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Effectiveness of Index-
linked Bond Issuance: A
Methodological Approach,
Illustrated Using UK
Examples

James Knight

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James Knight¹, UK Debt Management Office

JEL Classification: H63, G110, G32

Key words: Government bonds, treasury securities, risk hedging, risk return, financial risk management

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**ASSESSING THE COST EFFECTIVENESS OF INDEX-LINKED BOND ISSUANCE:
A METHODOLOGICAL APPROACH, ILLUSTRATED USING UK EXAMPLES**

James Knight, UK Debt Management Office

Abstract

Sovereign index-linked bond issuance has grown significantly since the early 1980s, with nearly \$2.5 trillion USD in bonds now in issue . Index-linked bonds have become a widely accepted part of the set of instruments that sovereign debt managers use for funding purposes and so the question of how to assess their cost effectiveness relative to other financing options is of increasing importance. This paper sets out a methodology for conducting such an analysis, the rationale behind it and ways in which such an approach could be further developed.

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1. Introduction

1. For those sovereigns that issue index-linked bonds, or are considering future issuance, determining their cost effectiveness is an important part in evaluating whether or not they should be included in any issuance strategy in the context of a debt management objective with a focus on cost minimisation, taking into account risk. There are a number of additional benefits² for an issuer that should be considered as part of this assessment (on both cost and risk grounds); however this note focuses solely on measuring the cost of index-linked issuance against alternative financing options.

2. In setting out a methodology for assessing the cost effectiveness of index-linked bond issuance in a debt management context, this paper focuses on (i) the rationale for this assessment, based on a comparative approach; (ii) the methodology used; (iii) the policy benefits of such an approach and the practical application of this methodology; and (iv) the limitations of the approach and potential areas for future development.

2. The rationale for assessing the cost effectiveness of index-linked bond issuance – a comparative approach using break-even inflation rates

3. For a sovereign debt manager, index-linked bonds are usually part of a wider set of issuance choices, including fixed-coupon bonds, floating rate notes (FRNs) and short-term paper (Treasury bills or sovereign Commercial Paper). Of these instruments, fixed-coupon bonds typically account for the largest part of sovereigns' issuance programmes and are therefore likely to be the most suitable benchmark against which index-linked issuance can be evaluated (in the same way in which an issuer might consider any other new financing instrument). It is nevertheless possible to evaluate the cost effectiveness of index-linked issuance against a wider set of instruments: for example, Fleckenstein et. al (2010) evaluate the cost effectiveness of US TIPS³ by examining the potential arbitrage opportunities available from the use of Treasuries and inflation swaps. However, for debt managers to whom such strategies are not available either on a policy or practical basis, they do not necessarily serve as an appropriate benchmark for this assessment.

4. To evaluate the cost effectiveness of index-linked bonds against fixed-coupon bonds, the most appropriate measure to use is the break-even inflation rate. This is because it is the rate of inflation that

² Such as the potential to diversify a sovereign's investor base, fiscal insurance benefits for the government and the creation of a market-based measure of inflation expectations to assist in the conduct of monetary policy.

³ Treasury Inflation Protected Securities, inflation-linked bonds issued by the US Treasury.

will equalise the return on an index-linked bond with that of a conventional bond of the same maturity. In essence, the break-even rate can be seen as the average rate of inflation, over the life of an issue, that will make an issuer indifferent on cost grounds between issuing either a fixed-coupon or an index-linked bond. At its most basic, it can be calculated by subtracting the yield of an index-linked bond from that of a conventional issue of the same maturity; more precisely, it is calculated using the Fisher identity (see Annex A).

5. While the break-even inflation rate is often interpreted as the market's expected view of inflation over a specific time period, in practice there are a number of factors that will cause it to deviate from this. Two key factors⁴ are:

- **an illiquidity premium:** index-linked bonds are typically less liquid than conventional bonds⁵, and so investors may require a premium for this illiquidity in order to hold them – this typically causes index-linked yields to be higher relative to fixed-coupon bonds, and the corresponding break-even inflation rate to be lower; and
- **an inflation risk premium:** if investors attach value to protection against inflation risk then they may be prepared to pay a premium for this protection – this will typically result in lower yields for index-linked bonds relative to fixed-coupon bonds, translating into a higher break-even inflation rate.

