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Non-Parametric Stochastic
Simulations to Investigate
Uncertainty around
the OECD Indicator Model
Forecasts

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**NON-PARAMETRIC STOCHASTIC SIMULATIONS TO INVESTIGATE UNCERTAINTY AROUND
THE OECD INDICATOR MODEL FORECASTS**

ECONOMICS DEPARTMENT WORKING PAPER No. 979

by Elena Rusticelli

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ABSTRACT/RÉSUMÉ

Non-Parametric Stochastic Simulations to Investigate Uncertainty around the OECD Indicator Model Forecasts

The forecasting uncertainty around point macroeconomic forecasts is usually measured by the historical performance of the forecasting model, using measures such as root mean squared forecasting errors (RMSE). This measure, however, has the major drawback that it is constant over time and hence does not convey any information on the specific source of uncertainty nor the magnitude and balance of risks in the immediate conjuncture. Moreover, specific parametric assumptions on the probability distribution of forecasting errors are needed in order to draw confidence bands around point forecasts. This paper proposes an alternative time-varying *simulated RMSE*, obtained by means of non-parametric stochastic simulations, which combines the uncertainty around the model's parameters and the structural errors term to construct asymmetric confidence bands around point forecasts. The procedure is applied, by way of example, to the short-term real GDP growth forecasts generated by the OECD Indicator Model for Germany. The empirical probability distributions of the GDP growth forecasts, derived through the bootstrapping technique, allow the *ex ante* probability of, for example, a negative GDP growth forecast for the current quarter to be estimated. The results suggest the presence of peaks of higher uncertainty related to economic recession events, with a balance of risks which became negative in the immediate aftermath of the global financial crisis.

JEL classification codes: C12; C15; C53

Keywords: Forecasting uncertainty; stochastic simulations; empirical probability distribution; GDP

Simulations stochastiques non-paramétriques pour étudier l'incertitude autour des prévisions du modèle d'indicateurs de l'OCDE

L'incertitude entourant les prévisions macro-économiques ponctuelles est généralement mesurée par la performance historique du modèle de prévision, à l'aide de mesures telles que la moyenne au carré des erreurs de prévisions (EQM). Cette mesure, a cependant l'inconvénient majeur d'être constante dans le temps et donc de ne transmettre aucune information ni sur la source spécifique de l'incertitude, ni sur l'ampleur et la balance des risques liée à la conjuncture immédiate. Par ailleurs, des hypothèses paramétriques spécifiques sur la distribution de probabilité des erreurs de prévision sont nécessaires afin de dessiner des bandes de confiance autour des prévisions ponctuelles. Cet article propose une *erreur quadratique moyenne simulé* variant dans le temps et obtenue au moyen de simulations stochastiques non-paramétriques, combinent l'incertitude autour des paramètres du modèle et le terme d'erreurs structurelles pour construire des bandes de confiance asymétrique autour des prévisions ponctuelles. La procédure est appliquée, à titre d'exemple, aux prévisions à court terme de la croissance du PIB réel générées par le modèle d'indicateurs de l'OCDE pour l'Allemagne. Les distributions empiriques de probabilité des prévisions de croissance du PIB, obtenues par la technique de bootstrap, permettent d'estimer la probabilité *ex ante* d'une croissance négative du PIB pour le trimestre en cours. Les résultats suggèrent la présence de pics d'incertitude liée aux événements de la récession économique, avec une balance des risques qui est devenue négative au lendemain de la crise financière mondiale.

Codes JEL : C12 ; C15 ; C53

Mots clés : Incertitude entourant des prévisions ; simulations stochastiques ; distribution empirique de probabilité ; PIB

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NON-PARAMETRIC STOCHASTIC SIMULATIONS TO INVESTIGATE UNCERTAINTY AROUND THE OECD INDICATOR MODEL FORECASTS

by

Elena Rusticelli¹

1. Introduction

1. Uncertainty is an intrinsic feature of any forecast and together with the assessment of the direction of risks to the given projection, it is recognised as an essential component when pronouncing on future economic developments, particularly in the near term. In fact, policy makers are not uniquely interested in the most likely evolution of the economic scenario, but also in the nature of risks surrounding the projection, their intensity and the probability of their materialisation. Moreover, conditional on the current state of the economy, the uncertainty around the forecast can change in amplitude and original source, and so its characterisation should include a time-varying dimension. In this framework, the purpose of this paper is to quantify a time-varying measure of forecasting uncertainty while distinguishing its different sources and asymmetric balance of risks. Furthermore, the paper derives the probability of specific events, like economic recessions, conditional on the empirical distribution of the point forecast.

2. The simplest and most common approach adopted by forecasters to estimate uncertainty considers root mean squared errors (RMSE) computed on the historical distribution of forecasting errors. This measure, while reproducing the average forecast uncertainty observed over history, does not account for the fact that the variance of the forecasting errors changes over time and that the uncertainty around the explanatory variables forecasts inflates the total error variance of the macroeconomic model. Moreover, due to its time-invariant and symmetric nature, the RMSE cannot provide any support to the policy-making process and the assessment of the main sources of risks to a given projection. This contrasts with the tendency, in recent decades, for many central banks to publish their views on the balance of risks around their macroeconomic projections as well as the uncertainty around point forecasts. In doing so, the most frequent approach has been the one originally developed by the Bank of England, summarising these two main characteristics of a forecasting model through fan-charts. More precisely, when the balance of risks to a forecast is judgmentally assessed by the Monetary Policy Committee, the future projection of the risk factors and the mechanism with which they propagate to the macroeconomic aggregate to be forecasted are also determined judgmentally and consequently the fan-chart assumes an asymmetric shape.

3. The characterisation of forecasting uncertainty itself varies substantially in the literature concerning macroeconomic forecasting models. Several sources of uncertainty can hamper the accuracy of a forecast, although their precise definition and measurement are not always clear. In general, four different sources of forecasting uncertainty are commonly analysed: the uncertainty from the forecasts of

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the *explanatory variables* determined outside the model, which are frequently assumed to be known with certainty or predetermined over the forecasting horizon in order to compute *ex post* forecasting errors; the uncertainty derived from unexpected random events which affect the *structural error terms* of the model, where the judgemental adjustment by the forecaster is usually applied;² the uncertainty stemming from the *coefficient estimates*, which rely strongly on asymptotic assumptions for the model variables and consistency of the estimation technique; and the possible *misspecification* of the model, for instance due to the risk of an incorrect choice of the functional form of the forecasting equation or dataset in use. While stochastic simulations can tackle the first three sources of forecast uncertainty, an estimate of the effect of model misspecification on forecast uncertainty is much harder and costly (Fair, 1980). In fact, any method for forecast accuracy comparison, for example the RMSE of the out-of-sample forecasting errors, relies on the assumption of constancy of the model's specification. For this reason, with the purpose of making comparisons between the standard OECD Indicator model in use and its simulated version, the assumption of constancy of the models characteristics is here adopted, although the number of chosen lags can vary across time. As a consequence, in this paper only the first three sources of uncertainty are considered since they can be separately investigated and then combined together by applying stochastic simulations. A further source of uncertainty concerns the discrepancy between flash and final estimates of the same economic aggregate (*e.g.* Lanser and Kranendok, 2008), although its treatment requires the availability of preliminary and revised datasets.

4. The original proposal of the Bank of England relies on a parametric approach, *i.e.* it assumes that the probability distribution of the risk factors and the variables of interest (inflation and GDP growth) follow a two-piece normal distribution, typically preferred to introduce skewness into the generated forecasts. However, Novo and Pinheiro (2003) showed that a linear combination of risk factors distributed as a two-piece normal distribution is not necessarily distributed as a two-piece normal.³ In general, the small sample distribution of both exogenous variables (predictors or risk factors) and endogenous variables (GDP or inflation) to be forecasted is unknown and specific assumptions on their asymptotic properties, as well as on the estimated residuals, are necessary to infer the model parameters and construct confidence intervals around point forecasts. Typical econometric studies of the properties of macroeconomic forecasts can rarely rely on a large set of observations, as these series are typically on an annual or quarterly periodicity. Similarly, when the linearity of the relation across the model variables is compromised by the presence of large outliers, then the resulting forecast is not normally distributed. Thus, non-parametric simulation methods, such as the bootstrapping, which are robust to the violation of the normality assumption represent the most suitable solution to make inference on forecasts distributions.

5. The common procedure in the classical literature on stochastic simulations applied on macroeconomic forecasting models, is to draw errors from estimated distributions, under the assumption of normality, following for example the Monte Carlo parametric approach (*e.g.* Van der Mensbrugghe *et al.*, 1990; Don, 1994; Garrat *et al.*, 2003; Meyermans and Van Brusselen, 2006; Kolsrud, 1993). However, the errors can also be drawn from the empirical distribution of the estimated residuals, thus not imposing any constraint on the distribution of the risk factors, the macroeconomic aggregate to be predicted and the forecasting errors. Indeed, the bootstrapping method preserves all moments of the series on which it is applied, such as the sample mean, the variance and skewness, if the assumption of independence is

2. Given the impossibility to forecast future structural shocks to the economy as well as the divergence from a zero mean probability distribution for the model disturbances, particularly on a very short and recent period preceding the forecasting horizon, it is common practice to adjust the point forecasts with an additional estimation of forecasting uncertainty. This latter should reflect the perception on the prevailing direction of risks as well as on the model misspecification.

3. Novo and Pinheiro (2003) demonstrate that under certain conditions on the degree of skewness and the correlation between variables, skewed generalised normal distributions instead are closed under linear combinations.

satisfied. Thus, when applied on the estimated residuals of the forecasting model the bootstrapping technique allows the empirical distribution of forecasts to be approximated through the construction of B simulated datasets of quarterly forecasts, which can be adopted to compute confidence bands. Moreover, the application of the bootstrapping method enables the different origins of forecasting uncertainty to be disentangled and analysed separately. Classical examples in this direction include Freedman and Peters (1985) and Fair (2003), whereas more recent developments of asymmetric bootstrapping techniques have been introduced by Miami and Siviero (2010) and Knüppel and Tödter (2007). Finally, the use of stochastic simulations permits the probabilities of specific events of interests to be estimated, such as the probability of an economic recession or deflation (Fair, 1991; Borbély and Meier, 2003).

