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**Research article****Further results on stability analysis of Takagi–Sugeno fuzzy time-delay systems via improved Lyapunov–Krasovskii functional****Rupak Datta<sup>1</sup>, Ramasamy Saravanakumar<sup>2,\*</sup>, Rajeeb Dey<sup>3</sup> and Baby Bhattacharya<sup>1</sup>**<sup>1</sup> Department of Mathematics, National Institute of Technology Agartala, Tripura 799046, India<sup>2</sup> Graduate School of Advanced Science and Engineering, Hiroshima University, 1-4-1 Kagamiyama, Higashi-Hiroshima 739-8527, Japan<sup>3</sup> Department of Electrical Engineering, National Institute of Technology Silchar, Assam 788010, India**\* Correspondence:** Email: saravanamaths30@gmail.com.

**Abstract:** The problem of delay-range-dependent (DRD) stability analysis for continuous time Takagi–Sugeno (T–S) fuzzy time-delay systems (TDSs) is addressed in this paper. An improved DRD stability criterion is proposed in an linear matrix inequality (LMI) framework by constructing an appropriate delay-product-type (DPT) Lyapunov–Krasovskii functional (LKF) to make use of Bessel–Legendre polynomial based relaxed integral inequality. The modification in the proposed LKF along with the judicious choice of integral inequalities helps to obtain a less conservative delay upper bound for a given lower bound. The efficacy of the obtained stability conditions is validated through the solution of three numerical examples.

**Keywords:** T–S fuzzy system; LKF; DRD stability; LMI; TDS**Mathematics Subject Classification:** 34D20, 34E05

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**1. Introduction**

Most of the physical models are nonlinear in nature, and they frequently involve many complex input-output relationships. Over the years, different nonlinear control techniques were developed to obtain the actual behavior of the nonlinear models [1, 2]. The Takagi–Sugeno (T–S) fuzzy model approach is recognized as an effective tool for approximation of complex nonlinear systems among several control methods [3]. The development and application of the T–S fuzzy model have greatly increased for the study of nonlinear systems. The universal approximation principle, which states that a T–S fuzzy model may estimate any smooth nonlinear system with any degree of certainty, made it possible to use a T–S fuzzy model to investigate nonlinear systems. The T–S can represent a nonlinear

system into local linear models through nonlinear membership functions so that established stability and control theories can be applied directly [4–6]. The major purpose of the T–S fuzzy control technique is that the stability and control architectures can be transformed into the linear matrix inequality (LMI) framework. [7].

Time delay is inherently present in most engineering systems, and it is one of the primary reasons of instability and performance degradation [8, 9]. Numerous results addressing the synthesis and analysis of T–S fuzzy systems with variable time-delay conditions have been obtained as a result of the industrial need [20–40]. The Lyapunov second approach can be used to analyze the stability of time-delay systems in two ways, (i) delay-dependent stability [8–14], and (ii) delay-independent stability [15]. A Lyapunov–Krasovskii functional (LKF) with an integral term is developed to obtain an efficient delay-dependent stability condition. Constructing a suitable LKF and estimating the integral term in the derivative of the LKF is the most common strategy for minimizing conservativeness. So far, various methods have been presented for processing the integral term and reducing the conservatism, such as model transformation method [10], free-weighting matrix approach was proposed in [11], Jensen’s inequality technique in [5, 8, 17], reciprocal convex lemma in [12], Wirtinger-based inequality [13], auxiliary-function-based inequality [14]. In order to further improve the stability conditions of time-delay systems, quadratic function negative-determination lemma was introduced in [16, 18].

Apart from the various bounding inequalities, constructing an appropriate LKF is another key point. In the recent years, delay-partitioning LKFs and augmented LKFs are broadly studied and successful results have been achieved [21, 22]. However, introducing too many free matrices makes computing LMIs immensely challenging, increasing processing complexity. Recently, a line integral fuzzy Lyapunov function was used to analyze the stability of T–S fuzzy systems in [23]. In the meantime, the concept was utilized for LMI based control design in [24]. Although the line integral Lyapunov function can be utilized to avoid time derivatives, it can also result in bilinear matrix inequalities in the controller design, which can be difficult to extrapolate for higher-order systems [25–30]. In the recent years, augmented LKF technique and Bessel-Legendre polynomial based integral inequality have been used to obtain the less conservative results for T–S fuzzy TDS in [31–40]. As a result, a less conservative criterion can be obtained by selecting a suitable LKF and developing a new integral inequality. A unique LKF construction method, called delay-product-type (DPT) functional approach, was recently introduced and analyzed in [31, 34, 36, 39], taking into account both conservatism and computational burden. Specifically, this LKF construction method multiplies time-varying delay terms with integral and non-integral terms, resulting in a LKF with additional time-delay information. Also, because the restriction on certain places is eased when using the DPT functional method, the LKF have a more comprehensive form. Thus, by combining the new Bessel-Legendre polynomial-based relaxed integral inequality with the augmented and DPT LKF, it is possible to obtain a less conservative admissibility condition for T–S fuzzy systems with time-varying delays. This is the motivation of this paper.

The aim of this paper is to study the stability of T–S fuzzy systems with time-varying delays. The main contributions of this study are listed as follows:

- (i) A less conservative stability condition is established for T–S fuzzy TDS by constructing a suitable DPT augmented LKF. The proposed LKF has the motivation that, situations of various delay derivative nature can be handled with ease and less complexity.
- (ii) Bessel-Legendre polynomial based relaxed integral inequality is used to estimate the integral

terms coming out from the derivative of LKF.

- (iii) The advantages of the proposed stability criteria are demonstrated using three numerical examples and a comparison of maximum delay upper bound results with various recent stability criteria.

*Notation:* Throughout this paper,  $A^T$  and  $A^{-1}$  stands for the transpose and the inverse of the matrix respectively,  $\mathbb{R}^n$  denotes the  $n$ -dimensional Euclidean space;  $\mathbb{R}^{n \times m}$  is the set of all  $n \times m$  real matrices;  $P > 0$  means that the matrix is positive definite;  $Sym(A)$  is defined as  $A + A^T$ ; for any square matrix  $A$  and  $B$ , define  $diag\{A, B\} = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$ . The notation  $I$  stands for the identity matrix and  $\begin{bmatrix} A & B \\ * & C \end{bmatrix}$  stands for  $\begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$ .

## 2. Problem formulation and useful lemmas

Consider the following nonlinear system with time delay as

$$\begin{aligned}\dot{\eta}(t) &= f(t, \eta(t), \eta(t - \gamma(t))), & t \geq 0, \\ \eta(t) &= \phi(t), & -\gamma_2 \leq t \leq 0,\end{aligned}\quad (2.1)$$

where  $\eta(t) \in \mathbb{R}^n$  is the state vector, ' $f$ ' is a non-linear function,  $\eta(t) = \phi(t)$  denote the initial condition on  $[-\gamma_2, 0]$  and  $\gamma(t)$  is the time-varying delay differential function.

The T-S fuzzy model of the system given in (2.1) can be described by following IF-THEN form as

**Rule i:** IF  $\theta_1(t)$  is  $\mathcal{G}_{i1}$  and .... and  $\theta_p(t)$  is  $\mathcal{G}_{ip}$  THEN

$$\begin{aligned}\dot{\eta}(t) &= \mathcal{A}_i \eta(t) + \mathcal{A}_{\gamma_i} \eta(t - \gamma(t)), & t \geq 0, \\ \eta(t) &= \phi(t), & -\gamma_2 \leq t \leq 0,\end{aligned}\quad (2.2)$$

where  $\theta_1(t), \theta_2(t), \dots, \theta_p(t)$  are the premises variables,  $\mathcal{G}_{ij}$  are the fuzzy membership functions with  $i = 1, 2, 3, \dots, r$ ,  $j = 1, 2, 3, \dots, p$ , the scalars  $r$  and  $p$  indicates the number of fuzzy IF-THEN rules and number of premise variable, respectively.  $\mathcal{A}_i, \mathcal{A}_{\gamma_i}$  are known system matrices of appropriate dimensions. The delay differential function  $\gamma(t)$  satisfy the following:

$$0 < \gamma_1 \leq \gamma(t) \leq \gamma_2, \quad \nu_1 \leq \dot{\gamma}(t) \leq \nu_2, \quad (2.3)$$

where  $\gamma_1, \gamma_2, \nu_1$  and  $\nu_2$  are given positive scalars represent the lower and upper bound of  $\gamma(t)$  and  $\dot{\gamma}(t)$ , respectively.

If  $\theta_j(t) = \theta_j^0$  are given, where  $\theta_j^0$  are singletons, then for each  $i^{th}$  fuzzy rule, the aggregation of the fuzzy rule using fuzzy 'min' operator can be expressed as

$$\lambda_i(\theta(t)) = (\mathcal{G}_{i1}(\theta_1(t)) \wedge \dots \wedge \mathcal{G}_{ip}(\theta_p(t))), \quad i = 1, 2, \dots, r, \quad (2.4)$$

where  $\mathcal{G}_{i1}(\theta_1(t)), \dots, \mathcal{G}_{ip}(\theta_p(t))$  is the grade of the membership of  $\theta_1(t), \dots, \theta_p(t)$  in  $\mathcal{G}_{ij}$ .

