Eur. Phys. J. C (2016) 76:341
DOI 10.1140/epjc/s10052-016-4165-y

THE EUROPEAN PHYSICAL JOURNAL C

CrossMark

Comment

Comment on "Quantum massive conformal gravity" by F. F. Faria

Yun Soo Myung[a]

Institute of Basic Sciences and Department of Computer Simulation, Inje University, Gimhae 621-749, Korea

Received: 1 May 2016 / Accepted: 30 May 2016 / Published online: 20 June 2016
© The Author(s) 2016. This article is published with open access at Springerlink.com

Abstract In a recent paper in EPJC doi:10.1140/epjc/s10052-016-4037-5, Faria has shown that quantum massive conformal gravity is renormalizable but has ghost states. We comment this paper on the aspect of renormalizability.

Recently, a paper in EPJC [1], Faria has insisted that quantum massive conformal gravity is renormalizable but has ghost states. The proposed action for this purpose is just the massive conformal gravity action (MCG) [2,3]

$$S_{\rm MCG} = -\frac{1}{12}\int d^4x\sqrt{-g}\left[(\phi^2 R + 6\partial_\mu\phi\partial^\mu\phi) - \frac{1}{m^2}C^2\right],$$
$$C^2 = C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma} \tag{1}$$

which is classically invariant under the conformal transformations as

$$g_{\mu\nu} \to \Omega^2(x) g_{\mu\nu}, \quad \phi \to \frac{\phi}{\Omega}. \tag{2}$$

Here $\Omega(x)$ is an arbitrary function of the spacetime coordinates. We note here that the MCG action ($S_{\rm MCG}$) is composed of the conformal dilation gravity ($S_{\rm CDG}$) and the conformal gravity ($S_{\rm CG}$). Since the integral of the Euler density ($E = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$) is a topological invariant quantity, $S_{\rm CG}$ reduces to the Weyl gravity

$$S_{\rm WG} = \frac{1}{6m^2}\int d^4x\sqrt{-g}\left[R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right]$$
$$\equiv \frac{1}{6m^2}\int d^4x\sqrt{-g}\tilde{C}^2. \tag{3}$$

It was argued that the massive conformal gravity (1) is a renormalizable quantum theory of gravity which has two massive ghost states. However, Faria's work is far from showing that (1) is a renormalizable quantum gravity because it was based on performing the canonical quantization of the second-order bilinear action. Concerning the renormalizability, Faria has mentioned that the graviton propagator of

$$\Psi_{\mu\nu} = h_{\mu\nu} - \eta_{\mu\nu}h/2,$$
$$D_\Psi^{\mu\nu,\alpha\beta} = -\frac{i}{2}(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - \eta^{\mu\nu}\eta^{\alpha\beta})$$
$$\times\int\frac{d^4p}{(2\pi)^4}\frac{m^2 e^{-ip\cdot(x-y)}}{(p^2 - i\chi)(p^2 + m^2 - i\chi)} \tag{4}$$

has a good behavior of $1/p^4$ at high momenta, making (1) power-counting renormalizable in the Minkowski spacetime. Unless the massive pole is shown to be unphysical, the MCG (1) is perturbatively meaningless because the graviton propagator (4) has ghost state. We wish to point out that this was a clear observation since the seminal work of Stelle released 40 years ago [4].

It was known that the MCG (1) is a promising quantum (gravity) model because the conformal symmetry restricts the number of counter-terms arising from the perturbative quantization of the scalar (dilaton) ϕ, while keeping the graviton fixed [5]. The inclusion of conformal symmetry is the reason for the cancelations. Explicitly, the one-loop counter term of the conformal dilation gravity ($S_{\rm CDG}$) is given by [6,7]

$$\Gamma_{\rm CDG} = -\frac{1}{960\pi^2(n-4)}\int d^nx\sqrt{-g}\left[R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right], \tag{5}$$

which is proportional to the $S_{\rm WG}$ (3), reflecting the conformal symmetry. We don't need any counter terms more. In the absence of conformally coupled term ($\phi^2 R$), however, quantizing a purely kinetic term requires (5) as well as R^2-term (conformally non-invariant term). Hence (1) plays the role of a proper quantization action when quantizing the dilation ϕ in the fixed curved background $g_{\mu\nu}$.

On the other hand, Stelle has proposed the conformally non-invariant Lagrangian of $\sqrt{-g}(R + a\tilde{C}^2 + bR^2)$ to improve the perturbative properties of Einstein gravity [4]. The first two terms could be derived from (1) by making use of the conformal transformation (2). Also, one includes R^2 term as a counter term of $g_{\mu\nu}$. If $ab \neq 0$, the renormalizabil-

[a] e-mail: ysmyung@inje.ac.kr

ity was achieved but the unitarity was violated, which shows that the renormalizability is not compatible with the unitarity. Although the Weyl-squared term ($\tilde{C}^2$) improves the ultraviolet divergence, it induces ghost excitations which spoil the unitarity simultaneously. Here, the price one has to pay for making the theory renormalizable is the loss of unitarity.

Up to now, there is no obvious way to attain the renormalizability without violating the unitarity in quantizing the gravity.

Open Access This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (http://creativecommons.org/licenses/by/4.0/), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made.
Funded by SCOAP^3.

References

1. F.F. Faria, Eur. Phys. J. C **76**, 188 (2016). doi:10.1140/epjc/s10052-016-4037-5. arXiv:1503.04355 [gr-qc]
2. F.F. Faria, Adv. High Energy Phys. **2014**, 520259 (2014). doi:10.1155/2014/520259. arXiv:1312.5553 [gr-qc]
3. Y.S. Myung, Phys. Lett. B **730**, 130 (2014). arXiv:1401.1890 [gr-qc]
4. K.S. Stelle, Phys. Rev. D **16**, 953 (1977). doi:10.1103/PhysRevD.16.953
5. G. 't Hooft, M.J.G. Veltman, Ann. Poincare Phys. Theor. A **20**, 69 (1974)
6. G. 't Hooft, Found. Phys. **41**, 1829 (2011). doi:10.1007/s10701-011-9586-8. arXiv:1104.4543 [gr-qc]
7. E. Alvarez, M. Herrero-Valea, C.P. Martin, JHEP **1410**, 115 (2014). doi:10.1007/JHEP10(2014)115. arXiv:1404.0806 [hep-th]