

# Analysis of the tensor–tensor type scalar tetraquark states with QCD sum rules

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**Abstract** In this article, we study the ground states and the first radial excited states of the tensor–tensor type scalar hidden-charm tetraquark states with the QCD sum rules. We separate the ground state contributions from the first radial excited state contributions unambiguously, and obtain the QCD sum rules for the ground states and the first radial excited states, respectively. Then we search for the Borel parameters and continuum threshold parameters according to four criteria and obtain the masses of the tensor–tensor type scalar hidden-charm tetraquark states, which can be confronted with the experimental data in the future.

## 1 Introduction

The attractive interaction induced by one-gluon exchange favors formation of diquark states in color antitriplet and disfavors formation of diquark states in color sextet. The antitriplet diquark states  $\varepsilon^{ijk}q_j^T C\Gamma q_k$  have five Dirac tensor structures, scalar  $C\gamma_5$ , pseudoscalar  $C$ , vector  $C\gamma_\mu\gamma_5$ , axial-vector  $C\gamma_\mu$  and tensor  $C\sigma_{\mu\nu}$ . The structures  $C\gamma_\mu$  and  $C\sigma_{\mu\nu}$  are symmetric, while the structures  $C\gamma_5$ ,  $C$  and  $C\gamma_\mu\gamma_5$  are antisymmetric. The scalar and axial-vector light diquark states have been studied with the QCD sum rules [1–4], the scalar and axial-vector heavy-light diquark states have also been studied with the QCD sum rules [5,6]. The calculations based on the QCD sum rules indicate that the scalar and axial-vector diquark states are more stable than the corresponding pseudoscalar and vector diquark states, respectively. We usually construct the  $C\gamma_5 \otimes \gamma_5 C$ -type and  $C\gamma_\mu \otimes \gamma^\mu C$ -type currents to study the lowest scalar light tetraquark states, hidden-charm or hidden-bottom tetraquark states [7–18], the corresponding  $C \otimes C$ -type and  $C\gamma_\mu\gamma_5 \otimes \gamma_5\gamma^\mu C$ -type scalar tetraquark states have much larger masses. The  $C\sigma_{\alpha\beta} \otimes \sigma^{\alpha\beta} C$ -type scalar hidden-charm or hidden-bottom tetraquark states have not been studied with the QCD sum

rules, so it is interesting to study them with the QCD sum rules.

The instantons play an important role in understanding the  $U_A(1)$  anomaly and in generating the spectrum of light hadrons [19]. The calculations based on the random instanton liquid model indicate that the most strongly correlated diquarks exist in the scalar and tensor channels [20]. The heavy-light tensor diquark states, although they differ from the light tensor diquark states due to the appearance of the heavy quarks, maybe play an important role in understanding the rich exotic hadron states, we should explore this possibility, the lowest hidden-charm and hidden-bottom tetraquark states maybe of the  $C\gamma_5 \otimes \gamma_5 C$ -type,  $C\gamma_\mu \otimes \gamma^\mu C$ -type or  $C\sigma_{\alpha\beta} \otimes \sigma^{\alpha\beta} C$ -type.

The QCD sum rules method provides a powerful theoretical tool in studying the hadronic properties, and it has been applied extensively to the study of the masses, decay constants, hadronic form-factors, coupling constants, etc. [21–23]. In this article, we construct the  $C\sigma_{\alpha\beta} \otimes \sigma^{\alpha\beta} C$ -type currents to study the scalar hidden-charm tetraquark states. There exist some candidates for the scalar hidden-charm tetraquark states. In Ref. [24], Lebed and Polosa propose that the  $X(3915)$  is the ground state scalar  $cs\bar{c}\bar{s}$  state based on lacking of the observed  $D\bar{D}$  and  $D^*\bar{D}^*$  decays, and attribute the single known decay mode  $J/\psi\omega$  to the  $\omega$ - $\phi$  mixing effect. Recently, the LHCb collaboration observed two new particles  $X(4500)$  and  $X(4700)$  in the  $J/\psi\phi$  mass spectrum with statistical significances  $6.1\sigma$  and  $5.6\sigma$ , respectively, and determined the quantum numbers to be  $J^{PC} = 0^{++}$  with statistical significances  $4.0\sigma$  and  $4.5\sigma$ , respectively [25,26]. The  $X(4500)$  and  $X(4700)$  are excellent candidates for the  $cs\bar{c}\bar{s}$  tetraquark states. In Refs. [27,28], we study the  $C\gamma_\mu \otimes \gamma^\mu C$ -type,  $C\gamma_\mu\gamma_5 \otimes \gamma_5\gamma^\mu C$ -type,  $C\gamma_5 \otimes \gamma_5 C$ -type, and  $C \otimes C$ -type scalar  $cs\bar{c}\bar{s}$  tetraquark states with the QCD sum rules. The numerical results support assigning the  $X(3915)$  to be the 1S  $C\gamma_5 \otimes \gamma_5 C$ -type or  $C\gamma_\mu \otimes \gamma^\mu C$ -type  $cs\bar{c}\bar{s}$  tetraquark state, assigning the  $X(4500)$  to be the 2S  $C\gamma_\mu \otimes \gamma^\mu C$ -type  $cs\bar{c}\bar{s}$  tetraquark state, assigning the

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$X(4700)$  to be the  $1S C\gamma_\mu\gamma_5 \otimes \gamma_5\gamma^\mu C$ -type  $c\bar{s}\bar{s}$  tetraquark state. For other possible assignments of the  $X(4500)$  and  $X(4700)$ , one can consult Refs. [29–34]. In this article, we study the  $C\sigma_{\alpha\beta} \otimes \sigma^{\alpha\beta} C$ -type hidden-charm tetraquark states with the QCD sum rules, and explore whether or not the  $X(3915)$ ,  $X(4500)$  and  $X(4700)$  can be assigned to be the  $C\sigma_{\alpha\beta} \otimes \sigma^{\alpha\beta} C$ -type tetraquark states.

The article is arranged as follows: we derive the QCD sum rules for the masses and pole residues of the ground state  $C\sigma_{\alpha\beta} \otimes \sigma^{\alpha\beta} C$ -type tetraquark states in Sect. 2; in Sect. 3, we derive the QCD sum rules for the masses and pole residues of the ground state and the first radial excited state of the  $C\sigma_{\alpha\beta} \otimes \sigma^{\alpha\beta} C$ -type tetraquark states; Sect. 4 is reserved for our conclusion.