By fuzzy blending technique, the final output of (2.2) is calculated as

$$\dot{\eta}(t) = \frac{\sum_{i=1}^r \lambda_i(\theta(t)) \{ \mathcal{A}_i \eta(t) + \mathcal{A}_{\gamma_i} \eta(t - \gamma(t)) \}}{\sum_{i=1}^r \lambda_i(\theta(t))} = \sum_{i=1}^r w_i(\theta(t)) \{ \mathcal{A}_i \eta(t) + \mathcal{A}_{\gamma_i} \eta(t - \gamma(t)) \}, \quad (2.5)$$

where  $w_i(\theta(t)) = \frac{\lambda_i(\theta(t))}{\sum_{i=1}^r \lambda_i(\theta(t))}$ ,  $\forall t$  and  $i = 1, 2, \dots, r$ , is called the fuzzy weighting function and  $\lambda_i(\theta(t)) = \prod_{j=1}^p \mathcal{G}_{ij}(\theta_j(t))$ . Since  $\lambda_i(\theta(t)) > 0$ , it holds that  $w_i(\theta(t)) \geq 0$  and  $\sum_{i=1}^r w_i(\theta(t)) = 1$  for all  $i = 1, 2, \dots, r$ . Further,  $w_i(\theta(t))$  will be denoted as  $w_i$  for simplicity.

The objective of this study is to derive a delay-range-dependent stability condition for T-S fuzzy time delay system (2.5). Following lemmas are used to obtain our main result.

**Lemma 1.** ([40]) *For a positive definite matrix  $R > 0$ , and any continuously differentiable function  $\eta(\cdot) : [\delta_1, \delta_2] \rightarrow \mathbb{R}^n$ , the following inequality holds*

$$\delta_{12} \int_{\delta_1}^{\delta_2} \dot{\eta}^T(\rho) R \dot{\eta}(\rho) d\rho \geq \Psi_1^T R \Psi_1 + 3 \Psi_2^T R \Psi_2 + 5 \Psi_3^T R \Psi_3 + 7 \Psi_4^T R \Psi_4, \quad (2.6)$$

where

$$\begin{aligned} \Psi_1 &= [\eta(\delta_2) - \eta(\delta_1)], \quad \delta_{12} = (\delta_2 - \delta_1), \\ \Psi_2 &= [\eta(\delta_2) + \eta(\delta_1) - \frac{2}{\delta_{12}} \int_{\delta_1}^{\delta_2} \eta(\rho) d\rho], \\ \Psi_3 &= [\eta(\delta_2) - \eta(\delta_1) + \frac{6}{\delta_{12}} \int_{\delta_1}^{\delta_2} \eta(\rho) d\rho - \frac{12}{\delta_{12}^2} \int_{\delta_1}^{\delta_2} \int_{\theta}^{\delta_2} \eta(\rho) d\rho d\theta], \\ \Psi_4 &= [\eta(\delta_2) + \eta(\delta_1) - \frac{12}{\delta_{12}} \int_{\delta_1}^{\delta_2} \eta(\rho) d\rho + \frac{60}{\delta_{12}^2} \int_{\delta_1}^{\delta_2} \int_{\theta}^{\delta_2} \eta(\rho) d\rho d\theta - \frac{120}{\delta_{12}^3} \int_{\delta_1}^{\delta_2} \int_{\theta}^{\delta_2} \int_{\lambda}^{\delta_2} \eta(\rho) d\rho d\lambda d\theta]. \end{aligned}$$

**Lemma 2.** ([40]) *Let  $\eta(t)$  be a continuously differentiable function,  $R \in \mathbb{R}^{n \times n}$  be real symmetric positive definite matrix and  $\psi_1(t), \psi_2(t) \in \mathbb{R}^{4n \times n}$  are real vectors,  $\epsilon_1, \epsilon_2 \in [0, 1]$ ,  $\delta_1 \leq \delta(t) \leq \delta_2$  are positive real scalars and  $\tau$  satisfies  $\tau \in (0, 1)$ . If there exists any real symmetric matrices  $M_1, M_2 \in \mathbb{R}^{4n \times 4n}$  and any appropriately dimensioned matrices  $X_1, X_2 \in \mathbb{R}^{4n \times 4n}$ ,*

$$\begin{bmatrix} \mathfrak{R} - M_1 & X_1 \\ * & \mathfrak{R} - \epsilon_2 M_1 \end{bmatrix} \geq 0, \quad \begin{bmatrix} \mathfrak{R} - \epsilon_1 M_2 & X_2 \\ * & \mathfrak{R} - M_2 \end{bmatrix} \geq 0, \quad (2.7)$$

such that the inequality

$$\begin{aligned} -\delta_{12} \int_{t-\delta_2}^{t-\delta_1} \dot{\eta}^T(\rho) R \dot{\eta}(\rho) d\rho &\leq -\chi^T(t) \left[ \psi_1^T \left( \mathfrak{R} + (1-\tau)M_1 + \epsilon_1 \frac{(1-\tau)^2}{\tau} M_2 \right) \psi_1 \right. \\ &\quad \left. - \text{Sym} \left\{ \psi_1^T (\tau X_1 + (1-\tau)X_2) \psi_2 \right\} \right. \\ &\quad \left. - \psi_2^T \left( \mathfrak{R} + \tau M_2 + \epsilon_2 \frac{\tau^2}{1-\tau} M_1 \right) \psi_2 \right] \chi(t), \end{aligned} \quad (2.8)$$

holds, where

$$\begin{aligned} \chi(t) &= [\eta^T(t - \delta_1), \eta^T(t - \delta(t)), \eta^T(t - \delta_2), \frac{1}{\delta(t) - \delta_1} \int_{t-\delta(t)}^{t-\delta_1} \eta^T(\rho) d\rho, \\ &\quad \frac{1}{\delta_1 - \delta(t)} \int_{t-\delta_2}^{t-\delta(t)} \eta^T(\rho) d\rho, \frac{2}{(\delta(t) - \delta_1)^2} \int_{t-\delta(t)}^{t-\delta_1} \int_{\theta}^{t-\delta_1} \eta^T(\rho) d\rho d\theta, \end{aligned}$$

$$\begin{aligned} & \frac{2}{(\delta_2 - \delta(t))^2} \int_{t-\delta_2}^{t-\delta(t)} \int_{\theta}^{t-\delta(t)} \eta^T(\rho) d\rho d\theta, \frac{6}{(\delta(t) - \delta_1)^3} \int_{t-\delta(t)}^{t-\delta_1} \int_{\theta}^{t-\delta_1} \int_{\lambda}^{t-\delta_1} \eta^T(\rho) d\rho d\lambda d\theta, \\ & \frac{6}{(\delta_2 - \delta(t))^3} \int_{t-\delta_2}^{t-\delta(t)} \int_{\theta}^{t-\delta(t)} \int_{\lambda}^{t-\delta(t)} \eta^T(\rho) d\rho d\lambda d\theta \Big], \\ \psi_1 &= [\sigma_1^T \ \sigma_2^T \ \sigma_3^T \ \sigma_4^T]^T, \ \psi_2 = [\sigma_5^T \ \sigma_6^T \ \sigma_7^T \ \sigma_8^T]^T, \\ \mathfrak{K} &= \text{diag}\{R, 3R, 5R, 7R\}, \ \sigma_1 = (\bar{e}_1 - \bar{e}_2), \\ \sigma_2 &= (\bar{e}_1 + \bar{e}_2 - 2\bar{e}_4), \sigma_3 = (\bar{e}_1 - \bar{e}_2 + 6\bar{e}_4 - 6\bar{e}_6), \sigma_5 = (\bar{e}_2 - \bar{e}_3), \sigma_6 = (\bar{e}_2 + \bar{e}_3 - 2\bar{e}_5), \\ \sigma_7 &= (\bar{e}_2 - \bar{e}_3 + 6\bar{e}_5 - 6\bar{e}_7), \sigma_4 = (\bar{e}_1 + \bar{e}_2 - 12\bar{e}_4 + 30\bar{e}_6 - 20\bar{e}_8), \\ \sigma_8 &= (\bar{e}_2 + \bar{e}_3 - 12\bar{e}_5 + 30\bar{e}_7 - 20\bar{e}_9), \\ \bar{e}_q &= [0_{n \times (q-1)n} \ I_{n \times n} \ 0_{n \times (9-q)n}]^T, \ q = 1, 2, \dots, 9. \end{aligned}$$

**Remark 1.** It is worth noting that Lemma 2 is coupled to the two predefined individual factors  $\epsilon_1$  and  $\epsilon_2$ .  $\epsilon_1$  and  $\epsilon_2$  can be determined independently because they are distinct of each other and unrestricted.

**Lemma 3.** ([19]) Let  $f(s) = a_2 s^2 + a_1 s + a_0$ , where  $s \in [h_1, h_2]$  and  $a_0, a_1, a_2 \in \mathbb{R}$ . Suppose that the following conditions are satisfied

$$(i) \ f(h_1) < 0, \ (ii) \ f(h_2) < 0, \ (iii) \ -a_2(h_2 - h_1)^2 + f(h_1) < 0.$$

Then,  $f(s) < 0$ .

