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Finite-time stabilization of switched nonlinear singular systems with asynchronous switching

Jing Wang[1*] and Xingtao Wang[1]

*Correspondence:
16b912020@stu.hit.edu.cn
[1]Department of Mathematics, Harbin Institute of Technology, Xidazhi Road, Harbin, 150000, P.R. China

Abstract

This paper is concerned with the finite-time stabilization of a class of switched nonlinear singular systems under asynchronous control. Asynchronism here refers to the delays in switching between the controller and the subsystem. First, the dynamic decomposition technique is used to prove that such a switched singular system is regular and impulse-free. Secondly, based on the state solutions of the closed-loop system in the matched time period and the mismatched time period of the system instead of constructing a Lyapunov function, the sufficient conditions for the finite-time stability of the asynchronous switched singular system are given, there is no limit to the stability of subsystems. Then, the mode-dependent state feedback controller that makes the original system stable is derived in the form of strict linear matrix inequalities. Finally, numerical examples are given to verify the feasibility and validity of the results.

Keywords: Switched singular systems; Finite-time stabilization; Asynchronous switching; Average dwell time; Linear matrix inequality

1 Introduction

A switched system is a class of hybrid system consisting of several continuous or discrete dynamic subsystems and a given switching rule. When simulating complex models, switched systems often have an advantage over a single system, so they are widely used in many fields such as switching power converters, aircraft and air-traffic control, see [1–5]. In recent years, many studies on switched systems have emerged, see [6] and [7, 8] and references therein. Most studies on switched systems are concerned with Lyapunov global asymptotic stability. However, in practice, we need the system to be stable within a finite-time interval instead of an infinite interval. The finite-time stability problem of a switched system has been discussed in [9–12]. Therefore, it is more valuable to study the transient performance of the system in a finite-time interval than Lyapunov asymptotic stability in some situations. The difference between the concept of finite-time stability and Lyapunov stability is mainly manifested in two aspects: one is that finite-time stability analyzes the system within a limited time interval; the other is that finite-time stability requires preset boundaries of system variables.

A switched singular system means that the system contains at least one singular subsystem. These systems widely exist in power systems, networked control systems, robotics and other practical systems [13–15]. Therefore, the study of switched singular systems has attracted the attention of many workers, and has achieved rich research results [16–18]. Compared with general switched systems, the stability analysis and controller design of switched singular systems are more complicated due to the problems of regularity, uniform initial state and impulse-mode cancelation. When more detailed and precise models are pursued, models of nonlinear rather than linear singular systems are established. It is inevitable that switching signals will take a certain amount of time in the transmission process, as even modern technology can not completely eliminate the time delay. Like the butterfly effect, even a small delay of the controller may have a great influence on the system. Thus, in order to simulate a more realistic real system, many workers focus their research on meaningful asynchronous controllers [19–22].

In the previous paper on switched singular systems [23], based on the equivalent dynamics-decomposition form, the exact description of the state jump is characterized at the moment of system switching. On the one hand, this state jump comes from the switching law of piecewise-constant values, and on the other hand, it comes from the constraint of algebraic equations. On the basis of the refined description for state jumps proposed above, the finite-time stabilization problem of switched linear singular systems has been considered in [24] without considering the occurrence of asynchronism. Some conditions to ensure that the state remains in a bounded region have been derived via the Lyapunov approach. The finite-time stability problem and finite-time bounded problem of switched singular systems with unstable subsystems have been presented by the authors in [25]. With the help of illustrative examples, the criterion given in [25] provides less conservative results than the approach given in [24]. For the vast majority of methods used to solve the finite-time stability of switched systems, Lyapunov methods have been proven to be one of the most efficient approaches [5, 26, 27]. Moreover, the Lyapunov function method is also a very effective tool when studying fractional-order systems, see [28–31]. The efficiency of those methods, however, depends crucially on appropriate construction of the Lyapunov–Krasovskii (L–K) functions. Since there is no uniform method to construct L–K functions, it is not easy to construct suitable L–K functions for different systems. Hence, we are curious about one thing: can we solve the problem of finite-time stability of switched singular systems under asynchronous control without using the Lyapunov function method? This is the first motivation of this research.

In fact, the solution of the state equation of the system is an intuitive and useful tool in studying the stability of the system, yet few workers use it. There are two main reasons for this phenomenon. On the one hand, the structure and state equation of the switching system are complex, the switching signals are constantly changing. Meanwhile, the subsystems are alternating so it is difficult to obtain the state solution of the system. On the other hand, even if the state solution is obtained, it is difficult to find effective analysis tools and methods. Thus, starting from the original solution of the system and combining the model with the mode-dependent average dwell time to study the asynchronous problem of a switched singular system has not been given enough attention, which is the second motivation of this paper.

The objective of this paper is twofold. The first is to find the appropriate switching law to make the system stable in finite time. The other is to find the specific form of an asynchronous controller that can be solved. Based on the problems raised above, the contributions of this paper are as follows.

(i) The regular and impulse-free properties of switched singular systems is proved based on the dynamic decomposition technique and there is no requirement that all subsystems must be stable. Then, the finite-time stability (FTS) problem of a switched singular system is transformed into the FTS problem of reduced-order switched systems.

(ii) In contrast to [24, 25, 32], we do not construct any Lyapunov functions in our research. Starting with the state-equation solution of the switched system with nonlinear disturbance and taking the switching time point as the boundary, the operation time period of each switched system is analyzed, and the state solutions of the closed-loop system in the matched time period and the mismatched time period are given, and the state solutions of the whole time period are obtained by alternating iterative derivation.

(iii) Based on the mathematical derivation and analysis of the state solution, and combined with the average dwell time method, the sufficient conditions for the FTS of the closed-loop switched singular system are obtained. Then, sufficient conditions for the system to be FTS are given in the form of strict linear matrix inequality and the gain matrix form of the controller is presented. Compared with [25], sufficient conditions with less conservatism can be obtained to determine the FTS of a switched singular system.

The rest of this paper is organized as follows. In Sect. 2, definitions and lemmas useful for the proof of theorems in this paper are listed. Section 3 presents the main results. Based on the decomposition transformation of the original system and taking the asynchronous controller into account, sufficient conditions for finite-time stability of switched singular systems are given. The proof process is concise and to the point. Two specific examples along with numerical and simulation results are provided in Sect. 4. Section 5 gives the conclusion of the work of this paper.

Notations: The notations used in this paper are fairly standard. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space over the reals, $\mathbb{R}^{m\times n}$ is the set of all $m \times n$ real matrices. N^+ represents all positive integer sets. "$*$" stands for the symmetric term in a symmetric matrix. $\mathrm{Re}(A)$ represents the real parts of the eigenvalues of matrix A. $P > 0$ ($P < 0$) means that P is real symmetric and positive-definite (negative-definite). Matrix $P > Q(P \geq Q)$ is equivalent to $P - Q > 0(P - Q \geq 0)$. $\lambda_{\max}(P)(\lambda_{\min}(P))$ denotes the maximum (minimum) eigenvalue of P, and $\|\cdot\|$ is the Euclidean norm.

2 Problem statement and preliminaries

Consider a class of nonlinear switched singular systems described by the following equation:

$$\begin{cases} E_{\sigma(t)}\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + f_{\sigma(t)}(t, x(t)), \\ x(t_0) = x_0. \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ are the state vector and control input, the index $\sigma(t) : [0, \infty) \to \mathcal{N} = \{1, 2, \ldots, \mathbf{N}\}$ is a piecewise right-continuous function of time t or $x(t)$, where $\mathbf{N} \in N^+$ is the number of subsystems. The switching sequence satisfies $t_0 < t_1 < t_2 < \cdots$, when $t \in [t_i, t_{i+1})$ and $\sigma(t) = l_i \in \mathcal{N}$, we say that subsystem l_i is activated. For all $\sigma(t) = l_i \in \mathcal{N}$, $E_{\sigma(t)}$, $A_{\sigma(t)}$, $B_{\sigma(t)}$ are known constant matrices with appropriate dimensions. Meanwhile,

$E_{\sigma(t)}$ is a singular matrix and satisfying rank$E_{\sigma(t)} = r < n$. $f_{\sigma(t)}(t,x(t))$ is a continuously differentiable nonlinear perturbation function on $x(t)$, and $f_{\sigma(t)}(t,0) = 0$ and satisfies the following quadratic constraint

$$f_{\sigma(t)}^{\mathrm{T}}\big(t,x(t)\big)f_{\sigma(t)}\big(t,x(t)\big) \le \omega^2 x^{\mathrm{T}}(t)W_{\sigma(t)}^{\mathrm{T}}W_{\sigma(t)}x(t). \tag{2}$$