6. While one can attempt to estimate the effect of these premia individually⁶, it is sufficient to take account of them in aggregate, if we consider them to be factors that can explain the overall relative demand between fixed-coupon and index-linked bonds. Assuming the existence of investor preferences (i.e. preferred habitats⁷) for specific types or maturities of bond (which may, amongst other factors, include a desire for liquidity or for inflation-linked cashflows), then yields on both fixed coupon and index-linked bonds may deviate from those based on future expectations of interest rates. In turn, break-even inflation

⁴ Other factors include the differential taxation treatment between fixed-coupon and index-linked bonds, as well as the different duration and convexity properties of the two instruments (in this paper we assume that a debt manager is interested in making relative issuance decisions based on the maturity of the instruments that it issues, rather than their duration characteristics). Also see Christensen et al. (2004) for a discussion on the potential for bias in the break-even rate when the term structure of inflation expectations is not flat.

⁵ Reflecting their different investor base, which typically includes a greater proportion of buy-and-hold investors, such as pension funds that purchase them for liability matching purposes, as well as a general lack of direct hedging instruments (e.g. index-linked bond futures).

⁶ See, for example, Christensen and Gillan (2011) for an attempt to estimate the inflation risk premium in the context of an assessment of the cost effectiveness of US TIPS issuance.

⁷ Based on segmented market theory, we assume that demand for government bonds is segmented, that is different types and maturities of bond are not substitutes for one another.

rates will deviate from inflation expectations because they capture the relative demand between these two types of instrument (in the context of a given level of supply).

7. As an example, if there are investors with strong demand (a preferred habitat) for long-dated inflation-linked cash flows, then the yield on index-linked bonds may be lower than that based on expectations of real interest rates alone. In this instance, if there is not equivalently strong demand for long-dated fixed-coupon bonds, then the break-even inflation rate will be higher than it would be otherwise on the basis of inflation expectations alone. That is, if investors are willing to pay for inflation protection, and do not discount the illiquidity of index-linked bonds too heavily and/or there is strong demand for index-linked bonds relative to fixed-coupon issuance, then there may be cost benefits from issuance of the former relative to the latter.

8. In essence, if the break-even inflation rate on a index-linked bond is higher than the actual inflation outturn over the life of the bond, then issuance will have been more cost effective than a fixed-coupon equivalent, and vice versa.

9. Finally, an issuer must be aware of the potential impact of “inflation surprise” on the cost of its index-linked issuance. If inflation outturns are meaningfully different from those expected at the time of issuance, then the relative cost of fixed-coupon and index-linked bonds will alter. For example, a positive inflation surprise will increase the relative cost of index-linked bonds outstanding, while a negative surprise will have the opposite effect. However, as Dudley et al. (2009) note, “over the long run, however, inflation surprises should not matter⁸. This is because investors are likely to learn from their mistakes and not repeat their forecast errors indefinitely. If investors incorporate all known information into their prediction, inflation surprises should be unbiased, with as many downward surprises in inflation performance as upward surprises.”

10. Nevertheless, there can be circumstances in which a policy-driven negative inflation surprise can generate significant short-run cost savings from index-linked issuance. Box 1 sets out an example of how such a surprise resulted in considerable cost savings for the UK Government in the 1980s.

⁸ There is a difference between smaller inflation forecasting errors, and major errors resulting from large positive (or negative) inflation shocks. In issuing index-linked bonds, particularly at longer maturities, an issuer will have to consider the additional inflation risk that it is assuming in the context of the rest of its balance sheet.

