

On thermodynamics of AdS black holes in M-theory

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Abstract Motivated by recent work on asymptotically AdS₄ black holes in M-theory, we investigate the thermodynamics and thermodynamical geometry of AdS black holes from M2- and M5-branes. Concretely, we consider AdS black holes in $AdS_{p+2} \times S^{11-p-2}$, where $p = 2, 5$ by interpreting the number of M2- (and M5-branes) as a thermodynamical variable. More precisely, we study the corresponding phase transition to examine their stabilities by calculating and discussing various thermodynamical quantities including the chemical potential. Then we compute the thermodynamical curvatures from the Quevedo metric for M2- and M5-branes geometries to reconsider the stability of such black holes. The Quevedo metric singularities recover similar stability results provided by the phase-transition program. It has been shown that similar behaviors are also present in the limit of large N .

1 Introduction

Recently, increasing interest has been shown in the study of black hole physics in connection with many subjects including string theory and famous thermodynamical models. The field has been explored to develop deeper relationships between the gravity theories and the thermodynamical physics using anti-de Sitter geometries. In this issue, the laws of black holes can be identified with the thermodynamic ones [1–5]. More precisely, the phase transition and the critical phenomena for various AdS black holes have been extensively investigated using different approaches [6–10]. In this way, certain equations of state, describing rotating black holes, have been identified with some known thermodynamical ones. In particular, serious efforts have been made to discuss the behavior of the Gibbs free energy in the fixed charge ensemble. This program has led to a nice interplay between the behavior of the AdS black hole systems and the

van der Waals fluids [11–18]. In fact, it has been shown that P–V criticality, the Gibbs free energy, the first order phase transition, and the behavior near the critical points can be associated with liquid–gas systems.

More recently, special focus has been put on the thermodynamics and thermodynamical geometry for a five-dimensional AdS black hole in the type IIB superstring background known as $AdS_5 \times S^5$ [19–21]. It is recalled that this geometry has been studied in many places in connection with AdS/CFT correspondence, providing a nice equivalence between gravitational theories in d-dimensional AdS geometries and conformal field theories (CFT) in a (d-1)-dimensional boundary of such AdS spaces [22–25]. In such black hole activities, the number of colors has been interpreted as a thermodynamical variable. In particular, the thermodynamic properties of black holes in $AdS_5 \times S^5$ have been investigated by considering the cosmological constant in the bulk as the number of colors. In fact, many thermodynamical quantities have been computed to discuss the stability behaviors of such black holes.

Motivated by these activities and recent work on asymptotically AdS₄ black holes in M-theory [26–29], we investigate the thermodynamics and thermodynamical geometry of AdS black holes from the physics of M2- and M5-branes. Concretely, we study AdS black holes in $AdS_{p+2} \times S^{11-p-2}$, where $p = 2, 5$ by viewing the number of M2- and M5-branes as a thermodynamical variable. To discuss the stability of such solutions, we examine first the corresponding phase transition by computing the relevant quantities including the chemical potential. Then we calculate the thermodynamical curvatures from the Quevedo metric for M2- and M5-brane geometries to reconsider the study of the stability.

The paper is organized as follows. We discuss the thermodynamic properties and the stability of the black holes in $AdS_{p+2} \times S^{11-p-2}$, where $p = 2, 5$ by viewing the number of M2- and M5-branes as a thermodynamical variable

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in Sects. 2 and 3. Similar results, which have been recovered using thermodynamical curvature calculations associated with the Quevedo metric, are presented in Sect. 4. The last section is devoted to the conclusions.

2 Thermodynamics of black holes in $AdS_4 \times S^7$ space

In this section, we investigate the phase transition of the AdS black holes in M-theory in the presence of solitonic objects. It is recalled that, at lower energy, M-theory describes an eleven-dimensional supergravity. This theory, which was proposed by Witten, can produce some non-perturbative limits of superstring models after its compactification on particular geometries [30].