### 3. Stability analysis

In this section, an improved DRD stability condition is established for the T-S fuzzy TDS (2.5). The following notations are defined for simplicity:

$$\begin{aligned} \xi(t) &= [\eta^T(t), \eta^T(t - \gamma_1), \eta^T(t - \gamma(t)), \eta^T(t - \gamma_2), \dot{\eta}^T(t), \dot{\eta}^T(t - \gamma_1), \dot{\eta}^T(t - \gamma(t)), \dot{\eta}^T(t - \gamma_2), \\ & \frac{2}{\gamma_1} \int_{t-\gamma_1}^t \eta^T(\rho) d\rho, \frac{1}{\gamma(t) - \gamma_1} \int_{t-\gamma(t)}^{t-\gamma_1} x^T(\rho) d\rho, \frac{1}{\gamma_2 - \gamma(t)} \int_{t-\gamma_2}^{t-\gamma(t)} \eta^T(\rho) d\rho, \\ & \frac{2}{\gamma_1^2} \int_{t-\gamma_1}^t \int_{\theta}^t \eta^T(\rho) d\rho d\theta, \frac{2}{(\gamma(t) - \gamma_1)^2} \int_{t-\gamma(t)}^{t-\gamma_1} \int_{\theta}^{t-\gamma_1} \eta^T(\rho) d\rho d\theta, \\ & \frac{2}{(\gamma_2 - \gamma(t))^2} \int_{t-\gamma_2}^{t-\gamma(t)} \int_{\theta}^{t-\gamma(t)} \eta^T(\rho) d\rho d\theta, \frac{6}{\gamma_1^3} \int_{t-\gamma_1}^t \int_{\theta}^t \int_{\lambda}^t \eta^T(\rho) d\rho d\lambda d\theta, \\ & \frac{6}{(\gamma(t) - \gamma_1)^3} \int_{t-\gamma(t)}^{t-\gamma_1} \int_{\theta}^{t-\gamma_1} \int_{\lambda}^{t-\gamma_1} \eta^T(\rho) d\rho d\lambda d\theta \\ & \frac{6}{(\gamma_2 - \gamma(t))^3} \int_{t-\gamma_2}^{t-\gamma(t)} \int_{\theta}^{t-\gamma(t)} \int_{\lambda}^{t-\gamma(t)} \eta^T(\rho) d\rho d\lambda d\theta]^T, \gamma_{12} = (\gamma_2 - \gamma_1), \alpha = \frac{\gamma(t) - \gamma_1}{\gamma_{12}}, \\ \bar{\xi}(t) &= [\eta^T(t), \eta^T(t - \gamma(t)), \eta^T(t - \gamma_2), \dot{\eta}^T(t), \dot{\eta}^T(t - \gamma(t)), \dot{\eta}^T(t - \gamma_2), \frac{1}{\gamma(t)} \int_{t-\gamma(t)}^t x^T(\rho) d\rho, \\ & \frac{1}{\gamma_2 - \gamma(t)} \int_{t-\gamma_2}^{t-\gamma(t)} \eta^T(\rho) d\rho, \frac{2}{(\gamma(t))^2} \int_{t-\gamma(t)}^t \int_{\theta}^t \eta^T(\rho) d\rho d\theta, \end{aligned}$$

$$\begin{aligned} & \frac{2}{(\gamma_2 - \gamma(t))^2} \int_{t-\gamma_2}^{t-\gamma(t)} \int_{\theta}^{t-\gamma(t)} \eta^T(\rho) d\rho d\theta \frac{6}{(\gamma(t))^3} \int_{t-\gamma(t)}^t \int_{\lambda}^t \int_{\lambda}^t \eta^T(\rho) d\rho d\lambda d\theta, \\ & \frac{6}{(\gamma_2 - \gamma(t))^3} \int_{t-\gamma_2}^{t-\gamma(t)} \int_{\theta}^{t-\gamma(t)} \int_{\lambda}^{t-\gamma(t)} \eta^T(\rho) d\rho d\lambda d\theta \Big]^T, \quad \bar{\alpha} = \frac{\gamma(t)}{\gamma_2}, \\ & \widehat{e}_p = \begin{bmatrix} 0_{n \times (p-1)n} & I_{n \times n} & 0_{n \times (6-p)n} \end{bmatrix}, \quad p = 1, 2, \dots, 6, \quad e_q = \begin{bmatrix} 0_{n \times (q-1)n} & I_{n \times n} & 0_{n \times (17-q)n} \end{bmatrix}, \quad q = 1, 2, \dots, 17, \\ & \widetilde{e}_r = \begin{bmatrix} 0_{n \times (r-1)n} & I_{n \times n} & 0_{n \times (3-r)n} \end{bmatrix}, \quad r = 1, 2, 3, \quad \bar{e}_s = \begin{bmatrix} 0_{n \times (s-1)n} & I_{n \times n} & 0_{n \times (12-s)n} \end{bmatrix}, \quad s = 1, 2, \dots, 12. \end{aligned}$$

**Theorem 1.** For given scalars  $\gamma_1, \gamma_2, v_1$  and  $v_2$ , the  $T$ - $S$  fuzzy TDS (2.5) with (2.3), is asymptotically stable if there exist symmetric positive definite matrices  $\mathcal{P}_1 \in \mathbb{R}^{6n \times 6n}$ ,  $\mathcal{P}_2 \in \mathbb{R}^{4n \times 4n}$ ,  $\mathcal{Q}_l \in \mathbb{R}^{2n \times 2n}$ , ( $l = 1, 2, 3, 4$ ),  $\mathcal{R}_k \in \mathbb{R}^{n \times n}$ , symmetric matrices  $\mathcal{M}_k \in \mathbb{R}^{4n \times 4n}$ , and any matrices  $\mathcal{Y}_k \in \mathbb{R}^{4n \times 4n}$  ( $k = 1, 2$ ) and  $N_p$  ( $p = 1, 2, 3$ ) with suitable dimension such that the following LMIs are satisfied for all  $\dot{\gamma}(t) \in [v_1, v_2]$ ,  $i = 1, 2, \dots, r$ :

$$\mathcal{P}_1 + \gamma_1 \Upsilon^T \mathcal{P}_2 \Upsilon > 0, \quad \mathcal{P}_1 + \gamma_2 \Upsilon^T \mathcal{P}_2 \Upsilon > 0, \quad (3.1)$$

$$\begin{bmatrix} \widehat{\mathcal{R}}_2 - \mathcal{M}_1 & \mathcal{Y}_1 \\ * & \widehat{\mathcal{R}}_2 - \mathcal{M}_1 \end{bmatrix} \geq 0, \quad \begin{bmatrix} \widehat{\mathcal{R}}_2 & \mathcal{Y}_2 \\ * & \widehat{\mathcal{R}}_2 - \mathcal{M}_2 \end{bmatrix} \geq 0, \quad (3.2)$$

$$\begin{bmatrix} \widehat{\mathcal{R}}_2 - \mathcal{M}_1 & \mathcal{Y}_1 \\ * & \widehat{\mathcal{R}}_2 \end{bmatrix} \geq 0, \quad \begin{bmatrix} \widehat{\mathcal{R}}_2 - \mathcal{M}_2 & \mathcal{Y}_2 \\ * & \widehat{\mathcal{R}}_2 - \mathcal{M}_2 \end{bmatrix} \geq 0, \quad (3.3)$$

and

$$f(0, \dot{\gamma}(t)) < 0, \quad f(1, \dot{\gamma}(t)) < 0, \quad (3.4)$$

$$-\Xi_2 + f(0, \dot{\gamma}(t)) < 0, \quad (3.5)$$

where

$$\begin{aligned} f(0, \dot{\gamma}(t)) &= f(\alpha, \dot{\gamma}(t))|_{\alpha=0}, \quad f(1, \dot{\gamma}(t)) = f(\alpha, \dot{\gamma}(t))|_{\alpha=1}, \\ f(\alpha, \dot{\gamma}(t)) &= \alpha^2 \Xi_2 + \alpha \Xi_1 + (\Xi_0 + \text{Sym}\{\Delta_{21} \Theta_i\}), \\ \Xi_0 &= \text{Sym}\{(\Delta_{1a}^T + \Delta_{1c}^T) \mathcal{P}_1 \Delta_2\} + \gamma_1 \text{Sym}\{(\Delta_{3a}^T + \Delta_{3c}^T) \mathcal{P}_2 \Delta_4\} + \dot{\gamma}(t) (\Delta_{3a}^T + \Delta_{3c}^T) \mathcal{P}_2 (\Delta_{3a} + \Delta_{3c}) \\ & \quad + e_5^T (\gamma_1^2 \mathcal{R}_1 + \gamma_{12}^2 \mathcal{R}_2) e_5 + \Delta_5^T (\mathcal{Q}_1 + \mathcal{Q}_2) \Delta_5 + \Delta_6^T (-\mathcal{Q}_1 + \mathcal{Q}_3) \Delta_6 + (1 - \dot{\gamma}(t)) \Delta_7^T (-\mathcal{Q}_3 + \mathcal{Q}_4) \Delta_7 \\ & \quad + \Delta_8^T (-\mathcal{Q}_2 - \mathcal{Q}_4) \Delta_8 - \Delta_9^T \mathcal{R}_1 \Delta_9 - 3 \Delta_{10}^T \mathcal{R}_1 \Delta_{10} - 5 \Delta_{11}^T \mathcal{R}_1 \Delta_{11} - 7 \Delta_{12}^T \mathcal{R}_1 \Delta_{12} \\ & \quad - \Psi_1^T (\widehat{\mathcal{R}}_2 + \mathcal{M}_1 + \mathcal{M}_2) \Psi_1 - \Psi_2^T \widehat{\mathcal{R}}_2 \Psi_2 - \text{Sym}\{\Psi_1^T \mathcal{Y}_2 \Psi_2\}, \\ \Xi_1 &= \text{Sym}\{(\Delta_{1b}^T - \Delta_{1c}^T) \mathcal{P}_1 \Delta_2\} + \gamma_1 \text{Sym}\{(\Delta_{3b}^T - \Delta_{3c}^T) \mathcal{P}_2 \Delta_4\} + \dot{\gamma}(t) \text{Sym}\{(\Delta_{3a}^T + \Delta_{3c}^T) \mathcal{P}_2 (\Delta_{3b} - \Delta_{3c})\} \\ & \quad + \gamma_{12} \text{Sym}\{(\Delta_{3a}^T + \Delta_{3c}^T) \mathcal{P}_2 \Delta_4\} - \Psi_1^T (-\mathcal{M}_1 - 2\mathcal{M}_2) \Psi_1 \\ & \quad - \Psi_2^T \mathcal{M}_2 \Psi_2 - \text{Sym}\{\Psi_1^T \mathcal{Y}_1 \Psi_2\} + \text{Sym}\{\Psi_1^T \mathcal{Y}_2 \Psi_2\}, \\ \Xi_2 &= \dot{\gamma}(t) (\Delta_{3b}^T - \Delta_{3c}^T) \mathcal{P}_2 (\Delta_{3b} - \Delta_{3c}) + \gamma_{12} \text{Sym}\{(\Delta_{3b}^T - \Delta_{3c}^T) \mathcal{P}_2 \Delta_4\} - \Psi_1^T \mathcal{M}_2 \Psi_1 - \Psi_2^T \mathcal{M}_1 \Psi_2, \\ \Upsilon &= [\widetilde{e}_1^T, \widetilde{e}_2^T, \widetilde{e}_3^T, \widetilde{e}_4^T]^T, \quad \widehat{\mathcal{R}}_2 = \text{diag}\{\mathcal{R}_2, 3\mathcal{R}_2, 5\mathcal{R}_2, 7\mathcal{R}_2\}, \quad \Delta_{1b} = [0, 0, \gamma_{12} e_{10}^T, 0, 0, 0]^T, \\ \Delta_{1a} &= [e_1^T, \gamma_1 e_9^T, 0, 0, \gamma_1 e_{12}^T, \gamma_1 e_{15}^T]^T, \quad \Delta_{1c} = [0, 0, 0, \gamma_{12} e_{11}^T, 0, 0]^T, \end{aligned}$$

