

ACOUSTICAL LETTER

Attempt to create unconventional tones of snare drums using numerical analysis

Shu Sekiguchi[1,*] and Toshiya Samejima[2]

[1] *Graduate School of Design, Kyushu University, 4–9–1 Shiobaru, Minami-ku, Fukuoka, 815–8540 Japan*
[2] *Faculty of Design, Kyushu University, 4–9–1 Shiobaru, Minami-ku, Fukuoka, 815–8540 Japan*

(*Received 19 August 2021, Accepted for publication 6 October 2021*)

Keywords: Physical modeling synthesis, Snare drums, Numerical analysis, Unheard instrument, Auditory impression

1. Introduction

1.1. About snare drums

A snare drum has two heads. The *batter head* is stretched on one side and is struck with the stick, and the *carry head* is stretched on the opposite side. A metal chain is stretched in contact with the carry head, and a characteristic rattling sound is added by the vibration of the carry head.

1.2. Attempt to create unheard instrument

Today, there is a huge variety of musical instruments in the world. When we hear a musical tone that we have already heard, we are able to discriminate what instrument produces it. For example, we describe a sound as a *trumpet-like* sound. At this point, the following natural question arises: is this classification exhaustive? In other words, will a given sound always be classified as an already existing instrument? This can be translated into the following question: will it ever be possible to create a new instrument that can produce sounds that we have never experienced before? This question is important, because if such an instrument could be created, the art and technology of music could experience a new breakthrough. Here, we emphasize that it is human subjectivity that determines whether or not we have ever experienced such an instrument before.

With the above discussion in mind, we propose another form of snare drum that has the potential to create unconventional timbres. The model is then analyzed numerically by the finite difference method. The purpose of this experiment is also to analyze in detail the effect of snare vibration on timbre, which has rarely been performed thus far. Because our experiments involve minute changes in physical parameters and the movement of objects that would not be possible with real objects, we chose to adopt numerical experiments instead of actual measurements.

2. The physical model adopted

The physical model used in this study is illustrated in Fig. 1. This model is similar to the model proposed by Bilbao [1]. Each component of the model is described in detail below.

2.1. Circular flexible membrane with fixed rim

The batter and carry heads were modeled as circular flexible membranes. Only vibrations in the direction perpendicular to the head surface, which is the direction of the z-axis, were modeled. The vibration displacements of the batter and carry heads are denoted as ζ_b and ζ_c, respectively. The governing equations are

$$\begin{aligned}\rho_{mb}H_b\frac{\partial^2\zeta_b}{\partial t^2} &= T_{mb}\Delta\zeta_b - 2\sigma_{0,mb}\frac{\partial\zeta_b}{\partial t} + 2\sigma_{1,mb}\Delta\frac{\partial\zeta_b}{\partial t}\\ &\quad + p_a - f_{st}\delta(r-r_{st},\theta-\theta_{st}),\end{aligned} \tag{1}$$

$$\begin{aligned}\rho_{mc}H_c\frac{\partial^2\zeta_c}{\partial t^2} &= T_{mc}\Delta\zeta_c - 2\sigma_{0,mc}\frac{\partial\zeta_c}{\partial t} + 2\sigma_{1,mc}\Delta\frac{\partial\zeta_c}{\partial t}\\ &\quad - p_a + f_s(r,\theta),\end{aligned} \tag{2}$$

where ρ_{mb}, ρ_{mc} are the volumetric mass densities of the heads, H_b, H_c are the thicknesses of the heads, T_{mb}, T_{mc} are the tensions of the heads, Δ is the Laplacian in polar coordinates, that is, $\Delta = \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$, $\sigma_{0,mb}, \sigma_{0,mc}$ are the frequency-independent damping coefficients, $\sigma_{1,mb}, \sigma_{1,mc}$ are the frequency-dependent damping coefficients, p_a is the pressure from the inner shell acoustic field (see below), f_{st} is the force from the stick (see below), $f_s(r,\theta)$ is the force from the snare (see below), and $\delta(r,\theta)$ is the Dirac delta function. r_{st}, θ_{st} are the coordinates of the position where the stick strikes. The boundary condition is a fixed rim.

According to Bilbao [1], in the case of thin heads, i.e., the effect of head stiffness is small, the mode frequency and mode shape change only slightly, even if the stiffness is taken into account, in the band where the frequency is not extremely high. Therefore, the bending stiffness of the heads was not considered in this study.

