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Multi-hop non-destructive qudit teleportation via non-maximally entangled GHZ channels

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Abstract

A scheme for the multihop non-destructive teleportation of an arbitrary single-qudit state based on various non-maximally entangled Greenberger–Horne–Zeilinger channels is proposed here. The parallel entanglement swap is used to form a direct d-dimensional ($d \geq 2$) entangled channel shared between the sender and the receiver. The authors' propose a novel idea that the third-party Charlie with high quantum information processing technology executes the channel modulation, which reduces the technical requirements of the sender and the receiver. In addition, if the teleportation fails, the unknown state will be restored by the sender. This work provides a feasible method for teleportation and ensures the integrity of the unknown quantum information.

1 | INTRODUCTION

Quantum mechanics is the foundation of quantum communication and provides some new ways for quantum information transmission, such as quantum key distribution [1, 2], quantum secure direct communication [3–6], quantum dialogue [7] and remote state preparation [8] etc. In 1993, quantum teleportation was first proposed by Bennett et al. [9]. Since then, scientists have made dramatic progress in the field of quantum teleportation. In 1997, Bouwmeester et al. [10] successfully demonstrated the first quantum teleportation experiment. Subsequently, quantum teleportation has been demonstrated by various experimental systems, such as photonic qubits [11, 12], atomic ensembles [13] and trapped atoms [14] etc. For instance, Barasiński et al. [15] demonstrated a tripartite controlled quantum teleportation on quantum optical devices. Rota et al. [16] utilised the non-local properties of entanglement to teleport three-photon and four-photon states in semiconductor quantum dots system. Ding et al. [17] demonstrated the inter-chip and intra-chip teleportation of single-qudit state with high fidelity, which had applications in chip-scale quantum communication and information processing. Raju et al. [18] reported the conditional quantum teleportation of time-bin qudits with fibre-coupled devices.

Quantum teleportation plays an important role in quantum communication and quantum information, which represents a fundamental ingredient of the development of many quantum technologies [19], such as quantum repeaters [20], quantum gate teleportation [21], measurement-based computing [22, 23] and port-based teleportation [24] etc. The procedure of quantum teleportation is as follows. In the beginning, a maximally entangled Bell channel is shared by both the sender Alice and the receiver Bob. Then, Alice performs a joint measurement on her two particles with the Bell basis and communicates Bob of her measurement result. Based on

Alice's information, Bob applies one of the four unitary operations onto his particle to recover the original information. If Alice can distinguish all four possible measurement outcomes, the teleportation can, in principle, be implemented with 100% success rate, which is called as deterministic teleportation [25]. The deterministic teleportation usually relies on a maximally entangled channel. However, in a realistic teleportation scenario, due to the inevitable coupling effect between the quantum states and its surrounding environment, most channels readily evolve from the maximally entangled state into the non-maximally entangled state. It will increase the risk of information loss and reduce the fidelity of the channel. Afterwards, several probabilistic teleportation schemes [26–29] were presented with various non-maximally entangled states as quantum channels such as Einstein–Podolsky–Rosen (EPR) state [28, 30], cluster state [31], W state [32], mixed state [33] and so on. For example, Li et al. [26] proposed a probabilistic teleportation scheme to transmit an unknown qudit state with various non-maximally entangled Bell states as the quantum channel. Chen et al. [34] proposed a probabilistic teleportation protocol with multiple partially entangled Bell states to teleport an unknown multi-particle Greenberger–Horne–Zeilinger (GHZ) state. Gao et al. [35] presented a scheme to teleport an unknown three-particle W state with two three-particle entangled W states as the quantum channel.

It is well known that performing unitary operation in the d-dimensional ($d \geq 2$) system is more challenging than performing unitary operation in the *two*-dimensional system. Recently, some teleportation protocols in the d-dimensional system have been proposed. For instance, Wei et al. [36–38] reported a multiple party sharing d-dimensional teleportation protocol. Zhou et al. [39] gave a general form of controlled teleportation of an arbitrary d-dimensional multi-qudit state. Long et al. [40] presented a scheme to transmit an arbitrary d-dimensional GHZ state. To the best of our knowledge, there have been no reports about non-destructive teleportation scheme in which all the involving nodes are linked by different d-dimensional GHZ channels in a network.

Noted that the remaining challenge is how to preserve the original unknown state when the probabilistic teleportation fails. In order to realise deterministic teleportation with multiple partially entangled channels, researchers introduced an auxiliary particle to assist the unknown qudit state transmission. For example, Roa et al. [41] proposed a scheme that the sender could still restore the original unknown state when the probabilistic teleportation failed. Later, Fu et al. [42] reported a multi-hop non-destructive teleportation protocol, which used multiple non-maximally entangled Bell pairs as the channel. In this paper, we propose a scheme for the multi-hop non-destructive teleportation with various d-dimensional non-maximally entangled GHZ states as the quantum channel.

In addition, it should be noted that the quantum logic gates are the building blocks of quantum circuits in quantum teleportation. Multi-qudit operation [43–46] requires a combination of single-qudit gates and two-qudit entangling gates, such as controlled-NOT ($CNOT$) gate or controlled-Z (CZ) gate [43]. In the past decades, several groups have demonstrated high fidelity single-qudit gates [47] and two-qudit gates in some systems [51], such as photons [16], ions [46, 48], superconducting qudits [49], and quantum dots [50]. For instance, the $CNOT$ gate has been realised between multi-photon qudits [46]. Barron et al. [51] demonstrated two-qudit gates in the presence of low-frequency noise in silicon double-quantum dots. Wang et al. [49] proposed a feasible fast adiabatic method for the CZ gate with two coupled superconducting Xmon qudits. Xu et al. [25] realised the controlled-phase gates between two logical qudits by dispersively coupling an ancillary superconducting qudit to these cavities. Yang et al. [48] proposed an efficient scheme for implementing the universal controlled-phase gate in a circuit quantum electrodynamical system.

The remainder of this paper is organised as follows. In Section 2, we introduce one point-to-point non-destructive teleportation scheme based on one non-maximally entangled GHZ channel. In Section 3, we generalise it to the scenario involving multiple intermediate nodes. In Section 4, we introduce an example of four-hop non-destructive teleportation. In Section 5, we analyse the performance of our scheme. Finally, we present a conclusion in Section 6.

2 | POINT-TO-POINT NON-DESTRUCTIVE TELEPORTATION WITH A d-DIMENSIONAL NON-MAXIMALLY ENTANGLED GHZ CHANNEL

For presenting the principle of our scheme clearly, we first describe one point-to-point scenario of the teleportation with a d-dimensional non-maximally entangled GHZ state as the quantum channel. Next, we generalise it to the case of multi-hop non-destructive teleportation.