$$\begin{aligned}
\Delta_2 &= \left[ e_5^T, (e_1 - e_2)^T, (e_2 - (1 - \dot{\gamma}(t))e_3)^T, ((1 - \dot{\gamma}(t))e_3 - e_4)^T, 2(e_1 - e_9)^T, 3(e_1 - e_{12})^T \right]^T, \\
\Delta_{3a} &= \left[ e_1^T, \gamma_1 e_9^T, 0, 0 \right]^T, \Delta_{3b} = \left[ 0, 0, \gamma_{12} e_{10}^T, 0 \right]^T, \Delta_{3c} = \left[ 0, 0, 0, \gamma_{12} e_{11}^T \right]^T, \\
\Delta_5 &= \left[ e_1^T, e_5^T \right]^T, \Delta_6 = \left[ e_2^T, e_6^T \right]^T, \Delta_7 = \left[ e_3^T, e_7^T \right]^T, \Delta_8 = \left[ e_4^T, e_8^T \right]^T, \\
\Delta_4 &= \left[ e_5^T, (e_1 - e_2)^T, (e_2 - (1 - \dot{\gamma}(t))e_3)^T, ((1 - \dot{\gamma}(t))e_3 - e_4)^T \right]^T, \\
\Delta_9 &= (e_1 - e_2), \Delta_{10} = (e_1 + e_2 - 2e_9), \Delta_{12} = (e_1 + e_2 - 12e_9 + 30e_{12} - 20e_{15}), \\
\Delta_{11} &= (e_1 - e_2 + 6e_9 - 6e_{12}), \Psi_1 = \left[ \Delta_{13}^T, \Delta_{14}^T, \Delta_{15}^T, \Delta_{16}^T \right]^T, \Psi_2 = \left[ \Delta_{17}^T, \Delta_{18}^T, \Delta_{19}^T, \Delta_{20}^T \right]^T, \\
\Delta_{13} &= (e_2 - e_3), \Delta_{14} = (e_2 + e_3 - 2e_{10}), \Delta_{15} = (e_2 - e_3 + 6e_{10} - 6e_{13}), \\
\Delta_{17} &= (e_3 - e_4), \Delta_{19} = (e_3 - e_4 + 6e_{11} - 6e_{14}), \Delta_{18} = (e_3 + e_4 - 2e_{11}), \\
\Delta_{16} &= (e_2 + e_3 - 12e_{10} + 30e_{13} - 20e_{16}), \Delta_{20} = (e_3 + e_4 - 12e_{11} + 30e_{14} - 20e_{17}), \\
\Delta_{21} &= \left[ e_1^T \mathcal{N}_1 + e_3^T \mathcal{N}_2 + e_5^T \mathcal{N}_3 \right], \Theta_i = \left[ \mathcal{A}_i e_1 + \mathcal{A}_{\gamma_i} e_3 - e_5 \right].
\end{aligned}$$

*Proof.* Choose the delay-dependent LKF as follows:

$$V(\eta_t) = \sum_{q=1}^4 V_q(\eta_t), \quad (3.6)$$

where

$$\begin{aligned}
V_1(\eta_t) &= \varpi_1^T(t) \mathcal{P}_1 \varpi_1(t), \quad V_2(\eta_t) = \gamma(t) \varpi_2^T(t) \mathcal{P}_2 \varpi_2(t), \\
V_3(\eta_t) &= \int_{t-\gamma_1}^t \varpi_3^T(s) \mathcal{Q}_1 \varpi_3(s) ds + \int_{t-\gamma_2}^t \varpi_3^T(s) \mathcal{Q}_2 \varpi_3(s) ds \\
&\quad + \int_{t-\gamma(t)}^{t-\gamma_1} \varpi_3^T(s) \mathcal{Q}_3 \varpi_3(s) ds + \int_{t-\gamma_2}^{t-\gamma(t)} \varpi_3^T(s) \mathcal{Q}_4 \varpi_3(s) ds, \\
V_4(\eta_t) &= \gamma_1 \int_{-\gamma_1}^0 \int_{t+\lambda}^t \dot{\eta}^T(s) \mathcal{R}_1 \dot{\eta}(s) ds d\lambda + \gamma_{12} \int_{-\gamma_2}^{-\gamma_1} \int_{t+\lambda}^t \dot{\eta}^T(s) \mathcal{R}_2 \dot{\eta}(s) ds d\lambda,
\end{aligned}$$

with

$$\begin{aligned}
\varpi_1(t) &= \left[ \eta^T(t), \int_{t-\gamma_1}^t \eta^T(s) ds, \int_{t-\gamma(t)}^{t-\gamma_1} \eta^T(s) ds, \int_{t-\gamma_2}^{t-\gamma(t)} \eta^T(s) ds, \frac{2}{\gamma_1} \int_{-\gamma_1}^0 \int_{t+\lambda}^t \eta^T(s) ds d\lambda, \right. \\
&\quad \left. \frac{6}{\gamma_1^2} \int_{-\gamma_1}^0 \int_{\lambda}^0 \int_{t+\theta}^t \eta^T(s) ds d\theta d\lambda \right]^T, \quad \varpi_3(t) = \left[ \eta^T(t), \dot{\eta}^T(t) \right]^T, \\
\varpi_2(t) &= \left[ \eta^T(t), \int_{t-\gamma_1}^t \eta^T(s) ds, \int_{t-\gamma(t)}^{t-\gamma_1} \eta^T(s) ds, \int_{t-\gamma_2}^{t-\gamma(t)} \eta^T(s) ds \right]^T.
\end{aligned}$$

In order to meet the stability condition of the fuzzy system (2.5) using the Lyapunov method, first we show the positive definiteness of  $V(\eta_t)$ . From the LKF  $V_1(\eta_t)$  and  $V_2(\eta_t)$ , we deduce

$$V_1(\eta_t) + V_2(\eta_t) = \varpi_1^T(t) \left[ \mathcal{P}_1 + \gamma(t) \Upsilon^T \mathcal{P}_2 \Upsilon \right] \varpi_1(t) = \varpi_1^T(t) \left[ \mathcal{P}_1 + (\gamma_1 + \alpha \gamma_{12}) \Upsilon^T \mathcal{P}_2 \Upsilon \right] \varpi_1(t), \quad (3.7)$$

where  $\mathcal{V}$  is defined after (3.5).

Therefore, if the LMIs (3.1) is holds; then  $V_1(\eta_t) + V_2(\eta_t) > \epsilon \|\eta_t\|^2$  should be fulfilled with  $\epsilon > 0$ . Hence, the positivity of  $V(\eta_t)$  ensure when the LMIs in (3.1) and  $\mathcal{Q}_l, \mathcal{R}_k > 0$  ( $l = 1, 2, 3, 4, k = 1, 2$ ) hold.