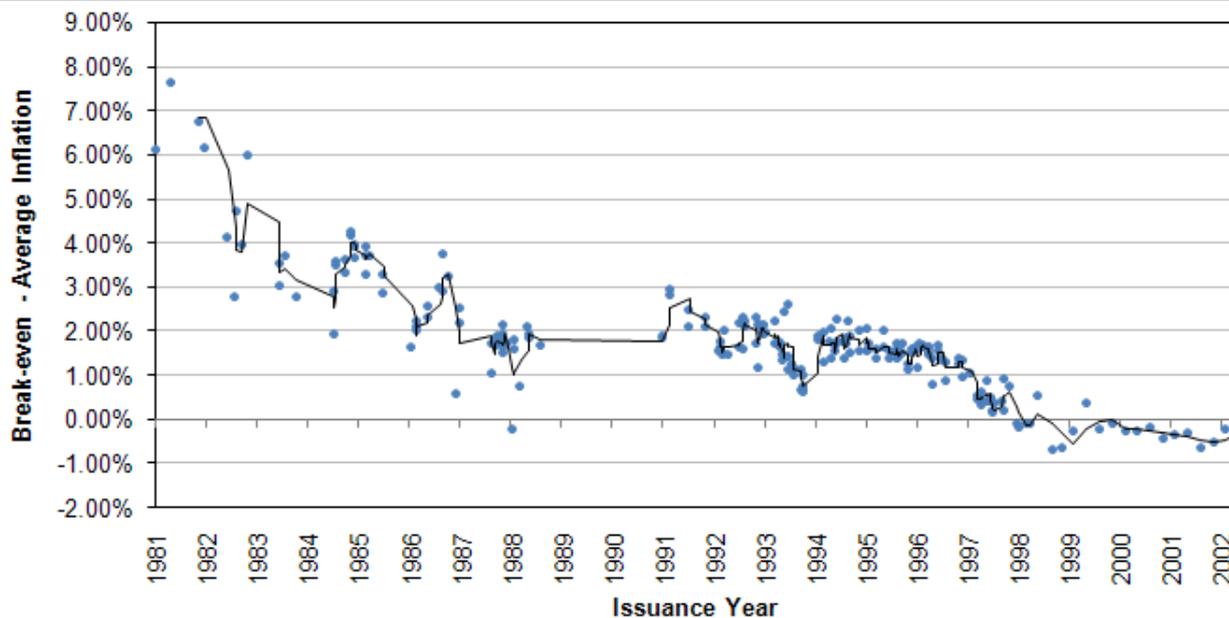
Box 1. Savings from the initial issuance of index-linked gilts in the UK⁹

In 1981, the UK's first index-linked gilt was issued with a break-even inflation rate of approximately 11.5%. The level of this break-even rate reflected, in part, the market's lack of belief in the Government's ambitions for inflationary discipline in the economy – i.e. it did not believe that the Government would be successful in reducing inflation significantly (the prevailing inflation rate at the time was 12.6%). However, the Government managed to bring inflation under control and the outturn average rate of inflation over the life of the bond was just 5.9%. As such, index-linked issuance turned out to be highly cost effective for the Government relative to its fixed-coupon equivalent, a consequence of this negative inflation surprise for the market.

While the largest savings were made in the early days of the UK's index-linked issuance programme, this trend continued for much of the issuance of index-linked gilts in the 1980s and early 1990s. Chart 1 below highlights the spread between the break-even inflation rate at issuance and the equivalent average rate of inflation (over the period from issue to maturity¹⁰) for each individual issue¹¹ of index-linked gilts and shows how the Government was able to make significant cost savings from issuance of index-linked gilts up until the late 1990s.

However, from 1997, the year in which the Bank of England was granted operational independence to set interest rates to meet the Government's inflation target, these savings have declined significantly. To the extent that the original savings made were a result of a negative inflation surprise for investors, they are unlikely to be repeated in an environment in which there is a credible central bank inflation target around which investors inflation expectations have become anchored.