6. In this paper, the bootstrapping approach is applied to distinguish different sources of forecasting uncertainty and draw confidence bands around the short-term GDP growth forecasts produced by the OECD's Indicator model (Sédillot and Pain, 2003) in the specific case of Germany. The combination of bootstrapped forecasts enables *simulated forecasts errors* to be estimated and used to compute a time-varying *simulated RMSE* and an asymmetric model-based balance of risks contingent on the state of the economy. This allows the identification of episodes of high forecasting uncertainty, which tend to correspond to economic recessions. The empirical probability distributions of the current and next quarter forecasts, obtained through bootstrapping, allows the likelihood of these recession events to be estimated as well as the *ex ante* probability of a negative GDP growth forecast for the current and next quarters of 2012.

7. The paper is organised as follows: section 2 presents the Indicator Model used at the OECD to forecast real GDP growth with particular reference to the model in use for Germany. Section 3 reviews the bootstrapping technique applied to dynamic simultaneous equations, *i.e.* the VAR models used to forecast the GDP monthly predictors. Section 4 considers the bootstrapping methodology to distinguish three different sources of forecasting uncertainty. Section 5 introduces the derivation of simulated forecast errors adopted to draw confidence bands around the GDP growth point forecasts. Sections 6 and 7 describe the empirical results and the estimation of probabilities for recession events respectively.

2. The OECD's Indicator model: the case of Germany

8. The short-term forecasting model used at the OECD to predict real GDP growth for the G7 countries and the euro area for the current and following quarter was devised by Sédillot and Pain (2003), and subsequently revised in following years. In particular, the set of monthly indicators and the sample period for the calculation of the historical root mean squared forecast errors have recently been updated. The Indicator models share the same modelling framework across all G7 countries and the euro area, differing only in terms of the set of monthly indicators chosen as predictors of real GDP growth.⁴

9. An important component of the Indicator model is a set of monthly Bayesian VAR models⁵ estimated to forecast the monthly indicators chosen as predictors of GDP growth.⁶ Since indicators of differing natures, *i.e.* surveys, financial variables and hard indicators, are adopted as predictors, three distinct monthly VARs have been implemented: one containing only soft indicators, one only hard

4. Given the monthly frequency of the real GDP series published by Statistics Canada, the indicator model to forecast quarterly GDP growth for Canada presents a different modelling approach than the other countries (Mourougane, 2006).

5. In order to avoid the over-parameterisation created by unrestricted vector autoregressions, a harmonic decay lag prior is set on a maximum number of lags equal to 6.

6. The selection procedure ranks the individual indicators on the basis of their explanatory power on GDP growth by means of adjusted coefficients of determination \bar{R}^2 from bivariate regressions.

indicators and one with soft and hard variables together. The form in which the indicators enter the VAR ensures stationarity, so hard indicators typically enter as growth rates, while soft and financial variables enter the VAR equations either in level or first difference. Details on the composition of the dataset used in the indicator model for Germany are presented in Table A1, which reports both the endogenous and the exogenous variables of the three VAR models. A conditional forecasting method is then applied to obtain six months of predictions given the indicators which end at different points in time.

10. The monthly information is then combined in single quarterly bridge equations in the form of an Autoregressive Distributed Lag model $ADL(a,b)$ of the form

$$\Delta \ln GDP_t = c + \sum_{j=1}^a \alpha_j \Delta \ln GDP_{t-j} + \sum_{j=0}^b \beta_{i,j} x_{i,t-j} \quad (1)$$

where the lag orders a and b are automatically selected through the Schwarz criterion⁷ and the x_i are the explanatory indicators (suitably transformed to be stationary). Table A2 reports the variable composition of the three bridge equations estimated from 1991 conditional on the current data for the monthly indicators, *i.e.* with the b lag starting from $j=0$.⁸ The bridge equation is recursively re-estimated at any period t and the forecasts for the current and following quarter are produced starting from 1998. In particular, the GDP forecast for the current quarter is obtained by filling the missing within quarter information with the monthly indicators forecasts from the VARs estimated previously. Moreover, since the monthly VARs are forecasted conditionally on the different availability of the indicators data, the bridge equations are also forecasted by taking into account the jagged flow of information within the forecasted quarter.⁹

11. An average forecast across the three bridge models is published twice per year at the time of the OECD Interim Outlook Press Conference. The RMSE of the forecasting errors are recursively computed over the entire historical time span, and those related to the period 1999 up to the last GDP data have typically been reported as a measure of forecasting uncertainty.¹⁰ Recently, a new approach to measure and convey information on forecasting uncertainty has been implemented. This latter is based on quintile-regressions as a way of estimating the distribution of forecasts and it uses the dispersion of the estimated quantiles for calculating an uncertainty index (Laurent and Kozluk, 2012).

3. The bootstrapping procedure

12. The bootstrapping technique was initially introduced by Efron (1979) with the purpose of computing variability measures for estimates obtained from a specific statistical procedure by applying this procedure repeatedly to artificial datasets constructed by re-sampling from the original observations. In essence, this nonparametric approach considers the variance of the observed sample to estimate the variance of the population from which the sample is drawn, by creating artificial random samples from the available sample. Freedman (1981) shows that with independently and identically distributed (*i.i.d.*)

7. The Schwarz criterion is applied also to evaluate the improvement to the predictive accuracy of the bridge equation model given by the inclusion of lagged GDP growth.

8. A wider set of forecasting models has been implemented considering bridge equations not conditional on the current quarter availability of monthly information, quarterly bridge equations and quarterly VARs, but they are not discussed here.

9. In order to overcome the possible model inadequacy, both quarterly forecasts are adjusted with the average of the estimation residuals computed over the last two years.

10. A correction recently implemented on the RMSE calculation considers the exclusion of outliers greater than two times the historical standard deviation of the forecast errors. Here, the outliers correction is not carried out for comparison purposes with the other measures of forecasting uncertainty.

disturbances, for increasing sample sizes and number of repetitions B , the bootstrap approximation of the estimator distribution converges to the OLS estimator distribution.

13. Consider a standard dynamic, linear, simultaneous equations model of the following form:

$$f_i(y_t, y_{t-1}, \dots, y_{t-p}, x_t, \alpha_i, \beta_i) = u_{it}, \quad i = 1, \dots, n \quad t = 1, \dots, T, \quad (2)$$

where y_t is an n -dimensional vector of endogenous variables, x_t is a vector of exogenous variables, and α_i is a K -dimensional vector and β_i is a $(m \times p)$ -dimensional matrix of coefficients. The elements of the vector of error terms $u_t = (u_{1t}, \dots, u_{nt})$ are assumed to be distributed *i.i.d* with mean zero. Thus, normality is not assumed and as consequence the OLS estimators $\hat{\beta}_i$ are not normally distributed. The function f_i may be nonlinear in variables and coefficients as well as differ across the n system equations, but in this study it has a linear form in both VAR and bridge equation models.

14. Let $\hat{\beta}_{i1}, \dots, \hat{\beta}_{im}$ denote the matrices of parameter estimates and $\hat{u}_{it} = (\hat{u}_{i1}, \dots, \hat{u}_{iT})$ the vector of errors for all available periods for equation i .¹¹ The bootstrap methodology proceeds as follows:

Step 1. From the empirical distribution of \hat{u}_{it} , B residuals \hat{u}_{it}^* , where B is a large number (here taken to be 1000), are drawn with replacement and uniform distribution for $t = 1, \dots, T$ and $i = 1, \dots, n$. The bootstrap procedure preserves all moments of the empirical distribution of the residuals, including the variance, the skewness and the contemporaneous correlations across all n equations since the random drawing is carried out in tandem.¹²

Step 2. The bootstrap residuals \hat{u}_i^* and the estimated matrices of parameters $\hat{\alpha}_i$ and $\hat{\beta}_i$ are applied to compute B artificial datasets recursively. Thus, the single equation i in (1) is computed as:

$$y_t^* = c_i + \sum_{j=1}^p \hat{\beta}_{1j} y_{1t-j}^* + \sum_{j=1}^p \hat{\beta}_{2j} y_{2t-j}^* + \dots + \sum_{j=1}^p \hat{\beta}_{mj} y_{mt-j}^* + \sum_{k=1}^K \hat{\alpha}_k x_{kt} + \hat{u}_t^* \quad (3)$$

for $i = 1, \dots, n$ and $t = 1, \dots, T$. All simulated datasets are conditional on the actual value of the endogenous variables before period $t = 1$ and the actual value of the exogenous variables for all periods.¹³

Step 3. Using the simulated bootstrapped variables y_t^* in (3) the model parameters are re-estimated to obtain $\hat{\beta}_1^*, \dots, \hat{\beta}_m^*$ and $\hat{\alpha}_1^*, \dots, \hat{\alpha}_K^*$. By repeating this procedure B times the empirical probability distributions of the variables y_t^* and the coefficients β_i^* at any time t are estimated and can be used to compute confidence intervals, standard deviations, etc. Moreover, the B models estimated can be applied for forecasting purposes at different h horizons.

11. Freedman (1981) demonstrates that a bootstrap re-sampling can fail in the case of the absence of a constant term if the residuals are not centred at zero. The zero mean distribution assumption has been tested and cannot be rejected at any conventional level of significance, moreover a constant term is included in both models considered. Hence, the residuals have not been rescaled here.

12. The hypothesis of zero autocorrelation of the estimated residuals $\hat{u}_{it} = (\hat{u}_{i1}, \dots, \hat{u}_{iT})$ has been tested across all equations and cannot be significantly rejected for any of them. Consequently, the bootstrapping is not performed in blocks of consecutive residuals in the VAR models.