Finding the time derivative of (3.6) along with the trajectory of (2.5), one can obtain

$$\dot{V}(\eta_t) = \sum_{q=1}^4 \dot{V}_q(\eta_t), \quad (3.8)$$

where

$$\begin{aligned} \dot{V}_1(\eta_t) &= 2\varpi_1^T(t)\mathcal{P}_1 \begin{bmatrix} \dot{\eta}(t) \\ \eta(t) - \eta(t - \gamma_1) \\ \eta(t - \gamma_1) - (1 - \dot{\gamma}(t))\eta(t - \gamma(t)) \\ (1 - \dot{\gamma}(t))\eta(t - \gamma(t)) - \eta(t - \gamma_2) \\ 2\eta(t) - \frac{2}{\gamma_1} \int_{t-\gamma_1}^t \eta(s)ds \\ 3\eta(t) - \frac{6}{\gamma_1^2} \int_{t-\gamma_1}^t \int_{\lambda}^t \eta(s)dsd\lambda \end{bmatrix} \\ &= 2\xi^T(t) \left[ (\mathcal{A}_{1a}^T + \alpha\mathcal{A}_{1b}^T + (1 - \alpha)\mathcal{A}_{1c}^T)\mathcal{P}_1\mathcal{A}_2 \right] \xi(t) \\ &= \xi^T(t) \left[ Sym \left\{ (\mathcal{A}_{1a}^T + \mathcal{A}_{1c}^T)\mathcal{P}_1\mathcal{A}_2 \right\} + \alpha Sym \left\{ (\mathcal{A}_{1b}^T - \mathcal{A}_{1c}^T)\mathcal{P}_1\mathcal{A}_2 \right\} \right] \xi(t). \end{aligned} \quad (3.9)$$

Similarly, we obtain

$$\begin{aligned} \dot{V}_2(\eta_t) &= \dot{\gamma}(t)\varpi_2^T(t)\mathcal{P}_2\varpi_2(t) + 2\gamma(t)\varpi_2^T(t)\mathcal{P}_2\dot{\varpi}_2(t) \\ &= \xi^T(t) \left[ \dot{\gamma}(t)(\mathcal{A}_{3a}^T + \alpha\mathcal{A}_{3b}^T + (1 - \alpha)\mathcal{A}_{3c}^T)\mathcal{P}_2(\mathcal{A}_{3a} + \alpha\mathcal{A}_{3b} + (1 - \alpha)\mathcal{A}_{3c}) \right. \\ &\quad \left. + (\gamma_1 + \alpha\gamma_{12})Sym \left\{ (\mathcal{A}_{3a}^T + \alpha\mathcal{A}_{3b}^T + (1 - \alpha)\mathcal{A}_{3c}^T)\mathcal{P}_2\mathcal{A}_4 \right\} \right] \xi(t) \\ &= \xi^T(t) \left[ \dot{\gamma}(t)(\mathcal{A}_{3a}^T + \mathcal{A}_{3c}^T)\mathcal{P}_2(\mathcal{A}_{3a} + \mathcal{A}_{3c}) + \gamma_1 Sym \left\{ (\mathcal{A}_{3a}^T + \mathcal{A}_{3c}^T)\mathcal{P}_2\mathcal{A}_4 \right\} \right. \\ &\quad \left. + \alpha \left\{ \gamma_1 Sym \left\{ (\mathcal{A}_{3b}^T - \mathcal{A}_{3c}^T)\mathcal{P}_2\mathcal{A}_4 \right\} \right. \right. \\ &\quad \left. \left. + \dot{\gamma}(t)Sym \left\{ (\mathcal{A}_{3a}^T + \mathcal{A}_{3c}^T)\mathcal{P}_2(\mathcal{A}_{3b} - \mathcal{A}_{3c}) \right\} + \gamma_{12}Sym \left\{ (\mathcal{A}_{3a}^T + \mathcal{A}_{3c}^T)\mathcal{P}_2\mathcal{A}_4 \right\} \right\} \right. \\ &\quad \left. + \alpha^2 \left\{ (\mathcal{A}_{3b}^T - \mathcal{A}_{3c}^T)\mathcal{P}_2(\mathcal{A}_{3b} - \mathcal{A}_{3c}) + \gamma_{12}Sym \left\{ (\mathcal{A}_{3b}^T - \mathcal{A}_{3c}^T)\mathcal{P}_2\mathcal{A}_4 \right\} \right\} \right] \xi(t), \end{aligned} \quad (3.10)$$

$$\begin{aligned} \dot{V}_3(\eta_t) &= \varpi_3^T(t)(\mathcal{Q}_1 + \mathcal{Q}_2)\varpi_3(t) + \varpi_3^T(t - \gamma_1)(-\mathcal{Q}_1 + \mathcal{Q}_3)\varpi_3(t - \gamma_1) \\ &\quad + (1 - \dot{\gamma}(t))\varpi_3^T(t - \gamma(t))(-\mathcal{Q}_3 + \mathcal{Q}_4)\varpi_3(t - \gamma(t)) + \varpi_3^T(t - \gamma_2)(-\mathcal{Q}_2 - \mathcal{Q}_4)\varpi_3(t - \gamma_2) \\ &= \xi^T(t) \left[ \mathcal{A}_5^T(\mathcal{Q}_1 + \mathcal{Q}_2)\mathcal{A}_5 + \mathcal{A}_6^T(-\mathcal{Q}_1 + \mathcal{Q}_3)\mathcal{A}_6 \right. \\ &\quad \left. + (1 - \dot{\gamma}(t))\mathcal{A}_7^T(-\mathcal{Q}_3 + \mathcal{Q}_4)\mathcal{A}_7 + \mathcal{A}_8^T(-\mathcal{Q}_2 - \mathcal{Q}_4)\mathcal{A}_8 \right] \xi(t), \end{aligned} \quad (3.11)$$

$$\dot{V}_4(\eta_t) = \dot{\eta}^T(t)(\gamma_1^2\mathcal{R}_1 + \gamma_{12}^2\mathcal{R}_2)\dot{\eta}(t) - \gamma_1 \int_{t-\gamma_1}^t \dot{\eta}^T(s)\mathcal{R}_1\dot{\eta}(s)ds - \gamma_{12} \int_{t-\gamma_2}^{t-\gamma_1} \dot{\eta}^T(s)\mathcal{R}_2\dot{\eta}(s)ds, \quad (3.12)$$

where  $\mathcal{A}_l$  ( $l = 2, 4, 5, \dots, 8$ ),  $\mathcal{A}_{1a}$ ,  $\mathcal{A}_{1b}$ ,  $\mathcal{A}_{1c}$ ,  $\mathcal{A}_{3a}$ ,  $\mathcal{A}_{3b}$ , and  $\mathcal{A}_{3c}$  can be found after (3.5).



The first integral term in the right hand side (RHS) of (3.12) contains only constant limits of integration, Lemma 1 yields

$$-\gamma_1 \int_{t-\gamma_1}^t \dot{\eta}^T(s) \mathcal{R}_1 \dot{\eta}(s) ds \leq -\xi^T(t) [\Delta_9^T \mathcal{R}_1 \Delta_9 + 3\Delta_{10}^T \mathcal{R}_1 \Delta_{10} + 5\Delta_{11}^T \mathcal{R}_1 \Delta_{11} + 7\Delta_{12}^T \mathcal{R}_1 \Delta_{12}] \xi(t). \quad (3.13)$$

Next, treating second integral term in the RHS of (3.13) containing uncertain limit of integration. According to Lemma 2, we choose  $\epsilon_1 = \alpha$  and  $\epsilon_2 = (1 - \alpha)$  ( $\alpha \in [0, 1]$ ).

If the LMIs  $\begin{bmatrix} \widehat{\mathcal{R}}_2 - \mathcal{M}_1 & \mathcal{Y}_1 \\ * & \widehat{\mathcal{R}}_2 - (1 - \alpha)\mathcal{M}_2 \end{bmatrix}$  and  $\begin{bmatrix} \widehat{\mathcal{R}}_2 - \alpha\mathcal{M}_1 & \mathcal{Y}_2 \\ * & \widehat{\mathcal{R}}_2 - \mathcal{M}_2 \end{bmatrix}$  is satisfied for all  $\alpha \in [0, 1]$ , then approximate the second integral terms in the RHS of (3.13) Lemma 2 is applied, yields

$$\begin{aligned} -\gamma_{12} \int_{t-\gamma_2}^{t-\gamma_1} \dot{\eta}^T(s) \mathcal{R}_2 \dot{\eta}(s) ds &\leq -\xi^T(t) \left[ \Psi_1^T (\widehat{\mathcal{R}}_2 + (1 - \alpha)\mathcal{M}_1 + (1 - \alpha)^2 \mathcal{M}_2) \Psi_1 \right. \\ &\quad \left. + Sym \{ \Psi_1^T (\alpha \mathcal{Y}_1 + (1 - \alpha)\mathcal{Y}_2) \Psi_2 \} + \Psi_2^T (\widehat{\mathcal{R}}_2 + \alpha \mathcal{M}_2 + \alpha^2 \mathcal{M}_1) \Psi_2 \right] \xi(t) \\ &= -\xi^T(t) \left[ \Psi_1^T (\widehat{\mathcal{R}}_2 + \mathcal{M}_1 + \mathcal{M}_2) \Psi_1 + \Psi_2^T \widehat{\mathcal{R}}_2 \Psi_2 + Sym \{ \Psi_1^T \mathcal{Y}_2 \Psi_2 \} \right. \\ &\quad \left. + \alpha \{ \Psi_1^T (-\mathcal{M}_1 - 2\mathcal{M}_2) \Psi_1 + \Psi_2^T \mathcal{M}_2 \Psi_2 + Sym \{ \Psi_1^T \mathcal{Y}_1 \Psi_2 \} \right. \\ &\quad \left. - Sym \{ \Psi_1^T \mathcal{Y}_2 \Psi_2 \} \right] + \alpha^2 (\Psi_1^T \mathcal{M}_2 \Psi_1 + \Psi_2^T \mathcal{M}_1 \Psi_2) \xi(t), \end{aligned} \quad (3.14)$$

where  $\Delta_l$  ( $l = 9, 10, \dots, 20$ ),  $\Psi_1$ ,  $\Psi_2$  and  $\widehat{\mathcal{R}}_2$  are defined after (3.5).