Chart 1. Spread between break-even inflation at issue and average inflation over the life of each tranche of index-linked gilt issuance from 1981-2001¹²



⁹ See Deacon and Knight (2008) for a discussion of the history of index-linked gilt issuance.

¹⁰ Taking into account the indexation lag.

¹¹ Both new issues and re-openings.

¹² For those tranches of issuance that have yet to redeem, a neutral long-run inflation assumption, where inflation returns to target in two-years, and remains there indefinitely, is used.

3. An approach for measuring the cost effectiveness of index-linked issuance

11. The approach for measuring the cost effectiveness of index-linked issuance in this paper takes the methodology developed by Sack and Elsasser (2002) as its starting point¹³. It uses the break-even inflation rate on an index-linked bond (as described in section 2), to create a counterfactual bond issue against which index-linked issuance is evaluated. Annex A sets out the specifics behind the calculations discussed in this section.

12. This approach can be used to analyse the cost effectiveness of either (i) previous index-linked issuance (a backward-looking analysis); or (ii) potential future index-linked issuance (a forward-looking analysis). Applying the methodology involves evaluating the cost effectiveness of one tranche of index-linked issuance at a time, replicating the analysis across other issues or maturities to produce an aggregate picture of the cost effectiveness of index-linked issuance.

13. Starting with the counterfactual bond, its coupon is set at the same rate as that on the index-linked bond being evaluated. However, the cashflows paid on the counterfactual will grow at a constant rate determined by the break-even at issue, while the cash flows on the index-linked bond grow in line with the prevailing rate of inflation. Both the coupons and redemption payment on the counterfactual bond are indexed to a Break-even Index (BEI), which grows at the rate set by the break-even inflation rate at issue, in contrast with the index-linked bond whose coupons and redemption payment are determined by the growth in the relevant inflation index. The difference in the cashflows on the counterfactual bond and the equivalent index-linked bond will determine cost effectiveness of each index-linked bond issue. As these cashflows arise at different points in time, it is necessary to discount each back to today to calculate a net present value of the costs or savings from each issue. We consider these key inputs to the calculation next.

3.1 Break-even Inflation Rate

14. There are two inputs to the calculation of the break-even inflation rate: the respective yields on the fixed-coupon and index-linked bonds. These can be either redemption yields on bonds in the secondary market or taken from yield curve models (or a mixture of both)¹⁴.

¹³ The methodology in this paper, developed at the UK DMO in 2006, is similar to that later developed independently by Roush (2008).

¹⁴ However, one of the issues to consider in using a yield curve model is that doing so can smooth out bond-specific distortions (e.g. due to liquidity, off-market coupon) from the analysis that it might be desirable to capture.

15. Using secondary market yields on index-linked bonds is appropriate when assessing the historical costs or savings from issuance of previously issued instruments, or when considering potential re-openings of existing instruments. However, for the choice of the comparator yield, depending on the precise maturity of any existing fixed-coupon issues and given potential maturity mismatches, it may be more appropriate to use a par yield derived from a nominal yield curve instead of a redemption yield on a fixed-coupon bond.

16. To assess the cost effectiveness of hypothetical index-linked issuance, using nominal and real yield curve data allow you to calculate the break-even inflation rate at set maturity points (e.g. examining potential 10 or 30-year index-linked issuance). Alternatively, data from an implied inflation curve can be used as a direct substitute for the break-even inflation rate.

3.2 The rate of inflation

17. One of the key inputs to the analysis of the cost effectiveness of index-linked issuance is the path of inflation itself. Depending upon whether the analysis is conducted on a forward or a backward-looking basis, a forecast of inflation may be required as one of the inputs to the calculations.

18. For backward-looking analysis, for those tranches of issuance that have already matured, an issuer will know exactly what inflation has been during their life and can compare this directly with the break-even inflation rate at issue to calculate savings or losses from issuance. However, for those bonds that remain in issue, a forecast of the path of inflation from the date of the calculations until their maturity will be required. Likewise for forward-looking analysis, evaluating potential future index-linked issuance will be completely reliant on an inflation forecast. Diagram 1 summarises this point.