In practical engineering applications, because the sensor identify subsystem and the corresponding controller will take some time, there will be a switching time delay in the controller, which results in switching asynchrony between them. Therefore, in this paper, the following form of controller is considered

$$u(t) = K_{\sigma(t-\tau(t))}x(t), \tag{3}$$

where $\tau(t)$ is the switching delay of the controller relative to the subsystem while meeting $0 < \tau(t_i) \le \bar{\tau} \le t_{i+1} - t_i$. Here, without loss of generality [33, 34], the upper bound of the switching delay is known in advance. By substituting this expression into formula (1), we get the following closed-loop system expression

$$\begin{cases} E_{\sigma(t)}\dot{x}(t) = (A_{\sigma(t)} + B_{\sigma(t)}K_{\sigma(t-\tau(t))})x(t) + f_{\sigma(t)}(t,x(t)), \\ x(t_0) = x_0. \end{cases} \tag{4}$$

The purpose here is to design a state feedback controller (3) such that the loop-closed system (4) is admissible. The switching time series of the controller is $t_0 < t_1 + \tau(t_1) < \cdots < t_i + \tau(t_i) < \cdots$. Meanwhile, $\tilde{t}_i$ is defined as $t_i + \tau(t_i)$. Further, system (4) can be written in the following form

$$E_{\sigma(t)}\dot{x}(t) = \begin{cases} \bar{A}_{\sigma(t_0)}x(t) + f_{\sigma(t_0)}(t,x(t)), & t \in [t_0,t_1), \\ \bar{A}_{\sigma(t_i)\sigma(t_{i-1})}x(t) + f_{\sigma(t_i)}(t,x(t)), & t \in [t_i,\tilde{t}_i), \\ \bar{A}_{\sigma(t_i)}x(t) + f_{\sigma(t_i)}(t,x(t)), & t \in [\tilde{t}_i,t_{i+1}). \end{cases} \tag{5}$$

For simplicity, we use the subscripts l_i and l_{i-1} to substitute for $\sigma(t_i)$ and $\sigma(t_{i-1})$. We use $\tilde{\sigma}(t_i) = \sigma(t_i)\sigma(t_{i-1}) = l_il_{i-1}$, $t \in [t_i,\tilde{t}_i)$, $\tilde{\sigma}(t_i) = \sigma(t_i) = l_i$, $t \in [\tilde{t}_i,t_{i+1})$, thus $\bar{A}_{l_il_{i-1}} = A_{l_i} + B_{l_i}K_{l_{i-1}}$, $\bar{A}_{l_i} = A_{l_i} + B_{l_i}K_{l_i}$. The above formula can be abbreviated as

$$E_{\tilde{\sigma}(t)}\dot{x}(t) = \bar{A}_{\tilde{\sigma}(t)}x(t) + f_{\sigma(t)}\big(t,x(t)\big). \tag{6}$$

In order to prove the theorem, we need some definitions and lemmas.

Definition 2.1 ([25]) For the switching signal $\sigma(t)$ of system (1) and any $t_2 > t_1 \ge 0$, let $N_{\sigma l_i}(t_1,t_2)$ denote the switched numbers of the l_ith subsystem over (t_1,t_2), $T_{l_i}(t_1,t_2)$ as the sum of the running time of the l_ith mode, if

$$N_{\sigma l_i}(t_1,t_2) \le N_{0l_i} + \frac{T_{l_i}(t_1,t_2)}{\tau_{al_i}}, \tag{7}$$

holds for $\tau_{al_i} > 0$, $N_{0l_i} \ge 0$, then τ_{al_i} is called the mode-dependent average dwell time and N_{0l_i} is called a chatter bound of the switching signal $\sigma(t)$.

Definition 2.2 ([35]) For every $l_i \in \mathcal{N}$, the pair (E_{l_i}, A_{l_i}) in (1) is said to be
(1) regular if $\det(sE_{l_i} - A_{l_i})$ is not identically zero;
(2) impulse-free if $\deg(\det(sE_{l_i} - A_{l_i})) = \operatorname{rank}(E_{l_i})$.

Definition 2.3 ([24]) For given three positive numbers c_1, c_2, T, with $c_1 < c_2$, a positive-definite matrix $R > 0$ and a given switching signal $\sigma(t) \in \mathcal{N}$, the switched nonlinear singular system (1) is said to be finite-time stabilized under an appropriate control input $u(t)$ with respect to (c_1, c_2, T, R, σ), if

$$x^{\mathrm{T}}(0)E^{\mathrm{T}}_{\sigma(t_0)}RE_{\sigma(t_0)}x(0) \le c_1 \quad \Rightarrow \quad x^{\mathrm{T}}(t)E^{\mathrm{T}}_{\sigma(t)}RE_{\sigma(t)}x(t) \le c_2, \quad \forall t \in [0, T]. \tag{8}$$

Lemma 2.1 *Let* $\mathcal{M} = \left[\begin{smallmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{smallmatrix}\right]$, *where* $\mathcal{A}, \mathcal{B}, \mathcal{C}$ *and* $\mathcal{D}$ *are any real given matrices with appropriate dimensions such that* $\mathcal{M} + \mathcal{M}^{\mathrm{T}} < 0$. *Then,* $\mathcal{D}$ *is nonsingular and* $\mathcal{A} + \mathcal{A}^{\mathrm{T}} - \mathcal{B}\mathcal{D}^{-1}\mathcal{C} - \mathcal{C}^{\mathrm{T}}\mathcal{D}^{-\mathrm{T}}\mathcal{B}^{\mathrm{T}} < 0$.

Lemma 2.2 ([36]) *Let u, v and w be nonnegative piecewise-continuous functions on* $[0, +\infty)$ *for which the inequality*

$$u(t) \le c + \int_a^t \bigl(u(s)v(s) + w(s)\bigr)\,\mathrm{d}s, \quad \forall t \ge a,$$

holds, where a and c are nonnegative constants. Then,

$$u(t) \le c\mathrm{e}^{\int_a^t v(s)\,\mathrm{d}s} + r\mathrm{e}^{\int_a^t (v(s) + \frac{w(s)}{r(s)})\,\mathrm{d}s}, \quad \forall t \ge a, \forall r > 0.$$

3 Main results

In this section, the decomposition technique and average dwell-time method are combined together to investigate the finite-time stabilization problems for the closed-loop system (6). Since $\operatorname{rank}E_{l_i} = r < n$, there exist two invertible matrices M_{l_i} and N_{l_i} such that

$$\begin{aligned} &x(t) = N_{l_i}\begin{bmatrix} \bar{x}_1(t) \\ \bar{x}_2(t) \end{bmatrix}, \qquad M_{l_i}E_{l_i}N_{l_i} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \qquad M_{l_i}\bar{A}_{l_i}N_{l_i} = \begin{bmatrix} A_{l_i11} & A_{l_i12} \\ A_{l_i21} & A_{l_222} \end{bmatrix}, \\ &M_{l_i}f_{l_i} = \begin{bmatrix} f_{l_i1} \\ f_{l_i2} \end{bmatrix}, \end{aligned} \tag{9}$$

where $\bar{x}_1(t) \in \mathbb{R}^r$, $\bar{x}_2(t) \in \mathbb{R}^{n-r}$. Then, equation (6) can be converted into

$$\begin{cases} \dot{\bar{x}}_1(t) = A_{\bar{\sigma}11}\bar{x}_1(t) + A_{\bar{\sigma}12}\bar{x}_2(t) + f_{\sigma1}(t, x(t)), \\ 0 = A_{\bar{\sigma}21}\bar{x}_1(t) + A_{\bar{\sigma}22}\bar{x}_2(t) + f_{\sigma2}(t, x(t)). \end{cases} \tag{10}$$

Suppose $A_{\bar{\sigma}22}$ is nonsingular, the following formula can be further obtained

$$\begin{cases} \dot{\bar{x}}_1(t) = A_{\bar{\sigma}11}\bar{x}_1(t) + A_{\bar{\sigma}12}\bar{x}_2(t) + f_{\sigma1}(t, x(t)), \\ \bar{x}_2(t) = -A^{-1}_{\bar{\sigma}22}A_{\bar{\sigma}21}\bar{x}_1(t) - A^{-1}_{\bar{\sigma}22}f_{\sigma2}(t, x(t)). \end{cases}$$

Thus, system (6) can be rewritten as

$$\begin{cases} \dot{\bar{x}}_1(t) = \bar{A}_{\tilde{\sigma}1}\bar{x}_1(t) + h_{\sigma 1}(t, x(t)), \\ \bar{x}_2(t) = \bar{A}_{\tilde{\sigma}2}\bar{x}_1(t) + h_{\sigma 2}(t, x(t)), \end{cases} \tag{11}$$

where $\bar{A}_{\tilde{\sigma}1} = A_{\tilde{\sigma}11} - A_{\tilde{\sigma}12}A_{\tilde{\sigma}22}^{-1}A_{\tilde{\sigma}21}$, $\bar{A}_{\tilde{\sigma}2} = -A_{\tilde{\sigma}22}^{-1}A_{\tilde{\sigma}21}$, $h_{\sigma 1} = -A_{\tilde{\sigma}12}A_{\tilde{\sigma}22}^{-1}f_{\sigma 2}(t, x(t)) + f_{\sigma 1}(t, x(t)), h_{\sigma 2} = -A_{\tilde{\sigma}22}^{-1}f_{\sigma 2}(t, x(t))$. At the same time, suppose there is a constant $\delta > 0$ such that $h_{\sigma(t)1}(t, x(t))$ satisfies the following inequality

$$\left\| h_{\sigma(t)1}\big(t, x(t)\big) \right\| \leq \delta \left\| \bar{x}_1(t) \right\|, \quad \forall \sigma(t) \in \mathcal{N}. \tag{12}$$

This article follows the previous definition, where $\bar{x}_1(t)$ is called the slow system variable and $\bar{x}_2(t)$ is called the fast subsystem variable.