13. For simplicity of notation the time span is here indicated to range from 1 to T , although there exists an initial estimation period $t = 1, \dots, t - 1$ on which either the bootstrapping or the forecasting procedures are conditioned on.

15. The bootstrapping procedure is applied here to approximate the empirical distributions of the monthly indicators and their forecasts in the Bayesian VAR models. In particular, at any replication the same prior distribution of the coefficients and its tightness parameter is kept unchanged and the model is estimated and forecasted iteratively.¹⁴ As consequence, since the bootstrapping technique is applied recursively over the estimation period, a set of B estimates and forecasts of the monthly indicators is available at any point in time t and used to compute GDP growth forecasts, also recursively, in the bridge equation.

4. Bootstrapping the different sources of forecasting uncertainty in a bridge equation

16. As described in the introduction, the forecast from a model is usually subject to four different sources of uncertainty: the uncertainty from the *explanatory variables forecasts*, the *structural error terms* of the model, the *coefficient estimates* and a possible *misspecification* of the model. However, only the first three sources will be here investigated through bootstrapping techniques.

17. Due to the model complexity and software restrictions, each of the three sources of uncertainty is investigated here separately, although, for example, a direct dependence of the error terms computation on the estimated parameters is inevitable. In fact, since the error terms are obtained conditional on the model parameters definition, it is likely that controlling for the parameters uncertainty will reduce the error terms variability. Hence, far from having an additive relationship, the four sources of uncertainty are treated here separately. However a combination of them is considered in the construction of simulated forecasting errors introduced in the next section.¹⁵

18. Measuring the first type of forecast uncertainty, *i.e.* on the *explanatory variables forecasts* of the GDP bridge equation, sets up a direct link between the monthly VAR models estimated to forecast the GDP indicators and the quarterly bridge equation itself. For this purpose, the bootstrapping technique recursively applied on the VARs estimation and forecast enables the construction of a set of B estimates and forecasts for each monthly indicator of interest at any forecasting horizon h . Subsequently, the following quarterly bridge equation is estimated recursively until time t and forecasted at two periods ahead ($h = 2$):

$$\Delta \ln GDP_{t+h|t}^b = c + \sum_{j=1}^a \hat{\alpha}_j \Delta \ln GDP_{t+h-j|t} + \sum_{j=0}^b \hat{\beta}_{i,j} x_{i,t+h-j|t}^b \quad (4)$$

14. Since the contemporaneous correlation across the residuals from the different equations should be considered, the Cholesky decomposition of the covariance matrix of the residuals $\hat{\Sigma} = E(\hat{u}_t \hat{u}_t')$ is applied and it gives the lower triangular matrix P such that $\hat{\Sigma} = PP'$. In this way the residuals can be transformed into independent errors $\hat{v}_t = P^{-1} \hat{u}_t$ since $E(\hat{v}_t \hat{v}_t') = P^{-1} \hat{\Sigma} P^{-1'} = I$. The matrix P is then used as a factor matrix to estimate the impulse responses needed in forecasting the VAR conditionally on the availability of some values for certain endogenous variables over the forecasting period.

15. Some authors (Wallis and Whitley, 1991; Feldstein 1971; Lanser and Kranendok, 2008) present an additive contribution of the different sources of uncertainty to the total error variance of the endogenous variable. But since not all types of uncertainty have been taken into account, then the final aggregated variance is lower than that of the real-time errors.

where $b = 1, \dots, B$ is the number of bootstrap simulations and x_i are the monthly indicators forecasted through monthly VARs converted to a quarterly frequency. The number of lags a and b as well as the inclusion of lagged GDP growth is automatically chosen by the Schwarz criterion. For this reason, at the forecasting horizon $h = 2$, the GDP growth forecast can include the forecast made at horizon $h = 1$. Since the coefficients $\hat{\alpha}_j$ and $\hat{\beta}_j$ are estimated on the original dataset and directly applied on the forecasted regressors, this approach allows the impact on forecasting uncertainty coming exclusively from the explanatory variables forecasts to be wholly considered and handled. Moreover, it enables the empirical distribution of the GDP growth forecasts at any forecasting horizon h and any historical period t to be derived.

19. The second source of uncertainty concerns the *structural error terms*, mainly affected by the occurrence of unpredictable random events external to the econometric model setting, often interpreted as random disturbances. In the forecasting process these events are unknown and thus uncertain by definition. As consequence, it is generally in the error terms that the expert opinion is judgementally added to the forecasting model. In order to distinguish this type of uncertainty, the bootstrapping methodology previously described in *step 1* and *2* of section 3 is adopted to create first a set of B bootstrapped residuals $\hat{\varepsilon}_{t+h|t}^*$ and then a set of B artificial forecasts for GDP growth at any horizon h as follows

$$\begin{aligned}\Delta \ln GDP_{t+h|t}^* &= c + \sum_{j=1}^a \hat{\alpha}_j \Delta \ln GDP_{t+h-j|t} + \sum_{j=0}^b \hat{\beta}_{i,j} x_{i,t+h-j|t} + \hat{\varepsilon}_{t+h|t}^* \\ &= \Delta \ln GDP_{t+h|t} + \hat{\varepsilon}_{t+h|t}^*\end{aligned}\quad (5)$$

where the set of B residuals $\hat{\varepsilon}_{t+h|t}^*$ are recursively and randomly drawn from the empirical distribution of the estimation errors $\hat{\varepsilon}_t$ obtained until time t . In order to take into account the possible presence of linear autocorrelation in the estimated errors, the moving blocks bootstrapping procedure has been performed at block size equal to 2, considering the forecasting horizon of 2 quarters. Thus, the term $\Delta \ln GDP_{t+h|t}$ represents the standard GDP growth forecast to which the effect of the structural error terms uncertainty is added.

20. Following *step 3* presented in the previous section, the bootstrapped residuals $\hat{\varepsilon}_t^*$ are applied also to compute a set of B artificial estimates for $\Delta \ln GDP_t^*$ and quarterlised monthly indicators $x_{i,t}^*$ ¹⁶ for $t = 1, \dots, T$ and $i = 1, \dots, n$. Successively, the generated artificial datasets are used to recursively re-estimate B times the bridge equation and obtain B estimates of the parameters sets $\hat{\alpha}_1^*, \dots, \hat{\alpha}_a^*$ and $\hat{\beta}_{i,1}^*, \dots, \hat{\beta}_{i,b}^*$ at any time span $t = 1, \dots, T$. The new bootstrapped parameters are then applied to compute B forecasts for GDP growth $\Delta \ln GDP_{t+h|t}^f$ as follows

$$\Delta \ln GDP_{t+h|t}^f = c + \sum_{j=1}^a \hat{\alpha}_j^* \Delta \ln GDP_{t+h-j|t} + \sum_{j=0}^b \hat{\beta}_{i,j}^* x_{i,t+h-j|t} \quad (6)$$

where the bridge equation in (6), containing bootstrapped parameters, is applied for forecasting purposes but using the original dataset for both GDP growth and the indicators series. In this way, only the effect of the *coefficient estimates* affects the forecasting uncertainty around GDP.

16. Since the bootstrapped technique is performed recursively on the monthly VARs over the entire historical time span, it allows both bootstrapped estimates and forecasts of the monthly indicators to be obtained.

5. Simulated forecast errors

21. The forecasting performance of a model is traditionally measured by means of the historical record of forecasting errors, which are conventionally synthesised by summary measures as the root mean squared error (RMSE) or the sample mean absolute error (MAE). The forecast error simply defined as $\epsilon_{t+h} = y_{t+h} - \hat{y}_{t+h|t}$, where y_{t+h} represents the actual realisation of y at time $t+h$ and $\hat{y}_{t+h|t}$ the forecast for y_{t+h} made at time t , incorporates all sources of forecasting uncertainty at the same time and it is constant over the entire time span. Moreover, the measure of uncertainty formulated for future forecasts is based on past forecast performance, thereby bringing an additional forecasting problem.

22. Consider a simple static regression model $y = X\beta + u$ with y being a $(T \times 1)$ vector, X a $(T \times k)$ matrix of explanatory variables and u a $(T \times 1)$ vector of random error values and β a $(k \times 1)$ vector of coefficients. Using the model, the forecast made at time t for horizon h is given by

$$\hat{y}_{t+h|t} = \hat{x}_{t+h|t}\hat{\beta} \quad (7)$$

where $\hat{x}_{t+h|t}$ denotes the vector of forecasts made for the exogenous variables at time t and $\hat{\beta}$ is the least square estimator. The *ex ante* forecast error can be disentangled in the sum of three components as

$$\begin{aligned} \epsilon_{t+h} &= y_{t+h} - \hat{y}_{t+h|t} = x_{t+h}\beta + u_{t+h} - \hat{x}_{t+h|t}\hat{\beta} + (x_{t+h}\hat{\beta} - x_{t+h}\hat{\beta}) \\ &= (x_{t+h} - \hat{x}_{t+h|t})\hat{\beta} + x_{t+h}(\beta - \hat{\beta}) + u_{t+h} \end{aligned} \quad (8)$$

where each term on the right side describes a source of forecasting uncertainty, *i.e.* the explanatory variables values, the coefficient estimate and the structural error term. On an *ex post* basis, the forecasting error is usually computed by resting on the strong assumption that the future values of the explanatory variables are known and exact, *i.e.* $\hat{x}_{t+h|t} = x_{t+h}$. As consequence, the root mean squared forecast error commonly reported equals

$$RMSE(\epsilon_{t+h}) = \sqrt{E(y_{t+h} - \hat{y}_{t+h|t})^2} = \sqrt{E[x_{t+h}(\beta - \hat{\beta}) + u_{t+h}]^2} = \sqrt{x_{t+h}\sigma_u^2(X^T X)^{-1}x_{t+h}^T + \sigma_u^2} \quad (9)$$

where σ_u^2 is the variance of the estimated residuals. Under the assumption of serial independence, the disturbances at time $t+h$, u_{t+h} , are independent from the disturbances in the estimation sample, therefore u_{t+h} and $\hat{\beta}$ are two independent random variables.