### 2 QCD sum rules for the $C\sigma_{\alpha\beta} \otimes \sigma^{\alpha\beta} C$ -type tetraquark states without including the first radial excited states

In the following, we write down the two-point correlation functions  $\Pi_{\bar{s}s/\bar{d}u}(p)$  in the QCD sum rules:

$$\Pi_{\bar{s}s/\bar{d}u}(p) = i \int d^4x e^{ip \cdot x} \langle 0 | T \left\{ J_{\bar{s}s/\bar{d}u}(x) J_{\bar{s}s/\bar{d}u}^\dagger(0) \right\} | 0 \rangle, \tag{1}$$

where

$$\begin{aligned} J_{\bar{s}s}(x) &= \varepsilon^{ijk} \varepsilon^{imn} s_j^T(x) C \sigma_{\alpha\beta} c_k(x) \bar{s}_m(x) \sigma^{\alpha\beta} C \bar{c}_n^T(x), \\ J_{\bar{d}u}(x) &= \varepsilon^{ijk} \varepsilon^{imn} u_j^T(x) C \sigma_{\alpha\beta} c_k(x) \bar{d}_m(x) \sigma^{\alpha\beta} C \bar{c}_n^T(x), \end{aligned} \tag{2}$$

where the  $i, j, k, m, n$  are color indices,  $C$  is the charge conjugation matrix.

At the hadronic side, we insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators  $J_{\bar{s}s/\bar{d}u}(x)$  into the correlation functions  $\Pi_{\bar{s}s/\bar{d}u}(p)$  to obtain the hadronic representation [21–23]. After isolating the ground state contributions of the scalar  $c\bar{s}\bar{s}/c\bar{u}\bar{d}$  tetraquark states  $X_{\bar{s}s/\bar{d}u}$ , we get the results,

$$\Pi_{\bar{s}s/\bar{d}u}(p) = \frac{\lambda_{\bar{s}s/\bar{d}u}^2}{M_{\bar{s}s/\bar{d}u}^2 - p^2} + \dots, \tag{3}$$

where the pole residues  $\lambda_{\bar{s}s/\bar{d}u}$  are defined by  $\langle 0 | J_{\bar{s}s/\bar{d}u}(0) | X_{\bar{s}s/\bar{d}u}(p) \rangle = \lambda_{\bar{s}s/\bar{d}u}$ .

In the following, we briefly outline the operator product expansion for the correlation functions  $\Pi_{\bar{s}s/\bar{d}u}(p)$  in perturbative QCD. We contract the  $u, d, s$  and  $c$  quark fields in the correlation functions  $\Pi_{\bar{s}s/\bar{d}u}(p)$  with Wick theorem, and obtain the results:

$$\begin{aligned} \Pi_{\bar{s}s}(p) &= i \varepsilon^{ijk} \varepsilon^{imn} \varepsilon^{i'j'k'} \varepsilon^{i'm'n'} \int d^4x e^{ip \cdot x} \\ &\quad \times \text{Tr} \left[ \sigma_{\mu\nu} C^{kk'}(x) \sigma_{\alpha\beta} C S^{jj'T}(x) C \right] \end{aligned}$$

$$\times \text{Tr} \left[ \sigma^{\alpha\beta} C^{n'n}(-x) \sigma^{\mu\nu} C S^{m'mT}(-x) C \right],$$

$$\begin{aligned} \Pi_{\bar{d}u}(p) &= i \varepsilon^{ijk} \varepsilon^{imn} \varepsilon^{i'j'k'} \varepsilon^{i'm'n'} \int d^4x e^{ip \cdot x} \\ &\quad \times \text{Tr} \left[ \sigma_{\mu\nu} C^{kk'}(x) \sigma_{\alpha\beta} C U^{jj'T}(x) C \right] \\ &\quad \times \text{Tr} \left[ \sigma^{\alpha\beta} C^{n'n}(-x) \sigma^{\mu\nu} C D^{m'mT}(-x) C \right], \end{aligned} \tag{4}$$

where  $S_{ij}(x)$ ,  $U_{ij}(x)$ ,  $D_{ij}(x)$  and  $C_{ij}(x)$  are the full  $s, u, d$  and  $c$  quark propagators, respectively,

$$\begin{aligned} S_{ij}(x) &= \frac{i\delta_{ij} \not{x}}{2\pi^2 x^4} - \frac{\delta_{ij} m_s}{4\pi^2 x^2} - \frac{\delta_{ij} \langle \bar{s}s \rangle}{12} + \frac{i\delta_{ij} \not{x} m_s \langle \bar{s}s \rangle}{48} \\ &\quad - \frac{\delta_{ij} x^2 \langle \bar{s} g_s \sigma G_s \rangle}{192} + \frac{i\delta_{ij} x^2 \not{x} m_s \langle \bar{s} g_s \sigma G_s \rangle}{1152} \\ &\quad - \frac{i g_s G_{\alpha\beta}^a t_{ij}^a (\not{x} \sigma^{\alpha\beta} + \sigma^{\alpha\beta} \not{x})}{32\pi^2 x^2} \\ &\quad - \frac{i\delta_{ij} x^2 \not{x} g_s^2 \langle \bar{s}s \rangle^2}{7776} \\ &\quad - \frac{\delta_{ij} x^4 \langle \bar{s}s \rangle \langle g_s^2 G G \rangle}{27648} - \frac{1}{8} \langle \bar{s}_j \sigma^{\mu\nu} s_i \rangle \sigma_{\mu\nu} \\ &\quad - \frac{1}{4} \langle \bar{s}_j \gamma^\mu s_i \rangle \gamma_\mu + \dots, \end{aligned} \tag{5}$$

$$U/D_{ij}(x) = S_{ij}(x) |_{m_s \rightarrow 0}, \langle \bar{s}s \rangle \rightarrow \langle \bar{q}q \rangle, \langle \bar{s} g_s \sigma G_s \rangle \rightarrow \langle \bar{q} g_s \sigma G q \rangle, \dots, \tag{6}$$