Remark 3.1 It should be noted that the dynamics-decomposition form is not unique because the choice of matrices M_{l_i}, N_{l_i} is not unique. According to the proof of Theorem 3.1 in reference [37], it can be seen that the properties of the system solution remain unchanged after the coefficient matrix of the system is transformed. Therefore, the regular and impulse-free nature of the solutions of (1) and (11) can be derived from each other. Some similar definitions about the pair (E_{l_i}, A_{l_i}) appear in Theorem 1 in [35] and Definition 1 in [38].

Remark 3.2 As stated in [39], finite-time stability and Lyapunov stability are two independent concepts. The former describes the local properties of the system state, and the latter describes the global asymptotic behavior of the system solution. These two properties cannot be deduced from each other. The upper bound T of the system running time is determined according to the specific situation in a practical application. Therefore, in this study, T is a known value given in advance. At the same time, the average dwell time should be as small as possible to reduce conservatism.

The proof will be divided into two steps. Let us start with the observation that system (6) is regular and impulse free.

Theorem 3.1 *Consider the switched singular system* (6), *given constants* $\alpha_{l_i} > 0$, *if there exist nonsingular matrices* P_{l_i}, $\forall l_i \in \mathcal{N}$ *such that*

$$E_{l_i}^{\mathrm{T}} P_{l_i} = P_{l_i}^{\mathrm{T}} E_{l_i} \geq 0, \tag{13}$$

$$\begin{bmatrix} \Phi_{11l_i} & P_{l_i}^{\mathrm{T}} & W_{l_i}^{\mathrm{T}} \\ * & -I & 0 \\ * & * & -\gamma I \end{bmatrix} < 0, \tag{14}$$

where $\Phi_{11l_i} = \bar{A}_{l_i}^{\mathrm{T}} P_{l_i} + P_{l_i}^{\mathrm{T}} \bar{A}_{l_i} - 2\alpha_{l_i} E_{l_i}^{\mathrm{T}} P_{l_i}$, $\gamma = \omega^{-2}$ *hold, then the pair* (E_{l_i}, A_{l_i}) *in system* (6) *is regular and impulse free and system* (6) *has a unique solution in the neighborhood of an equilibrium point.*

Proof From condition (13), we have

$$N_{l_i}^{\mathrm{T}} E_{l_i}^{\mathrm{T}} M_{l_i}^{\mathrm{T}} \big(M_{l_i}^{\mathrm{T}}\big)^{-1} P_{l_i} N_{l_i} = N_{l_i}^{\mathrm{T}} P_{l_i}^{\mathrm{T}} M_{l_i}^{-1} M_{l_i} E_{l_i} N_{l_i} \geq 0.$$

Taking $\bar{P}_{l_i} = (M_{l_i}^{\mathrm{T}})^{-1} P_{l_i} N_{l_i} = \left[\begin{smallmatrix} P_{l_i11} & P_{l_i12} \\ P_{l_i21} & P_{l_i22} \end{smallmatrix}\right]$. Thus, from $\left[\begin{smallmatrix} I_r & 0 \\ 0 & 0 \end{smallmatrix}\right] \bar{P}_{l_i} = \bar{P}_{l_i}^{\mathrm{T}} \left[\begin{smallmatrix} I_r & 0 \\ 0 & 0 \end{smallmatrix}\right]$, we can obtain $P_{l_i11} > 0$, $P_{l_i12} = 0$. We can conclude from (14) that

$$N_{l_i}^{\mathrm{T}} \bar{A}_{l_i}^{\mathrm{T}} M_{l_i}^{\mathrm{T}} \big(M_{l_i}^{\mathrm{T}}\big)^{-1} P_{l_i} N_{l_i} + N_{l_i}^{\mathrm{T}} P_{l_i}^{\mathrm{T}} M_{l_i}^{-1} M_{l_i} \bar{A}_{l_i} N_{l_i} - 2\alpha_{l_i} N_{l_i}^{\mathrm{T}} E_{l_i}^{\mathrm{T}} M_{l_i}^{\mathrm{T}} \big(M_{l_i}^{\mathrm{T}}\big)^{-1} P_{l_i} N_{l_i} < 0.$$

Substituting (9) into the above formula, we have

$$\begin{aligned} &\begin{bmatrix} A_{l_i11} & A_{l_i12} \\ A_{l_i21} & A_{l_i22} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} P_{l_i11} & 0 \\ P_{l_i21} & P_{l_i22} \end{bmatrix} + \begin{bmatrix} P_{l_i11} & 0 \\ P_{l_i21} & P_{l_i22} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} A_{l_i11} & A_{l_i12} \\ A_{l_i21} & A_{l_i22} \end{bmatrix} \\ &\quad - 2\alpha_{l_i} \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P_{l_i11} & 0 \\ P_{l_i21} & P_{l_i22} \end{bmatrix} < 0, \end{aligned}$$

and finally that

$$\begin{bmatrix} \mathrm{Sym}(A_{l_i11}^{\mathrm{T}} P_{l_i11}) + \mathrm{Sym}(A_{l_i21}^{\mathrm{T}} P_{l_i21}) - 2\alpha_{l_i} P_{l_i11} & A_{l_i21}^{\mathrm{T}} P_{l_i22} + P_{l_i21}^{\mathrm{T}} A_{l_i22} + P_{l_i11}^{\mathrm{T}} A_{l_i12} \\ A_{l_i22}^{\mathrm{T}} P_{l_i21} + P_{l_i22}^{\mathrm{T}} A_{l_i21} + A_{l_i12}^{\mathrm{T}} P_{l_i11} & A_{l_i22}^{\mathrm{T}} P_{l_i22} + P_{l_i22}^{\mathrm{T}} A_{l_i22} \end{bmatrix} < 0.$$

Let

$$\mathcal{M} = \begin{bmatrix} P_{l_i11}^{\mathrm{T}} A_{l_i11} + P_{l_i21}^{\mathrm{T}} A_{l_i21} - \alpha_{l_i} P_{l_i11} & A_{l_i21}^{\mathrm{T}} P_{l_i22} \\ A_{l_i22}^{\mathrm{T}} P_{l_i21} + A_{l_i12}^{\mathrm{T}} P_{l_i11} & A_{l_i22}^{\mathrm{T}} P_{l_i22} \end{bmatrix},$$

according to Lemma 2.1, it follows that

$$\bar{A}_{l_i1}^{\mathrm{T}} P_{l_i11} + P_{l_i11}^{\mathrm{T}} \bar{A}_{l_i1} - \alpha_{l_i} P_{l_i11} < 0, \tag{15}$$

and $A_{l_i22}^{\mathrm{T}} P_{l_i22}$ is nonsingular. Therefore, A_{l_i22} is nonsingular, by [35] and Definition 2.2, system (6) is regular and impulse free. In the neighborhood of an equilibrium point $x(t) = 0$, $f_{\sigma(t)}(t, x(t))$ can be written as $f_{\sigma(t)}(t, x(t)) = W_{\sigma(t)0}(t)x(t) + l_i(t, x(t))$. Thus, system (6) can be rewritten as $E_{\sigma(t)}\dot{x}(t) = (\bar{A}_{\sigma(t)} + W_{\sigma(t)0}(t))x(t) + l_i(t, x(t))$. Then, from [38], we can obtain that

$$W_{l_i0}^{\mathrm{T}}(t) W_{l_i0}(t) \leq \omega^2 W_{l_i}^{\mathrm{T}} W_{l_i}. \tag{16}$$