For any free weighting matrices  $\mathcal{N}_p$  ( $p = 1, 2, 3$ ) with suitable dimension, the following zero equation holds

$$\begin{aligned} 0 &= 2 \sum_{i=1}^r w_i \left[ \eta^T(t) \mathcal{N}_1 + \eta^T(t - \gamma(t)) \mathcal{N}_2 + \dot{\eta}^T(t) \mathcal{N}_3 \right] \times \left[ \mathcal{A}_i \eta(t) + \mathcal{A}_{\gamma_i} \eta(t - \gamma(t)) - \dot{\eta}(t) \right] \\ &= \sum_{i=1}^r w_i \xi^T(t) \left[ Sym \{ \Delta_{21} \Theta_i \} \right] \xi(t), \end{aligned} \quad (3.15)$$

where

$$\Delta_{21} = [e_1^T \mathcal{N}_1 + e_3^T \mathcal{N}_2 + e_5^T \mathcal{N}_3], \quad \Theta_i = [\mathcal{A}_i e_1 + \mathcal{A}_{\gamma_i} e_3 - e_5].$$

Then, by substituting (3.9)–(3.15) in (3.8), we obtain

$$\begin{aligned} &\dot{V}(\eta_t) \\ &\leq \sum_{i=1}^r w_i \xi^T(t) \left[ Sym \{ (\Delta_{1a}^T + \Delta_{1c}^T) \mathcal{P}_1 \Delta_2 \} + \gamma_1 Sym \{ (\Delta_{3a}^T + \Delta_{3c}^T) \mathcal{P}_2 \Delta_4 \} + \dot{\gamma}(t) (\Delta_{3a}^T + \Delta_{3c}^T) \mathcal{P}_2 (\Delta_{3a} + \Delta_{3c}) \right. \\ &\quad + e_5^T (\gamma_1^2 \mathcal{R}_1 + \gamma_{12}^2 \mathcal{R}_2) e_5 + \Delta_5^T (\mathcal{Q}_1 + \mathcal{Q}_2) \Delta_5 + \Delta_6^T (-\mathcal{Q}_1 + \mathcal{Q}_3) \Delta_6 + (1 - \dot{\gamma}(t)) \Delta_7^T (-\mathcal{Q}_3 + \mathcal{Q}_4) \Delta_7 \\ &\quad + \Delta_8^T (-\mathcal{Q}_2 - \mathcal{Q}_4) \Delta_8 - \Delta_9^T \mathcal{R}_1 \Delta_9 - 3\Delta_{10}^T \mathcal{R}_1 \Delta_{10} - 5\Delta_{11}^T \mathcal{R}_1 \Delta_{11} - 7\Delta_{12}^T \mathcal{R}_1 \Delta_{12} + Sym \{ \Delta_{21} \Theta_i \} \\ &\quad - \Psi_1^T (\widehat{\mathcal{R}}_2 + \mathcal{M}_1 + \mathcal{M}_2) \Psi_1 - \Psi_2^T \widehat{\mathcal{R}}_2 \Psi_2 - Sym \{ \Psi_1^T \mathcal{Y}_2 \Psi_2 \} + \alpha \{ Sym \{ (\Delta_{1b}^T - \Delta_{1c}^T) \mathcal{P}_1 \Delta_2 \} \\ &\quad \left. + \gamma_1 Sym \{ (\Delta_{3b}^T - \Delta_{3c}^T) \mathcal{P}_2 \Delta_4 \} + \dot{\gamma}(t) Sym \{ (\Delta_{3a}^T + \Delta_{3c}^T) \mathcal{P}_2 (\Delta_{3b} - \Delta_{3c}) \} + \gamma_{12} Sym \{ (\Delta_{3a}^T + \Delta_{3c}^T) \mathcal{P}_2 \Delta_4 \} \right] \end{aligned}$$

$$\begin{aligned}
& -\Psi_1^T(-\mathcal{M}_1 - 2\mathcal{M}_2)\Psi_1 - \Psi_2^T\mathcal{M}_2\Psi_2 - \text{Sym}\{\Psi_1^T\mathcal{Y}_1\Psi_2\} + \text{Sym}\{\Psi_1^T\mathcal{Y}_2\Psi_2\} \\
& + \alpha^2\left\{\dot{\gamma}(t)(\mathcal{A}_{3b}^T - \mathcal{A}_{3c}^T)\mathcal{P}_2(\mathcal{A}_{3b} - \mathcal{A}_{3c}) + \gamma_{12}\text{Sym}\{(\mathcal{A}_{3b}^T - \mathcal{A}_{3c}^T)\mathcal{P}_2\mathcal{A}_4\} - \Psi_1^T\mathcal{M}_2\Psi_1 - \Psi_2^T\mathcal{M}_1\Psi_2\right\}\xi(t) \\
& = \sum_{i=1}^r w_i \xi^T(t) \left[ \underbrace{\alpha^2\Xi_2 + \alpha\Xi_1 + (\Xi_0 + \text{Sym}\{\mathcal{A}_{21}\Theta_i\})}_{f(\alpha, \dot{\gamma}(t))} \right] \xi(t),
\end{aligned} \tag{3.16}$$

where  $f(\alpha, \dot{\gamma}(t))$  is defined after (3.5).

Note that, the RHS of (3.16) depends on the two parameters  $\alpha \in [0, 1]$  and  $\dot{\gamma}(t) \in [\nu_1, \nu_2]$ . Since  $\sum_{i=1}^r w_i = 1$  and the RHS of (3.16) is quadratic with respect to  $\alpha$ , so by Lemma 3 we can easily obtain the LMIs in (3.4) and (3.5). Thus, if the LMIs (3.4) and (3.5) along with constraint (3.1)–(3.3) are holds, then it implies that  $\dot{V}(\eta_t) < -\epsilon\|\eta_t\|^2$ , for  $\epsilon > 0$ , which in turn guaranteed the asymptotic stability of the fuzzy system (2.5) as per Lyapunov–Krasovskii Theorem. This completes the proof of Theorem 1.

**Remark 2.** If  $\gamma_1 = 0$ , then  $\gamma_{12} = \gamma_2$ , then Theorem 1 is no more applicable to find the maximum delay upper bound  $\gamma_2$  for stability of the T–S fuzzy TDS (2.5). The following Corollary is formulated to deal with this circumstance.

**Corollary 1.** Given scalars  $\gamma_2, \nu_1$  and  $\nu_2$ , the T–S fuzzy TDS (2.5) with  $0 \leq \gamma(t) \leq \gamma_2$ ,  $\nu_1 \leq \dot{\gamma}(t) \leq \nu_2$  is asymptotically stable if there exist symmetric positive definite matrices  $\bar{\mathcal{P}}_1, \bar{\mathcal{P}}_2 \in \mathbb{R}^{3n \times 3n}$ ,  $\bar{\mathcal{Q}}_l \in \mathbb{R}^{2n \times 2n}$ , ( $l = 1, 2, 3$ ),  $\bar{\mathcal{R}} \in \mathbb{R}^{n \times n}$ , symmetric matrices  $\bar{\mathcal{M}}_k \in \mathbb{R}^{4n \times 4n}$ , and any matrices  $\bar{\mathcal{Y}}_k \in \mathbb{R}^{4n \times 4n}$  ( $k = 1, 2$ ) and  $\bar{\mathcal{N}}_p$  ( $p = 1, 2, 3$ ) with suitable dimension such that the following LMIs are satisfied for all  $\dot{\gamma}(t) \in [\nu_1, \nu_2]$ ,  $i = 1, 2, \dots, r$ :

$$\bar{\mathcal{P}}_1 + \gamma_2 \bar{\mathcal{Y}}^T \bar{\mathcal{P}}_2 \bar{\mathcal{Y}} > 0, \tag{3.17}$$

$$\begin{bmatrix} \widehat{\mathcal{R}} - \bar{\mathcal{M}}_1 & \bar{\mathcal{Y}}_1 \\ * & \widehat{\mathcal{R}} - \bar{\mathcal{M}}_1 \end{bmatrix} \geq 0, \quad \begin{bmatrix} \widehat{\mathcal{R}} & \bar{\mathcal{Y}}_2 \\ * & \widehat{\mathcal{R}} - \bar{\mathcal{M}}_2 \end{bmatrix} \geq 0, \tag{3.18}$$

$$\begin{bmatrix} \widehat{\mathcal{R}} - \bar{\mathcal{M}}_1 & \bar{\mathcal{Y}}_1 \\ * & \widehat{\mathcal{R}} \end{bmatrix} \geq 0, \quad \begin{bmatrix} \widehat{\mathcal{R}} - \bar{\mathcal{M}}_2 & \bar{\mathcal{Y}}_2 \\ * & \widehat{\mathcal{R}} - \bar{\mathcal{M}}_2 \end{bmatrix} \geq 0, \tag{3.19}$$

and

$$\bar{f}(0, \dot{\gamma}(t)) < 0, \quad \bar{f}(1, \dot{\gamma}(t)) < 0, \tag{3.20}$$