23. Thus, following this approach (as in Peters and Freedman, 1985 or Calzolari, 1987), in order to account for both sources of forecast uncertainty, *i.e.* the coefficient estimates and the structural error terms, equations (5) and (6) are combined to obtain the *simulated forecast errors* as

$$\Delta \ln GDP_{t+h|t}^* - \Delta \ln GDP_{t+h|t}^f = \sum_{j=1}^a (\hat{\alpha}_j - \hat{\alpha}_j^*) \Delta \ln GDP_{t+h-j|t} + \sum_{j=0}^b (\hat{\beta}_{i,j} - \hat{\beta}_{i,j}^*) x_{i,t+h-j|t} + \hat{\epsilon}_{t+h|t}^* \quad (10)$$

for $t = 1, \dots, T$ and $h=2$. These forecast errors are the sum of two stochastic components: the first is the sampling error in the estimated coefficients $\hat{\alpha}_j$ and $\hat{\beta}_{i,j}$ which are random since $x_{i,t+h}$ are assumed to be known; the second is the structural disturbance term that will be drawn at time $t+h$. The B simulated forecast errors for period $t+h$ can be used to approximate an empirical probability distribution of forecast errors at any time and construct error bands around point forecasts.

6. Empirical results

24. The Indicator model for Germany has been run on the dataset downloaded on 29 February 2012, which implies the availability of two months of information in the first quarter on the IFO business surveys and none on real activity indicators. More precisely, weak hard indicators (especially the industrial production index and construction in December 2011) have significantly contributed to the negative GDP growth rate in the fourth quarter, whereas business confidence has increased in February 2012 above consensus expectations. An official series for real GDP growth is available until the last quarter of 2011, hence forecasts for 2012Q1 and 2012Q2 are produced.

25. The application of the bootstrap technique, as well as the consistency of OLS parameter estimates, relies on the assumption that the estimated residuals are independent. For this purpose, the Durbin-Watson statistic can be computed to test for first-order serial correlation of the residuals, although it is not appropriate in presence of lagged dependent variables in the right-hand side of the equation. In this case, all three bridge equations estimated until 2011Q4 and used for real-time forecasting do not include lagged GDP, but it could be possible that the Schwarz criterion has automatically included it in the previous estimation iterations. Thus, both the Durbin-Watson statistic and the Breusch-Godfrey test for higher orders serial dependence have been performed on the residuals. The ranges in which the value of the Durbin-Watson statistic falls across all estimation iterations is [1.93, 2.41] and [1.80, 2.29] for the soft and the hard indicators bridge equations respectively (the mixed indicators bridge equation is set similarly to the hard indicators one). The Breusch-Godfrey test rejects the null hypothesis of absence of serial autocorrelation against the alternative of autocorrelation until order 4 at a level of significance lower than 10% only in 7% of the cases. Moreover, the normality of the residuals was tested using the Jarque-Bera statistics, which rejects the null hypothesis at a significance level lower than 10% in 25% of the cases. These results tend to support the adoption of a non-parametric approach to define the uncertainty around the GDP growth point forecasts. However, to err on the side of caution, the moving blocks bootstrapping technique with block size 2 has been preferred.¹⁷

26. The results reported in this section are based on $B = 1000$ bootstrap repetitions applied either on the monthly VAR models or on the quarterly bridge equations.¹⁸ Table 1 presents the bootstrap distribution of the OLS estimators from the three bridge equations estimated until 2011Q4, and compares the bootstrapped standard errors with those obtained under the OLS asymptotic normality assumption. Overall, the bootstrapped coefficients are quite close to the original OLS estimates¹⁹ with the exception of the business expectations indicator in the soft bridge equation. This variable is in fact very volatile, with a varying lag structure due to it leading predicting power on GDP growth. As consequence, the set of artificial values of the simulated business expectations indicator is characterised by high variance and standard errors of the bootstrapped coefficients doubling those of the OLS estimates. In general, OLS and bootstrapped parameter estimates statistically differ by less than 1%, implying that the bootstrap distribution is a good approximation of the estimators' distributions, and consequently that the generated bootstrapped parameters can be applied for forecasting purposes. The standard deviations of the

17. Normality and independence of residuals have been also tested on the three monthly VAR models estimated to forecast the indicators. The results, not reported here, suggest even more strongly the need for a non-parametric approach to stochastic simulations.

18. Despite the similarity of the hard and mixed indicators bridge equations, the bootstrapped coefficients are obtained including the simulated indicators from the hard and mixed indicators VAR models, therefore the estimates in the two bridge equations differ.

19. In the majority of the cases the average bootstrapped coefficient is slightly smaller than the OLS estimate, suggesting that there is a little bias in the OLS estimator (Efron and Tibshirani, 1993).

bootstrapped coefficients tend to be slightly larger than the asymptotic standard errors highlighting the inadequacy of the asymptotic theory with small samples.

Table 1. Bootstrap distribution of the parameter estimators

Indicator	OLS Estimates		Bootstrapped Estimates	
	Coefficient	Std. Error	Avg. Coefficient	Std. Error
<i>SOFT BRIDGE EQUATION</i>				
Constant	-0.093	0.015	-0.110	0.096
Business expectations/100	-0.021	0.045	0.100	0.079
Business expectations (-1)/100	0.117	0.042	0.013	0.083
Orders on hand vs. last month/100	0.059	0.017	0.054	0.004
<i>HARD BRIDGE EQUATION</i>				
Constant	0.002	0.000	0.003	0.000
Industrial production index	0.271	0.021	0.277	0.021
Retail sales	0.159	0.040	0.106	0.037
Ind. prod. index construction	0.050	0.008	0.047	0.008
<i>MIXED BRIDGE EQUATION</i>				
Constant	0.002	0.000	0.003	0.000
Industrial production index	0.271	0.021	0.257	0.040
Retail sales	0.159	0.040	0.087	0.061
Ind. prod. index construction	0.050	0.008	0.043	0.012

Source: OECD calculations.

27. Table 2 presents the set of GDP growth forecasts obtained from the standard bridge equations and the bootstrapped versions introduced in section 3, where the three different sources of forecasting uncertainty are considered. The 95% confidence interval for the standard forecasts is computed by means of the RMSE of the forecasting errors computed over the sample 1999Q1-2011Q4. The percentile bootstrap procedure (Efron, 1982) is instead applied to estimate the $(1 - \alpha)$ confidence interval for the bootstrapped forecasts, implying that for $\alpha = 5\%$ and $B = 1000$ the lower and upper confidence bounds are the $B(\alpha/2) = 25$ and $B(1 - \alpha/2) = 975$ -th ordered elements of the bootstrapped dataset respectively. The average forecast, usually published, is also reported and for the bootstrapped versions it is computed over a dataset of 3000 simulated forecasts.

28. The difference in the GDP forecasts across the three types of bridge equations is mainly due to the various data availability in the indicators set. In fact, the lack of data on real activity on the current quarter, in addition to the slowdown in both the industrial production index and the manufacturing orders at the end of 2011, contributes to the negative forecasts obtained from the hard bridge equations in both methodologies. Indeed, when leading survey indicators are used to predict real activity series in the mixed VAR model, the divergence with estimates from the soft bridge equation is reduced. Moreover, when new hard indicators data become available on the current quarter, the variability across estimates from different models and the associated root mean squared errors diminish significantly.²⁰

20. The results are not reported here, but they are available upon request.

29. As expected, the bootstrapped forecasts computed considering the uncertainty around the indicators forecasts present the highest absolute deviation from the standard point forecast. In fact, they are directly affected by the predictive performance of the VARs models, which can add quite a large dispersion around the expected indicator point forecast $\hat{x}_{t+h|t}$ included in the standard GDP forecast. Conversely, the widest confidence intervals are mainly associated with the bootstrapped forecasts based on bootstrapped structural errors. These confidence intervals are negatively skewed in the case of the soft bridge equation and positively skewed for the hard and mixed indicators bridge equations, implying that the most likely actual value for the survey indicators might be lower than expected, and for the hard indicators higher than expected.

Table 2. Forecasts and confidence intervals of the annualised quarter on quarter percentage GDP growth rates for 2012Q1 (current quarter) and 2012Q2 (next quarter)

Quarter	OLS Estimates		Bootstrapped Explanatory Variables		Bootstrapped Structural Errors		Bootstrapped Coefficients	
	Standard Forecast	95% interval	Avg. Forecast	95% interval	Avg. Forecast	95% interval	Avg. Forecast	95% interval
<i>SOFT BRIDGE EQUATION</i>								
Current	1.97	[-3.63; 7.82]	1.89	[1.17; 2.65]	2.03	[-3.12; 6.34]	1.98	[0.49; 3.45]
Next	2.42	[-3.21; 8.29]	1.66	[-0.94; 4.08]	2.34	[-2.87; 6.83]	3.04	[2.26; 3.81]
<i>HARD BRIDGE EQUATION</i>								
Current	-0.44	[-5.80; 5.14]	-1.16	[-3.77; 1.34]	-0.43	[-3.03; 4.15]	-0.22	[-0.54; 0.12]
Next	1.17	[-5.69; 8.39]	1.27	[-0.65; 3.02]	1.15	[-1.50; 5.80]	1.40	[1.16; 1.65]
<i>MIXED BRIDGE EQUATION</i>								
Current	0.56	[-4.61; 5.94]	-1.03	[-3.67; 1.73]	0.48	[-2.06; 3.84]	0.88	[0.55; 1.40]
Next	3.36	[-2.61; 9.61]	3.15	[0.22; 6.59]	3.39	[0.65; 8.06]	3.34	[2.33; 3.78]
<i>AVERAGE</i>								
Current	0.69	[-4.71; 6.33]	-0.11	[-3.49; 2.44]	0.69	[-2.47; 5.52]	0.88	[-0.45; 3.04]
Next	2.31	[-4.15; 9.09]	2.03	[-0.60; 5.50]	2.29	[-1.47; 6.71]	2.59	[1.21; 3.74]

Source: OECD calculations.