From (14), we have

$$\Phi_{11l_i} + P_{l_i}^{\mathrm{T}} P_{l_i} + \omega^2 W_{l_i}^{\mathrm{T}} W_{l_i} < 0. \tag{17}$$

According to (16) and (17), we can obtain $\Phi_{11l_i} + P_{l_i}^{\mathrm{T}} P_{l_i} + W_{l_i0}^{\mathrm{T}}(t) W_{l_i0}(t) < 0$. Further, it can be obtained that

$$\big(\bar{A}_{l_i}^{\mathrm{T}} + W_{l_i0}^{\mathrm{T}}(t)\big) P_{l_i} + P_{l_i}^{\mathrm{T}} \big(\bar{A}_{l_i} + W_{l_i0}(t)\big) - 2\alpha_{l_i} E_{l_i}^{\mathrm{T}} P_{l_i} < 0. \tag{18}$$

Therefore, from the proof process of the first half, the approximation system $E_{\sigma(t)}\dot{x}(t) = (\bar{A}_{\sigma(t)} + W_{\sigma(t)0}(t))x(t)$ is regular and impulse free. The rest of the proof is the same as in reference [38], it can be concluded that system (6) has a unique solution in the neighborhood of an equilibrium point. □

Theorem 3.2 *Consider the switched singular system* (6), *given constants* $0 < c_1 < c_2$, $\alpha_{l_i} > 0$, $\alpha_{l_i l_{i-1}} > 0$, $T > 0$, $\delta > 0$, *and matrix* $R > 0$, *if there exist nonsingular matrices* P_{l_i}, $\forall l_i \in \mathcal{N}$ *such that* (13), (14) *and*

$$\begin{bmatrix} \Phi_{11 l_i l_{i-1}} & P_{l_i}^{\mathrm{T}} & W_{l_i}^{\mathrm{T}} \\ * & -I & 0 \\ * & * & -\gamma I \end{bmatrix} < 0, \tag{19}$$

$$\frac{\lambda_{\min}(R_1)c_2}{\lambda_{\max}(\bar{R}_1)c_1} > \mathrm{e}^{\eta + 2\delta T}, \tag{20}$$

where $\Phi_{11 l_i l_{i-1}} = \bar{A}_{l_i l_{i-1}}^{\mathrm{T}} P_{l_i} + P_{l_i}^{\mathrm{T}} \bar{A}_{l_i l_{i-1}} - 2\alpha_{l_i l_{i-1}} E_{l_i}^{\mathrm{T}} P_{l_i}$ *hold, then the average dwell time of the switching signal that guarantees the regular, impulse-free nature and stability of system* (6) *in finite time satisfies the following formula*

$$\tau_{al_i} \geq \tau_{al_i}^{*} = \frac{6\theta T}{\ln(\frac{\lambda_{\min}(R_1)c_2}{\lambda_{\max}(\bar{R}_1)c_1}) - \eta - 2\delta T}, \tag{21}$$

where $\eta = \sum_{k=0}^{i}(\alpha_{l_k}(t_{k+1} - t_k) + (\alpha_{l_k l_{k-1}} - \alpha_{l_k}) T_{l_k l_{k-1}}(0, t))$.

Proof It remains to prove that system (1) is finite-time stabilized. By virtue of (15) and repeating the previous argument and using (19) leads to

$$\mathrm{Re}(\bar{A}_{l_i 1}) < \frac{1}{2}\alpha_{l_i}, \qquad \mathrm{Re}(\bar{A}_{l_i l_{i-1} 1}) < \frac{1}{2}\alpha_{l_i l_{i-1}}. \tag{22}$$

By the definition of a matrix eigenvalue, it can be shown that there exist invertible matrices S_{l_i} and $S_{l_i l_{i-1}}$ such that

$$\begin{aligned} &S_{l_i}^{-1} \bar{A}_{l_i 1} S_{l_i} = J(\bar{A}_{l_i 1}), \\ &S_{l_i l_{i-1}}^{-1} \bar{A}_{l_i l_{i-1} 1} S_{l_i l_{i-1}} = J(\bar{A}_{l_i l_{i-1} 1}), \end{aligned} \tag{23}$$

where $J(\bar{A}_{l_i 1}), J(\bar{A}_{l_i l_{i-1} 1})$ are the Jordan forms of $\bar{A}_{l_i 1}$ and $\bar{A}_{l_i l_{i-1} 1}$, respectively, $\lambda_{l_i 1}, \lambda_{l_i 2}, \ldots, \lambda_{l_i n}$ are the eigenvalues of the matrix $\bar{A}_{l_i 1}$, $\lambda_{l_i l_{i-1} 1}, \lambda_{l_i l_{i-1} 2}, \ldots, \lambda_{l_i l_{i-1} n}$ are the eigenvalues of the matrix $\bar{A}_{l_i l_{i-1} 1}$.

Combining (22) with (23), we deduce that

$$\left\| \mathrm{e}^{\bar{A}_{l_i 1} t} \right\| \leq \mathrm{e}^{\theta_{l_i} + \frac{1}{2}\alpha_{l_i} t}, \qquad \left\| \mathrm{e}^{\bar{A}_{l_i l_{i-1} 1} t} \right\| \leq \mathrm{e}^{\theta_{l_i l_{i-1}} + \frac{1}{2}\alpha_{l_i l_{i-1}} t}, \tag{24}$$

where $\theta_{l_i} = \ln[\lambda_{\max}(S_{l_i})/\lambda_{\min}(S_{l_i})]$, $\theta_{l_i l_{i-1}} = \ln[\lambda_{\max}(S_{l_i l_{i-1}})/\lambda_{\min}(S_{l_i l_{i-1}})]$, use $\lambda_{\max}(S_{l_i})$ to represent the maximum eigenvalue of matrix S_{l_i}, $\lambda_{\max}(S_{l_i l_{i-1}})$ denotes the maximum of all eigenvalues of matrix $S_{l_i l_{i-1}}$.

Denoting $\max_{l_i,l_{i-1}\in\mathcal{N}}\{\theta_{l_i},\theta_{l_il_{i-1}}\}$ briefly by θ with the notation $\alpha = \max_{l_i,l_{i-1}\in\mathcal{N}}\{\alpha_{l_i},\alpha_{l_il_{i-1}}\}$ and using equation (24), we obtain

$$\left\| e^{\bar{A}_{l_i1}t}\right\| \le e^{\theta+\frac{1}{2}\alpha_{l_i}t}, \qquad \left\| e^{\bar{A}_{l_il_{i-1}1}t}\right\| \le e^{\theta+\frac{1}{2}\alpha_{l_il_{i-1}}t}, \quad \forall t\ge 0. \tag{25}$$

For any $t\in[t_0,t_1)$, according to the theoretical knowledge of the solution of the differential equation, when the initial state satisfies $x_1(t_0^-) = x_1(0)$, the solution of equation (6) is

$$\bar{x}_1(t) = e^{\bar{A}_{\sigma(t_0)1}t}x_1\left(t_0^-\right) + \int_0^t e^{\bar{A}_{\sigma(t_0)1}(t-s)}h_{\sigma(t_0)1}(s)\,ds. \tag{26}$$

Furthermore, for any $t\in[t_1,\tilde{t}_1)$ and noting $x_1(t_1) = x_1(t_1^-)$, combined with (26), yields

$$\begin{aligned}\bar{x}_1(t) &= e^{\bar{A}_{\sigma(t_1)\sigma(t_0)1}(t-t_1)}x_1(t_1) + \int_{t_1}^t e^{\bar{A}_{\sigma(t_1)\sigma(t_0)1}(t-s)}h_{\sigma(t_1)1}(s)\,ds\\ &= e^{\bar{A}_{\sigma(t_1)\sigma(t_0)1}(t-t_1)}\left[e^{\bar{A}_{\sigma(t_0)1}t_1}x_1(0) + \int_0^{t_1} e^{\bar{A}_{\sigma(t_0)1}(t-s)}h_{\sigma(t_0)1}(s)\,ds\right]\\ &\quad + \int_{t_1}^t e^{\bar{A}_{\sigma(t_1)\sigma(t_0)1}(t-s)}h_{\sigma(t_1)1}(s)\,ds.\end{aligned} \tag{27}$$