$$\begin{aligned} C_{ij}(x) &= \frac{i}{(2\pi)^4} \int d^4k e^{-ik \cdot x} \left\{ \frac{\delta_{ij}}{\not{k} - m_c} \right. \\ &\quad - \frac{g_s G_{\alpha\beta}^n t_{ij}^n \sigma^{\alpha\beta} (\not{k} + m_c) + (\not{k} + m_c) \sigma^{\alpha\beta}}{4(k^2 - m_c^2)^2} \\ &\quad + \frac{g_s D_\alpha G_{\beta\lambda}^n t_{ij}^n (f^{\lambda\beta\alpha} + f^{\lambda\alpha\beta})}{3(k^2 - m_c^2)^4} \\ &\quad - \frac{g_s^2 (t^a t^b)_{ij} G_{\alpha\beta}^a G_{\mu\nu}^b (f^{\alpha\beta\mu\nu} + f^{\alpha\mu\beta\nu} + f^{\alpha\mu\nu\beta})}{4(k^2 - m_c^2)^5} \\ &\quad \left. + \dots \right\}, \end{aligned} \tag{7}$$

$$\begin{aligned} f^{\lambda\alpha\beta} &= (\not{k} + m_c) \gamma^\lambda (\not{k} + m_c) \gamma^\alpha (\not{k} + m_c) \gamma^\beta (\not{k} + m_c), \\ f^{\alpha\beta\mu\nu} &= (\not{k} + m_c) \gamma^\alpha (\not{k} + m_c) \gamma^\beta (\not{k} + m_c) \gamma^\mu (\not{k} + m_c) \\ &\quad \times \gamma^\nu (\not{k} + m_c), \end{aligned} \tag{8}$$

and  $t^n = \frac{\lambda^n}{2}$ , the  $\lambda^n$  is the Gell-Mann matrix,  $D_\alpha = \partial_\alpha - i g_s G_\alpha^n t^n$  [23]. Then we compute the integrals both in the coordinate space and the momentum space, and obtain the correlation functions  $\Pi_{\bar{s}s/\bar{d}u}(p)$  at the quark level, therefore the QCD spectral densities through dispersion relation. In this article, we calculate the contributions of the vacuum condensates up to dimension 10 in a consistent way, for technical details, one can consult Ref. [35].

Once the analytical QCD spectral densities are obtained, we take the quark-hadron duality below the continuum

thresholds  $s_{\bar{s}s/\bar{d}u}^0$  and perform Borel transform with respect to the variable  $P^2 = -p^2$  to obtain the QCD sum rules:

$$\lambda_{\bar{s}s/\bar{d}u}^2 \exp\left(-\frac{M_{\bar{s}s/\bar{d}u}^2}{T^2}\right) = \int_{4m_c^2}^{s_{\bar{s}s/\bar{d}u}^0} ds \rho_{\bar{s}s/\bar{d}u}(s) \exp\left(-\frac{s}{T^2}\right), \tag{9}$$

where

$$\begin{aligned} \rho_{\bar{s}s}(s) &= \rho_0(s) + \rho_3(s) + \rho_4(s) + \rho_5(s) + \rho_6(s) + \rho_7(s) \\ &\quad + \rho_8(s) + \rho_{10}(s), \\ \rho_{\bar{d}u}(s) &= \rho_{\bar{s}s}(s) \mid_{m_s \rightarrow 0, (\bar{s}s) \rightarrow (\bar{q}q), (\bar{s}g_s \sigma Gs) \rightarrow (\bar{q}g_s \sigma Gq)}, \end{aligned} \tag{10}$$

$$\begin{aligned} \rho_0(s) &= \frac{1}{64\pi^6} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz yz (1-y-z)^3 (s - \bar{m}_c^2)^2 \\ &\quad \times (7s^2 - 6s\bar{m}_c^2 + \bar{m}_c^4) \\ &\quad + \frac{1}{32\pi^6} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz yz (1-y-z)^2 (s - \bar{m}_c^2)^3 \\ &\quad \times (3s - \bar{m}_c^2), \end{aligned} \tag{11}$$

$$\begin{aligned} \rho_3(s) &= \frac{m_s \langle \bar{s}s \rangle}{2\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz yz (1-y-z) \\ &\quad \times (10s^2 - 12s\bar{m}_c^2 + 3\bar{m}_c^4) \\ &\quad + \frac{m_s \langle \bar{s}s \rangle}{\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz yz (s - \bar{m}_c^2) (2s - \bar{m}_c^2) \\ &\quad - \frac{3m_s m_c^2 \langle \bar{s}s \rangle}{\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz (s - \bar{m}_c^2), \end{aligned} \tag{12}$$

$$\begin{aligned} \rho_4(s) &= -\frac{m_c^2}{48\pi^4} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( \frac{z}{y^2} + \frac{y}{z^2} \right) \\ &\quad \times (1-y-z)^3 \left\{ 2s - \bar{m}_c^2 + \frac{s^2}{6} \delta(s - \bar{m}_c^2) \right\} \\ &\quad - \frac{m_c^2}{48\pi^4} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( \frac{z}{y^2} + \frac{y}{z^2} \right) \\ &\quad \times (1-y-z)^2 (3s - 2\bar{m}_c^2) \\ &\quad - \frac{1}{864\pi^4} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz (1-y-z)^3 \\ &\quad \times (10s^2 - 12s\bar{m}_c^2 + 3\bar{m}_c^4) \\ &\quad + \frac{7}{288\pi^4} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz (1-y-z)^2 \\ &\quad \times (s - \bar{m}_c^2) (2s - \bar{m}_c^2) \\ &\quad + \frac{7}{576\pi^4} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz (y+z) \end{aligned}$$

$$\begin{aligned} &\times (1-y-z)^2 (10s^2 - 12s\bar{m}_c^2 + 3\bar{m}_c^4) \\ &\quad - \frac{1}{72\pi^4} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz (y+z) \\ &\quad \times (1-y-z) (s - \bar{m}_c^2) (2s - \bar{m}_c^2) \\ &\quad - \frac{1}{144\pi^4} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz yz (1-y-z) \\ &\quad \times (10s^2 - 12s\bar{m}_c^2 + 3\bar{m}_c^4) \\ &\quad + \frac{7}{144\pi^4} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz yz (s - \bar{m}_c^2) \\ &\quad \times (2s - \bar{m}_c^2), \end{aligned} \tag{13}$$