Diagram 1. The path of inflation – the degree of forecasting required

← Backward-looking analysis		Forward-looking analysis →
Matured Issuance	Issuance Outstanding	Future Issuance
Path of actual inflation is known to calculate all cashflows on index-linked bonds	Path of inflation is partially known: known for past cashflows paid, forecast required for future cashflows	Path of inflation is unknown: forecast required for all cashflows on potential issuance

19. Where should this forecast of inflation come from? This is what gives the model flexibility – it is possible to use either a central inflation assumption, or to undertake various scenario analyses by varying the path of future inflation to analyse the cost performance of index-linked bonds against their fixed-coupon counterparts. Using a range of inflation scenarios may also enable the model to be used for basic risk analysis, as it can show the exposure of past and future index-linked issuance to changes in inflation.

20. For those countries with an inflation targeting regime, setting the long-run average rate of inflation in the model equal to the central bank's inflation target¹⁵ (provided that the central bank targets the same index that is used to uplift the index-linked bond cash flows¹⁶) can be considered to be an appropriate choice of inflation rate, as it implies the government assumes that the central bank will keep inflation on target *on average* in the long run. The use of this assumption avoids the risk of an issuer being opportunistic and seeking to out-predict the market on the future path of inflation.

3.3 Discount rates

21. The final input to the calculation is a set of discount rates to present value each cash flow to the point in time at which the calculations are performed. Cash flows occurring before the calculation date will need to be scaled up, while those in the future will need to be discounted back. Depending on the availability of data, the discount rate(s) used can either be a fixed factor or a set of rates derived from a nominal yield curve.

4. The policy benefits of the approach to measuring index-linked cost effectiveness

22. As has been noted earlier, the analysis of index-linked cost effectiveness can either be conducted on a backward or forward-looking basis. Backward-looking analysis seeks to answer the question as to whether the cost of past index-linked issuance has exceeded the cost of equivalent fixed-coupon issuance. In contrast, forward-looking analysis considers whether future index-linked issuance is likely to be a cost effective source of financing.

23. From a policy perspective, forward-looking analysis is likely to be a more useful tool for the debt manager than an assessment of the sunk gains or costs from past issuance, because it provides an issuer with information that can help it to decide whether or not index-linked bonds should form part of its issuance programme, how much issuance should be considered and at what maturities it should be directed.

¹⁵ Allowing for some transition period for inflation from its current level to the target. For example, in the case of the UK, a path from the rate of inflation today to an inflation target fixed two years in the future.

¹⁶ In the case of the UK, the Bank of England targets the Consumer Prices Index (CPI), whereas the cashflows on index-linked gilts are linked to the Retail Prices Index (RPI). As such, an assumption has to be made about the long-run difference between the two inflation indices.

In addition, the results of backward-looking analysis may be influenced by the period over which the analysis is conducted, particularly if there is not a long history of issuance to analyse¹⁷.

24. Box 2 illustrates the use of forward-looking analysis in the UK and how it feeds into determining the overall shape of the UK’s financing remit, as well as individual index-linked gilt issuance choices.

Box 2. Analysing the potential cost effectiveness of index-linked gilt issuance in the UK

The UK DMO evaluates the potential cost effectiveness of index-linked gilt issuance using the methodology set out above and this analysis is one of a number of inputs used in providing HM Treasury with advice on the appropriate amount of index-linked gilts to be issued in the year ahead.

Charts 2 and 3 highlight how data from the DMO’s nominal and real yield curves, combined with a range of paths for inflation, can be used to estimate potential costs or savings from issuance of index-linked gilts across the maturity spectrum at a set point in time. Chart 2 shows these potential paths of future inflation, while Chart 3 translates them into cost effectiveness forecasts for index-linked gilt issuance by maturity, where the cost effectiveness of an index-linked gilt at any maturity point is the present value of all the future cashflows on the gilt from the date of the calculation until it matures.