The point-to-point non-destructive teleportation has three communication parties, the sender Alice, the third-party Charlie and the receiver Bob. The sender intends to teleport an unknown d-dimensional single-qudit state $|\chi\rangle_t$ to Bob. Suppose the original unknown state $|\chi\rangle_t$ can be expressed as

$$|\chi\rangle_t = \sum_{j=0}^{d\text{-}1} a_j |j\rangle_t. \tag{1}$$

All $a_j (j = 0, 1, \ldots, d-1)$ are complex and satisfy the normalisation condition $\sum_{j=0}^{d-1} |a_j|^2 = 1$. The non-maximally entangled channel can be expressed as

$$|\varphi\rangle_{ABC} = \sum_{k=0}^{d-1} b_k |kkk\rangle_{ABC}. \tag{2}$$

Here $b_k (k = 0, 1, \ldots,\ d-1)$ are the channel parameters and satisfy the relationship $\sum_{k=0}^{d-1} |b_k|^2 = 1$. Without the loss of generality, we assume that $b_{\min} = \min\{b_0, b_1, \ldots, b_{d-1}\}$.

Alice possesses particles t and A. The third-party Charlie holds particle B, while Bob possesses particle C. In the beginning, the third-party Charlie introduces an auxiliary particle e with the initial state $|0\rangle_e$. The quantum circuit is described in Figure 1. The classical communication channel labelled with double line is equipped among the three parties.

The total state of the particles t, A, B, C and e can be written as

$$\begin{aligned} |\varphi\rangle_{tABCe} &= |\chi\rangle_t \otimes |\varphi\rangle_{ABC} \otimes |0\rangle_e \\ &= \left(\sum_{j=0}^{d-1} a_j |j\rangle_t\right) \otimes \left(\sum_{k=0}^{d-1} b_k |kkk\rangle_{ABC}\right) \otimes |0\rangle_e \\ &= \sum_{k=0}^{d-1}\left[\left(\sum_{j=0}^{d-1} a_j |jk\rangle_{tA}\right) \otimes b_k |kk0\rangle_{BCe}\right]. \end{aligned} \tag{3}$$

In order to directly teleport the unknown single-qudit state $|\chi\rangle_t$ from the sender Alice to the receiver Bob, our protocol is divided into three steps.

Step 1: Alice performs a generalised controlled-NOT gate ($GCNOT$) operation [40] on her particles A and t. Similar to the *two*-dimensional system, the $GCNOT$ operation can be expressed as

$$GCNOT|mn\rangle = |m\rangle|(m+n)\mathrm{mod}\ d\rangle\ . \tag{4}$$

Next, the generalised Hadamard gate (GH) operation [26, 43, 49] is performed by Alice on the particle t. The gate GH has the form of

$$GH = \frac{1}{\sqrt{d}} \sum_{k,k'=0}^{d-1} e^{\frac{2\pi i}{d}kk'} |k\rangle\langle k'|\ . \tag{5}$$

The gate GH can also be written in matrix form

$$GH = \frac{1}{\sqrt{d}} \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{2\pi i/d} & \cdots & e^{2(d-1)\pi i/d} \\ 1 & e^{4\pi i/d} & \cdots & e^{4(d-1)\pi i/d} \\ \vdots & \vdots & \cdots & \vdots \\ 1 & e^{2(d-1)\pi i/d} & \cdots & e^{2(d-1)^2\pi i/d} \end{pmatrix}. \tag{6}$$

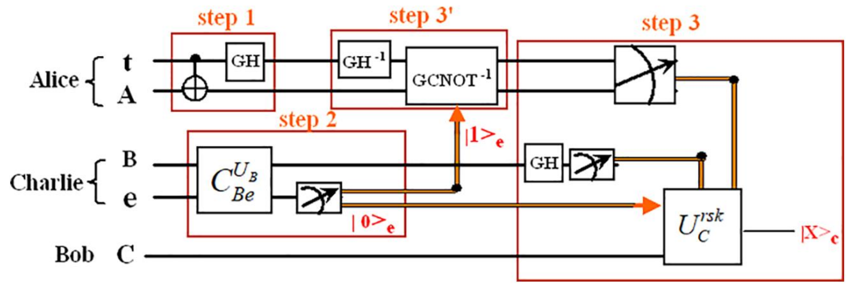
After Alice performs the $GCNOT$ and GH operations, the state of particles t, A, B, C and e can be written as

$$\begin{aligned} |\varphi'\rangle_{tABCe} &= (GH)_t (GCNOT)_{tA} |\varphi\rangle_{tABCe} \\ &= \frac{1}{\sqrt{d}} \sum_{s=0}^{d-1}\left(\sum_{r=0}^{d-1} a_r \left(\sum_{j=0}^{d-1} e^{\frac{2\pi i}{d}jr} |j, (r+s)\mathrm{mod}\ d\rangle_{tA}\right)\right. \\ &\quad \left. \otimes\, b_s |ss\rangle_{BC}\right) |0\rangle_e\ . \end{aligned} \tag{7}$$

Step 2: Meanwhile, the third-party Charlie performs channel modulation operation $C_{Be}^{U_B}$ on his two particles B and e [39]. The matrix $C_{Be}^{U_B}$ is given by

$$C_{Be}^{U_B} = \begin{pmatrix} \frac{b_{\min}}{b_0} & 0 & \cdots & 0 & \sqrt{1-\left(\frac{b_{\min}}{b_0}\right)^2} & 0 & \cdots & 0 \\ 0 & \frac{b_{\min}}{b_1} & \cdots & 0 & 0 & \sqrt{1-\left(\frac{b_{\min}}{b_1}\right)^2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{b_{\min}}{b_{d-1}} & 0 & 0 & \cdots & \sqrt{1-\left(\frac{b_{\min}}{b_{d-1}}\right)^2} \\ \sqrt{1-\left(\frac{b_{\min}}{b_0}\right)^2} & 0 & \cdots & 0 & -\frac{b_{\min}}{b_0} & 0 & \cdots & 0 \\ 0 & \sqrt{1-\left(\frac{b_{\min}}{b_1}\right)^2} & \cdots & 0 & 0 & -\frac{b_{\min}}{b_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sqrt{1-\left(\frac{b_{\min}}{b_{d-1}}\right)^2} & 0 & 0 & \cdots & -\frac{b_{\min}}{b_{d-1}} \end{pmatrix}. \tag{8}$$

FIGURE 1 The quantum circuit diagram for point-to-point non-destructive teleportation



After the third-party Charlie performs the channel modulation operation, the entire five-qudit state will be transformed into

$$|\varphi''\rangle_{tABeC} = C_{Be}^{U_B}|\varphi'\rangle_{tABeC}$$

$$= \frac{1}{\sqrt{d}}\sum_{s=0}^{d-1}\left[\begin{array}{l}\sum_{r=0}^{d-1} a_r\left(\sum_{j=0}^{d-1} e^{\frac{2\pi i}{d}jr}|j,(r+s)\bmod d\rangle_{tA}\right)\\ \otimes\left(b_{\min}|s0s\rangle_{BeC} + \sqrt{b_s^2 - b_{\min}^2}\,|s1s\rangle_{BeC}\right)\end{array}\right]. \tag{9}$$

Subsequently, the third-party Charlie performs a single-qudit measurement on his auxiliary particle e in the basis of $\{|0\rangle, |1\rangle\}$. There are two possible measurement results $|0\rangle_e$ or $|1\rangle_e$. If the measurement result is $|0\rangle_e$, step 3 will be executed to recover the original unknown state by the receiver Bob. Otherwise, step $3'$ will be implemented to preserve the original information on the sending location.