30. The dispersion of the empirical distribution of the forecasts generated through the bootstrapping methodology represents a measure of uncertainty around the point forecast. More precisely, the standard error computed across all bootstrapped forecasts for the current and the next quarter is compared with the historical RMSE computed on the time span 1999Q1-2010Q4 as a proxy for the unknown forecast standard error. The well-known predictive superiority of hard indicators respect to survey data, when at least one month of information is available on the quarter to be forecasted, is confirmed here by a general lower uncertainty coming from the hard and mixed indicators bridge equations compared to the soft indicators one, with an adjusted \bar{R}^2 equal to 0.79 and 0.47 respectively.

31. As frequently found in the literature on simulations of macroeconometrics models (see Fair, 1991; Lanser and Kranendonk, 2008) the effect of the coefficient uncertainty on the forecast standard error is small. The uncertainty associated with the explanatory variables forecasts is particularly low in case of

the soft indicators bridge equation when, as in the case discussed here, two months of data are already available on the current quarter. Conversely, the uncertainty increases abruptly when no within quarter information is provided to both VAR and bridge equation models on hard indicators. Furthermore, the typical mean-reverting nature of VAR models appears in the hard indicators bridge equations, where the next quarter forecasts (*i.e.* six months of forecasts) is characterised by a lower dispersion than the current quarter one (*i.e.* three months of forecasts). As expected, the structural error terms represent an important source of forecasting uncertainty, hence the common practice of residuals adjustment to point forecasts or removal of big outliers from the calculation of the RMSE seems justified.

32. The skewness coefficient is also reported as a measure of asymmetric balance of risks. The D'Agostino's K^2 test on the skewness measure rejects the normality assumption at the 5% significance level in most of the cases, indicating a significant asymmetry in the generated empirical distributions. The sign of the asymmetry is reflected in the confidence intervals in Table 2 although it is quite diversified across the three sources of uncertainty.

Table 3. Forecasting uncertainty and balance of risks

Quarter	OLS Estimates	Bootstrapped Explanatory Variables		Bootstrapped Structural Errors		Bootstrapped Coefficients	
	RMSE	Std. Error	Skewness	Std. Error	Skewness	Std. Error	Skewness
<i>SOFT BRIDGE EQUATION</i>							
Current quarter	0.72	0.09	0.17	0.59	-0.15	0.18	0.05
Next quarter	0.72	0.31	-0.13	0.61	-0.35	0.10	-0.06
<i>HARD BRIDGE EQUATION</i>							
Current quarter	0.70	0.33	-0.10	0.39	0.67	0.04	0.66
Next quarter	0.89	0.23	-0.22	0.39	0.51	0.03	0.14
<i>MIXED BRIDGE EQUATION</i>							
Current quarter	0.67	0.33	-0.07	0.37	0.60	0.05	0.99
Next quarter	0.76	0.39	0.32	0.39	0.57	0.08	-2.68
<i>AVERAGE</i>							
Current quarter	0.70	0.45	-0.24	0.53	0.58	0.25	0.56
Next quarter	0.83	0.37	0.62	0.53	-0.03	0.22	-0.40

Source: OECD calculations.

33. In order to combine different sources of forecasting uncertainty and construct error bands around the GDP growth point forecasts, the *ex post* simulated forecast errors accounting for structural errors and coefficient uncertainty presented in section 4 are computed and compared with the forecast errors commonly published with the characteristics of historical out-of-sample root mean squared errors. More precisely, in case of the average forecast from the three bridge equations, the RMSE is computed on GDP forecasts obtained recursively from 1991Q1 to 2011Q4 for the two forecasting horizons $h=1$ and $h=2$ as follows:

$$RMSE(\varepsilon_{t+h})_{t+h} = \sqrt{\frac{\sum_{t=1}^{52} (y_{t+h} - \hat{y}_{t+h|t})^2}{52}} \quad (11)$$

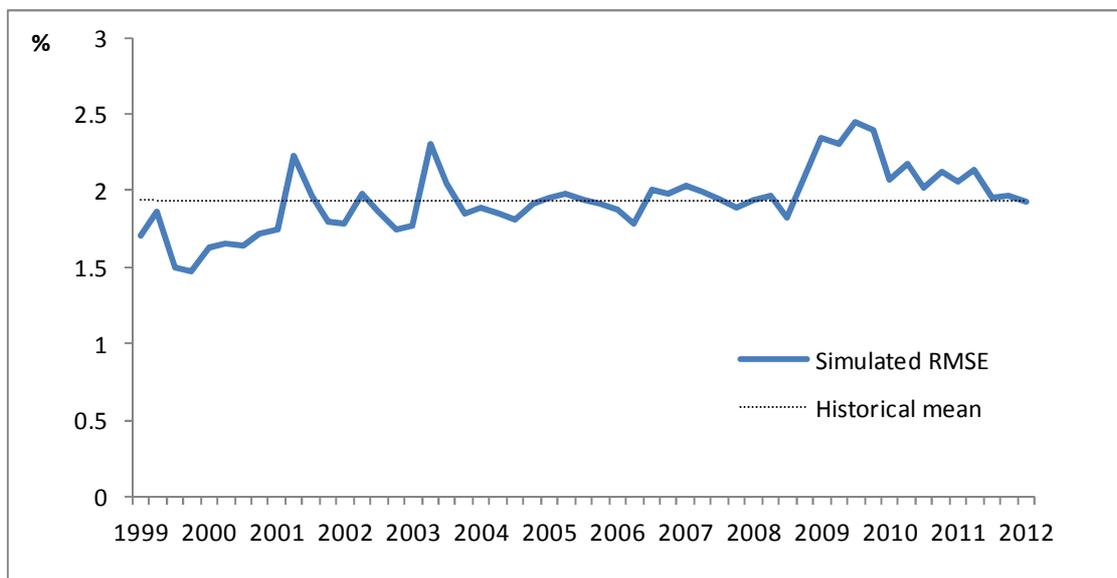
where $\hat{y}_{t+h|t}$ is the average forecast from the three models made at time t . As consequence, this latter does not vary over the forecasting period and can be computed only once the unknown GDP value forecasted y_{t+h} is published, *i.e.* only at time $t+h$. The simulated forecast error, shown in equation (10), can instead be computed at the same time t when the forecast is made and allows computing a *simulated RMSE* of the form

$$RMSE(\varepsilon_{t+h})_t = \sqrt{\frac{\sum_{b=1}^B (\hat{y}_{t+h|t}^{*b} - \hat{y}_{t+h|t}^{fb})^2}{B}} \quad (12)$$

where $B = 3000$ in case of the average forecast from the three bridge equations, $\hat{y}_{t+h|t}^{*b}$ is the simulated forecast based on bootstrapped structural errors in equation (5) and $\hat{y}_{t+h|t}^{fb}$ is the simulated forecast based on bootstrapped coefficients in equation (6). On this purpose, it is important to remind that bootstrapped parameters have been estimated by means of the generated artificial datasets of monthly indicators from the bootstrapped VAR models, hence to some extent they contain part of the uncertainty linked to the explanatory variables forecasts.

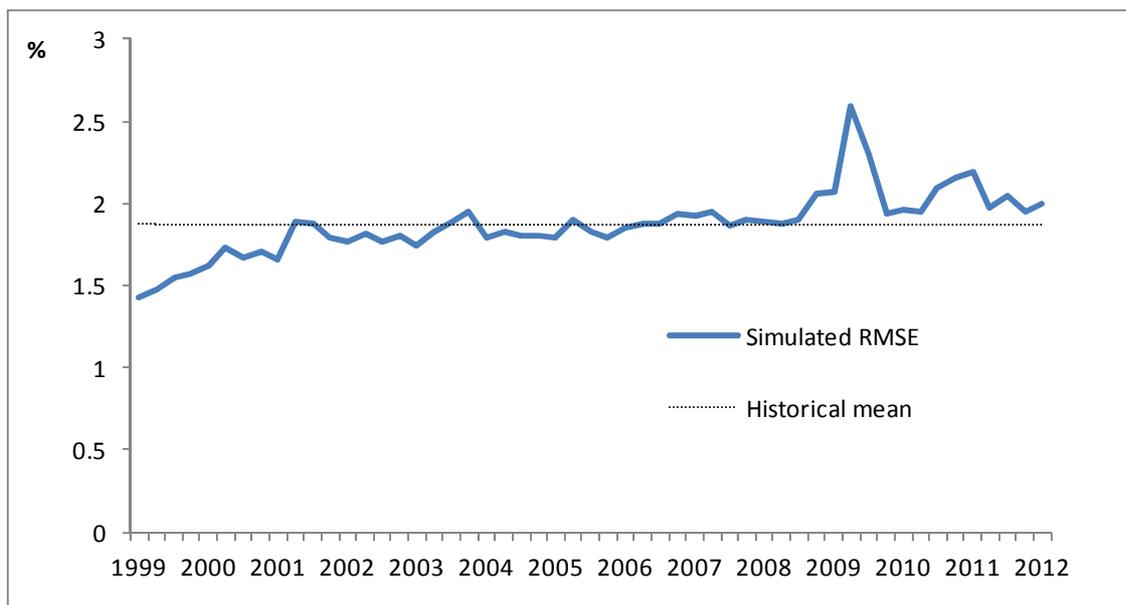
34. Figures 1 and 2 display the *simulated RMSE* reported in equation (12) computed from 1991Q1 to 2012Q2 for the current and the next quarter forecasts. The results can be compared with the much higher standard historical RMSE of the model, usually published, that annualised equals 2.83 and 3.36 for the current and the next quarter forecasts respectively. In both figures, the simulated RMSEs are not constant over the forecasting time and when plotted together with their respective historical means few specific periods of high forecasting uncertainty are recognisable. In particular, in case of the current quarter forecasts, three peaks of uncertainty occur at the end of the three episodes of economic recession which Germany experienced in the last 13 years, *i.e.* at the end of year 2000, in the first half of 2003 and during the recent financial crisis. The forecasting uncertainty is particularly high at the end of a recession also due to the composition of the indicators set used to forecast GDP in this paper. In fact, two months of survey data are available within the current quarter forecasted but no information is available on the real activity indicators. In consequence, good news from leading survey indicators combined with lagged negative growth rates from the hard indicators create particularly high forecasting uncertainty. Moreover, a statistically significant upward time-trend can be identified for both current and next quarter forecasts. In fact, dynamic models typically imply time-dependent forecast uncertainty, usually also increasing with the forecasting horizon. Forecasting errors tend to cumulate over the time, reflecting the fact that the bootstrapping methodology, applied iteratively at any time t , draws residuals from a wider sets $\hat{u}_{it} = (\hat{u}_{i1}, \dots, \hat{u}_{it})$ at any subsequent recursion. As a consequence, the variance of both the structural error terms and the coefficient estimates inflates over the time with the accumulation of economic shocks creating a deterministic upward trend in the *simulated RMSE*.