In the study by Bilbao [1], the heads were modeled in the Cartesian coordinate system to avoid the band-limit effect described below. In this study, the polar coordinate system was adopted to represent the circular shape more accurately.

2.2. Inner shell acoustic field

When the size of the inner shell acoustic field is small compared with the wavelength of the sound waves in air, the sound pressure of the inner shell acoustic field can be considered to be uniform.

First, the volume change of air, $\mathit{\Delta}V$, is determined by

$$\mathit{\Delta}V = \int_0^R\int_0^{2\pi}\zeta_b\,rd\theta dr - \int_0^R\int_0^{2\pi}\zeta_c\,rd\theta dr, \tag{3}$$

where R is the radius of the heads, ζ_b is the displacement

*e-mail: sekiguchi.shu.847@s.kyushu-u.ac.jp
[doi:10.1250/ast.43.117]

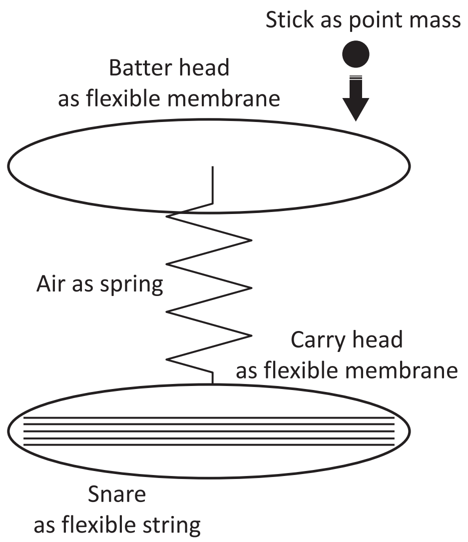

Fig. 1 Physical model of a snare drum.

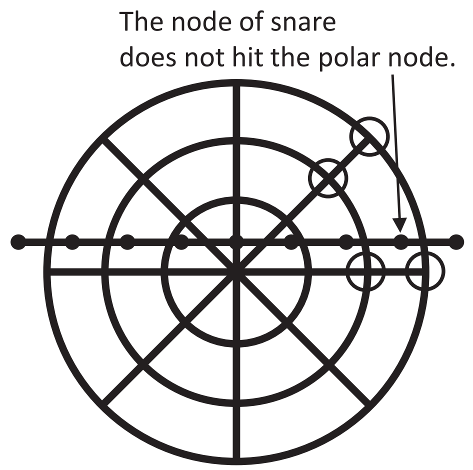

Fig. 2 Relationship between polar and one-dimensional nodes.

of the batter head, and ζ_c is the displacement of the carry head.

Next, we use ΔV to find p_a, which is the uniform pressure in the shell,

$$p_a = -K_a \frac{\Delta V}{V}, \tag{4}$$

where K_a is the bulk modulus of air and V is the volume of the shell.

2.3. Fixed-ends flexible strings

The snare is wound in a spiral shape, so it behaves similarly to a string in the low-frequency range [1]. Several strings were placed parallel to each other and in contact with the carry head. The boundary conditions were fixed at both ends. In the study by Bilbao [1], both ends of the strings were modeled to vibrate coupled with the carry head. However, in an actual snare drum, the snare is firmly fixed to the shell. Therefore, we consider our assumption to be closer to reality. The governing equation of one string is

$$\begin{aligned}\rho_s A \frac{\partial^2 \zeta_s}{\partial t^2} \\ = T_s \frac{\partial^2 \zeta_s}{\partial x^2} - 2\sigma_{0,s} \frac{\partial \zeta_s}{\partial t} + 2\sigma_{1,s} \frac{\partial^3 \zeta_s}{\partial t \partial x^2} - f_s(x),\end{aligned} \tag{5}$$

where ζ_s is the displacement of the string, ρ_s is the volumetric mass density of the string, A is the cross-sectional area of the string, T_s is the tension of the string, $\sigma_{0,s}$ is the frequency-independent damping coefficient, and $\sigma_{1,s}$ is the frequency-dependent damping coefficient. f_s was obtained as

$$f_s = K_s\{[\zeta_s(x) - \zeta_c(r,\theta)]^+\}^{\alpha_s}, \tag{6}$$

where K_s is the nonlinear stiffness when the interaction between the carry head and the snare is regarded as a nonlinear spring, α_s is the nonlinear exponent, and $[\cdot]^+$ is the ramp function. Which x on the string corresponds to which r, θ on the carry head is determined by the snare arrangement (see above). Since there is not always a calculation node at any certain r, θ, as shown in Fig. 2, a linear interpolation was performed using the surrounding four nodes. The corresponding four nodes are indicated by small circles in Fig. 2. This equation means that the restoring force is generated for the length of the contraction only while the spring, which models the snare, is contracting.