In this example¹⁸, the charts demonstrate the potential cost effectiveness of longer-dated index-linked gilts relative to those at shorter maturities; however, they also highlight the greater dispersion in potential costs or savings from longer-dated index-linked issuance (depending on the inflation assumption used), consistent with the longer horizon over which these bonds are in issue.

Chart 2. Inflation: Actual and Forecast

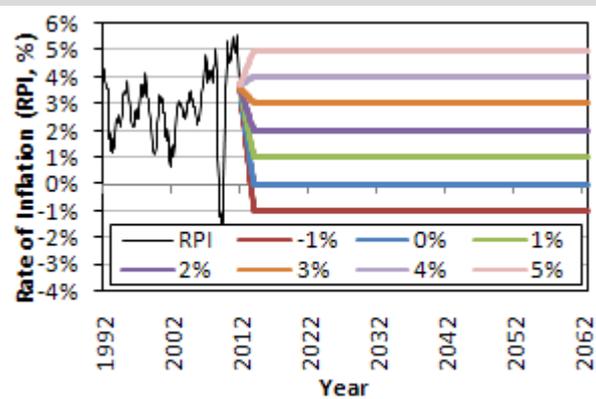
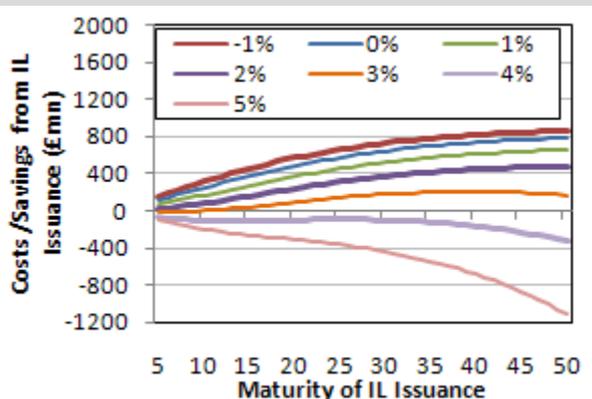


Chart 3. Costs/Savings from Issuance (per £bn)



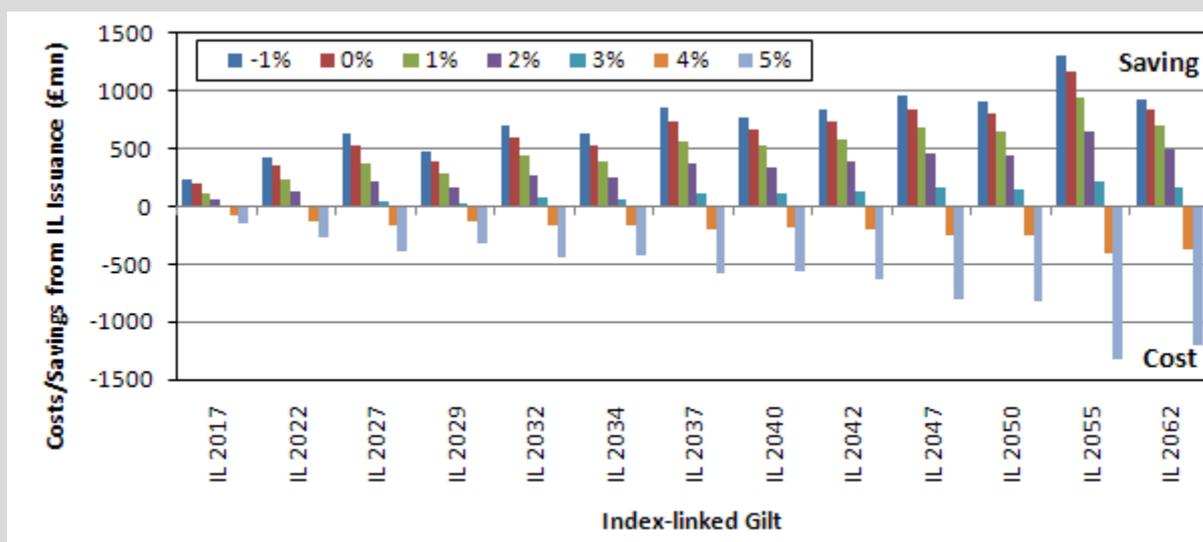
This approach can be repeated across a series of dates to show the evolution of the forecast cost effectiveness of index-linked issuance by maturity through time.