Step 3: If Charlie's measurement result is $|0\rangle_e$, the joint state of particles t, A, B and C collapses into

$$|\varphi_0\rangle_{tABC}$$

$$= \frac{1}{\sqrt{d}}\sum_{s=0}^{d-1}\left[\sum_{r=0}^{d-1} a_r\left(\sum_{j=0}^{d-1} e^{\frac{2\pi i}{d}jr}|j,(r+s)\bmod d\rangle_{tA}\right)|ss\rangle_{BC}\right]. \tag{10}$$

In order to successfully teleport the original unknown state $|\chi\rangle_t$, the third-party Charlie performs the GH operation on his particle B. The state of particles t, A, B and C is written as

$$|\varphi_0{}'\rangle_{tABC}$$

$$= \frac{1}{d}(GH)_B\sum_{r,s=0}^{d-1}\left(|rs\rangle_{tA}\otimes\sum_{k,j=0}^{d-1} e^{\frac{2\pi i}{d}jr}a_j|k,(s-j)\bmod d\right)_{BC}$$

$$= \frac{1}{d}\sum_{r,s,k=0}^{d-1}\left[|rsk\rangle_{tAB}\otimes\left(\sum_{j=0}^{d-1} e^{\frac{2\pi i}{d}jr}e^{\frac{2\pi i}{d}(s-j)k}a_j|(s-j)\bmod d\rangle_C\right)\right]. \tag{11}$$

Then, Alice performs measurement operations with the orthogonal basis of $\{|rs\rangle\}(r,\ s = 0,\ 1,\ 2,\ \ldots,\ d-1)$ on particles t and A. There are d^2 kinds of possible measurement results obtained by Alice. At the same time, the third-party Charlie performs a single-particle measurement on particle B in the basis of $\{|k\rangle\}(k = 0,\ 1,\ 2,\ \ldots,\ d-1)$. There are d kinds of possible measurement results obtained by the third-party Charlie. After measurements, both Alice and the third-party Charlie inform Bob of their measurement outcomes. According to their measurement results, Bob performs a local unitary operation U_C^{rsk} on his particle C to recover the initial unknown state. The unitary operation can be expressed as

$$U_C^{rsk} = U_{rs}\otimes U_k$$

$$= \left(\begin{array}{l}\sum_{j=0}^{d-1} e^{-\frac{2\pi i}{d}jr}|j\rangle\langle(j+s)\bmod d|\\ \otimes\sum_{s-j=0}^{d-1} e^{-\frac{2\pi i}{d}(s-j)k}|(j+s)\bmod d\rangle\langle(s-j)\ \bmod\ d|\end{array}\right)$$

$$= \sum_{s-j=0}^{d-1} e^{-\frac{2\pi i}{d}jr}e^{-\frac{2\pi i}{d}(s-j)k}|j\rangle\langle(s-j)\ \bmod\ d|. \tag{12}$$

Now, the state of Bob's particle becomes

$$|\varphi\rangle_C = \sum_{j=0}^{d-1} a_j|j\rangle_C. \tag{13}$$

We can see that the initial unknown state $|\chi\rangle_t$ is successfully teleported from Alice to Bob as shown in Equation (13).

Step 3': However, if the third-party Charlie's measurement result is $|1\rangle_e$, the teleportation fails. The state of the left particles t, A, B and C becomes

$$|\varphi_1\rangle_{\mathrm{tABC}} = \frac{1}{\sqrt{d}}\sum_{s=0}^{d-1}\left[\begin{array}{l}\sum_{r=0}^{d-1}\left(a_r\sum_{j=0}^{d-1} e^{\frac{2\pi i}{d}jr}|j,(r+s)\bmod d\rangle_{tA}\right)\\ \otimes\left(\sqrt{b_s^2 - b_{\min}^2}\,|ss\rangle_{BC}\right)\end{array}\right]. \tag{14}$$

Under this circumstance, Alice performs an inverse GH gate operation GH^{-1} on her first particle t, and then performs an inverse $GCNOT$ gate operation $GCNOT^{-1}$ on his particles t and A as follows

$$|\varphi_1{}'\rangle_{tABC} = (GCNOT)^{-1}{}_{\mathrm{tA}}(GH)_t{}^{-1}|\varphi_1{}'\rangle_{\mathrm{tABC}} = \sum_{r=0}^{d-1}(a_r|r\rangle_t) \otimes \left(\sum_{s=0}^{d-1}\sqrt{b_s^2 - b_{\min}^2}|sss\rangle_{ABC}\right), \tag{15}$$

with $GH^{-1} = \frac{1}{\sqrt{d}}\sum_{k,k'=0}^{d-1} e^{-2\pi ikk'/d}\,|k'\rangle\langle k|$ and $GCNOT^{-1}\,|mn\rangle = |m\rangle\,|\,(n-m)\bmod d\rangle$. From Equation (15), we can know that the unknown state is completely reconstructed by the sender.

In this section, we introduce one point-to-point non-destructive quantum teleportation scheme with a non-maximally entangled GHZ channel among three parties. The third-party Charlie introduces an auxiliary particle e and then performs the channel modulation and a single-particle measurement. If the measurement result of the auxiliary particle is $|0\rangle_e$, the receiver needs to perform the corresponding unitary operation to recover the initial state. Otherwise, the sender requires to perform GH^{-1} and $GCNOT^{-1}$ operations to rebuild the unknown state.

3 | NON-DESTRUCTIVE QUDIT TELEPORTATION WITH MULTIPLE RELAY NODES

In quantum network, if the sender and the receiver are linked by a direct quantum channel, this network is called as a single-hop network. However, in long-distance communication, there is usually no direct entangled state shared between the sender Alice and the receiver Bob. In this case, it is necessary to introduce multiple intermediate nodes to form one indirect channel between the source node and the destination node for implementing the long-distance teleportation. In this way, the quantum network consisting of multiple participants to form the indirect channel can be regarded a multi-hop communication network.