It has been shown that the corresponding eleven-dimensional supergravity involves a cubic R^4 one-loop UV divergence [31], which has been obtained using a specific cutoff motivated by string theory [32,33]. This calculation gives the following correction of the Einstein action:

$$I = - \int d^{11}x \sqrt{g} \left(\frac{1}{2\kappa_{11}} R + \frac{1}{\kappa_{11}^{2/3}} \zeta W + \dots \right), \quad (1)$$

where κ_{11} is related to the Planck length by $\kappa_{11}^2 = 2^4 \pi^5 \ell_{11}^2$ and $\zeta = \frac{2\pi^2}{3}$. W can be given in terms of the Ricci tensor as follows: $W \sim RRRR$ [34,35]. Roughly speaking, M-theory contains two fundamental objects called M2- and M5-branes coupled in eleven dimensions to 3- and 6-forms, respectively. The near horizon of such black objects is defined by the product of AdS spaces and spheres

$$AdS_{p+2} \times S^{11-p-2}, \quad p = 2, 5. \quad (2)$$

To start, let us consider the case of M2-brane. The corresponding geometry is $AdS_4 \times S^7$. In such a geometric background, the line element of the black M2-brane metric is given by [34,36]

$$ds^2 = \frac{r^4}{L^4} \left(-f dt^2 + \sum_{i=1}^2 dx_i^2 \right) + \frac{L^2}{r^2} f^{-1} dr^2 + L^2 d\Omega_7^2, \quad (3)$$

where $d\Omega_7^2$ is the metric of a seven-dimensional sphere with unit radius. In this solution, the metric function reads as follows:

$$f = 1 - \frac{m}{r} + \frac{r^2}{L^2}, \quad (4)$$

where L is the AdS radius and m is an integration constant. The cosmological constant is $\Lambda = -6/L^2$. From the M-theory point of view, the eleven-dimensional spacetime Eq. (3) can be interpreted as the near horizon geometry of N coincident configurations of M2-branes. In this background,

the AdS radius L is linked to the M2-brane number N via the relation [34,37]

$$L^9 = N^3 / 2 \frac{\kappa_{11}^2 \sqrt{2}}{\pi^5}. \quad (5)$$

According to the proposition reported in [19–21], we consider the cosmological constant as the number of coincident M2-branes in the M-theory background and its conjugate quantity as the associated chemical potential.

The event horizon r_h of the corresponding black hole is determined by solving the equation $f = 0$. Exploring Eq. (4), the mass of the black hole can be written as¹

$$M_4 = \frac{m\omega_2}{8\pi\mathcal{G}_4} = \frac{r\omega_2(L^2 + r^2)}{8\pi\mathcal{G}_4 L^2}. \quad (6)$$

The Bekenstein–Hawking entropy formula of the black hole produces

$$S = \frac{A}{4\mathcal{G}_4} = \frac{\omega_2 r^2}{4\mathcal{G}_4}. \quad (7)$$

It is recalled that the four-dimensional Newton gravitational constant is related to the eleven-dimensional one by

$$\mathcal{G}_4 = \frac{3\mathcal{G}_{11}}{2\pi\omega_2 L^4}. \quad (8)$$

For simplicity reasons, we take in the rest of this paper $\mathcal{G}_{11} = \kappa_{11} = 1$. In this way, the mass of the black hole can be expressed as a function of N and S ,

$$M_4(S, N) = \frac{\sqrt{S} (16N + 3 \cdot 2^{2/3} \sqrt[3]{\pi} S)}{8 \cdot 2^{7/18} \sqrt{3\pi}^{11/18} N^{2/3}}. \quad (9)$$

Using the standard thermodynamic relation $dM = TdS + \mu dN$, the corresponding temperature takes the following form:

$$T_4 = \frac{\partial M_4(S, N)}{\partial S} \Big|_N = \frac{8\sqrt[3]{2}N + 9\sqrt[3]{\pi}S}{8 \cdot 2^{13/18} \sqrt{3\pi}^{11/18} N^{2/3} \sqrt{S}}. \quad (10)$$

This quantity can be identified with the Hawking temperature of the black hole. Using Eq. (9), the chemical potential μ conjugate to the number of M2-branes is given by

$$\mu_4 = \frac{\partial M_4(S, N)}{\partial N} \Big|_S = \frac{\sqrt{S} (8N - 3 \cdot 2^{2/3} \sqrt[3]{\pi} S)}{12 \cdot 2^{7/18} \sqrt{3\pi}^{11/18} N^{5/3}}. \quad (11)$$

It defines the measure of the energy cost to the system when one increases the variable N . In terms of these quantities, the Gibbs free energy reads

$$G_4(T, N) = M_4 - T_4 S = \frac{\sqrt{S} (8\sqrt[3]{2}N - 3\sqrt[3]{\pi}S)}{8 \cdot 2^{13/18} \sqrt{3\pi}^{11/18} N^{2/3}}. \quad (12)$$

Having calculated the relevant thermodynamical quantities, we investigate the corresponding phase transition. To do so,

¹ Where $\omega_d = \frac{2\pi^{\frac{d+1}{2}}}{\Gamma(\frac{d+1}{2})}$.