$$\begin{aligned} \rho_5(s) &= -\frac{m_s \langle \bar{s}g_s \sigma Gs \rangle}{2\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz yz \\ &\quad \times \left\{ 2s - \bar{m}_c^2 + \frac{s^2}{6} \delta(s - \bar{m}_c^2) \right\} \\ &\quad - \frac{m_s \langle \bar{s}g_s \sigma Gs \rangle}{6\pi^4} \int_{y_i}^{y_f} dy y(1-y) (3s - 2\bar{m}_c^2) \\ &\quad + \frac{3m_s m_c^2 \langle \bar{s}g_s \sigma Gs \rangle}{4\pi^4} \int_{y_i}^{y_f} dy \\ &\quad - \frac{m_s m_c^2 \langle \bar{s}g_s \sigma Gs \rangle}{24\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( \frac{1}{y} + \frac{1}{z} \right), \end{aligned} \tag{14}$$

$$\begin{aligned} \rho_6(s) &= \frac{2m_c^2 \langle \bar{s}s \rangle^2}{\pi^2} \int_{y_i}^{y_f} dy + \frac{2g_s^2 \langle \bar{s}s \rangle^2}{27\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz yz \\ &\quad \times \left\{ 2s - \bar{m}_c^2 + \frac{s^2}{6} \delta(s - \bar{m}_c^2) \right\} \\ &\quad + \frac{2g_s^2 \langle \bar{s}s \rangle^2}{81\pi^4} \int_{y_i}^{y_f} dy y(1-y) (3s - 2\bar{m}_c^2) \\ &\quad - \frac{g_s^2 \langle \bar{s}s \rangle^2}{162\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz (1-y-z) \\ &\quad \times \left\{ \frac{10}{3} \left( \frac{z}{y^2} + \frac{y}{z^2} \right) m_c^2 [2 + s \delta(s - \bar{m}_c^2)] \right. \\ &\quad \left. + 28(y+z) \left[ 2s - \bar{m}_c^2 + \frac{s^2}{6} \delta(s - \bar{m}_c^2) \right] \right. \\ &\quad \left. + 9 \left( \frac{z}{y} + \frac{y}{z} \right) (3s - 2\bar{m}_c^2) \right\}, \end{aligned} \tag{15}$$

$$\begin{aligned} \rho_7(s) &= -\frac{m_s m_c^2 \langle \bar{s}s \rangle}{18\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_0^1 dy \int_0^{1-y} dz \left( \frac{z}{y^2} + \frac{y}{z^2} \right) \\ &\quad \times (1-y-z) \left( 1 + \frac{s}{T^2} + \frac{s^2}{2T^4} \right) \delta(s - \bar{m}_c^2) \\ &\quad - \frac{m_s m_c^2 \langle \bar{s}s \rangle}{18\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_0^1 dy \int_0^{1-y} dz \left( \frac{z}{y^2} + \frac{y}{z^2} \right) \end{aligned}$$

$$\begin{aligned}
 & \times \left(1 + \frac{s}{T^2}\right) \delta\left(s - \bar{m}_c^2\right) \\
 & + \frac{m_s m_c^4 \langle \bar{s}s \rangle}{6\pi^2 T^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_0^1 dy \int_0^{1-y} dz \left(\frac{1}{y^3} + \frac{1}{z^3}\right) \\
 & \times \delta\left(s - \bar{m}_c^2\right) \\
 & - \frac{m_s m_c^2 \langle \bar{s}s \rangle}{2\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_0^1 dy \int_0^{1-y} dz \left(\frac{1}{y^2} + \frac{1}{z^2}\right) \\
 & \times \delta\left(s - \bar{m}_c^2\right) \\
 & - \frac{m_s \langle \bar{s}s \rangle}{18\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz (1 - y - z) \\
 & \times \left\{1 + \left(\frac{2s}{3} + \frac{s^2}{6T^2}\right) \delta\left(s - \bar{m}_c^2\right)\right\} \\
 & + \frac{m_s \langle \bar{s}s \rangle}{9\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_0^1 dy \left\{1 + \frac{s}{2} \delta\left(s - \tilde{m}_c^2\right)\right\} \\
 & + \frac{7m_s \langle \bar{s}s \rangle}{72\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz (y + z) \\
 & \times \left\{1 + \left(\frac{2s}{3} + \frac{s^2}{6T^2}\right) \delta\left(s - \bar{m}_c^2\right)\right\} \\
 & - \frac{2m_s m_c^2 \langle \bar{s}s \rangle}{9\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} \\
 & \times dz \frac{1}{yz} \delta\left(s - \bar{m}_c^2\right) \\
 & - \frac{m_s m_c^2 \langle \bar{s}s \rangle}{12\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_0^1 dy \left(1 + \frac{s}{T^2}\right) \\
 & \times \delta\left(s - \tilde{m}_c^2\right), \tag{16}
 \end{aligned}$$

$$\begin{aligned}
 \rho_8(s) &= \frac{\langle \bar{s}s \rangle \langle \bar{s}g_s \sigma Gs \rangle}{18\pi^2} \int_0^1 dy s \delta\left(s - \tilde{m}_c^2\right) \\
 & - \frac{m_c^2 \langle \bar{s}s \rangle \langle \bar{s}g_s \sigma Gs \rangle}{\pi^2} \int_0^1 dy \left(1 + \frac{s}{T^2}\right) \delta\left(s - \tilde{m}_c^2\right), \tag{17}
 \end{aligned}$$

$$\begin{aligned}
 \rho_{10}(s) &= \frac{m_c^2 \langle \bar{s}g_s \sigma Gs \rangle^2}{8\pi^2 T^6} \int_0^1 dy s^2 \delta\left(s - \tilde{m}_c^2\right) \\
 & - \frac{m_c^4 \langle \bar{s}s \rangle^2}{9T^4} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_0^1 dy \left\{\frac{1}{y^3} + \frac{1}{(1-y)^3}\right\} \\
 & \times \delta\left(s - \tilde{m}_c^2\right) \\
 & + \frac{m_c^2 \langle \bar{s}s \rangle^2}{3T^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_0^1 dy \left\{\frac{1}{y^2} + \frac{1}{(1-y)^2}\right\} \\
 & \times \delta\left(s - \tilde{m}_c^2\right) \\
 & + \frac{4 \langle \bar{s}s \rangle^2}{27T^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_0^1 dy s \delta\left(s - \tilde{m}_c^2\right) \\
 & + \frac{11 \langle \bar{s}g_s \sigma Gs \rangle^2}{36\pi^2 T^2} \int_0^1 dy s \delta\left(s - \tilde{m}_c^2\right)
 \end{aligned}$$