In this section, we further generalise the above non-destructive scheme to the multi-hop scenario. Similarly, the third-party Charlie should introduce an auxiliary particle e and perform the channel modulation operation. As shown in Figure 2, in the beginning, the sender Alice possesses the original qudits t and A_1 and the receiver Bob holds the qudit C_{p+1}. The third-party Charlie possesses the particles B_{p+1} and an auxiliary particle e. The qth $(q = 1, 2, \ldots, p)$ intermediate node $\mathrm{Relay_q}$ holds particles A_{q+1}, B_q and C_q. The particle distribution is shown in Figure 2.

The initial state of all particles $A_1, B_1, C_1, A_2, \ldots, A_{P+1}, B_{P+1}, C_{P+1}$ can be expressed as

$$\begin{aligned}
&|\varphi\rangle_{A_1B_1C_1\ldots A_{P+1}B_{P+1}C_{P+1}} \\
&= \otimes_{q=1}^{P+1}\left(\sum_{j=0}^{d-1}\beta_{qj}|jjj\rangle_{A_qB_qC_q}\right) \\
&= \left(\frac{1}{\sqrt{d}}\right)^{2P}\left\{\begin{array}{l}
\displaystyle\sum_{x_1=0}^{d-1}\cdots\sum_{x_p=0}^{d-1}|H_{x_1}\rangle_{B_1}\ldots|H_{x_p}\rangle_{B_p}\sum_{m_1,n_1=0}^{d-1}\ \sum_{m_2,n_2=0}^{d-1}\cdots\sum_{m_P,n_P=0}^{d-1}|\phi_{m_1n_1}\rangle_{C_1A_2}|\phi_{m_2n_2}\rangle_{C_2A_3}\ldots|\phi_{m_Pn_P}\rangle_{C_PA_{P+1}} \\
\displaystyle\left(U_{x_1}^{-1}\right)_{A_1}\left(U_{m_1n_1}^{-1}\right)_{A_1}\left(U_{x_2}^{-1}\right)_{A_1}\left(U_{m_2n_2}^{-1}\right)_{A_1}\cdots\cdots\left(U_{x_p}^{-1}\right)_{A_1}\left(U_{m_pn_p}^{-1}\right)_{A_1}\sum_{j=0}^{d-1}\left[\left(\prod_{q=P+1}^{1}\beta_{q((k_q+j)\bmod d)}\right)|jjj\rangle_{A_1B_{P+1}C_{P+1}}\right]
\end{array}\right\},
\end{aligned} \tag{16}$$

with $|H_{x_q}\rangle = \frac{1}{\sqrt{d}}\sum_{j=0}^{d-1} e^{\frac{2\pi i}{d}jx_q}|j\rangle$, $|U_{x_q}^{-1}\rangle = \sum_{j=0}^{d-1} e^{-\frac{2\pi i}{d}jx_q}|j\rangle\langle j|$, $|\phi_{m_qn_q}\rangle = \frac{1}{\sqrt{d}}\sum_{j=0}^{d-1} e^{\frac{2\pi i}{d}jm_q}|j\rangle|(j+n_q)\bmod d\rangle$ and $U_{m_qn_q}^{-1} = \sum_{j=0}^{d-1} e^{\frac{-2\pi i}{d}jm_q}|(j+n_q)\bmod d\rangle\langle j|$. The initial condition of subscript $k_{p+1} = 0$. Here, we ignore the global phases. $\prod_{q=P+1}^{1}\beta_{q((k_q+j)\bmod d)}$ is the entangled channel parameter and satisfies the constraint equation $(k_q + j)\bmod d = (k_{q+1} - n_q)\bmod d$.

Our multi-hop non-destructive teleportation protocol is expressed by the quantum circuit as shown in Figure 3.

In order to form a direct entangled channel among communicating parties, all the intermediate nodes $\mathrm{Relay_q}$ $(q = 1, 2, \ldots, p)$ simultaneously perform d-dimensional Bell

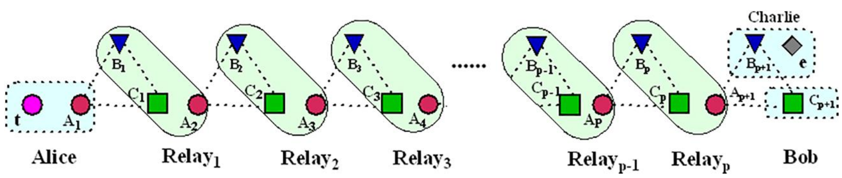


FIGURE 2 Particle distribution of multi-hop probabilistic non-destructive teleportation

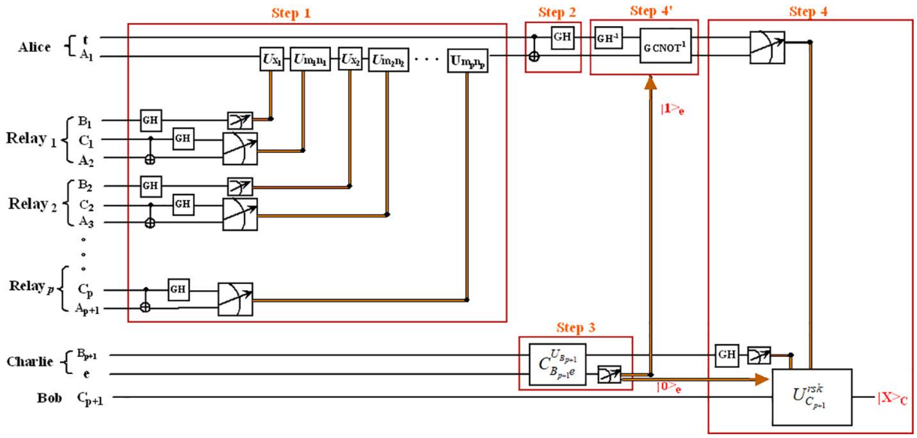


FIGURE 3 Quantum circuit diagram for our multi-hop non-destructive teleportation scheme

state measurement on their own particles C_i and A_{i+1} $(i = 1, 2, \ldots, p)$ in the basis of $|\phi_{m_q n_q}\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} e^{\frac{2\pi i}{d} j m_q} |j\rangle|(j + n_q)\mathrm{mod}\, d\rangle$ $(m_q, n_q = 0, 1, \ldots, d-1)$. There are d^2 kinds of possible measurement result obtained by Relay$_q$. Meanwhile, GH measurement is performed by each intermediate node Relay$_q$ on particles $B_i (i = 1, 2, \ldots, p)$ in the basis of $|H_{x_k}\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} e^{\frac{2\pi i}{d} j x_k} |j\rangle (x_k = 0, 1, \ldots, d-1)$. Relay$_q$ can receive d kinds of possible measurement result. Then, they inform Alice of all the measurement results through the classic channel. According to the measurement results, Alice carries out the corresponding unitary transformation on the particle A_1. The unitary operation can be expressed as