¹⁷ In the case of analysing US TIPS issuance, Dudley et al. (2009) argue that an “ex-post” approach is not appropriate because there is only a small sample of data available, which does not allow for an averaging out of inflation forecast errors (as discussed in section 2).

¹⁸ Data as at 21 March 2012.

Alongside this curve-based analysis, the DMO evaluates the cost effectiveness of individual index-linked gilts. This analysis is also used, alongside other inputs, as part of the advice to HM Treasury on the potential amount of index-linked issuance for the year ahead, but also as an input throughout the year to decisions on which specific index-linked gilts to issue on a quarterly basis. Chart 4 shows the potential costs or savings from individual index-linked gilt issues (from the date of calculation to maturity), based on a range of long-term inflation paths.

Chart 4. Cost effectiveness of Index-linked Gilts (per £bn of issuance)



As with the data in Chart 3, Chart 4 shows the potential savings to be made from index-linked issuance were generally increasing with maturity. Conducting the analysis on a bond-by-bond basis takes into account the idiosyncratic factors that may make specific issues more cost effective than others (e.g. varying degrees of liquidity, anticipated forthcoming supply, etc.)

5. The limitations of the model

25. As with most modelling exercises, the calculations rely heavily on the quality of the inputs used. As set out above, the model requires a significant amount of data to undertake the necessary calculations. In particular, for a forward-looking analysis, it requires up-to-date break-even inflation rates at differing maturities, which can either be taken directly from existing index-linked bonds in the secondary market, or from an implied inflation curve calculated from fitted nominal and real yield curves (these nominal yield curve data are also necessary for discounting the future differences in cash flows).

26. For an issuer that has not yet issued index-linked bonds, or does not have a actively traded nominal and/or real yield curve at the maturities it wishes to analyse, it may be difficult to make use of this framework – that is the model is most useful once an issuer has been issuing index-linked bonds and has readily available price sources in the secondary market rather than for a new issuer considering potential issuance.

27. One of the main limitations of the methodology, as currently specified, is its static nature – it evaluates the cost effectiveness of index-linked issuance at a fixed point in time, i.e. it does not forecast the potential evolution of fixed-coupon or index-linked yields (and thus the evolution of break-even inflation rates). Further, as was noted in Section 2, neither does it consider the contribution of the components of fixed-coupon and index-linked yields to the break-even inflation rate, such as the liquidity premium or inflation risk premium and thus their individual impact on the relative demand for the two types of instrument. Both of these issues could be addressed by incorporating the methodology into a stochastic debt simulation model, which forecasts the future path of conventional and index-linked yields and simulates potential changes in the underlying yield premia. A stochastic debt simulation model could also provide a distribution of inflation outcomes, more readily allowing for an analysis of the impact of different inflation scenarios on index-linked issuance.

28. Finally, the calculations do not take into account the potential elasticity of supply for fixed-coupon or index-linked bonds, or how the market might react to major shifts in issuance between both types of bond. As such, the methodology can be seen to be more appropriate for considering marginal, rather than wholesale, changes in issuance patterns.

6. Conclusion

29. The approach set out in this paper should allow an issuer to either estimate the cost effectiveness of past index-linked issuance at a set point in time, or to consider the potential cost effectiveness of future issuance. The methodology set out in this paper can also act as a starting point for a more sophisticated approach to evaluating issuance of index-linked bonds, either by undertaking a decomposition of the constituent parts of the break-even inflation rate or by using a stochastic model over a multi-period horizon.