Similarly, $x_1(\tilde{t}_1) = x_1(\tilde{t}_1^-)$ holds, and we have that for any $t\in[\tilde{t}_1,t_2)$

$$\begin{aligned}\bar{x}_1(t) &= e^{\bar{A}_{\sigma(t_1)1}(t-\tilde{t}_1)}x_1(\tilde{t}_1) + \int_{\tilde{t}_1}^t e^{\bar{A}_{\sigma(t_1)1}(t-s)}h_{\sigma(t_1)1}(s)\,ds\\ &= e^{\bar{A}_{\sigma(t_1)1}(t-\tilde{t}_1)}\left[e^{\bar{A}_{\sigma(t_1)\sigma(t_0)1}\tau(t_1)}\left[e^{\bar{A}_{\sigma(t_0)1}t_1}x_1(0) + \int_0^{t_1} e^{\bar{A}_{\sigma(t_0)1}(t-s)}h_{\sigma(t_0)1}(s)\,ds\right]\right.\\ &\quad \left. + \int_{t_1}^{\tilde{t}_1} e^{\bar{A}_{\sigma(t_1)\sigma(t_0)1}(t-s)}h_{\sigma(t_1)1}(s)\,ds\right] + \int_{\tilde{t}_1}^t e^{\bar{A}_{\sigma(t_1)\sigma(t_0)1}(t-s)}h_{\sigma(t_1)1}(s)\,ds\\ &= e^{\bar{A}_{\sigma(t_1)1}(t-\tilde{t}_1)}e^{\bar{A}_{\sigma(t_1)\sigma(t_0)1}\tau(t_1)}e^{\bar{A}_{\sigma(t_0)1}t_1}x_1(0)\\ &\quad + e^{\bar{A}_{\sigma(t_1)1}(t-\tilde{t}_1)}e^{\bar{A}_{\sigma(t_1)\sigma(t_0)1}\tau(t_1)}\int_0^{t_1} e^{\bar{A}_{\sigma(t_0)1}(t-s)}h_{\sigma(t_0)1}(s)\,ds\\ &\quad + e^{\bar{A}_{\sigma(t_1)1}(t-\tilde{t}_1)}\int_{t_1}^{\tilde{t}_1} e^{\bar{A}_{\sigma(t_1)\sigma(t_0)1}(t-s)}h_{\sigma(t_1)1}(s)\,ds\\ &\quad + \int_{\tilde{t}_1}^t e^{\bar{A}_{\sigma(t_1)\sigma(t_0)1}(t-s)}h_{\sigma(t_1)1}(s)\,ds.\end{aligned} \tag{28}$$

Under the conditions stated above, when $t\in[t_i,\tilde{t}_i)$, we infer that

$$\begin{aligned}\bar{x}_1(t) &= e^{\bar{A}_{\sigma(t_i)\sigma(t_{i-1})1}(t-t_i)}e^{\bar{A}_{\sigma(t_{i-1})1}(t_i-\tilde{t}_{i-1})}\dots e^{\bar{A}_{\sigma(t_0)1}t_1}x_1(0)\\ &\quad + \int_0^{t_1} e^{\bar{A}_{\sigma(t_i)\sigma(t_{i-1})1}(t-t_i)}e^{\bar{A}_{\sigma(t_{i-1})1}(t_i-\tilde{t}_{i-1})}\dots e^{\bar{A}_{\sigma(t_0)1}(t_1-s)}h_{\sigma(t_0)1}(s)\,ds\\ &\quad + \int_{t_1}^{\tilde{t}_1} e^{\bar{A}_{\sigma(t_i)\sigma(t_{i-1})1}(t-t_i)}e^{\bar{A}_{\sigma(t_{i-1})1}(t_i-\tilde{t}_{i-1})}\dots e^{\bar{A}_{\sigma(t_1)\sigma(t_0)1}(\tilde{t}_1-s)}h_{\sigma(t_1)1}(s)\,ds\end{aligned} \tag{29}$$

$$+\cdots+\int_{\tilde{t}_{i-1}}^{t_i}\mathrm{e}^{\bar{A}_{\sigma(t_i)\sigma(t_{i-1})1}(t-t_i)}\mathrm{e}^{\bar{A}_{\sigma(t_{i-1})1}(t_i-s)}h_{\sigma(t_{i-1})1}(s)\,\mathrm{d}s$$
$$+\int_{t_i}^{t}\mathrm{e}^{\bar{A}_{\sigma(t_i)\sigma(t_{i-1})1}(t-s)}h_{\sigma(t_i)1}(s)\,\mathrm{d}s.$$

Thus, $\|\bar{x}_1(t)\|$ can be bounded by

$$\begin{aligned}\left\|\bar{x}_1(t)\right\| \le{}& \mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_il_{i-1}}(t-t_i)}\mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_{i-1}}(t_i-\tilde{t}_{i-1})}\ldots\mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_0}t_1}\left\|x_1(0)\right\| \\ &+\int_0^{t_1}\mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_il_{i-1}}(t-t_i)}\mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_{i-1}}(t_i-\tilde{t}_{i-1})}\ldots\mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_0}(t_1-s)}\left\|h_{\sigma(t_0)1}(s)\right\|\,\mathrm{d}s \\ &+\int_{t_1}^{\tilde{t}_1}\mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_il_{i-1}}(t-t_i)}\mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_{i-1}}(t_i-\tilde{t}_{i-1})}\ldots\mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_1l_0}(\tilde{t}_1-s)}\left\|h_{\sigma(t_1)1}(s)\right\|\,\mathrm{d}s \\ &+\cdots+\int_{\tilde{t}_{i-1}}^{t_i}\mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_il_{i-1}}(t-t_i)}\mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_{i-1}}(t_i-s)}\left\|h_{\sigma(t_{i-1})1}(s)\right\|\,\mathrm{d}s \\ &+\int_{t_i}^{t}\mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_il_{i-1}}(t-s)}\left\|h_{\sigma(t_k)1}(s)\right\|\,\mathrm{d}s.\end{aligned} \tag{30}$$

Using (12), (30) shows that

$$\begin{aligned}\left\|\bar{x}_1(t)\right\| \le{}& \mathrm{e}^{2N_\sigma(0,t)\theta+\frac{1}{2}\sum_{k=0}^{i}(\alpha_{l_k}T_{l_k}(0,t)+\alpha_{l_kl_{k-1}}T_{l_kl_{k-1}}(0,t))}\left\|x_1(0)\right\| \\ &+\int_0^t\mathrm{e}^{(2N_\sigma(s,t)+1)\theta+\frac{1}{2}\sum_{k=0}^{i}(\alpha_{l_k}T_{l_k}(s,t)+\alpha_{l_kl_{k-1}}T_{l_kl_{k-1}}(s,t))}\delta\left\|\bar{x}_1(s)\right\|\,\mathrm{d}s.\end{aligned} \tag{31}$$

Moreover, for $t\in[\tilde{t}_i,t_{i+1})$,

$$\begin{aligned}\bar{x}_1(t) ={}& \mathrm{e}^{\bar{A}_{\sigma(t_i)1}(t-\tilde{t}_i)}\mathrm{e}^{\bar{A}_{\sigma(t_i)\sigma(t_{i-1})1}\tau(t_i)}\ldots\mathrm{e}^{\bar{A}_{\sigma(t_0)1}t_1}x_1(0) \\ &+\int_0^{t_1}\mathrm{e}^{\bar{A}_{\sigma(t_i)1}(t-\tilde{t}_i)}\mathrm{e}^{\bar{A}_{\sigma(t_i)\sigma(t_{i-1})1}\tau(t_i)}\ldots\mathrm{e}^{\bar{A}_{\sigma(t_0)1}(t_1-s)}h_{\sigma(t_0)1}(s)\,\mathrm{d}s \\ &+\int_{t_1}^{\tilde{t}_1}\mathrm{e}^{\bar{A}_{\sigma(t_i)1}(t-\tilde{t}_i)}\mathrm{e}^{\bar{A}_{\sigma(t_i)\sigma(t_{i-1})1}\tau(t_i)}\ldots\mathrm{e}^{\bar{A}_{\sigma(t_i)\sigma(t_{i-1})1}(\tilde{t}_1-s)}h_{\sigma(t_1)1}(s)\,\mathrm{d}s \\ &+\cdots+\int_{\tilde{t}_{i-1}}^{t_i}\mathrm{e}^{\bar{A}_{\sigma(t_i)1}(t-\tilde{t}_i)}\mathrm{e}^{\bar{A}_{\sigma(t_i)\sigma(t_{i-1})1}(\tilde{t}_i-s)}h_{\sigma(t_{i-1})1}(s)\,\mathrm{d}s \\ &+\int_{t_i}^{t}\mathrm{e}^{\bar{A}_{\sigma(t_i)1}(t-s)}h_{\sigma(t_i)1}(s)\,\mathrm{d}s.\end{aligned} \tag{32}$$