$$U_{A_1}^{oper} = \prod_{q=1}^{P} U_{m_q n_q} \prod_{k=1}^{p} U_{x_k}$$

$$= \prod_{q=1}^{P} \left(\sum_{j=0}^{d-1} e^{\frac{2\pi i}{d} j m_q} |j\rangle\langle (j + \mathrm{n}_q) \mathrm{mod}\, d| \right) \prod_{k=1}^{p} \left(\sum_{j=0}^{d-1} e^{\frac{2\pi i}{d} j x_k} |j\rangle\langle j| \right), \tag{17}$$

with $U_{m_q n_q} = \sum_{j=0}^{d-1} e^{\frac{2\pi i}{d} j m_q} |j\rangle\langle (j + n_q)\mathrm{mod}\, d|$, $|U_{x_k}\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} e^{\frac{2\pi i}{d} j x_k} |j\rangle\langle j|$. If the global factor is ignored, the state of remaining particles A_1, B_{p+1} and C_{P+1} collapses into

$$|\varphi'\rangle_{A_1 B_{P+1} C_{P+1}} = \sum_{j=0}^{d-1} \left[\left(\prod_{q=P+1}^{1} \beta_{q((k_q + j)\mathrm{mod} d)} \right) |jjj\rangle_{A_1 B_{P+1} C_{P+1}} \right]. \tag{18}$$

To simplify our equation, we assume $p_j = \prod_{q=P+1}^{1} \beta_q ((k_q + j)\mathrm{mod} d)$. Finally, the direct entangled state can be rewritten as

$$|\varphi'\rangle_{A_1 B_{P+1} C_{P+1}} = \sum_{j=0}^{d-1} \left(p_j |jjj\rangle_{A_1 B_{P+1} C_{P+1}} \right). \tag{19}$$

With the above operations, we can establish a direct entangled channel shown in Equation (19). The multi-hop non-destructive teleportation scheme has been transformed into the point-to-point scenario. Therefore, we take the same measurement operations as shown in Section 2 to teleport the unknown state via one non-maximally entangled GHZ channel.

For most communication scenarios, the receiver or sender is only responsible for sending or receiving quantum information. In fact, if the receiver or the sender has the higher information processing technology, they can also perform the channel modulation operation. According to all measurement results, the receiver or the sender needs to perform the corresponding unitary transformation to reconstruct the unknown quantum state and these operations undoubtedly increase the workload of the receiver or sender. In addition, it is unrealistic that each communication party has higher quantum information processing technology such as multi-qudit operations [52–58]. Therefore, it is a better choice for the third-party Charlie to perform channel modulation operations.

4 | EXAMPLE OF FOUR-HOP QUANTUM NON-DESTRUCTIVE TELEPORTATION

To make our multi-hop teleportation scheme more clearly, a four-hop example for teleporting an unknown *three*-dimensional single-qudit is illustrated. The sender Alice holds the particles t and A_1, while the receiver Bob holds particle C_3. The third-party Charlie possesses particle B_3 and an auxiliary particle e. The remaining particles belong to the intermediate nodes Relay$_1$ and Relay$_2$. Figure 4 describes the quantum circuit diagram of this example.

As shown in Figure 4, Alice wishes to teleport an unknown state $|\chi\rangle_t$ to the receiver Bob which is expressed as

$$|\chi\rangle_t = \sum_{j=0}^{2} c_j |j\rangle. \tag{20}$$

Here, $c_j (j = 0, 1, 2)$ are complex and satisfy the relationship $\sum_{j=0}^{2} |c_j|^2 = 1$. The non-maximally entangled GHZ states shared between neighbouring nodes are given by

$$\begin{aligned} |\varphi_1\rangle_{A_1B_1C_1} &= (a_{10}|000\rangle + a_{11}|111\rangle + a_{12}|222\rangle)_{A_1B_1C_1}, \\ |\varphi_2\rangle_{A_2B_2C_2} &= (a_{20}|000\rangle + a_{21}|111\rangle + a_{22}|222\rangle)_{A_2B_2C_2}, \\ |\varphi_3\rangle_{A_3B_3C_3} &= (a_{30}|000\rangle + a_{31}|111\rangle + a_{32}|222\rangle)_{A_3B_3C_3}. \end{aligned} \tag{21}$$

$a_{kj} (k = 1, 2, 3\ j = 0, 1, 2)$are the quantum channel parameters that satisfy the relationship $\sum_{j=0}^{2} |a_{kj}|^2 = 1$. Initially, the state of the nine-qudit quantum channel can be written as

$$\begin{aligned} |\varphi\rangle_{A_1B_1C_1A_2B_2C_2A_3B_3C_3} &= \otimes_{q=1}^{3} |\varphi_q\rangle_{A_qB_qC_q} \\ &= \sum_{j_3=0}^{2}\sum_{j_2=0}^{2}\sum_{j_1=0}^{2} a_{3j_3}a_{2j_2}a_{1j_1} |j_1j_1j_1j_2j_2j_2j_3j_3j_3\rangle_{A_1B_1C_1A_2B_2C_2A_3B_3C_3} \\ &= \left(\frac{1}{\sqrt{3}}\right)^4 \left\{ \begin{array}{l} \sum_{x_1=0}^{2}\sum_{x_2=0}^{2} |H_{x_1}\rangle_{B_1} |H_{x_2}\rangle \sum_{m_1,n_1=0}^{2} \sum_{m_2=0,n_2=0}^{2} |\phi_{m_1n_1}\rangle_{C_1A_2} |\phi_{m_2n_2}\rangle_{C_2A_3} \\ \left(U_{x_1}^{-1}\right)_{A_1} \left(U_{m_1n_{n_1}}^{-1}\right)_{A_1} \left(U_{x_2}^{-1}\right)_{A_1} \left(U_{m_{m_2}n_{n_2}}^{-1}\right)_{A_1} \sum_{j=0}^{2} \left[\left(\prod_{q=3}^{1} a_{q\left(\left(k_q+j\right)\bmod 3\right)} \right) |jjj\rangle_{A_1B_3C_3} \right] \end{array} \right\}, \end{aligned} \tag{22}$$

with the initial condition of subscript $k_{p+1} = 0$. Here, we ignore the global phases. $\prod_{q=3}^{1} a_{q((k_q+j)\bmod 3)} (j = 0, 1, 2)$ are the entangled channel parameters and satisfy the constraint equation $(k_q + j)\bmod d = (k_{q+1} - n_q)\bmod d (q = 2, 1; j = 0, 1, 2)$. The intermediate nodes Relay$_\mathrm{q}$ $(q = 1, 2)$ and the third-party Charlie perform generalised Bell state measurement on particles C_1, A_2 and C_2, A_3 with the basis of $|\phi_{m_qnq}\rangle = \frac{1}{\sqrt{3}} \sum_{j=0}^{2} e^{\frac{2\pi i}{3} jm_q} |j\rangle |(j + n_q)\bmod d\rangle\ (m_q, n_q = 0, 1, 2)$.