2.4. One-degree-of-freedom spring with mass

Striking with a stick was modeled by impinging an elastic point mass on the batter head. The governing equation is

$$f_{st} = K_{st}\{[\zeta_b(r_{st},\theta_{st}) - \zeta_{st}]^+\}^{\alpha_{st}}, \tag{7}$$

where ζ_{st} is the displacement of the stick, K_{st} is the nonlinear stiffness when the interaction between the batter head and the stick is considered as a nonlinear spring, and α_{st} is the nonlinear exponent. The point mass is assumed to move at a constant velocity and then collide with the batter head.

3. Numerical experiments

3.1. Numerical conditions for testing another arrangement of snares

As mentioned earlier, the governing equations for the coupled stick and batter head, the coupled batter head and inner shell acoustic field, the coupled inner shell acoustic field and carry head, and the coupled carry head and snare were converted to difference equations, and the time evolution was calculated by the time domain finite difference method. The implementation was based on that in Ref. [2]. The vibration velocity of the batter head was used as the output sound. The parameters used in the calculation are shown in Table 1. These parameters are basically the same as those in Ref. [1]. For parameters that do not appear in Ref. [1], we adjusted the parameters so that the calculated results are subjectively close to the actual measurements. However, since our numerical experiment is to observe the relative change in timbre, this adjustment need not be very strict.

3.2. Results of numerical experiments

First, the spectrograms of the output sounds are shown. The spectrograms without and with the snare are shown in Figs. 3 and 4, respectively. It can be seen that a high-frequency noise component is added when the snare is present.

Table 1 Parameters used in the calculation.

Parameter name	Value
Sample rate	$48{,}000 \times 3\,\mathrm{Hz}$
Grid spacing of r-axis	$3.1 \times 10^{-3}\,\mathrm{m}$
Grid spacing of θ-axis	2.1×10^{-2}
Grid spacing of x-axis	$4.2 \times 10^{-4}\,\mathrm{m}$
Initial velocity of stick	$-16.75\,\mathrm{m/s}$
K_{st}	$1.0 \times 10^{11}\,\mathrm{N/m}^{\alpha_{st}}$
α_{st}	3.0
Mass of point mass	0.8 kg
R	0.15 m
$\sqrt{T_{mb}/\rho_{mb}H_b}$	102.8 m/s
$\sqrt{T_{mc}/\rho_{mc}H_c}$	102.8 m/s
$\sigma_{0,mb}$	$11.0\,\mathrm{kg/(m^2{\cdot}s)}$
$\sigma_{1,mb}$	$6.0 \times 10^{-4}\,\mathrm{kg/s}$
$\sigma_{0,mc}$	$10.5\,\mathrm{kg/(m^2{\cdot}s)}$
$\sigma_{1,mc}$	$1.0 \times 10^{-4}\,\mathrm{kg/s}$
Position of sound output	$r, \theta = 45\,\mathrm{mm},\ 1.4\pi$
Position of adding force	$r, \theta = 8.3\,\mathrm{mm},\ 1.7\pi$
K_a/V	$1.0 \times 10^{4}\,\mathrm{Pa/m^3}$
$\sqrt{T_s/\rho_s A}$	40.0 m/s
$\sigma_{0,s}$	$20.0\,\mathrm{kg/(m{\cdot}s)}$
$\sigma_{1,s}$	$0\,\mathrm{kg{\cdot}m/s}$
Width of snare	50 mm
K_s	$1.0 \times 10^{3}\,\mathrm{N/m}^{\alpha_s}$
α_s	1.0
Number of strings	18

