= o(1)\lambda t\overline{F}\left(\frac{\delta x}{\Gamma}\right) \\ &= o(1)\lambda t\overline{F}(x). \end{aligned} \tag{4.3}$$

Hence, we substitute (4.2) and (4.3) into (4.1) to prove that the relation (2.2) holds uniformly for all $x \in [\gamma t, \Gamma t]$. □

Proof of Theorem 2.2. For any, but small, $0 < \delta < 1$, we have

$$\begin{aligned} P(S(t) - \mu\lambda t > x) \leq\ & P\left(\sum_{i=1}^{(1+\delta)\lambda t} X_i - \mu\lambda t > x,\ N(t) \leq (1+\delta)\lambda t\right) \\ & + P(N(t) > (1+\delta)\lambda t) \end{aligned}$$

$$:= \quad I_3(x,t) + I_4(t). \tag{4.4}$$

For $I_3(x,t)$, we take a small δ such that $\gamma - \delta\mu\lambda > 0$, and we use Lemma 3.1 with $n = \lfloor(1+\delta)\lambda t\rfloor$ to show that, uniformly for all $x \geq \gamma t$,

$$\begin{aligned} I_3(x,t) &\leq P\left(\sum_{i=1}^{\lfloor(1+\delta)\lambda t\rfloor} X_i - \mu\lfloor(1+\delta)\lambda t\rfloor > x + \mu\lambda t - \mu\lfloor(1+\delta)\lambda t\rfloor\right) \\ &\lesssim \lfloor(1+\delta)\lambda t\rfloor\overline{F}\left(x + \mu\lambda t - \mu\lfloor(1+\delta)\lambda t\rfloor\right) \\ &\leq (1+\delta)\lambda t\overline{F}\left(x\left(1 - \frac{\delta\mu\lambda}{\gamma}\right)\right), \end{aligned} \tag{4.5}$$

where $\lfloor(1+\delta)\lambda t\rfloor$ denotes the integer part of a real number $(1+\delta)\lambda t$. For $I_4(t)$, by going along the same lines of the proof of $I_2(t)$ with some slight modifications, it holds uniformly for all $x \leq \Gamma t$ that

$$\begin{aligned} I_4(t) &\leq P\left(\sum_{i=1}^{(1+\delta)\lambda t} \theta_i \leq t\right) \\ &= P\left(\sum_{i=1}^{(1+\delta)\lambda t}\left(\theta_i - \frac{1}{\lambda}\right) \leq -\delta t\right) \\ &= P\left(\left|\sum_{i=1}^{(1+\delta)\lambda t}\left(\theta_i - \frac{1}{\lambda}\right)\right| \geq \delta t\right) \\ &= o(1)\lambda t\overline{F}(\delta t) \\ &= o(1)\lambda t\overline{F}(x), \end{aligned} \tag{4.6}$$

where the second last step is due to Lemma 3.3. Then, by (4.4)–(4.6), $F \in \mathscr{C} \subset \mathscr{D}$ and the arbitrariness of $0 < \delta < 1$, we obtain the uniformity of the relation (2.3) for all $x \in [\gamma t, \Gamma t]$. □

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Conflict of interest

The authors declare no conflict of interest.

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