Accordingly, we can obtain

$$\begin{aligned}\left\|\bar{x}_1(t)\right\| \le{}& \mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_i}(t-\tilde{t}_i)}\mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_il_{i-1}}(\tilde{t}_i-t_i)}\ldots\mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_0}t_1}\left\|x_1(0)\right\| \\ &+\int_0^{t_1}\mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_i}(t-\tilde{t}_i)}\mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_il_{i-1}}(\tilde{t}_i-t_i)}\ldots\mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_0}(t_1-s)}\left\|h_{\sigma(t_0)1}(s)\right\|\,\mathrm{d}s \\ &+\int_{t_1}^{\tilde{t}_1}\mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_i}(t-\tilde{t}_i)}\mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_il_{i-1}}(\tilde{t}_i-t_i)}\ldots\mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_1l_0}(\tilde{t}_1-s)}\left\|h_{\sigma(t_1)1}(s)\right\|\,\mathrm{d}s \\ &+\cdots+\int_{\tilde{t}_{i-1}}^{t_i}\mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_i}(t-\tilde{t}_i)}\mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_il_{i-1}}(\tilde{t}_i-s)}\left\|h_{\sigma(t_{i-1})1}(s)\right\|\,\mathrm{d}s\end{aligned} \tag{33}$$

$$+\int_{t_i}^{t} \mathrm{e}^{\theta+\frac{1}{2}\alpha_{l_i}(t-s)} \left\| h_{\sigma(t_i)1}(s) \right\| \mathrm{d}s.$$

Combining (12) and (33) leads to

$$\begin{aligned} \left\|\bar{x}_1(t)\right\| &\le \mathrm{e}^{(2N_\sigma(0,t)+1)\theta+\frac{1}{2}\sum_{k=0}^{i}(\alpha_{l_k}T_{l_k}(0,t)+\alpha_{l_kl_{k-1}}T_{l_kl_{k-1}}(0,t))} \left\|x_1(0)\right\| \\ &\quad + \int_0^t \mathrm{e}^{(2N_\sigma(s,t)+1)\theta+\frac{1}{2}\sum_{k=0}^{i}(\alpha_{l_k}T_{l_k}(s,t)+\alpha_{l_kl_{k-1}}T_{l_kl_{k-1}}(s,t))}\delta \left\|\bar{x}_1(s)\right\| \mathrm{d}s. \end{aligned} \tag{34}$$

On account of the above discussion, formula (34) holds for any $t \in [t_i, t_{i+1})$. Since $1 \le N_\sigma(0,t) \le \frac{t}{\tau_a}$, we have

$$\begin{aligned} \left\|\bar{x}_1(t)\right\| &\le \mathrm{e}^{(\frac{2t}{\tau_a}+1)\theta+\frac{1}{2}\sum_{k=0}^{i}(\alpha_{l_k}T_{l_k}(0,t)+\alpha_{l_kl_{k-1}}T_{l_kl_{k-1}}(0,t))} \left\|x_1(0)\right\| \\ &\quad + \int_0^t \mathrm{e}^{(\frac{2(t-s)}{\tau_a}+1)\theta+\frac{1}{2}\sum_{k=0}^{i}(\alpha_{l_k}T_{l_k}(s,t)+\alpha_{l_kl_{k-1}}T_{l_kl_{k-1}}(s,t))}\delta \left\|\bar{x}_1(s)\right\| \mathrm{d}s \\ &\le \mathrm{e}^{\frac{3t}{\tau_a}\theta+\frac{1}{2}\sum_{k=0}^{i}(\alpha_{l_k}T_{l_k}(0,t)+\alpha_{l_kl_{k-1}}T_{l_kl_{k-1}}(0,t))} \left\|x_1(0)\right\| \\ &\quad + \int_0^t \mathrm{e}^{\frac{3(t-s)}{\tau_a}\theta+\frac{1}{2}\sum_{k=0}^{i}(\alpha_{l_k}T_{l_k}(s,t)+\alpha_{l_kl_{k-1}}T_{l_kl_{k-1}}(s,t))}\delta \left\|\bar{x}_1(s)\right\| \mathrm{d}s. \end{aligned} \tag{35}$$

On multiplying both sides of (35) by $\mathrm{e}^{-\frac{3t}{\tau_a}\theta-\frac{1}{2}\sum_{k=0}^{i}(\alpha_{l_k}T_{l_k}(0,t)+\alpha_{l_kl_{k-1}}T_{l_kl_{k-1}}(0,t))}$, we obtain

$$\begin{aligned} &\mathrm{e}^{-\frac{3t\theta}{\tau_a}-\frac{1}{2}\sum_{k=0}^{i}(\alpha_{l_k}T_{l_k}(0,t)+\alpha_{l_kl_{k-1}}T_{l_kl_{k-1}}(0,t))} \left\|\bar{x}_1(t)\right\| \\ &\quad \le \left\|\bar{x}_1(0)\right\| + \int_0^t \mathrm{e}^{-\frac{3s\theta}{\tau_a}-\frac{1}{2}\sum_{k=0}^{i}(\alpha_{l_k}T_{l_k}(0,s)+\alpha_{l_kl_{k-1}}T_{l_kl_{k-1}}(0,s))}\delta \left\|\bar{x}_1(s)\right\| \mathrm{d}s. \end{aligned} \tag{36}$$

Then, it can be deduced from Lemma 2.2 that

$$\mathrm{e}^{-\frac{3t\theta}{\tau_a}-\frac{1}{2}\sum_{k=0}^{i}(\alpha_{l_k}T_{l_k}(0,t)+\alpha_{l_kl_{k-1}}T_{l_kl_{k-1}}(0,t))} \left\|\bar{x}_1(t)\right\| \le \left\|\bar{x}_1(0)\right\| \mathrm{e}^{\delta t}.$$

That is,

$$\left\|\bar{x}_1(t)\right\| \le \mathrm{e}^{(\frac{3\theta}{\tau_a}+\delta)t+\frac{1}{2}\sum_{k=0}^{i}(\alpha_{l_k}T_{l_k}(0,t)+\alpha_{l_kl_{k-1}}T_{l_kl_{k-1}}(0,t))} \left\|\bar{x}_1(0)\right\|. \tag{37}$$

Using the expressions of

$$M_{\sigma(0)}^{-\mathrm{T}} R M_{\sigma(0)}^{-1} = \begin{bmatrix} R_1 & R_2 \\ R_3 & R_4 \end{bmatrix}, \qquad M_{\sigma(t)}^{-\mathrm{T}} R M_{\sigma(t)}^{-1} = \begin{bmatrix} \bar{R}_1 & \bar{R}_2 \\ \bar{R}_3 & \bar{R}_4 \end{bmatrix}. \tag{38}$$

Noting that $x(t) = N_{\sigma(t)}\bar{x}(t)$, we can show that

$$\begin{aligned} &x^{\mathrm{T}}(0)E_{\sigma(0)}^{\mathrm{T}} R E_{\sigma(0)} x(0) \\ &\quad = x^{\mathrm{T}}(0) N_{\sigma(0)}^{-\mathrm{T}} N_{\sigma(0)}^{\mathrm{T}} E_{\sigma(0)}^{\mathrm{T}} M_{\sigma(0)}^{\mathrm{T}} M_{\sigma(0)}^{-\mathrm{T}} R M_{\sigma(0)}^{-1} M_{\sigma(0)} E_{\sigma(0)} N_{\sigma(0)} N_{\sigma(0)}^{-1} x(0) \\ &\quad = \begin{bmatrix} \bar{x}_1^{\mathrm{T}}(0) & \bar{x}_2^{\mathrm{T}}(0) \end{bmatrix} \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} R_1 & R_2 \\ R_3 & R_4 \end{bmatrix} \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{x}_1(0) \\ \bar{x}_2(0) \end{bmatrix} \\ &\quad = \bar{x}_1^{\mathrm{T}}(0) R_1 \bar{x}_1(0) \le c_1. \end{aligned} \tag{39}$$

Similarly, $x^{\mathrm{T}}(t)E^{\mathrm{T}}_{\sigma(t)}RE_{\sigma(t)}x(t) \le c_2$ can be derived from $\bar{x}_1^{\mathrm{T}}(t)\bar{R}_1\bar{x}_1(t) \le c_2$. It follows from (39) that $\|\bar{x}_1(0)\|^2 \le \frac{c_1}{\lambda_{\min}(R_1)}$. By switching signal (21) and (37), it holds that

$$\begin{aligned}\bar{x}_1^{\mathrm{T}}(t)\bar{R}_1\bar{x}_1(t) &\le \lambda_{\max}(\bar{R}_1)\bar{x}_1^{\mathrm{T}}(t)\bar{x}_1(t)\\ &\le \lambda_{\max}(\bar{R}_1)\mathrm{e}^{2T(\frac{3\theta}{\tau_a}+\delta)+\sum_{k=0}^{i}(\alpha_{l_k}T_{l_k}(0,t)+\alpha_{l_kl_{k-1}}T_{l_kl_{k-1}}(0,t))}\frac{c_1}{\lambda_{\min}(R_1)}\\ &= \mathrm{e}^{2T(\frac{3\theta}{\tau_a}+\delta)+\sum_{k=0}^{i}(\alpha_{l_k}(t_{k+1}-t_k)+(\alpha_{l_kl_{k-1}}-\alpha_{l_k})T_{l_kl_{k-1}}(0,t))}\frac{c_1\lambda_{\max}(\bar{R}_1)}{\lambda_{\min}(R_1)}\\ &\le c_2.\end{aligned} \tag{40}$$

Thus, the proof is completed. □

Remark 3.3 In reference [24], the authors' proof is given on the basis of assuming that the subsystem is regular, impulse free and stable. The proof in this paper removes this hypothesis and proves the regularity and nonpulsation of the switching system through the known constraints. It is worth noting that the matrix $E_{\sigma(t)}$ in this paper varies with different subsystems, so the conclusion has stronger applicability. When $E_{\sigma(t)}$ is a nonsingular matrix, the system studied in this paper degenerates into a general switching system.