30. As has been noted earlier, this methodology can provide a useful policy role, on a forward-looking basis, in helping to determine a debt manager's approach to the issuance of index-linked bonds. However, rather than being undertaken in isolation, it should form part of a wider evaluation of the costs and risks of issuing index-linked bonds.

ANNEX A. COST EFFECTIVENESS CALCULATIONS

This annex sets out the calculations that underpin the methodology described in this paper.

1. Calculating a break-even index (BEI)

A1. The starting point for the calculation is a value for the break-even inflation rate (BEIR). If the BEIR is to be calculated using relative bond yields or yield curve data (as considered in section 3.1) then the Fisher identity¹⁹ can be used:

$$BEIR = \left(\frac{1 + \frac{y}{2}}{1 + \frac{r}{2}} \right)^2 - 1$$

where y = nominal yield on fixed-coupon bond (or nominal yield from yield curve)

r = real yield on index-linked bond (or real yield from yield curve)

A2. The BEIR is used to calculate a ‘break-even inflation index’ (BEI), which continues the inflation series after the point of issuance of the tranche at a constant rate equal to the break-even inflation rate.

A3. The BEI runs for months $i=(1,2,\dots,t,t+1,\dots,n)$, where t is the month of the index relating to the issuance of the tranche and n is the month of the index relating to the redemption payment (both lagged by the correct number of months in line with the indexation lag of the bond), $BEIR$ is the calculated break-even inflation rate (as a decimal) and RPI_i is the price index value for month i :

$$BEI_i = RPI_i I_{i \leq t} + RPI_{i-1} (1 + BEIR)^{\frac{1}{12}} I_{i=t+1} + BEI_{i-1} (1 + BEIR)^{\frac{1}{12}} I_{i > t+1}$$

$$\text{where } I_A(x) = \begin{cases} 1 & \text{if A is true} \\ 0 & \text{otherwise} \end{cases}$$

A4. The BEI data can be used to calculate Reference BEIs (RefBEI) for any calendar day²⁰:

¹⁹ This example assumes semi-annual coupon paying bonds.

²⁰ See also UK DMO (2005:32).

$$\text{RefBEI}_d = \text{RefBEI}_M + \left(\frac{d-1}{D} \right) (\text{RefBEI}_{M+1} - \text{RefBEI}_M) I_{d=1}$$

$$\text{where } I_A(x) = \begin{cases} 0 & \text{if A is true} \\ 1 & \text{otherwise} \end{cases}$$

D = number of days in the calendar month in which the given date falls

t = the calendar day corresponding to the given date

RefBEI_M = Reference BEI for the first day of the calendar month in which the given date falls

RefBEI_{M+1} = Reference BEI for the first day of the calendar month immediately following the given date.

A5. The reference BEI for the first calendar day of any calendar month is the BEI for the calendar month falling three months earlier. The reference BEI for any other day in the month is calculated by linear interpolation.

A6. These RefBEIs are then used to calculate a series of break-even index ratios (BIR), where each BIR is calculated as:

$$\text{BIR}_d = \left[\frac{\text{RefBEI}_{\text{date}}}{\text{RefBEI}_{\text{first issue date}}} \right]$$

A7. Where $\text{RefBEI}_{\text{first issue date}}$ for a given bond remains constant over its life.

2. Calculating the costs or savings from index-linked issuance

For three-month index-linked gilts the cost saving is the sum of the discounted differences between the cashflows on the counterfactual and the index linked gilt from the first dividend (d) to maturity (m), d=1...m.

$$\sum_{d=1}^m \left((d_d \{BIR_d - IR_d\}) \frac{n}{100} \delta_d \right) + (\{BIR_m - IR_m\} n \delta_m)$$

Where:

BIR = the index-ratio calculated using the BEI

IR = the index-ratio calculated using the actual inflation index

d_d = the dividend payment on the relevant date

δ_d = the discount factor applicable to the cashflow

n = the nominal amount of the index-linked bond being considered

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