**Table 1** The input parameters in the QCD sum rules, the values in the bracket denote the energy scales  $\mu = 1, 2 \text{ GeV}$  and  $m_c$ , respectively

Parameters	Values
$\langle \bar{q}q \rangle (1 \text{ GeV})$	$-(0.24 \pm 0.01 \text{ GeV})^3$ [21–23,36]
$\langle \bar{s}s \rangle (1 \text{ GeV})$	$(0.8 \pm 0.1) \langle \bar{q}q \rangle (1 \text{ GeV})$ [21–23,36]
$\langle \bar{q}g_s \sigma Gq \rangle (1 \text{ GeV})$	$m_0^2 \langle \bar{q}q \rangle (1 \text{ GeV})$ [21–23,36]
$\langle \bar{s}g_s \sigma Gs \rangle (1 \text{ GeV})$	$m_0^2 \langle \bar{s}s \rangle (1 \text{ GeV})$ [21–23,36]
$m_0^2 (1 \text{ GeV})$	$(0.8 \pm 0.1) \text{ GeV}^2$ [21–23,36]
$\langle \frac{\alpha_s GG}{\pi} \rangle$	$(0.33 \text{ GeV})^4$ [21–23,36]
$m_c(m_c)$	$(1.275 \pm 0.025) \text{ GeV}$ [37]
$m_s(2 \text{ GeV})$	$(0.095 \pm 0.005) \text{ GeV}$ [37]

$$\begin{aligned}
 & - \frac{\langle \bar{s}g_s \sigma Gs \rangle^2}{72\pi^2 T^4} \int_0^1 dy s^2 \delta\left(s - \tilde{m}_c^2\right) \\
 & + \frac{m_c^2 \langle \bar{s}s \rangle^2}{9T^6} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_0^1 dy s^2 \delta\left(s - \tilde{m}_c^2\right), \tag{18}
 \end{aligned}$$

$y_f = \frac{1 + \sqrt{1 - 4m_c^2/s}}{2}$ ,  $y_i = \frac{1 - \sqrt{1 - 4m_c^2/s}}{2}$ ,  $z_i = \frac{ym_c^2}{ys - m_c^2}$ ,  
 $\bar{m}_c^2 = \frac{(y+z)m_c^2}{yz}$ ,  $\tilde{m}_c^2 = \frac{m_c^2}{y(1-y)}$ ,  $\int_{y_i}^{y_f} dy \rightarrow \int_0^1 dy$ ,  $\int_{z_i}^{1-y} dz \rightarrow \int_0^{1-y} dz$ , where the  $\delta$  functions  $\delta(s - \bar{m}_c^2)$  and  $\delta(s - \tilde{m}_c^2)$  appear.

We differentiate Eq. (9) with respect to  $\frac{1}{T^2}$ , then eliminate the pole residues  $\lambda_{\bar{s}s/\bar{d}u}$ , and obtain the QCD sum rules for the ground state masses  $M_{\bar{s}s/\bar{d}u}$  of the  $C\sigma_{\alpha\beta} \otimes \sigma^{\alpha\beta} C$ -type scalar hidden-charm tetraquark states,

$$M_{\bar{s}s/\bar{d}u}^2 = - \frac{\int_{4m_c^2}^{s_{\bar{s}s/\bar{d}u}^0} ds \frac{d}{d(1/T^2)} \rho_{\bar{s}s/\bar{d}u}(s) \exp\left(-\frac{s}{T^2}\right)}{\int_{4m_c^2}^{s_{\bar{s}s/\bar{d}u}^0} ds \rho_{\bar{s}s/\bar{d}u}(s) \exp\left(-\frac{s}{T^2}\right)}. \tag{19}$$

The input parameters are shown explicitly in Table 1. The quark condensates, mixed quark condensates, and  $\overline{MS}$  masses evolve with the renormalization group equation; we take into account the energy-scale dependence according to the following equations:

$$\begin{aligned}
 \langle \bar{q}q \rangle(\mu) &= \langle \bar{q}q \rangle(Q) \left[ \frac{\alpha_s(Q)}{\alpha_s(\mu)} \right]^{\frac{4}{9}}, \\
 \langle \bar{s}s \rangle(\mu) &= \langle \bar{s}s \rangle(Q) \left[ \frac{\alpha_s(Q)}{\alpha_s(\mu)} \right]^{\frac{4}{9}}, \\
 \langle \bar{q}g_s \sigma Gq \rangle(\mu) &= \langle \bar{q}g_s \sigma Gq \rangle(Q) \left[ \frac{\alpha_s(Q)}{\alpha_s(\mu)} \right]^{\frac{27}{27}}, \\
 \langle \bar{s}g_s \sigma Gs \rangle(\mu) &= \langle \bar{s}g_s \sigma Gs \rangle(Q) \left[ \frac{\alpha_s(Q)}{\alpha_s(\mu)} \right]^{\frac{27}{27}}, \\
 m_c(\mu) &= m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{12}{25}}
 \end{aligned}$$

$$m_s(\mu) = m_s(2\text{GeV}) \left[ \frac{\alpha_s(\mu)}{\alpha_s(2\text{GeV})} \right]^{\frac{4}{9}},$$

$$\alpha_s(\mu) = \frac{1}{b_0 t} \left[ 1 - \frac{b_1 \log t}{b_0^2 t} + \frac{b_1^2(\log^2 t - \log t - 1) + b_0 b_2}{b_0^4 t^2} \right], \tag{20}$$

where  $t = \log \frac{\mu^2}{\Lambda^2}$ ,  $b_0 = \frac{33-2n_f}{12\pi}$ ,  $b_1 = \frac{153-19n_f}{24\pi^2}$ ,  $b_2 = \frac{2857 - \frac{5033}{9}n_f + \frac{325}{27}n_f^2}{128\pi^3}$ ,  $\Lambda = 213, 296$  and  $339$  MeV for the flavors  $n_f = 5, 4$  and  $3$ , respectively [37]. Furthermore, we set  $m_u = m_d = 0$ .