Remark 3.4 In the proof of the above theorem, no Lyapunov function is constructed. Instead, starting with the state-equation solution of the switched system with nonlinear disturbance and taking the switching time point as the boundary, the operation time period of each switched system is analyzed. The state solutions of the closed-loop system in the matched time period and the mismatched time period are given, and the state solutions of the whole time period are obtained by alternating iterative derivation. The condition that all subsystems must be stable is removed.

Theorem 3.3 *Consider the switched singular system* (6), *given constants* $0 < c_1 < c_2$, $\alpha > 0$, $T > 0$, $\delta > 0$, *matrix* $R > 0$ *and a full column rank matrix* $X_{l_i} \in \mathbb{R}^{n\times(n-r)}$ *satisfies* $E_{l_i}X_{l_i} = 0$, *if there exist matrices* $F_{l_i} > 0$, S_{l_i}, G_{l_i}, $\forall l_i \in \mathcal{N}$ *such that* (20) *and the following LMIs hold*

$$\begin{bmatrix}\Gamma_{11l_i} & I & \Omega_{l_i}^{\mathrm{T}}W_{l_i}^{\mathrm{T}}\\ * & -I & 0\\ * & * & -\gamma I\end{bmatrix} < 0, \tag{41}$$

$$\begin{bmatrix}\Gamma_{11l_il_{i-1}} & I & \Omega_{l_i}^{\mathrm{T}}W_{l_i}^{\mathrm{T}}\\ * & -I & 0\\ * & * & -\gamma I\end{bmatrix} < 0. \tag{42}$$

where $\Gamma_{11l_i} = A_{l_i}\Omega_{l_i} + B_{l_i}G_{l_i} + \Omega_{l_i}^{\mathrm{T}}A_{l_i}^{\mathrm{T}} + G_{l_i}^{\mathrm{T}}B_{l_i}^{\mathrm{T}} - 2\alpha_{l_i}\Omega_{l_i}^{\mathrm{T}}E_{l_i}^{\mathrm{T}}$, $\Gamma_{11l_il_{i-1}} = A_{l_i}\Omega_{l_i} + B_{l_i}G_{l_il_{i-1}} + \Omega_{l_i}^{\mathrm{T}}A_{l_i}^{\mathrm{T}} + G_{l_il_{i-1}}^{\mathrm{T}}B_{l_i}^{\mathrm{T}} - 2\alpha_{l_il_{i-1}}\Omega_{l_i}^{\mathrm{T}}E_{l_i}^{\mathrm{T}}$. *Then, the average dwell time of the switching signal that guarantees the finite-time stabilization of system* (6) *satisfies* (21). *Moreover, the controller gain is given by*

$$\begin{aligned}u(t) &= G_{l_i}\Omega_{l_i}^{-1}x(t),\\ \Omega_{l_i} &= F_{l_i}E_{l_i}^{\mathrm{T}} + X_{l_i}S_{l_i}.\end{aligned} \tag{43}$$

Proof In order to obtain the controller gain, we denote $D_{l_i}^{\mathrm{T}} = P_{l_i}^{-\mathrm{T}}$, $D_{l_i} = P_{l_i}^{-1}$. From (5), we have $\bar{A}_{l_i} = A_{l_i} + B_{l_i}K_{l_i}$, $\bar{A}_{l_i l_{i-1}} = A_{l_i} + B_{l_i}K_{l_{i-1}}$. Pre- and postmultiplying (14) and (19) by $\operatorname{diag}\{D_{l_i}^{\mathrm{T}}, I, I\}$ and its transpose, respectively, and using the definition of $G_{l_i} = K_{l_i}D_{l_i}$, it follows that

$$\begin{bmatrix} \Sigma_{11l_i} & I & D_{l_i}^{\mathrm{T}} W_{l_i}^{\mathrm{T}} \\ * & -I & 0 \\ * & * & -\gamma I \end{bmatrix} < 0, \tag{44}$$

where $\Sigma_{11l_i} = D_{l_i}^{\mathrm{T}} A_{l_i}^{\mathrm{T}} + G_{l_i}^{\mathrm{T}} B_{l_i}^{\mathrm{T}} + A_{l_i} D_{l_i} + B_{l_i} G_{l_i} - 2\alpha_{l_i} D_{l_i}^{\mathrm{T}} E_{l_i}^{\mathrm{T}}$. Similarly, we obtain

$$\begin{bmatrix} \Sigma_{11l_i l_{i-1}} & I & D_{l_i}^{\mathrm{T}} W_{l_i}^{\mathrm{T}} \\ * & -I & 0 \\ * & * & -\gamma I \end{bmatrix} < 0, \tag{45}$$

where $\Sigma_{11l_i l_{i-1}} = D_{l_i}^{\mathrm{T}} A_{l_i}^{\mathrm{T}} + G_{l_i l_{i-1}}^{\mathrm{T}} B_{l_i}^{\mathrm{T}} + A_{l_i} D_{l_i} + B_{l_i} G_{l_i l_{i-1}} - 2\alpha_{l_i l_{i-1}} D_{l_i}^{\mathrm{T}} E_{l_i}^{\mathrm{T}}$. Substituting $D_{l_i} = \Omega_{l_i} = F_{l_i} E^{\mathrm{T}} + X_{l_i} S_{l_i}$ into (44) and (45), respectively, and denoting $G_{l_i} = K_{l_i}\Omega_{l_i}$, (41)((42)) is equivalent to (44)((45)). □

Remark 3.5 The proof of the theorem does not require the stability of the subsystem and parameter α can take different values α_{l_i} for different subsystems so it is less conservative. Compared with Theorem 3.1 in [25], the constraint conditions of equations (17) and (18) are discarded. The subsystem and the corresponding controller are one-to-one corresponding. Therefore, the design of the controller is only related to the subscript l_i and not dependent on l_{i-1}.

When the switching delay is not considered, that is, when the operation of the controller and the corresponding subsystem is synchronous, we can obtain the following corollary. It is worth noting that the controller of the system becomes

$$u(t) = K_{\sigma(t)} x(t). \tag{46}$$

Corollary 3.1 *Consider the switched singular system* (6) *with control input* (46), *given constants* $0 < c_1 < c_2$, $\alpha > 0$, $T > 0$, $\delta > 0$, *matrix* $R > 0$ *and a full column rank matrix* $X_{l_i} \in \mathbb{R}^{n\times(n-r)}$ *satisfies* $E_{l_i} X_{l_i} = 0$, *if there exist matrices* $F_{l_i} > 0$, S_{l_i}, G_{l_i}, $\forall l_i \in \mathcal{N}$ *such that*

$$\frac{\lambda_{\min}(R_1)c_2}{\lambda_{\max}(\bar{R}_1)c_1} > (\alpha + 2\delta)T, \tag{47}$$

and (41) *hold. Then, the average dwell time of the switching signal that guarantees the finite-time stabilization of system* (6) *with respect to* (c_1, c_2, T, R, σ) *satisfies*

$$\tau_{al_i} \geq \tau_{al_i}^{*} = \frac{6\theta T}{\ln(\frac{\lambda_{\min}(R_1)c_2}{\lambda_{\max}(\bar{R}_1)c_1}) - (\alpha + 2\delta)T}, \tag{48}$$

where $\alpha = \max_{l_i, l_{i-1} \in \mathcal{N}}\{\alpha_{l_i}, \alpha_{l_i l_{i-1}}\}$. *Moreover, the controller gain is given by* (43).