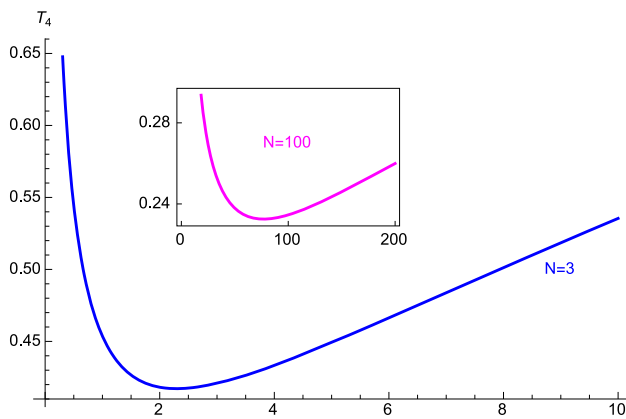


Fig. 1 The temperature as a function of the entropy S , with $N = 3$ and $N = 100$

we study the variation of the Hawking temperature as a function of the entropy. This variation is plotted in Fig. 1.

It follows from Fig. 1 that the Hawking temperature is not a monotonic function. In fact, it involves a minimum at the point

$$S_{4,1} = \frac{8}{9} \sqrt[3]{\frac{2}{\pi}} N, \quad (13)$$

which corresponds to the minimal temperature $T_{4,\min} = \frac{\sqrt{3}}{2^{18/5} \pi^{4/9} \sqrt[5]{N}}$. It is observed that for such a temperature no black hole can exist. Otherwise, two branches are shown up. Indeed, the first branch, associated with small values of the entropy S , is thermodynamically unstable. However, the second phase, corresponding to a large entropy S , is considered as a thermodynamical stable one. It has been shown that similar behaviors are also present in the large limit of N .

It is observed from the Gibbs free energy, given in Eq. (12), that the Hawking–Page phase transition occurs where the corresponding phase-transition temperature is

$$T_{HP4} = \frac{1}{\sqrt[18]{2\pi^{4/9} \sqrt[5]{N}}}. \quad (14)$$

It is verified that this quantity is larger than the temperature $T_{4,\min} = T|_{S=S_{4,1}}$. At the Hawking–Page transition, the associated entropy takes the following form:

$$S_{4,2} = \frac{8}{3} \sqrt[3]{\frac{2}{\pi}} N. \quad (15)$$

In Fig. 2, we illustrate the Gibbs free energy as a function of the Hawking temperature T for some fixed values of N .

It is noted that the down branch Gibbs free energy for a fixed N changes its sign at the point $S = S_{4,2}$, which corresponds to the Hawking–Page transition point. Moreover, there is observed a minimum temperature $T_{4,\min}$ for which no black holes ($T < T_{4,\min}$) can survive. However, above this temperature, two branches of the black holes are shown up. Indeed, the upper branch describes an unstable small

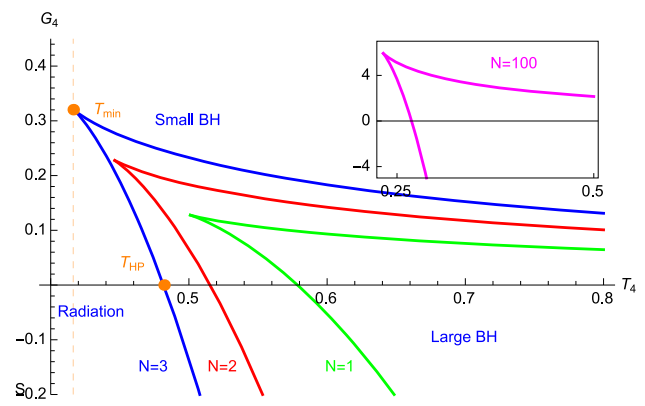


Fig. 2 The Gibbs free energy as a function of the temperature T_4 , for $N = 1, 2, 3, 100$