There are nine possible measurement results obtained by each intermediate node. Simultaneously, the intermediate nodes Relay$_1$ and Relay$_2$ perform GH measurement operation on their particles B_1 and B_2 with the basis of $|H_{xq}\rangle = \frac{1}{\sqrt{3}} \sum_{j=0}^{2} e^{\frac{2\pi i}{3} jx_q} |j\rangle\ (x_q = 0, 1, 2)$. There are three possible measurement results obtained by Relay$_\mathrm{q}$. For instance, suppose that the measurement outcomes by them are $|\phi_{22}\rangle_{C_1A_2}$, $|\phi_{12}\rangle_{C_2A_3}$, $|H_1\rangle_{B_1}$ and $|H_0\rangle_{B_2}$, respectively. According to the measurement results, Alice performs the unitary operation $U_{A_1}^{oper}$ to form a direct entangled channel. The unitary operation can be expressed

$$\begin{aligned} U_{A_1}^{oper} &= \prod_{q=1}^{2} \left(U_{m_qn_q}\right)_{A_1} \prod_{k=1}^{2} \left(U_{x_k}\right)_{A_1} \\ &= \left[\begin{array}{l} \left(\sum_{j=0}^{2} e^{\frac{4\pi i}{3} j} |(j+2)\bmod 3\rangle_{A_1}\langle j| \right) \\ \left(\sum_{j=0}^{2} e^{\frac{2\pi i}{3} j} |(j+2)\bmod 3\rangle_{A_1}\langle j| \right) \left(\sum_{j=0}^{2} e^{\frac{2\pi i}{3} j} |j\rangle_{A_1}\langle j| \right) \left(\sum_{j=0}^{2} |j\rangle_{A_1}\langle j| \right) \end{array} \right] \\ &= |0\rangle_{A_1}\langle 2| + e^{\frac{2\pi i}{3}} |1\rangle_{A_1}\langle 0| + e^{\frac{4\pi i}{3}} |2\rangle_{A_1}\langle 1|. \end{aligned} \tag{23}$$

If the global factor is ignored, the direct entangled channel composed of particles A_1, B_3 and C_3 becomes

$$\begin{aligned} |\varphi'\rangle_{A_1B_3C_3} &= \sum_{j=0}^{2} \left[\left(\prod_{q=3}^{1} a_{q\left(\left(k_q+j\right)\bmod 3\right)} \right) |jjj\rangle_{A_1B_3C_3} \right] \\ &= \left[\begin{array}{l} a_{30}a_{2k_2}a_{1k_1} |000\rangle_{A_1B_3C_3} \\ + a_{31}a_{2[(k_2+1)\bmod 3]}a_{1[(k_1+1)\bmod 3]} |111\rangle_{A_1B_3C_3} \\ + a_{32}a_{2[(k_2+2)\bmod 3]}a_{1[(k_1+2)\bmod 3]} |222\rangle_{A_1B_3C_3} \end{array} \right]. \end{aligned} \tag{24}$$

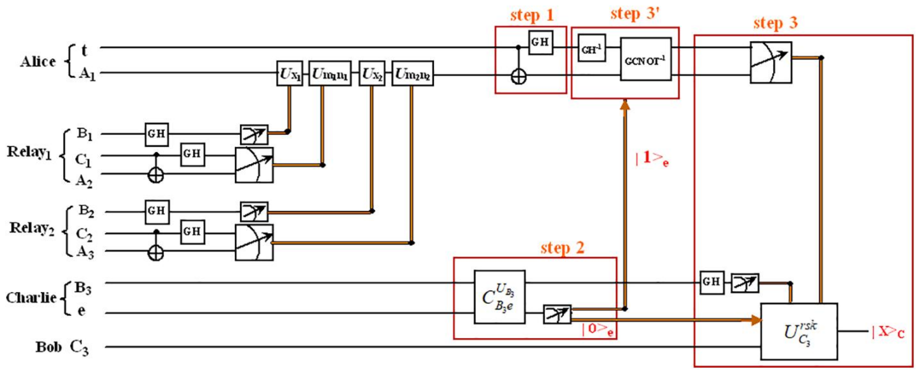


FIGURE 4 Four-hop quantum teleportation with channel modulation performed by the third-party Charlie

According to the above measurement outcomes from the intermediate nodes and the third-party Charlie, we can deduce the direct channel parameters and obtain the entangled state

$$|\varphi'\rangle_{A_1B_3C_3} = (a_{30}a_{22}a_{10}|000\rangle + a_{31}a_{20}a_{11}|111\rangle + a_{32}a_{21}a_{12}|222\rangle)_{A_1B_3C_3}. \tag{25}$$

In order to simplify the formula calculation, assume $p_0 = a_{30}a_{22}a_{10}, p_1 = a_{31}a_{20}a_{11}$ and $p_2 = a_{32}a_{21}a_{12}$. Now the direct entangled channel can be rewritten as

$$|\varphi\rangle_{A_1B_3C_3} = \left(p_0|000\rangle + p_1|111\rangle + p_2|222\rangle\right)_{A_1B_3C_3}. \tag{26}$$

Step 1: In the beginning, the third-party Charlie introduces an auxiliary particle e with the initial state $|0\rangle_e$. The state of particles t, A_1, B_3, C_3 and e can be written as

$$|\varphi\rangle_{tA_1B_3C_3e} = \sum_{j=0}^{2} c_j|j\rangle_t \otimes \sum_{k=0}^{2} p_k|kkk\rangle_{A_1B_3C_3} \otimes |\rangle_e. \tag{27}$$

After Alice performs the 3-dimensional Bell state measurement on the particles t and A_1, the state of particles B_3, e and C_3 becomes

$$\begin{aligned} |\varphi'\rangle_{B_3eC_3} &= {}_{A_1}\langle\phi_{mn}|\varphi\rangle\ t_{A_1B_3C_3et} \\ &= \frac{1}{\sqrt{3}} \sum_{j=0}^{2} e^{-\frac{2\pi i}{3}jm} c_j p_{j+n} |(j+n), 0, (j+n) \mathrm{mod}\ d\rangle_{B_3eC_3}, \end{aligned} \tag{28}$$

with $m, n = \{0, 1, 2\}$. Assume the measurement result of the particles t and A_1 is $|\phi_{11}\rangle_{tA_1}$, the state of particles B_3, e and C_3 can be expressed as

$$|\varphi'\rangle_{B_3eC_3} = \left(c_0p_1|101\rangle + e^{\frac{-2\pi i}{3}} c_1 p_2 |202\rangle + e^{\frac{-4\pi i}{3}} a_2 p_0 |000\rangle\right)_{B_3eC_3}. \tag{29}$$