In the diquark–antidiquark type tetraquark system  $Qq\bar{Q}\bar{q}'$ , the  $Q$ -quark serves as a static well potential and combines with the light quark  $q$  to form a heavy diquark  $\mathcal{D}$  in color antitriplet, while the  $\bar{Q}$ -quark serves as another static well potential and combines with the light antiquark  $\bar{q}'$  to form a heavy antidiquark  $\bar{\mathcal{D}}$  in color triplet; the  $\mathcal{D}$  and  $\bar{\mathcal{D}}$  combine to form a compact tetraquark state [16–18, 35, 38, 39]. For such diquark–antidiquark type tetraquark systems, we suggest an energy-scale formula  $\mu = \sqrt{M_{X/Y/Z}^2 - (2\mathbb{M}_Q)^2}$  to determine the energy scales of the QCD spectral densities, where the  $X, Y$  and  $Z$  are the hidden-charm or hidden-bottom tetraquark states  $Qq\bar{Q}\bar{q}'$ , the  $\mathbb{M}_Q$  are the effective heavy quark masses. In this article, we choose the updated value  $\mathbb{M}_c = 1.82$  GeV [40].

Now we search for the Borel parameters  $T^2$  and continuum threshold parameters  $s_{\bar{s}s/\bar{d}u}^0$  according to the four criteria:

1. Pole dominance at the hadron side;
2. Convergence of the operator product expansion;
3. Appearance of the Borel platforms;
4. Satisfying the energy scale formula.

We cannot obtain reasonable Borel parameters  $T^2$  and continuum threshold parameters  $s_{\bar{s}s/\bar{d}u}^0$ , if the energy gap between the ground state and the first radial excited state is about  $0.3 - 0.7$  GeV.

### 3 QCD sum rules for the $C\sigma_{\alpha\beta} \otimes \sigma^{\alpha\beta}C$ -type tetraquark states including the first radial excited states

Now we take into account both the ground state contribution and the first radial excited state contribution at the hadronic side of the QCD sum rules [41]. First of all, we introduce the notations  $\tau = \frac{1}{T^2}$ ,  $D^n = \left(-\frac{d}{d\tau}\right)^n$ , and use the subscripts 1 and 2 to denote the ground state and the first radial excited state of the  $C\sigma_{\alpha\beta} \otimes \sigma^{\alpha\beta}C$ -type tetraquark states, respectively. Then the QCD sum rules can be written as

$$\lambda_1^2 \exp(-\tau M_1^2) + \lambda_2^2 \exp(-\tau M_2^2) = \Pi_{\text{QCD}}(\tau); \tag{21}$$

the subscript QCD denotes the QCD side of the correlation functions  $\Pi_{\bar{s}s/\bar{d}u}(\tau)$ . We differentiate both sides of the QCD sum rules in Eq. (21) with respect to  $\tau$  and obtain

$$\lambda_1^2 M_1^2 \exp(-\tau M_1^2) + \lambda_2^2 M_2^2 \exp(-\tau M_2^2) = D\Pi_{\text{QCD}}(\tau). \tag{22}$$

Then we solve the two equations and obtain the QCD sum rules,

$$\lambda_i^2 \exp(-\tau M_i^2) = \frac{(D - M_j^2) \Pi_{\text{QCD}}(\tau)}{M_i^2 - M_j^2}, \tag{23}$$

where  $i \neq j$ . We differentiate both sides of the QCD sum rules in Eq. (23) with respect to  $\tau$  and obtain

$$M_i^2 = \frac{(D^2 - M_j^2 D) \Pi_{\text{QCD}}(\tau)}{(D - M_j^2) \Pi_{\text{QCD}}(\tau)},$$

$$M_i^4 = \frac{(D^3 - M_j^2 D^2) \Pi_{\text{QCD}}(\tau)}{(D - M_j^2) \Pi_{\text{QCD}}(\tau)}. \tag{24}$$

The squared masses  $M_i^2$  satisfy the following equation:

$$M_i^4 - bM_i^2 + c = 0, \tag{25}$$

where

$$b = \frac{D^3 \otimes D^0 - D^2 \otimes D}{D^2 \otimes D^0 - D \otimes D},$$

$$c = \frac{D^3 \otimes D - D^2 \otimes D^2}{D^2 \otimes D^0 - D \otimes D},$$

$$D^j \otimes D^k = D^j \Pi_{\text{QCD}}(\tau) D^k \Pi_{\text{QCD}}(\tau), \tag{26}$$

$i = 1, 2, j, k = 0, 1, 2, 3$ . We solve Eq. (25) and obtain two solutions,

$$M_1^2 = \frac{b - \sqrt{b^2 - 4c}}{2}, \tag{27}$$

$$M_2^2 = \frac{b + \sqrt{b^2 - 4c}}{2}. \tag{28}$$

The ground state contributions are separated from the first radial excited state contributions unambiguously, and we obtain the QCD sum rules for the ground states and the first radial excited states, respectively.

Again, we search for the Borel parameters  $T^2$  and continuum threshold parameters  $s_{\bar{s}s/\bar{d}u}^0$  according to the four criteria:

1. Pole dominance at the hadron side;
2. Convergence of the operator product expansion;
3. Appearance of the Borel platforms;
4. Satisfying the energy scale formula.

The resulting Borel parameters  $T^2$  and continuum threshold parameters  $s_{ss/\bar{d}u}^0$  are

$$\begin{aligned} X_{\bar{d}u} : T^2 &= (2.0 - 2.4) \text{ GeV}^2, \quad s_{\bar{d}u}^0 = (4.8 \pm 0.1 \text{ GeV})^2, \\ X_{\bar{s}s} : T^2 &= (2.1 - 2.5) \text{ GeV}^2, \quad s_{\bar{s}s}^0 = (4.8 \pm 0.1 \text{ GeV})^2. \end{aligned} \tag{29}$$