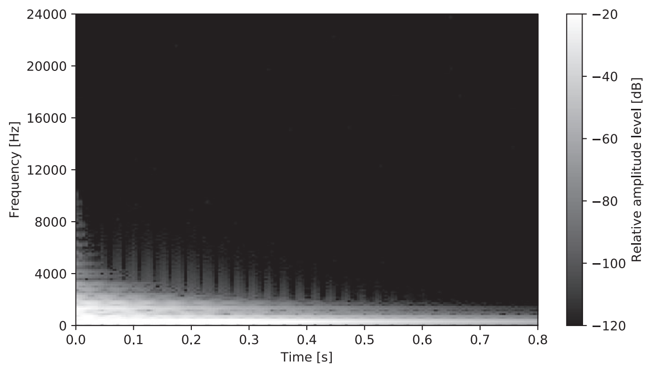

Fig. 3 Spectrogram of the vibrational velocity of the batter head without snare.

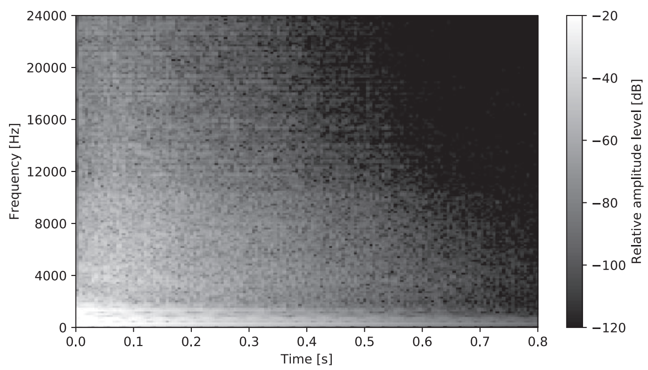

Fig. 4 Spectrogram of the vibrational velocity of the batter head with snare.

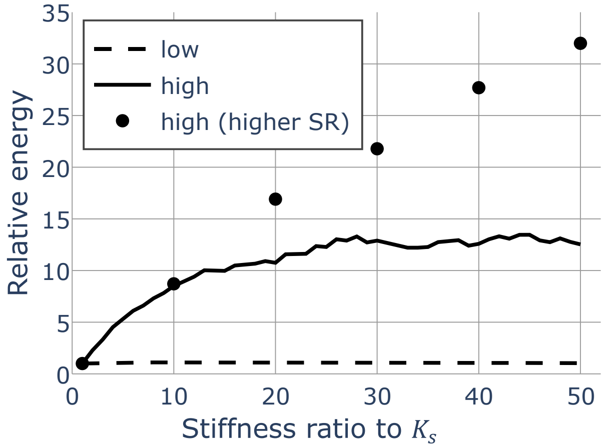

Fig. 5 Effect of varying the stiffness K_s of the snare.

Next, we will show the effect of varying the stiffness K_s of the snare on the tone. As an auditory impression, it is easy to imagine that the larger the K_s value, the crispier the tone, i.e., the higher frequency component increases. Figure 5 shows the variations of the energy in the band between 0 Hz and 5 kHz (dashed line) and the energy in the band between 5 and 10 kHz (solid line) of the output sound with stiffness. It can be seen that the energy in the lower band is almost unchanged, while the energy in the higher band increases with stiffness. However, it can be seen that the increasing trend of the solid line stops at a certain point. This is due to the band-limit effect [2], which is a characteristic of the time-domain finite difference method. Since the components in a higher range do not appear in the calculation results, the increase in energy in the high frequency range appears to stop. As evidence of this, when the sample rate was set to $48{,}000 \times 15\,\mathrm{Hz}$, the energy in the high frequency range did not reach a ceiling, as shown by the dots in the figure. As the stiffness of the snare increases, the contact duration with the carry head decreases. The tendency of the high-frequency energy to increase as the contact duration decreases is consistent with the inference based on the theoretical analysis of struck strings [3].

Next, we will examine the effects of changing the geometric arrangement and number of snares. In ordinary snare drums, snares are usually stretched in parallel. The purpose of this study is to observe how the tone would be affected when this *convention* is broken. Schematic diagrams of the conventional arrangement and the one we adopted are shown in Fig. 6. For convenience, we will refer to the former as *parallel* and the latter as *crossed*. The crossed arrangement is one of the many non-parallel arrangements. We chose the crossed arrangement because it is necessary to cross the snares in order to obtain the maximal angle between a certain number of snares. As a preliminary study, the snares are modeled not to collide with each other, because we wanted to focus only on the effect of changing the direction of the snares; the effect of collision behavior, which does not arise when the snares are parallel, is disturbing.