4 Numerical example

In this section, two numerical examples are provided to demonstrate the validity and feasibility of the above results.

Example 1 Consider the switched nonlinear singular system (1) with two subsystems and matrix parameters as follows:

Subsystem1:

$$E_1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} -0.7 & -2.6 & 1 \\ -1.5 & 0 & -1.5 \\ 1 & -1 & -2 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ 1 & 2 \\ 0 & 1 \end{bmatrix}, \qquad f_1 = \begin{bmatrix} 0.1\sin(x_1(t)) \\ 0.1\sin(x_2(t)) \\ 0 \end{bmatrix},$$

Subsystem2:

$$E_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -1.7 & 0 & -1.5 \\ 0.8 & -2.1 & 2.2 \\ 3 & -0.6 & -1 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 0 & 1 \end{bmatrix}, \qquad f_2 = \begin{bmatrix} \sin(0.1x_1(t)) \\ 0 \\ \cos(x_2(t)) \end{bmatrix}.$$

Choosing two sets of matrices as follows that can transform matrices E_1 and E_2 into a unit matrix, respectively. Selecting X_1, X_2 that satisfy equation $EX_1 = EX_2 = 0$ as follows

$$M_1 \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad N_1 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad M_2 = I_3,$$

$$N_2 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad X_1 = X_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix},$$

$$R = I, \qquad M_1^{-\mathrm{T}} R M_1^{-1} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad M_2^{-\mathrm{T}} R M_2^{-1} = I_3.$$

Suppose $W_1 = W_2 = [1\ 1\ 1]$, $\omega = 1$, $\alpha_1 = 0.47$, $\alpha_2 = 0.14$, $c_1 = 0.01$, $c_2 = 1.5 \times 10^{21}$, $T = 10$, $\theta = 2.4680$, $\delta = 0.1$. Then, by solving the linear matrix inequality (41), the corresponding state feedback controller gains are obtained as follows

$$K_1 = \begin{bmatrix} 4.4995 & -5.2703 & -2.7031 \\ 3.4235 & -9.4527 & -4.9857 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} -14.7294 & 1.8166 & -0.9038 \\ -2.3569 & -0.5674 & -3.3576 \end{bmatrix}.$$

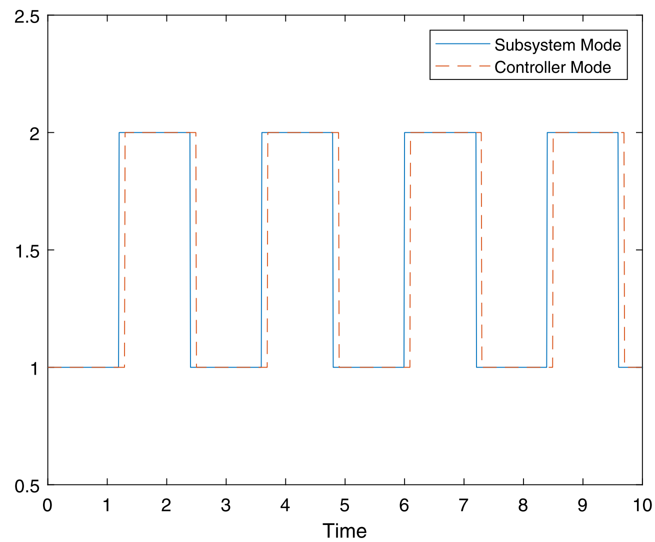

Figure 1 The switching signal

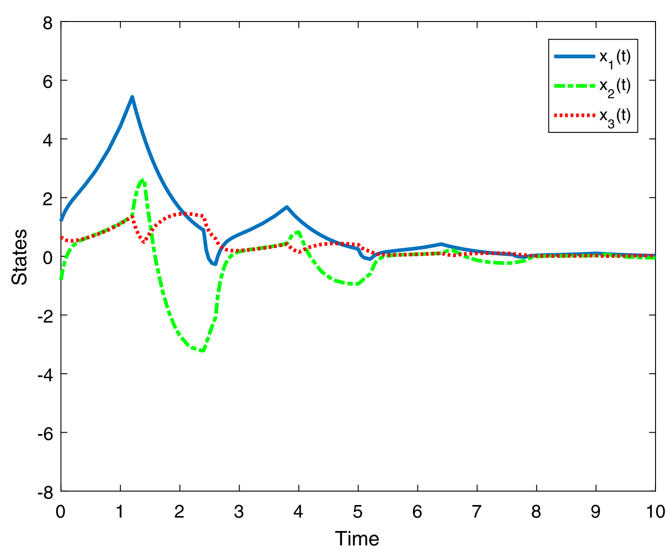

Figure 2 State response of the closed-loop system with $\tau_a = 1.2$, $\bar{\tau}_t = 0.1$

It can be calculated that $\tau_a = 1.2$. The switching signals of the subsystems and the controllers are plotted in Fig. 1, respectively. Choose the initial state response as $x_0 = [1.2, -0.8, 0.6]$, then the state response of switched singular system (1) under the action of asynchronous controller (3) is depicted in Fig. 2. From the curve in this figure, it can be seen that the three state variables of the system tend to be stable in a finite-time interval under the action of the switching signal designed by Theorem 3.2.

Example 2 Consider a set of 2-dimensional switched singular systems selected from the numerical simulation in reference [25]. The corresponding matrix coefficients are shown

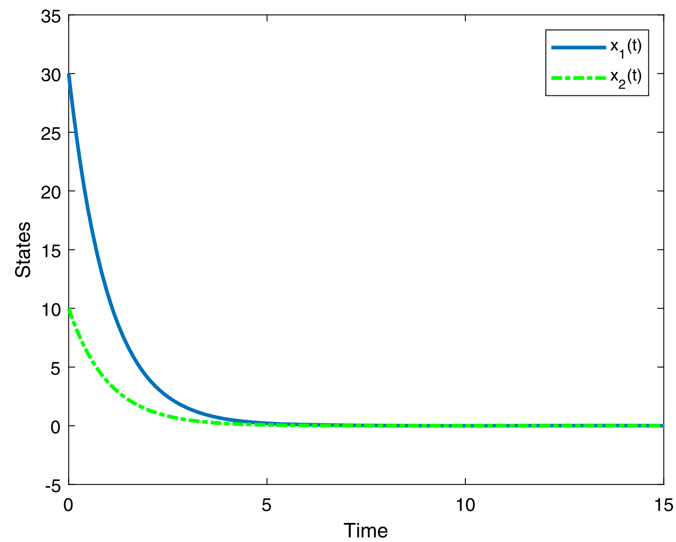

Figure 3 State response of the closed-loop system with $\tau_a = 0.3$

Table 1 Comparisons of τ_{a1} between [25] and Theorem 3.2

α_1	0.17	0.26	0.47	0.52	0.58	0.68
τ_{a1} in [25]	0.3224	0.3357	0.3744	0.3852	0.3991	0.4248
τ_{a1} in this paper	0.2656	0.2782	0.2953	0.3084	0.3365	0.3671

below:

$$E_1 \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}, \qquad E_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}.$$

It is easy to verify that subsystem 1 is a stable system and subsystem 2 is an unstable system. The authors of [25] investigated the finite-time stabilization of switched singular linear systems via the Lyapunov approach. In this paper, we study the finite-time stability of linear switched singular systems based on the form of the initial solution of the equation. We choose

$$M_1 = M_2 = I_2, \qquad N_1 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, \qquad N_2 = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}.$$

Let $\alpha_1 = 1.5$, $\alpha_2 = 5$, $c_1 = 1$, $c_2 = 30$. The average residence time was calculated to be 0.3, i.e., less than the time given in [25] of 0.38. The corresponding state responses of the 2-dimensional linear switched singular system are illustrated in Fig. 3. See Table 1 for more comparison of calculation results. It can be seen that tighter dwell bounds are obtained as long as we choose appropriate parameters.

5 Conclusion

Finite-time stabilization problems for a class of switched nonlinear singular systems have been discussed in this paper. A controller describing asynchronism has been presented and considered in the analysis. By decomposing the system, the regular and impulse-free nature of the switched system is proved to be valid. Without the help of the Lyapunov method, combining the average dwell-time method with differential equation theory, some necessary conditions for finite-time stabilization of systems are given in the form of linear matrix inequalities. In addition, the conditions for solving the parameters of the controller have been obtained. Finally, two numerical examples have been given to verify the effectiveness and correctness of the method presented in this paper. The extensions of the derived results to the finite-time stabilization problem of a fractional-order switched system will be our future investigation.

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The authors equally conceived of the study, participated in its design and coordination, drafted the manuscript, and read and approved the final manuscript.

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