The pole contributions are

$$\begin{aligned} X_{\bar{d}u}^-(1S + 2S) : \text{pole} &= (68 - 89)\% \text{ at } \mu = 1.20 \text{ GeV}, \\ X_{\bar{d}u}^-(1S + 2S) : \text{pole} &= (79 - 95)\% \text{ at } \mu = 2.45 \text{ GeV}, \end{aligned} \tag{30}$$

$$\begin{aligned} X_{\bar{s}s}^-(1S + 2S) : \text{pole} &= (64 - 87)\% \text{ at } \mu = 1.20 \text{ GeV}, \\ X_{\bar{s}s}^-(1S + 2S) : \text{pole} &= (76 - 93)\% \text{ at } \mu = 2.50 \text{ GeV}, \end{aligned} \tag{31}$$

the pole dominance condition is well satisfied, the criterion 1 is satisfied. The contributions come from the vacuum condensates of dimension 10  $D_{10}$  are

$$\begin{aligned} X_{\bar{d}u}^-(1S + 2S) : D_{10} &= (9 - 25)\% \text{ at } \mu = 1.20 \text{ GeV}, \\ X_{\bar{d}u}^-(1S + 2S) : D_{10} &= (4 - 11)\% \text{ at } \mu = 2.45 \text{ GeV}, \end{aligned} \tag{32}$$

$$\begin{aligned} X_{\bar{s}s}^-(1S + 2S) : D_{10} &= (5 - 14)\% \text{ at } \mu = 1.20 \text{ GeV}, \\ X_{\bar{s}s}^-(1S + 2S) : D_{10} &= (2 - 6)\% \text{ at } \mu = 2.50 \text{ GeV}, \end{aligned} \tag{33}$$

for the central values of the continuum threshold parameters, the operator product expansion is convergent, the criterion 2 is satisfied.

Now we take into account the uncertainties of all the input parameters, and obtain the masses and pole residues of the  $C\sigma_{\alpha\beta} \otimes \sigma^{\alpha\beta} C$ -type tetraquark states,

$$\begin{aligned} M_{\bar{d}u,1S} &= 3.82 \pm 0.16 \text{ GeV}, \\ M_{\bar{s}s,1S} &= 3.84 \pm 0.16 \text{ GeV}, \\ \lambda_{\bar{d}u,1S} &= (5.20 \pm 1.35) \times 10^{-2} \text{ GeV}^5, \\ \lambda_{\bar{s}s,1S} &= (4.87 \pm 1.25) \times 10^{-2} \text{ GeV}^5, \end{aligned} \tag{34}$$

at the energy scale  $\mu = 1.20 \text{ GeV}$ ,

$$\begin{aligned} M_{\bar{d}u,2S} &= 4.38 \pm 0.09 \text{ GeV}, \\ \lambda_{\bar{d}u,2S} &= (2.12 \pm 0.31) \times 10^{-1} \text{ GeV}^5, \end{aligned} \tag{35}$$

at the energy scale  $\mu = 2.45 \text{ GeV}$ ,

$$\begin{aligned} M_{\bar{s}s,2S} &= 4.40 \pm 0.09 \text{ GeV}, \\ \lambda_{\bar{s}s,2S} &= (2.14 \pm 0.33) \times 10^{-1} \text{ GeV}^5, \end{aligned} \tag{36}$$

at the energy scale  $\mu = 2.50 \text{ GeV}$ . The central values of the predicted masses satisfy the energy-scale formula, the criterion 4 is satisfied.

In Fig. 1, we plot the predicted masses  $M_{\bar{d}u/\bar{s}s}$  with variations of the Borel parameters  $T^2$ . From the figure, we can see that the plateaus are rather flat, the criterion 3 is satisfied.

The four criteria are all satisfied, we expect to make reliable predictions.

The energy gaps between the ground states and the first radial excited states are

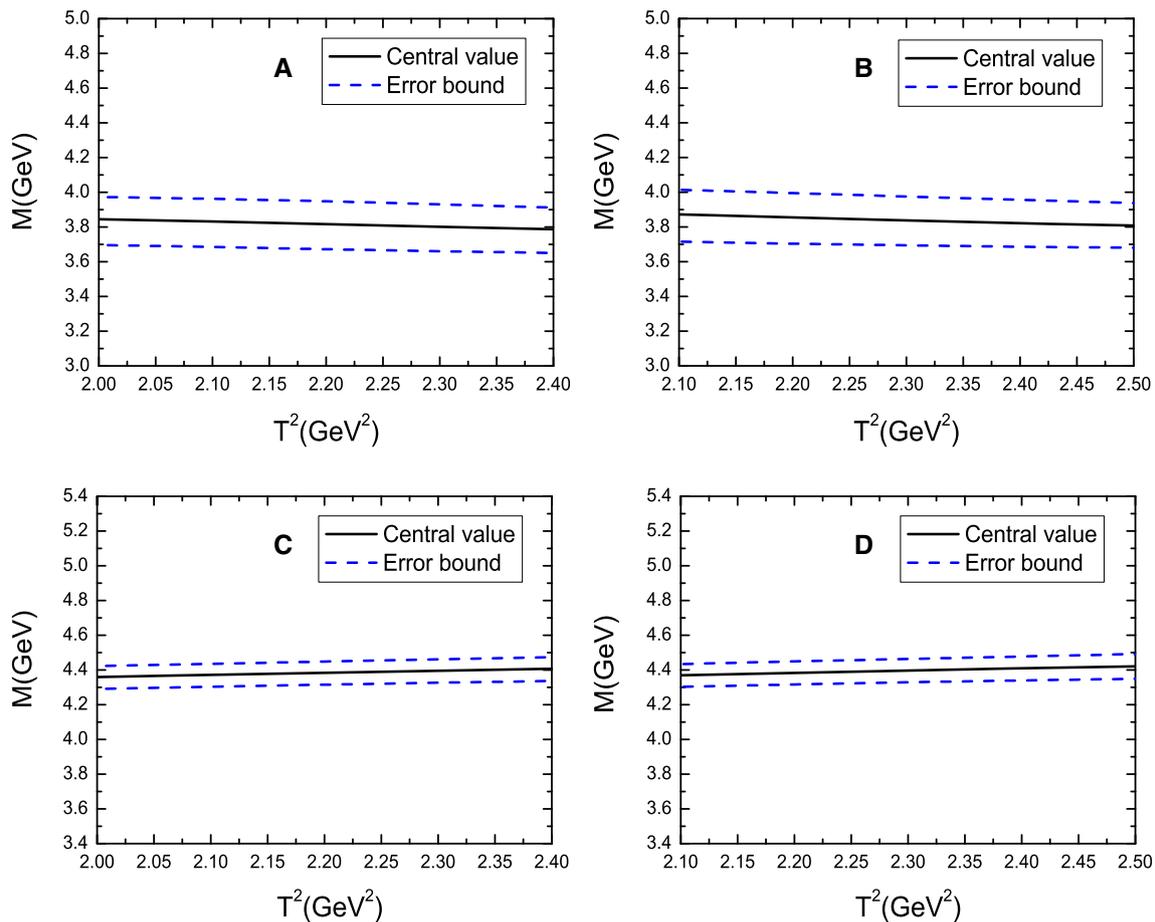
$$\begin{aligned} M_{\bar{d}u,2S} - M_{\bar{d}u,1S} &= 0.56 \text{ GeV}, \\ M_{\bar{s}s,2S} - M_{\bar{s}s,1S} &= 0.56 \text{ GeV}. \end{aligned} \tag{37}$$