In the auditory impression, the crossed arrangement seemed to produce a brighter tone. Therefore, we examined the spectral centroids of the output sounds. Figure 7 shows the

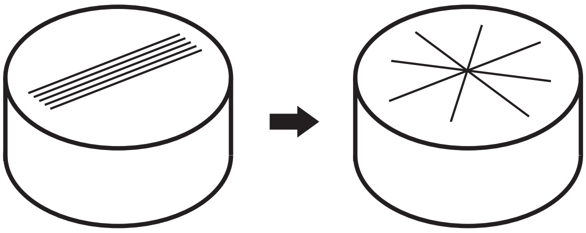

Fig. 6 Illustration of a new snare arrangement.

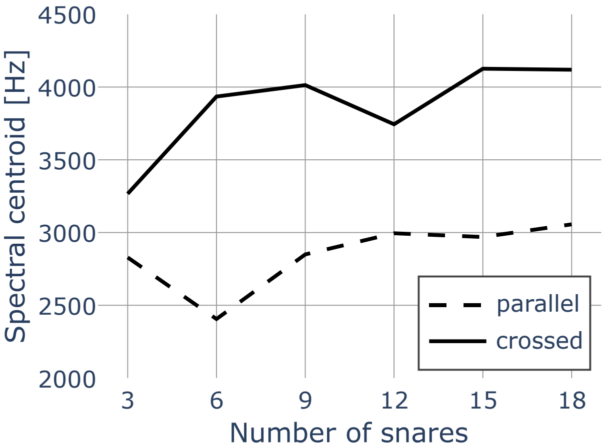

Fig. 7 Effect of changing the geometric arrangement and number of snares.

spectral centroids of the output sounds when the number of snares was changed in both configurations. First, it is clear that the spectral centroid of the crossed configuration is higher than that of the parallel configuration. This is consistent with the auditory impression. In addition, the spectral centroid tends to increase as the number of snares increases. The above results can probably be explained as follows. In the parallel configuration, each snare approximately passes through the nodal line of the membrane, making it difficult to excite higher-order modes. In contrast, in the crossed configuration, higher-order modes can be excited. This relationship is illustrated in Fig. 8.

4. Conclusions

The reason for experimenting with the crossed arrangement of snares was to explore the possibility of creating an unheard tone, but in our subjective opinion, the tone did not seem to deviate from the realm of existing snare drums. However, we were able to obtain the following findings regarding the configuration of the snare:

(1) A stiffer snare material will increase the energy in the high frequency range.
(2) If the snares are placed crosswise, a brighter rattling sound can be produced.

In the introduction, we mentioned the usefulness of constructing unheard musical instruments. However, it is impossible to talk about musical instruments without mentioning human sensitivity. At present, however, physicists have yet to develop a mathematical model of human sensitivity. We will continue our research to develop a mathematical model that includes the way in which human emotions are swayed by music.

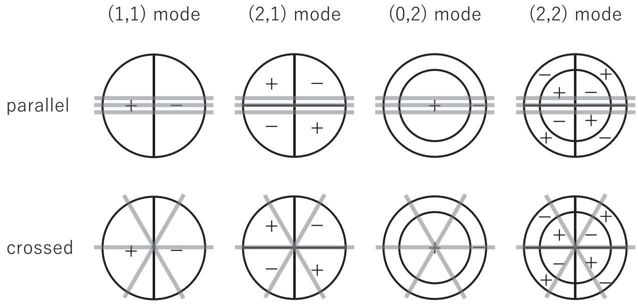

Fig. 8 Relationship between the snare arrangement and the mode shape.

Acknowledgment

This work was supported by JSPS KAKENHI Grant Number JP19H04126.

References

[1] S. Bilbao, "Time domain simulation and sound synthesis for the snare drum," *J. Acoust. Soc. Am.*, **131**, 914–925 (2012).
[2] S. Bilbao, *Numerical Sound Synthesis* (John Wiley & Sons, Hoboken, NJ, 2009).
[3] T. Ito, "Free vibration of strings struck with a hammer," in *Principles of Acoustical Engineering* (Corona Publishing, Tokyo, 1957), Subsect. 2.1.5 (in Japanese).