Step 2: Next, the third-party Charlie performs channel modulation $C_{B_3e}^{U_{B_3}}$ on the particles B_3 and e. Suppose $P_{\min} = \min\{p_0, p_1, p_2\}$ The system collapses into

$$\begin{aligned} |\varphi''\rangle_{B_3eC_3} &= C_{B_3e}^{U_{B_3}} |\varphi'\rangle_{B_3eC_3} \\ &= \Big(c_0 p_{\min}|101\rangle + c_0\sqrt{p_1^2 - p_{\min}^2}|111\rangle + e^{\frac{-2\pi i}{3}} c_1 p_{\min}|202\rangle \\ &\quad + e^{\frac{-2\pi i}{3}} c_1 \sqrt{p_2^2 - p_{\min}^2}|212\rangle + e^{\frac{-4\pi i}{3}} c_2 p_{\min}|000\rangle \\ &\quad + e^{\frac{-4\pi i}{3}} c_2 \sqrt{p_0^2 - p_{\min}^2}|010\rangle \Big)_{B_3eC_3}. \end{aligned} \tag{30}$$

Subsequently, the third-party Charlie measures his auxiliary particle e in the basis of $\{|0\rangle, |1\rangle\}$. There are two possible measurement results $|0\rangle_e$ or $|1\rangle_e$. If the measurement result is $|0\rangle_e$, step 3 will be executed. Otherwise, step 3' will be implemented.

Step 3: If the measurement result of auxiliary particle e is $|0\rangle_e$, the receiver may successfully recover the unknown state. Then, the third-party Charlie carries out GH operation on his particle B_3. The state of particles B_3 and C_3 can be written as

$$\begin{aligned} |\varphi_0\rangle_{C_3} &= (GH)_{B_3}\left(c_0|11\rangle + e^{\frac{-2\pi i}{3}} c_1|22\rangle + e^{\frac{-4\pi i}{3}} c_2|00\rangle\right)_{B_3C_3} \\ &= \frac{1}{\sqrt{3}} \left[\begin{array}{l} \left(|H_0\rangle_{B_3} \otimes \left(c_0|1\rangle + e^{\frac{-2\pi i}{3}} c_1|2\rangle + e^{\frac{-4\pi i}{3}} c_2|0\rangle\right)_{C_3} \right. \\ + |H_1\rangle_{B_3} \otimes \left(e^{\frac{-2\pi i}{3}} c_0|1\rangle + e^{\frac{-6\pi i}{3}} c_1|2\rangle + e^{\frac{-4\pi i}{3}} c_2|0\rangle\right)c_3 \\ \left. + |H_2\rangle_{B_3} \otimes \left(e^{\frac{-4\pi i}{3}} c_0|1\rangle + e^{\frac{-10\pi i}{3}} c_1|2\rangle + e^{\frac{-4\pi i}{3}} c_2|0\rangle\right)c_3\right) \end{array} \right], \end{aligned} \tag{31}$$

Next, the third-party Charlie performs a single-qudit measurement on the particle B_3 in the basis of $\{|k\rangle\}(k = 0, 1, 2)$. There are three kinds of possible measurement outcomes obtained by him. For instance, assume the measurement outcome of the third-party Charlie is $|H_0\rangle_{B_3}$ and the measurement result obtained by Alice is $|\phi_{11}\rangle_{tA_1}$. According to their measurement results, Bob performs a unitary operation $U_{C_3}^{110}$ to recover the initial unknown state. The state of Bob's qudit C_3 collapses into

$$|\varphi_0{}'\rangle_{C_3} = U_{C_3}^{110} \otimes |\varphi_0\rangle_{C_3} = (c_0|0\rangle + c_1|1\rangle + c_2|2\rangle)_{C_3}, \tag{32}$$

with $U_{C_3}^{110} = |0\rangle\langle 1| + e^{\frac{2\pi i}{3}}|1\rangle\langle 2| + e^{\frac{4\pi i}{3}}|2\rangle\langle 0|$. From Equation (32), we can see that Alice successfully transmits the unknown state $|\chi\rangle_t$ to Bob.

Step 3': If the measurement results of the auxiliary particle e is $|1\rangle_e$, the teleportation fails. Alice performs a GH^{-1} on particle t, and then performs the $GCNOT^{-1}$ operation on his two particles t and A_1. The state of particles t, A_1, B_3 and C_3 collapses into

$$\begin{aligned}|\varphi_1{}'\rangle_{tA_1B_3C_3} &= \left(GCNOT^{-1}\right)_{tA_1}\left(GH^{-1}\right)_t|\varphi\rangle_{tA_1B_3C_3} \\ &= \frac{1}{\sqrt{3}}\left((c_0|0\rangle + c_1|1\rangle + c_2|2\rangle)_t \otimes \begin{pmatrix}\sqrt{p_0^2 - p_{\min}^2}|000\rangle \\ + \sqrt{p_1^2 - p_{\min}^2}|111\rangle + \sqrt{p_2^2 - p_{\min}^2}|222\rangle\end{pmatrix}_{A_1C_3B_3}\right).\end{aligned} \tag{33}$$

From Equation (33), we can know that the final quantum state of qudit t is

$$|\varphi_1\rangle_t = (c_0|0\rangle + c_1|1\rangle + c_2|2\rangle)_t. \tag{34}$$

By the comparing Equation (20) and Equation (34), we know that the original unknown state $|\chi\rangle_t$ is preserved in qudit t held by the sender Alice.

5 | PERFORMANCE ANALYSIS AND COMPARISON

In this section, we discuss the performance of our scheme from three aspects: classical communication cost, quantum communication delay and success probability. We also compare it with the previous teleportation schemes.

5.1 | Classical communication cost

Communication cost is defined as the amount of data transmission required in the communication network. The classical communication cost is relevant to both the single-hop communication cost and the hop number. In our scheme shown in Figure 3, the channel is composed of $p+1$ d-dimensional GHZ states. All the intermediate nodes need to send their d-dimensional Bell measurement outcomes of particles C_iA_{i+1} $(i = 1, 2, \ldots, p)$ and GH measurement results of particles B_i to the sender or the receiver. These operations cost $2p[\log_2(d+1)]$ classical bits and $p+1[\log_2(d+1)]$ bits, respectively. The measurement result of particle e costs $\log_2(d+1)$ classical bits. In addition, Alice requires to inform Bob of her measurement result on particles t and A_1 which costs $2[\log_2(d+1)]$ bits. Therefore, the classical communication cost can be calculated as

$$C_{total}^{c-bit} = [3(p+1)+1]\left[\log_2(d+1)\right](bits) \; . \tag{35}$$

For example, in the routing path of Figure 4, $d = 3$ and $p = 2$. The classical communication cost of Alice, Relay$_1$, Relay$_2$ and the third-party Charlie is calculated as $C_{total}^{c-bit} = [3(p+1)+1][\log_2(d+1)](bits) = (9+1)(\log_2 4)(bits) = 20(bits)$in total.