Figure 1. Simulated RMSE of the annualised current quarter forecast in per cent



Source: OECD calculations.

Figure 2. Simulated RMSE of the annualised next quarter forecast in per cent

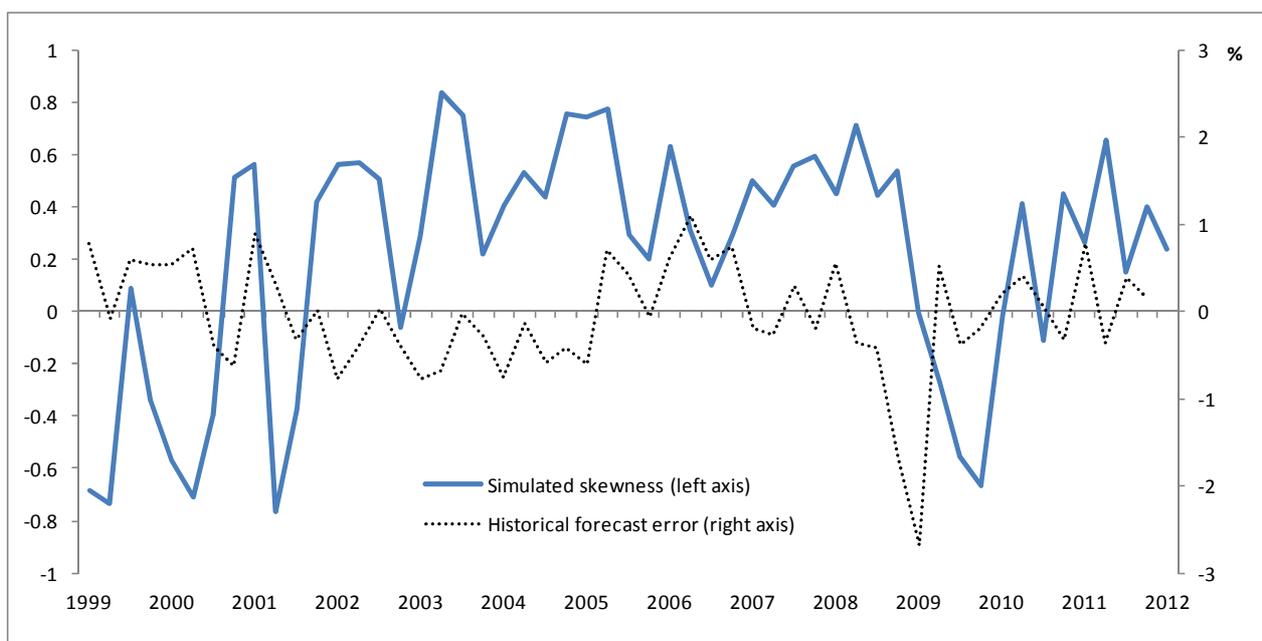


Source: OECD calculations.

35. The balance of risks around the point forecast can be approximated by the degree of asymmetry of its confidence bands. Hence, the skewness coefficient computed across the set of B simulated forecast errors at any time t gives an indication of the risk around the GDP point forecast at time t . Figures 3 and 4 present the *simulated skewness* plotted together with the historical forecasting error for the current and the next quarter forecasts. A moderate asymmetry, varying across time, characterises the empirical distribution of the simulated forecast errors, which tend to be mostly positive, *i.e.* with the actual GDP growth y_{t+h}

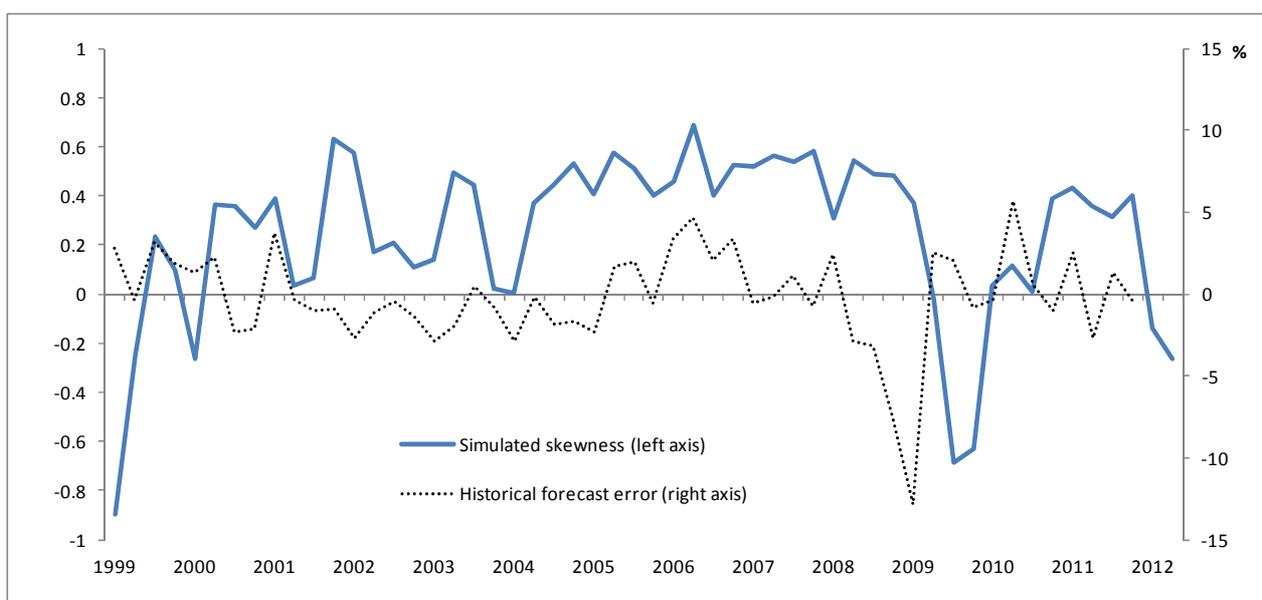
higher than its forecast $\hat{y}_{t+h|t}$, in case of positive skewness or mostly negative otherwise. This is particularly true in early 2001, from 2005 to 2007 and at the end of the sample, where a positive forecasting error from the model is associated with a positive simulated skewness. Conversely, at the time of the recent financial crisis in 2009, a negatively skewed balance of risks indicates the higher likelihood of a downward revision to the current point forecast.

Figure 3. Simulated skewness of the current quarter forecast



Source: OECD calculations.

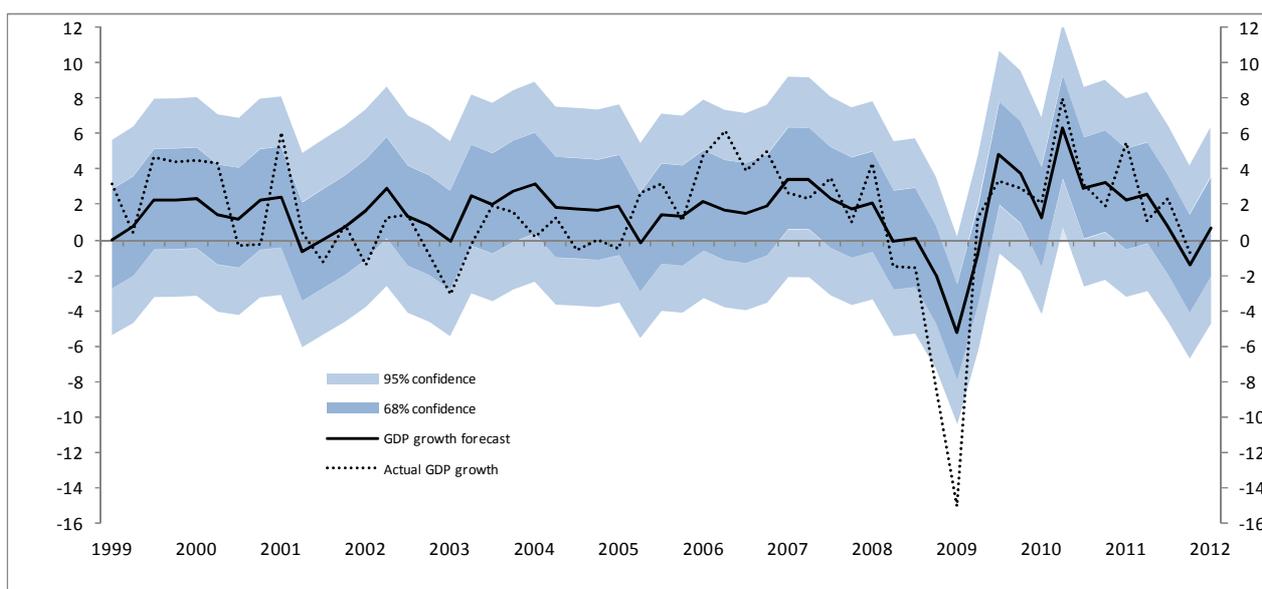
Figure 4. Simulated skewness of the next quarter forecast



Source: OECD calculations.