$Z(4430)$  is assigned to be the first radial excitation of  $Z_c(3900)$  according to the analogous decays,

$$\begin{aligned} Z_c(3900)^\pm &\rightarrow J/\psi\pi^\pm, \\ Z(4430)^\pm &\rightarrow \psi'\pi^\pm, \end{aligned} \tag{38}$$

and the mass differences  $M_{Z(4430)} - M_{Z_c(3900)} = 576 \text{ MeV}$  and  $M_{\psi'} - M_{J/\psi} = 589 \text{ MeV}$  [42–44]. The energy gaps  $M_{\bar{d}u,2S} - M_{\bar{d}u,1S}$ ,  $M_{\bar{s}s,2S} - M_{\bar{s}s,1S}$ ,  $M_{Z(4430)} - M_{Z_c(3900)}$  are compatible with each other. The widths  $\Gamma_{Z_c(3900)} = 46 \pm 10 \pm 20 \text{ MeV}$  [45] and  $\Gamma_{Z(4430)} = 172 \pm 13_{-34}^{+37} \text{ MeV}$  [46] are not broad, the QCD sum rules for the ground state  $Z_c(3900)$  alone or without including the first radial excited state  $Z(4430)$  work well [35]. If both the ground state  $Z_c(3900)$  and the first radial excited state  $Z(4430)$  are included in, the continuum threshold parameter  $\sqrt{s_0} = 4.8 \pm 0.1 \text{ GeV} = M_{Z(4430)} + (0.2 \sim 0.4) \text{ GeV}$ , the lower bound of the  $\sqrt{s_0} - M_{Z(4430)}$  is about 0.2 GeV, which is large enough to take into account the contribution of the  $Z(4430)$  [44]. In the present case, the lower bound of the  $\sqrt{s_0} - M_{\bar{d}u/\bar{s}s,2S}$  are about 0.3 GeV, which indicates that the widths of the first radial excited states of the  $C\sigma_{\alpha\beta} \otimes \sigma^{\alpha\beta} C$ -type tetraquark states are rather large. According to the energy gaps between the ground states and the first radial excited states, the continuum threshold parameters should be chosen as large as  $\sqrt{s_0} - M_{\bar{d}u/\bar{s}s,1S} = 0.5 \pm 0.1 \text{ GeV}$  without including the first radial excited states  $X_{\bar{d}u/\bar{s}s,2S}$  explicitly, however, for such large continuum thresholds, the contributions of the  $X_{\bar{d}u/\bar{s}s,2S}$  are already included in due to their large widths. So the QCD sum rules in which only the ground state  $C\sigma_{\alpha\beta} \otimes \sigma^{\alpha\beta} C$ -type tetraquark states are taken into account cannot work.

The predicted mass  $M_{\bar{s}s,1S} = 3.84 \pm 0.16 \text{ GeV}$  overlaps with the experimental value  $M_{X(3915)} = 3918.4 \pm 1.9 \text{ MeV}$  slightly [37], the  $X(3915)$  cannot be a pure  $C\sigma_{\alpha\beta} \otimes \sigma^{\alpha\beta} C$ -type  $cs\bar{c}\bar{s}$  tetraquark state. The predicted mass  $M_{\bar{s}s,2S} = 4.40 \pm 0.09 \text{ GeV}$  overlaps with the experimental value  $M_{X(4500)} = 4506 \pm 11_{-15}^{+12} \text{ MeV}$  slightly [25, 26], the  $X(4500)$  cannot be a pure  $C\sigma_{\alpha\beta} \otimes \sigma^{\alpha\beta} C$ -type  $cs\bar{c}\bar{s}$  tetraquark state. As the central values of the  $M_{\bar{s}s,1S}$  and  $M_{\bar{s}s,2S}$  differ from the central values of the  $M_{X(3915)}$  and  $M_{X(4500)}$  significantly, it is difficult to assign the  $M_{X(3915)}$  and  $M_{X(4500)}$  to be the  $C\sigma_{\alpha\beta} \otimes \sigma^{\alpha\beta} C$ -type  $cs\bar{c}\bar{s}$  tetraquark states. The  $X_{\bar{s}s,1S}$  and  $X_{\bar{s}s,2S}$  are new particles, the present predictions can be confronted to the experimental data in the future.



**Fig. 1** The masses with variations of the Borel parameters, where the **a**, **b**, **c** and **d** denote the  $X_{\bar{d}u,1S}$ ,  $X_{\bar{s}s,1S}$ ,  $X_{\bar{d}u,2S}$  and  $X_{\bar{s}s,2S}$ , respectively

## 4 Conclusion

In this article, we study the ground states and the first radial excited states of the  $C\sigma_{\alpha\beta} \otimes \sigma^{\alpha\beta}C$ -type hidden-charm tetraquark states with the QCD sum rules by calculating the contributions of the vacuum condensates up to dimension 10 in a consistent way. We separate the ground state contributions from the first radial excited state contributions unambiguously, and obtain the QCD sum rules for the ground states and the first radial excited states, respectively. Then we search for the Borel parameters and continuum threshold parameters according to the four criteria: (1) pole dominance at the hadron side; (2) convergence of the operator product expansion; (3) appearance of the Borel platforms; (4) satisfying the energy-scale formula. Finally, we obtain the masses and pole residues of the  $C\sigma_{\alpha\beta} \otimes \sigma^{\alpha\beta}C$ -type hidden-charm tetraquark states. The masses can be confronted to the experimental data in the future, while the pole residues can be used to study the relevant processes with the three-point QCD sum rules or the light-cone QCD sum rules.

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