5.2 | Quantum communication delay

Considering the restriction of the quantum device's efficiency and the classical channel bandwidth, the quantum communication delay is inevitable in a quantum network. Our teleportation scheme goes through four steps and we define the delay of each step as D_1, D_2, D_3, D_4 or $D_{4'}$, respectively. In step 1, d-dimensional Bell measurement, GH gate operation, classical information transmission and unitary operation are implemented and these operations delay are D_{GBSM}, D_{GH}, $D_{trans}^{(1)}$ and D_{uoper}, respectively. In step 2, the delay D_{GBSM} occurs when performing d-dimensional Bell measurement by Alice. In step 3, the time consumption includes channel modulation delay $D_{C_{B_{p+1}e}}$, single-qudit measurement delay $D_{e-means}$, and the classical information transmission delay $D^{(2)}_{trans}$. When considering the time delay in step 2 and step 3, we notice that they can be executed in parallel. In realistic quantum operations, step 3 usually takes longer than step 2. Therefore, the time delay for step 2 and step 3 can be regarded as D_3. If the measurement result of auxiliary qudit e is $|0\rangle_e$, we perform step 4 to recover the original unknown state where the unitary operation delay, GH operation delay, and the classical information transmission delay are $D_{U^{rsk}_{C_{p+1}}}$, D_{GH}, $D^{(3)}_{trans}$, respectively. Otherwise, we perform step 4′ to rebuild the original information in which GH^{-1} and $GCNOT^{-1}$ operation delays are $D_{GH^{-1}}$ and $D_{GCNOT^{-1}}$, respectively. The total delay of the entire scheme could be expressed as

$$\begin{aligned} & D_{total} \\ & = D_1 + D_3 + \max(D_4, D_{4'}) \\ & = \left[\begin{array}{l} \left(D_{GBM} + D_{GH} + D^{(1)}_{trans} + D_{uoper}\right) \\ + \left(D_{C_{B_{p+1}e}} + D_{e\text{-}meas} + D^{(2)}_{trans}\right) \\ + \max\left(D_{GH} + D^{(3)}_{trans} + D_{U^{rsk}_{C_{p+1}}}, D_{GH^{-1}} + D_{GCNOT^{-1}}\right) \end{array}\right]. \end{aligned} \tag{36}$$

5.3 | Success probability

In our scheme, if the measurement result of the auxiliary qudit e is $|0\rangle_e$, it allows the sender to teleport an unknown state successfully. In the point-to-point teleportation scheme, the probability of success is $\sum_{r,s,k=0}^{d-1}(1/dP_{\min})^2 = dp^2_{\min}$. Obviously, when $P_{\min}$ goes to $1/\sqrt{d}$, the probabilistic teleportation becomes a deterministic teleportation. On the other hand, if the measurement outcome of the auxiliary qudit is $|1\rangle_e$, the teleportation fails with probability $1 - dp^2_{\min}$ and the sender can rebuild the initial unknown state. The success probability in the multi-hop teleportation scheme can be calculated in a similar way.

5.4 | Comparison

We compare our protocol with the previous teleportation schemes in the following aspects: scheme architecture, channel type, information integrity and space dimension, as summarised in Table 1.

In the standard teleportation scheme, maximally entangled states [40, 59, 61, 62] were utilised as the quantum channels and it ignores the coupling effect between quantum states and their surroundings. Based on this, some researchers considered to teleport the unknown state by non-maximally entangled channels [35, 41, 60, 63]. However, in their schemes, the initial state could not be teleported with a certain probability 100% [35, 40, 60, 63] which led to the risk of data loss. Therefore, it is important to build a teleportation network that could keep the integrity of the initial state. Our scheme aims to address the problem of information loss. When the probabilistic teleportation fails, the sender can rebuild the unknown state. Compared with Refs. [35, 41, 59, 60] we consider the scenario that consists of multiple intermediate nodes between the sender and the receiver. In addition, compared with Refs. [40, 59, 61, 62], different non-maximally entangled channels are distributed among the participants in our scheme, leading to the reduction of the quantum channel requirement. Moreover,

TABLE 1 Comparison between previous teleportation schemes and ours

Protocol	Scheme architecture	Channel type	Information integrity	Space dimension
Gao et al [35]	Point-to-point network	Non-maximally entangled W state	Destructive	*Two*-dimensional
Roa et al [41]	Point-to-point network	Non-maximally entangled Bell state	Non-destructive	*Two*-dimensional
Wang et al [59]	Two-hop network	Maximally entangled GHZ state	Non-destructive	*Two*-dimensional
Yan et al [60]	Three-hop network	Non-maximally entangled Bell state	Destructive	*Three*-dimensional
Li et al [61]	Multi-hop network	Maximally entangled GHZ state	Non-destructive	*Two*-dimensional
Binayak et al [62]	Multi-hop network	Maximally entangled Bell state	Non-destructive	*Two*-dimensional
Juan et al [63]	Multi-hop network	Non-maximally entangled GHZ state	Destructive	*d*-dimensional
Long et al [40]	Multi-hop network	Maximally entangled GHZ state	Destructive	*d*-dimensional
Our protocol	Multi-hop network	Non-maximally entangled GHZ state	Non-destructive	*d*-dimensional

Abbreviation: GHZ, Greenberger–Horne–Zeilinger.

compared with most previous probabilistic teleportation [35, 41, 59, 61, 62], our scheme can be used to teleport a *d*-dimensional unknown state, which means that this system can transmit more information.

6 | CONCLUSION

In summary, we propose a multi-hop non-destructive teleportation scheme via various non-maximally entangled GHZ channels. Here, the sender transmits an arbitrary *d*-dimensional unknown state to the receiver by a direct entangled channel among the three parties. In this quantum network, first, all intermediate nodes perform generalised Bell measurement and GH operation, respectively, to build the direct entangled channels. Next, the third-party Charlie with high quantum information processing technology performs channel modulation and measures the auxiliary qudit e. If the measurement result is $|0\rangle_e$, the receiver needs to perform the corresponding unitary operation to recover the initial unknown state. Otherwise, the sender performs GH^{-1} and $GCNOT^{-1}$ operations to rebuild the unknown state information at the sending location.

Our solution has some unique benefits. First, a quantum network composed of various non-maximally entangled GHZ channels is provided to realise quantum teleportation. The *d*-dimensional GHZ channels can transmit more information than *two*-dimensional channels. Moreover, when the teleportation fails, the information of the unknown state can still be preserved. Finally, we compare our scheme with previous teleportation schemes in detail. In conclusion, we hope to provide an efficient qudit teleportation scheme for teleporting an unknown quantum state without information loss in a quantum network composed of multi-particle channels such as GHZ state, W state and cluster states and so on.

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CONFLICT OF INTEREST

The authors declare that there is no conflict of interest.

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