36. An empirical distribution of the simulated forecasting errors is available at any time between 1991Q1 to 2012Q2 and it is applied to approximate the confidence bands around the annualised average point forecast of the Indicator model for the current and next quarters displayed in Figures 5 to 8. The confidence bands drawn with the simulated errors are narrower than those drawn applying the historical RMSE, moreover they show a certain degree of asymmetry. For this purpose, the median forecast re-computed through the distribution of the simulated forecast errors is also reported and contributes to approximate the balance of risks to the model forecasts. In the period preceding the financial crisis, when the skewness of the error band is particularly pronounced (e.g. early 2001 and 2002 and from 2005 to 2007), *i.e.* when the deviation of the average point forecast from the median is larger, the actual GDP growth value falls exactly on the side of the distribution where the risk is defined. Conversely, in correspondence of the crisis, the nonlinearity introduced by the big extreme observation in the first quarter of 2009, seems to negatively affect the risk balance of the empirical distribution which keeps on persisting downside biased.²¹

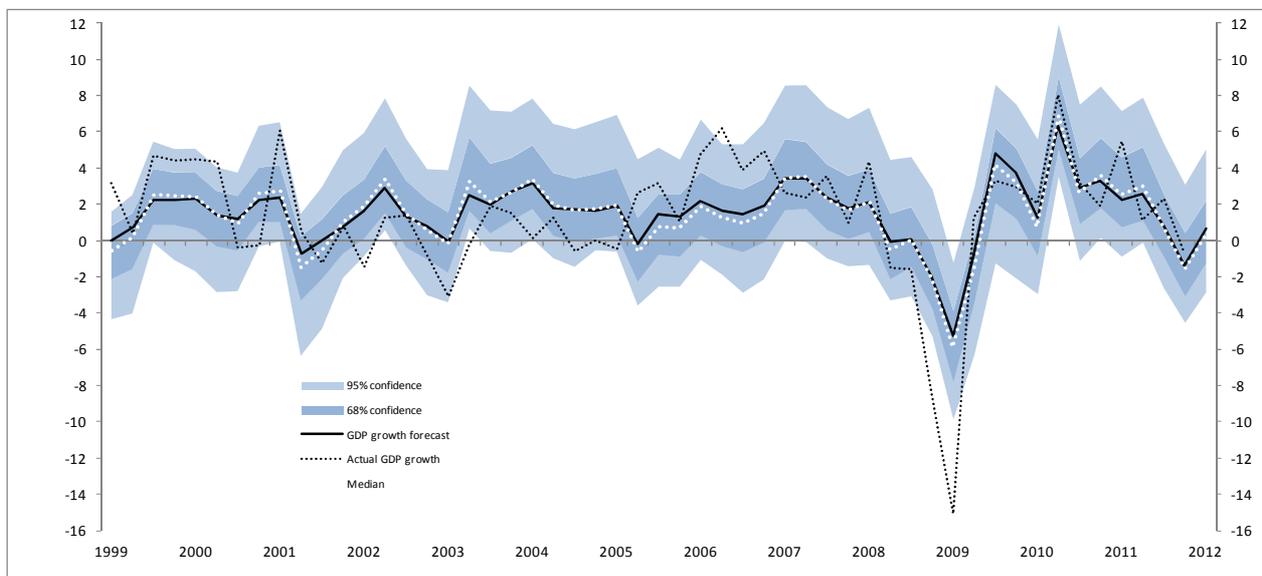
Figure 5. Confidence bands around the current quarter forecast computed with the historical RMSE



Source: OECD calculations.

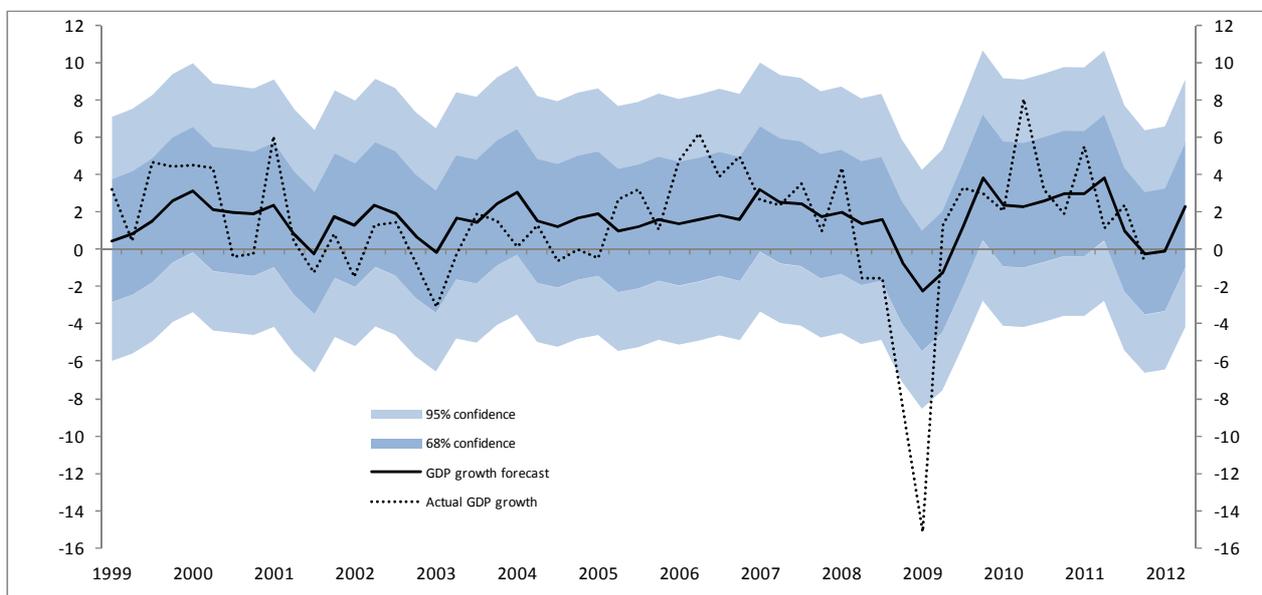
21. When an outlier correction is carried out on the computation of the standard deviation of the forecast errors, this bias to the skewness is reduced.

Figure 6. Confidence bands around the current quarter forecast computed with the simulated forecast errors

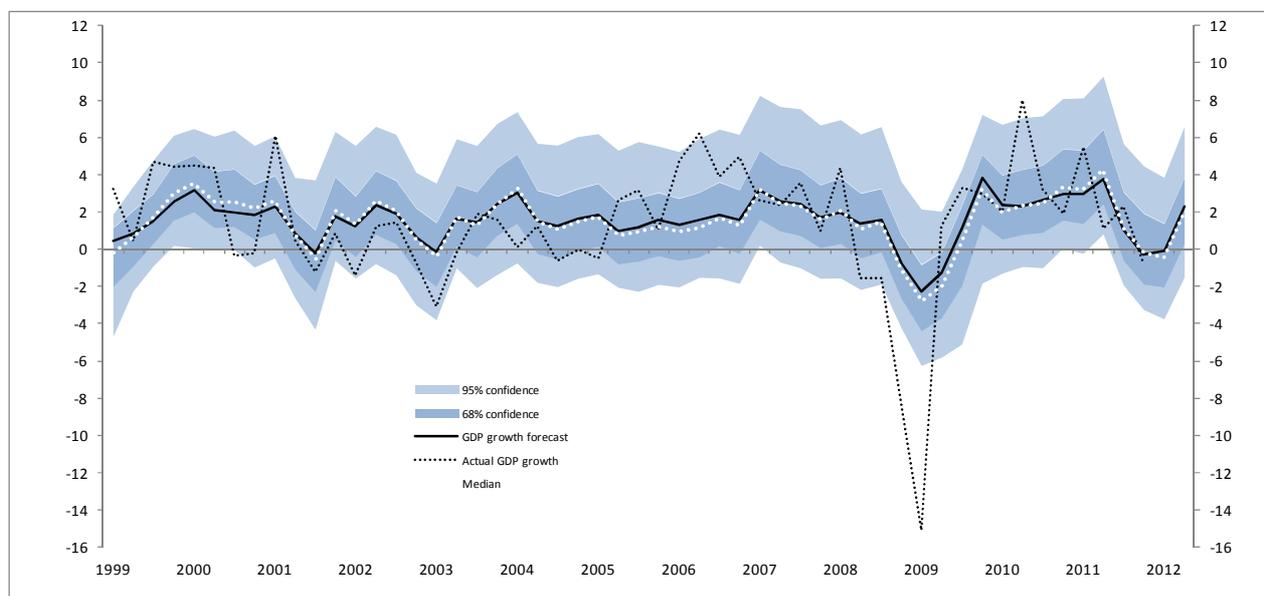


Source: OECD calculations.

Figure 7. Confidence bands around the next quarter forecast computed with the historical RMSE



Source: OECD calculations.

Figure 8. Confidence bands around the next quarter forecast computed with the simulated forecast errors

Source: OECD calculations.

7. Estimation of events probabilities

37. The bootstrapping simulation technique enables the likelihood estimation of events of particular interest over the forecast period. In this paper, these events probabilities are calculated as in Fair (1991) or Borbély and Meier (2003) by recording at each bootstrapping replication whether or not the event has occurred. The probability of the event is simply the number of times it occurred divided by the number of B repetitions.

38. For this purpose, the probability of an economic recession, defined as at least two consecutive quarters of negative real GDP growth, has been estimated. More precisely, in order to test the accuracy of the Indicator model on both forecasting horizons $h = 1$ and $h = 2$, two probabilities of recessions have been estimated:

Probability Event 1

$$PE_1 = Pr[\hat{y}_{t+1|t} < 0 | y_t < 0]$$

that is the probability of negative GDP growth at the forecasting horizon $h = 1$, *i.e.* at time $t + 1$ conditional on the negative growth at time t .