$$-\bar{\Xi}_2 + \bar{f}(0, \dot{\gamma}(t)) < 0, \tag{3.21}$$

where

$$\begin{aligned}
& \bar{f}(0, \dot{\gamma}(t)) = \bar{f}(\bar{\alpha}, \dot{\gamma}(t))|_{\bar{\alpha}=0}, \quad \bar{f}(1, \dot{\gamma}(t)) = \bar{f}(\bar{\alpha}, \dot{\gamma}(t))|_{\bar{\alpha}=1}, \\
& \bar{f}(\bar{\alpha}, \dot{\gamma}(t)) = \bar{\alpha}^2 \bar{\Xi}_2 + \bar{\alpha} \bar{\Xi}_1 + (\bar{\Xi}_0 + \text{Sym}\{\bar{\mathcal{A}}_{14} \bar{\Theta}_i\}), \\
& \bar{\Xi}_0 = \text{Sym}\left\{(\bar{\mathcal{A}}_{1a}^T + \bar{\mathcal{A}}_{1c}^T) \bar{\mathcal{P}}_1 \bar{\mathcal{A}}_2\right\} + \dot{\gamma}(t)(\bar{\mathcal{A}}_{1a}^T + \bar{\mathcal{A}}_{1c}^T) \bar{\mathcal{P}}_2 (\bar{\mathcal{A}}_{1a} + \bar{\mathcal{A}}_{1c}) + \bar{e}_4^T (\gamma_2^2 \bar{\mathcal{R}}) \bar{e}_4 + \bar{\mathcal{A}}_3^T (\bar{\mathcal{Q}}_1 + \bar{\mathcal{Q}}_2) \bar{\mathcal{A}}_3 \\
& + (1 - \dot{\gamma}(t)) \bar{\mathcal{A}}_4^T (-\bar{\mathcal{Q}}_2 + \bar{\mathcal{Q}}_3) \bar{\mathcal{A}}_4 + \bar{\mathcal{A}}_5^T (-\bar{\mathcal{Q}}_1 - \bar{\mathcal{Q}}_3) \bar{\mathcal{A}}_5 \\
& - \bar{\Psi}_1^T (\widehat{\mathcal{R}} + \bar{\mathcal{M}}_1 + \bar{\mathcal{M}}_2) \bar{\Psi}_1 - \bar{\Psi}_2^T \widehat{\mathcal{R}} \bar{\Psi}_2 - \text{Sym}\left\{\bar{\Psi}_1^T \bar{\mathcal{Y}}_2 \bar{\Psi}_2\right\},
\end{aligned}$$

$$\begin{aligned}
\bar{\Xi}_1 &= Sym \left\{ (\bar{A}_{1b}^T - \bar{A}_{1c}^T) \bar{\mathcal{P}}_1 \bar{A}_2 \right\} + \dot{\gamma}(t) Sym \left\{ (\bar{A}_{1a}^T + \bar{A}_{1c}^T) \bar{\mathcal{P}}_2 (\bar{A}_{1b} - \bar{A}_{1c}) \right\} - \bar{\Psi}_2^T \bar{\mathcal{M}}_2 \bar{\Psi}_2 \\
&\quad + \gamma_2 Sym \left\{ (\bar{A}_{1a}^T + \bar{A}_{1c}^T) \bar{\mathcal{P}}_2 \bar{A}_2 \right\} - \bar{\Psi}_1^T (-\bar{\mathcal{M}}_1 - 2\bar{\mathcal{M}}_2) \bar{\Psi}_1 - Sym \left\{ \bar{\Psi}_1^T (\bar{\mathcal{Y}}_1 - \bar{\mathcal{Y}}_2) \bar{\Psi}_2 \right\}, \\
\bar{\Xi}_2 &= \dot{\gamma}(t) (\bar{A}_{1b}^T - \bar{A}_{1c}^T) \bar{\mathcal{P}}_2 (\bar{A}_{1b} - \bar{A}_{1c}) + \gamma_2 Sym \left\{ (\bar{A}_{1b}^T - \bar{A}_{1c}^T) \bar{\mathcal{P}}_2 \bar{A}_2 \right\} - \bar{\Psi}_1^T \bar{\mathcal{M}}_2 \bar{\Psi}_1 - \bar{\Psi}_2^T \bar{\mathcal{M}}_1 \bar{\Psi}_2, \\
\bar{\mathcal{R}} &= [\bar{e}_1^T, \bar{e}_2^T, \bar{e}_3^T]^T, \quad \widehat{\mathcal{R}} = diag \{ \mathcal{R}, 3\mathcal{R}, 5\mathcal{R}, 7\mathcal{R} \}, \\
\bar{A}_{1a} &= [\bar{e}_1^T, 0, 0]^T, \quad \bar{A}_{1b} = [0, \gamma_2 \bar{e}_7^T, 0]^T, \quad \bar{A}_{1c} = [0, 0, \gamma_2 \bar{e}_8^T]^T, \\
\bar{A}_2 &= [\bar{e}_4^T, (\bar{e}_1 - (1 - \dot{\gamma}(t))\bar{e}_2)^T, ((1 - \dot{\gamma}(t))\bar{e}_2 - \bar{e}_3)^T]^T, \quad \bar{A}_3 = [\bar{e}_1^T, \bar{e}_4^T]^T, \quad \bar{A}_4 = [\bar{e}_2^T, \bar{e}_5^T]^T, \\
\bar{A}_5 &= [\bar{e}_3^T, \bar{e}_6^T]^T, \quad \bar{\Psi}_1 = [\bar{A}_6^T, \bar{A}_7^T, \bar{A}_8^T, \bar{A}_9^T]^T, \quad \bar{\Psi}_2 = [\bar{A}_{10}^T, \bar{A}_{11}^T, \bar{A}_{12}^T, \bar{A}_{13}^T]^T, \quad \bar{A}_6 = (\bar{e}_1 - \bar{e}_2), \\
\bar{A}_7 &= (\bar{e}_1 + \bar{e}_2 - 2\bar{e}_7), \quad \bar{A}_8 = (\bar{e}_1 - \bar{e}_2 + 6\bar{e}_7 - 6\bar{e}_9), \quad \bar{A}_9 = (\bar{e}_1 + \bar{e}_2 - 12\bar{e}_7 + 30\bar{e}_9 - 20\bar{e}_{11}), \\
\bar{A}_{10} &= (\bar{e}_2 - \bar{e}_3), \quad \bar{A}_{11} = (\bar{e}_2 + \bar{e}_3 - 2\bar{e}_8), \quad \bar{A}_{13} = (\bar{e}_2 + \bar{e}_3 - 12\bar{e}_8 + 30\bar{e}_{10} - 20\bar{e}_{12}), \\
\bar{A}_{12} &= (\bar{e}_2 - \bar{e}_3 + 6\bar{e}_8 - 6\bar{e}_{10}), \quad \bar{A}_{14} = [\bar{e}_1^T \mathcal{N}_1 + \bar{e}_2^T \mathcal{N}_2 + \bar{e}_4^T \mathcal{N}_3], \quad \bar{\Theta}_i = [\mathcal{A}_i \bar{e}_1 + \mathcal{A}_{\gamma_i} \bar{e}_2 - \bar{e}_4].
\end{aligned}$$

*Proof.* Let us consider the LKF as follows:

$$\bar{V}(\eta_t) = \sum_{q=1}^4 \bar{V}_q(\eta_t), \quad (3.22)$$

where

$$\begin{aligned}
\bar{V}_1(\eta_t) &= \bar{\omega}_1^T(t) \bar{\mathcal{P}}_1 \bar{\omega}_1(t), \quad \bar{V}_2(\eta_t) = \gamma(t) \bar{\omega}_1^T(t) \bar{\mathcal{P}}_2 \bar{\omega}_1(t), \\
\bar{V}_3(\eta_t) &= \int_{t-\gamma_2}^t \bar{\omega}_2^T(s) \bar{\mathcal{Q}}_1 \bar{\omega}_2(s) ds + \int_{t-\gamma(t)}^t \bar{\omega}_2^T(s) \bar{\mathcal{Q}}_2 \bar{\omega}_2(s) ds + \int_{t-\gamma_2}^{t-\gamma(t)} \bar{\omega}_2^T(s) \bar{\mathcal{Q}}_3 \bar{\omega}_2(s) ds, \\
\bar{V}_4(\eta_t) &= \gamma_2 \int_{-\gamma_2}^0 \int_{t+\lambda}^t \dot{\eta}^T(s) \mathcal{R} \dot{\eta}(s) ds d\lambda,
\end{aligned}$$

with

$$\bar{\omega}_1(t) = \left[ \eta^T(t), \int_{t-\gamma(t)}^t \eta^T(s) ds, \int_{t-\gamma_2}^{t-\gamma(t)} \eta^T(s) ds \right]^T, \quad \bar{\omega}_2(t) = \left[ \eta^T(t), \dot{\eta}^T(t) \right]^T.$$

Now, take the similar steps as in Theorem 1 yields the LMIs in (3.17)–(3.21). The analysis is skipped here because it is straightforward.

#### 4. Numerical examples

Three numerical examples are given in this section to demonstrate the reduction of conservativeness of our proposed method numerically.

**Example 1.** Consider the following T–S fuzzy TDS [31]:

$$\dot{\eta}(t) = \sum_{i=1}^2 w_i \left\{ \mathcal{A}_i \eta(t) + \mathcal{A}_{\gamma_i} \eta(t - \gamma(t)) \right\}, \quad (4.1)$$

where

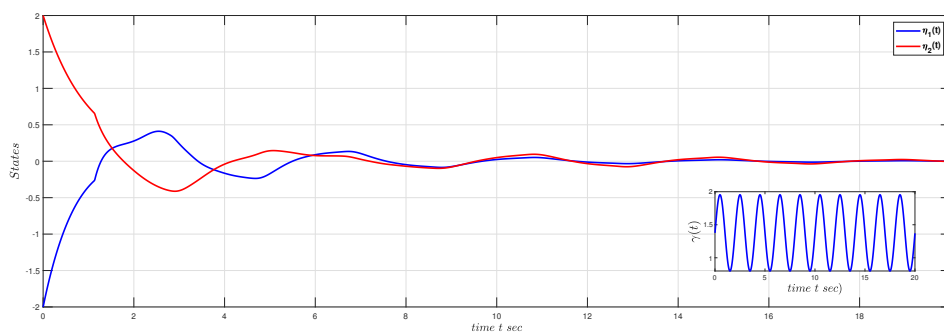
$$\mathcal{A}_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \mathcal{A}_2 = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix}, \mathcal{A}_{\gamma_1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \mathcal{A}_{\gamma_2} = \begin{bmatrix} -1 & 0 \\ 0.1 & -1 \end{bmatrix},$$

and the membership functions are defined as  $w_1 = \frac{1}{1+e^{-2\eta_1(t)}}$ ,  $w_2 = 1 - w_1$ .