Probability Event 2

$$PE_2 = Pr[\hat{y}_{t+2|t} < 0 | \hat{y}_{t+1|t} < 0]$$

that is the probability of a negative GDP growth at the forecasting horizon $h = 2$ given a negative growth on the previous forecasting horizon $t + 1$. The probabilities have been estimated using the three different sets of bootstrapped average forecasts from 1999Q1 to 2012Q2. As previously mentioned, during the last 13 years Germany experienced three economic recessions: a first period of recession taking place in the

second half of the year 2000; a second recession occurred in the first half of 2003; and the recent financial crisis, which generated four consecutive quarters of negative GDP growth from 2008Q2 to 2009Q1.²² In addition, in consideration of the latest negative real GDP growth in the fourth quarter of 2011, the *ex ante* probability of a negative forecast for the first quarter of 2012 has been also estimated.

39. The results are reported in Table 4 and show a quite high predictive performance of the Indicator model when the uncertainty related to the monthly indicators forecasts is considered. But while this probability is particularly high in case of a recession event involving the first forecasting horizon $h = 1$, it decreases in case of the event occurring in the second horizon $h = 2$. In fact, in case of a recession involving the second quarter forecasted, GDP forecasts made considering the structural errors uncertainty tend to be more accurate. For what concerns the likelihood of a second consecutive quarter of negative GDP growth occurring in 2012Q1, the probability ranges from 31% to 54%.

Table 4. Estimated probability of economic recessions in per cent (with two months of available data on survey indicators and none on hard indicators)

Economic recession periods	Bootstrapped Explanatory Variables		Bootstrapped Structural Errors		Bootstrapped Coefficients	
	PE_1	PE_2	PE_1	PE_2	PE_1	PE_2
2000Q4	15	2	8	0	0	0
2003Q1	54	3	51	7	50	0
2003Q2	1	0	7	1	23	0
2008Q3	49	16	44	29	34	34
2008Q4	68	36	79	67	71	65
2009Q1	100	45	97	77	99	65
<i>Actual time</i>						
2012Q1	54	3	39	6	31	0

Source: OECD calculations.

22. The first period of technical recessions took place in the second half of the year 2000, due to an increase in oil and import prices associated to a contraction of about 0.2% (quarter on quarter annualised) of GDP. The second recession occurred in the first half of 2003, when the strengthening of the euro caused a fall in the German exports of goods which could not be offset by a stronger domestic consumption. Finally the recent financial crisis, which generated four consecutive quarters of negative GDP growth from 2008Q2 to 2009Q1.

40. Table 5 presents the estimated probabilities for the same two events but computed on a set of average forecasts obtained with the availability of one month of additional within quarter information on the hard indicators for both VAR and bridge equation models. In order to isolate the effect of added data on hard indicators, the survey data availability has been kept equal to two months within the current quarter. As expected, there is a general improvement on the forecasting performance in correspondence of all three bootstrapped bridge equations. This is particularly evident in the case of the financial crisis and the second horizon of forecast.

Table 5. Estimated probability of economic recessions in per cent (with two months of available data on survey indicators and one month on hard indicators)

Economic recession periods	Bootstrapped Explanatory Variables		Bootstrapped Structural Errors		Bootstrapped Coefficients	
	PE_1	PE_2	PE_1	PE_2	PE_1	PE_2
2000Q4	19	4	13	0	0	0
2003Q1	39	3	53	5	19	0
2003Q2	1	0	7	1	23	0
2008Q3	94	37	73	47	97	64
2008Q4	99	48	93	81	98	98
2009Q1	100	57	97	81	99	66

Source: OECD calculations.

41. The estimated probabilities can be used to evaluate the accuracy of the three bootstrapped bridge models, where forecasting accuracy means the closeness, on average, of predicted probabilities to observed frequencies. For this purpose, the *quadratic probability score* QPS (Diebold and Rudebusch, 1989) is computed in order to compare the estimated probabilities of economic recession with the actual outcomes on the historical series of real GDP growth for Germany. This metric has been commonly applied for the detection of turning points in business cycle studies (Filardo, 1994; Layton, 1997; Layton and Smith, 2000) with a small score indicating higher closeness to the NBER chronology. The QPS statistic is equal to:

$$QPS = \frac{1}{T} \sum_{t=1}^T 2(PE_t - R_t)^2 \quad (13)$$

where PE_t denotes the model estimate of the probability of the event recession and R_t represents the actual outcome on the time series of realisations $\{R_t\}_{t=1}^T$ and equals 1 if the event occurred and 0 otherwise. The QPS statistic ranges from 0 to 2, with a score of 0 corresponding to perfect accuracy.

42. Table 6 displays the results on the two datasets ranging from 1999Q1 to 2011Q4. The scores, varying from 0.12 to 0.19 out of 2, prove a particularly good accuracy on all bootstrapped models. These findings confirm the results showed in Tables 4 and 5, since they exhibit a higher predictive accuracy obtained in correspondence of the first forecasting horizon $h = 1$. Overall, there is not a significant improvement when adding one more month of within quarter information on the hard indicators.

Moreover, when the uncertainty around the explanatory variables forecasts is included, the bootstrapped bridge equations seem to fail more frequently in recognising the first negative quarter of a recession on the next forecasting horizon $h = 2$, when the GDP growth forecast was positive on $h = 1$. The reason is mainly due to the mean-reverting nature of the VAR models, which weakens the forecast of a next quarter of sign opposite to the current one.

Table 6. Quadratic probability scores

	Bootstrapped Explanatory Variables		Bootstrapped Structural Errors		Bootstrapped Coefficients	
	PE_1	PE_2	PE_1	PE_2	PE_1	PE_2
Data availability						
Survey indic. (2 months)						
Hard indic. (0 months)	0.12	0.18	0.13	0.15	0.12	0.15
Survey indic. (2 months)						
Hard indic. (1 month)	0.13	0.19	0.12	0.18	0.14	0.17

Source: OECD calculations.

8. Conclusions

43. This paper shows that the application of non-parametric bootstrapping simulations can be useful to define different sources of forecasting uncertainty around the Indicator model in use at the OECD to forecast real GDP growth. This has been illustrated in the particular case of Germany. The paper quantifies a measure of dispersion and a degree of asymmetry around standard point forecasts through the estimation of an empirical probability distribution for these forecasts. Moreover, through a combination of bootstrapped forecasts, *ex post simulated forecast errors* accounting for structural errors and coefficient uncertainty are obtained and can be applied to draw confidence bands around the Indicator model forecasts. These confidence bands show a moderate degree of skewness which approximates a time-varying balance of risks around the GDP point forecasts. Furthermore, differently from the classical RMSE, these simulated forecasting errors generate a *simulated RMSE* which also varies across time and allows identifying few periods of peaking uncertainty in the historical series of GDP forecasts for Germany. In particular, the degree of uncertainty as measured by the *simulated RMSE* increases in the aftermath of an economic recession. Furthermore, the pre-crisis distribution of the forecasts shows positive skewness (as *ex-post* forecast errors), while in the immediate aftermath of the crisis the skewness is negative. These findings, *e.g.* the existence of distributional skewness or peaks of higher uncertainty in correspondence of economic recession events, may be even more exaggerated for countries more affected by crisis.

44. The empirical probability distributions obtained through bootstrapping enable the estimation of the probability of specific events, like economic recession, with a quite high degree of forecasting accuracy. In particular, the probability of a positive or negative GDP growth can be directly computed also for the current and next quarter forecast, *i.e.* 2012Q1 and 2012Q2. The computation of *quadratic probability scores* shows a quite high predictive accuracy of all bootstrapped models, in particular on the current quarter forecasted.

45. Further developments of this approach aim at including the third source of forecasting uncertainty concerning the explanatory variables forecasts in the estimation of the *simulated RMSE*. As a matter of fact, in the classical literature on *ex post* macroeconomic forecasts and forecast errors decomposition, explanatory variables forecasts are usually assumed as exact. However, the consideration of this third source of uncertainty could establish a direct correlation between the simulated balance of risks around the GDP point forecasts and the forecasted pattern of the monthly indicators. Moreover, it would enable the probability estimation that the occurrence of an extreme event on a specific risk factor, *i.e.* the monthly indicator, has to affect the forecasting errors.

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ANNEX

Table A1. Bayesian VAR models and data availability on 29 February

Indicator	Type of variable	Availability
<i>Soft indicators VAR</i>		
Business expectations	endogenous	February 2012
Orders on hand vs. last month	endogenous	February 2012
Production vs. last month	endogenous	February 2012
Survey of construction ind. (unfavourable weather)	exogenous	February 2012
<i>Hard indicators VAR</i>		
Industrial production index	endogenous	December 2011
Industrial production index on construction	endogenous	December 2011
Retail sales	endogenous	December 2011
Manufacturing orders	endogenous	December 2011
Redemption yield on the benchmark bond	exogenous	January 2012
Dax index	exogenous	January 2012
<i>Mixed indicators VAR</i>		
Industrial production index	endogenous	December 2011
Industrial production index on construction	endogenous	December 2011
Retail sales	endogenous	December 2011
Manufacturing orders	endogenous	December 2011
Business expectations	endogenous	February 2012
Survey of construction ind. (unfavourable weather)	exogenous	February 2012
Dax index	exogenous	January 2012

Source: OECD's Indicator Model (Sédillot and Pain, 2003).

Table A2. Quarterly bridge models

Only soft indicators (in levels and difference)

Business expectations

Orders on hand vs. last month

Only hard indicators (in growth rates)

Industrial production index

Industrial production index on construction

Retail sales

Mixed indicators (in growth rates)¹

Industrial production index

Industrial production index on construction

Retail sales

1. In case of Germany, the mixed bridge equation contains soft indicators only through the VAR forecasts.

Source: OECD's Indicator Model (Sédillot and Pain, 2003).

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