For given  $\nu_1 = 0, \nu_2 = 0.1$ , delay upper bound  $\gamma_2$  is calculated by using Theorem 1 with different values  $\gamma_1$ . The obtained delay bound results are presented in Table 1 along with the results of some recent stability conditions. Further, numerical simulation for the system (4.1) is carried out and it is consider that  $\gamma(t) = 0.5768 \sin(\omega t) + 1.3768$  is a very slow varying sine signal ( $\omega = 3.141 \text{ rad/sec}$  or  $f = 0.5 \text{ Hz}$ ). Figure 1 validates the fact that, the system states are asymptotically stable for the initial condition  $\eta(0) = [-2, 2]^T$ .

**Table 1.** Admissible upper bound  $\gamma_2$  for various  $\gamma_1$  and  $\nu = 0.1$  for Example 1.

$\gamma_1$ /Method	[5]	[6]	[20]	[30]	[31]	Theorem 1
0.8	1.6539	1.7633	1.7828	1.857	NA	<b>1.9536</b>
1.0	1.8069	1.7718	1.8106	1.868	1.9405	<b>1.9956</b>

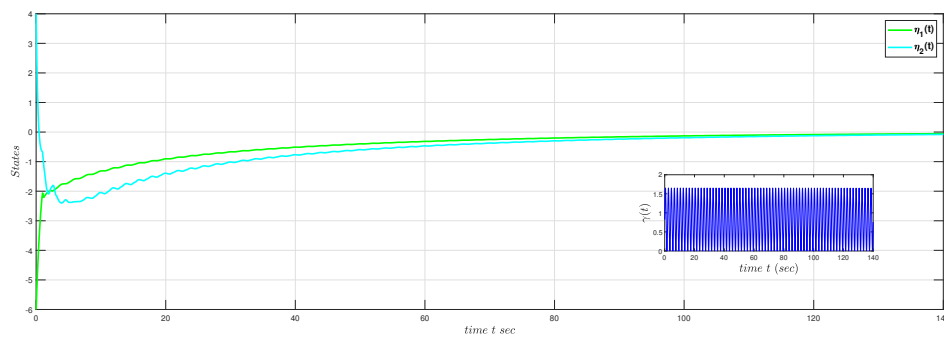


**Figure 1.** State responses of the system given in Example 1 with  $\gamma_1 = 0.8$ ,  $\gamma_2 = 1.9536$ .

**Example 2.** Consider the T–S fuzzy TDS (2.5) with a two plant rule and the parameters are as follows:

$$\mathcal{A}_1 = \begin{bmatrix} -3.2 & 0.6 \\ 0 & -2.1 \end{bmatrix}, \mathcal{A}_2 = \begin{bmatrix} -1 & 0 \\ 1 & -3 \end{bmatrix}, \mathcal{A}_{\gamma_1} = \begin{bmatrix} -1 & 0.9 \\ 0 & 2 \end{bmatrix}, \mathcal{A}_{\gamma_2} = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix}.$$

The membership functions are chosen as  $w_1 = \frac{1}{1+e^{-2\eta_1(t)}}$ ,  $w_2 = 1 - w_1$ . Let  $\gamma_1 = 0$  and  $\nu = \nu_2 = -\nu_1$ . For given various  $\nu$ , the maximum allowable upper bounds of  $\gamma_2$  for Corollary 1 are obtained according to Remark 2. The results of Corollary 1 are given in Table 2 along with several recent results from the literatures in [32–36]. By choosing  $\gamma_1 = 0$ ,  $\gamma_2 = 1.6519$  and  $\eta(0) = [-6, 4]^T$ , Figure 2 shows the state trajectory  $\eta(t)$ . The state responses clearly indicate that T–S fuzzy system considered in Example 2 is asymptotically stable.



**Figure 2.** State responses of the system given in Example 2.

**Table 2.** Maximum delay bound  $\gamma_2$  for various  $\nu$  and  $\gamma_1 = 0$  for Example 2.

Method	$\nu = 0.03$	$\nu = 0.1$	$\nu = 0.5$
[32]	0.8771	0.7687	0.7584
[33]	0.9281	0.8092	0.7671
[34] ( $R_1 = R_2 = 0$ )	1.8328	1.3857	1.2186
[34]	1.9137	1.4354	1.3123
[35]	2.4291	1.7493	1.6355
[36] (Theorem 1 with (I))	2.9931	1.8916	1.4594
[36] (Theorem 2 with (I))	2.6160	1.6084	1.3409
<b>Corollary 1</b>	<b>3.0130</b>	<b>1.6519</b>	<b>0.9925</b>

**Remark 3.** Based on the results presented in Table 1 and Table 2, it can be seen that the proposed Theorem 1 and Corollary 1 of this paper are less conservative than the existing results [5, 6, 20, 30–36], which shows the effectiveness and superiority of the method.

**Example 3.** Consider the T–S fuzzy TDS given in [27] as

$$\dot{\eta}(t) = \sum_{i=1}^2 w_i \{ \mathcal{A}_i \eta(t) + \mathcal{A}_{\gamma_i} \eta(t - \gamma(t)) \}, \quad (4.2)$$

where

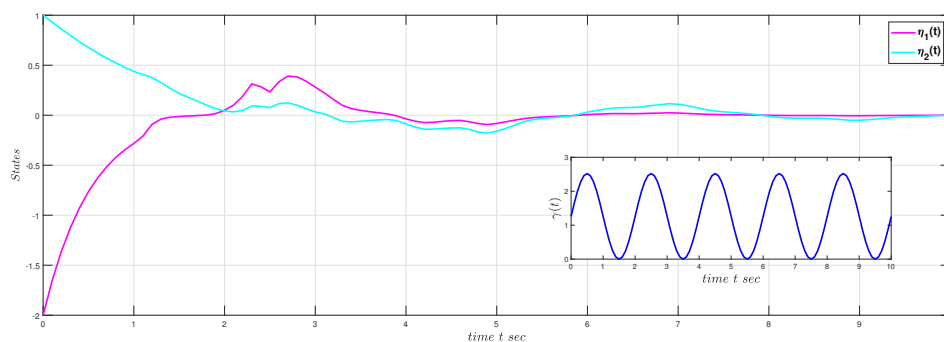
$$\mathcal{A}_1 = \begin{bmatrix} -2.1 & 0.1 \\ -0.2 & -0.9 \end{bmatrix}, \quad \mathcal{A}_2 = \begin{bmatrix} -1.9 & 0.0 \\ -0.2 & -1.1 \end{bmatrix}, \quad \mathcal{A}_{\gamma_1} = \begin{bmatrix} -1.1 & 0.1 \\ -0.8 & -0.9 \end{bmatrix}, \quad \mathcal{A}_{\gamma_2} = \begin{bmatrix} -0.9 & 0.0 \\ -1.1 & -1.2 \end{bmatrix},$$

and the membership functions are  $w_1 = \frac{1}{1+e^{-2\eta_1(t)}}$ ,  $w_2 = 1 - w_1$ .

By solving the LMIs in Corollary 1, delay upper bounds ( $\gamma_2$ ) obtained for given different values of  $\nu$  with  $\gamma_1 = 0$  and the results are given in Table 3. From Table 3, it is found that the results presented in this paper is less conservative than previous researches [27, 34, 37–39]. Figure 3 show the state responses of the fuzzy system given in Example 3 with  $\gamma_1 = 0$ ,  $\gamma_2 = 2.5198$  and  $\eta(0) = [-2, 1]^T$ .

**Table 3.** Delay bound  $\gamma_2$  with various  $\nu = \nu_2 = -\nu_1$  and  $\gamma_1 = 0$  for Example 3.

Method	$\nu = 0.1$	$\nu = 0.5$
[34]	3.42	2.02
[27]	3.5518	2.3204
[37]	4.2044	2.0685
[38]	4.324	2.226
[39]	5.2300	3.3454
<b>Corollary 1</b>	<b>5.4985</b>	<b>2.5198</b>

**Figure 3.** State responses of the system given in Example 3.

**Remark 4.** The Lyapunov conditions for finite-time stability of impulsive systems is proposed in [41], where the settling-time is well estimated via impulsive signals. In [42], the authors addressed a class of nonlinear systems with delayed impulses, where the double effects (i.e., negative and positive effects) of time delays in impulses are fully and systematically considered. In the framework of Lyapunov conditions, in [43], the authors proposed a novel Zeno-free event-triggered impulsive control strategy for uniform stability and asymptotic stability, where a class of forced impulse sequences was introduced freely. Also, singular systems have found widespread use in circuits, power systems, economic models, interconnected systems, and neural network models in recent years [44–47]. Further research topics would be considered to extend the main results of this paper to design an event-triggered control scheme and filter design for the T–S fuzzy TDS or singular network systems with induced network delays.

## 5. Conclusions

An improved DRD stability criteria in a LMI framework has been proposed in this paper for T–S fuzzy TDS. A new stability condition that successfully reduces conservativeness is obtained by constructing a suitable DPT LKF and estimating the derivative of LKF using the Bessel–Legendre polynomial-based relaxed integral inequality. Moreover, three numerical examples are given that compare maximum acceptable delay bounds to highlight the advantages and usefulness of the proposed criteria.

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## Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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