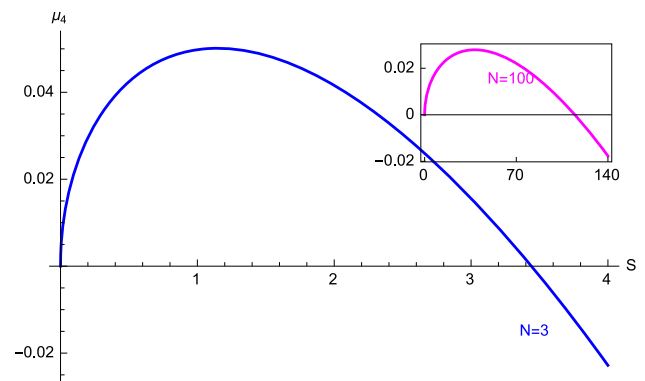


Fig. 3 The chemical potential μ as a function of the entropy for $N = 3, 100$

(Schwarzschild-like) black hole associated with a negative specific heat. For $T > T_{4,\min}$, the black holes, at the lower branch, can be considered as a stable solution corresponding to a positive specific heat. Since the Hawking–Page temperature $T_{4,HP}$ is associated with vanishing values of the Gibbs free energy, the black hole Gibbs free energy becomes negative for $T > T_{4,HP}$. As reported in [1,8,10], at $T = T_{4,HP}$, a first order Hawking–Page phase transition occurs between the thermal radiations and the large black holes.

To study the phase transition, we vary the chemical potential in terms of the entropy. In Fig. 3, we plot such a variation for a fixed value of N .

For small values of S , the chemical potential is positive. However, it changes becoming negative when S is large. Moreover, the chemical potential changes its sign at

$$S_{4,3} = \frac{4}{3} \sqrt[3]{\frac{2}{\pi}} N. \quad (16)$$

It is easy to check the following constraint:

$$S_{4,3} < S_{4,2} < S_{4,1}. \quad (17)$$

It turns out that the vanishing of the chemical potential appears in the unstable branch.

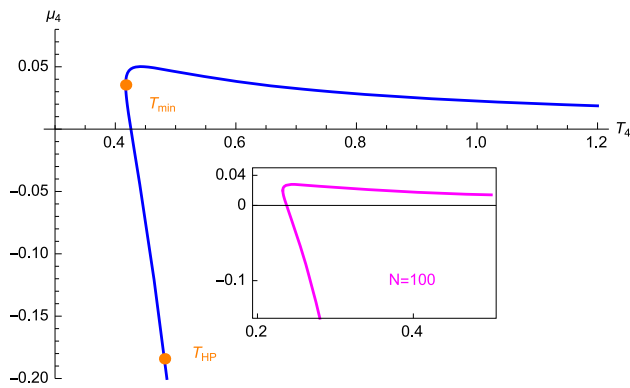


Fig. 4 The chemical potential μ as a function of the temperature T_4 , with $N = 3, 100$

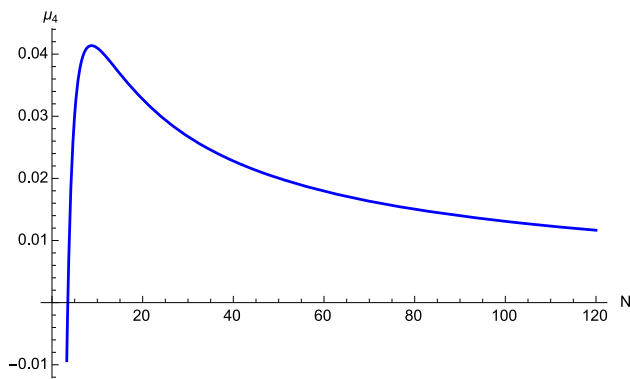


Fig. 5 The chemical potential μ as a function of N ; we have set $S_4 = 4$

In Fig. 4, we plot the chemical potential as a function of temperature T_4 for a fixed N .

From Fig. 4, we can find the Hawking–Page temperature. On the branch below this point, the black holes are stable. Such a point resides in the negative region of the chemical potential. However, the upper branch, which corresponds to unstable black hole solutions, lives in the positive region of the chemical potential.

To see the effect of the number of M2-branes, we discuss the behavior of the chemical potential μ in terms of such a variable. The calculation is illustrated in Fig. 5.

It is observed that the chemical potential μ presents a maximum at

$$N_{4\max} = \frac{15}{8} \sqrt[3]{\frac{\pi}{2}} S, \quad \text{namely } S_{4,4} = \frac{8}{15} \sqrt[3]{\frac{2}{\pi}} N. \quad (18)$$

It is noted that $S_{4,4}$ is also less than $S_{4,1}$. It is remarked that this is quite different from the classical gas, having a negative chemical potential. In the case where the chemical potential approaches zero or becomes positive, quantum effects should be considered and should be relevant in the discussion [21].

Having discussed the case of M2-branes, let us move to a higher-dimensional case provided by M-theory. It is shown

that in eleven dimensions the dual magnetic analogs of M2-branes are M5-branes. In the following, we investigate the black holes in such magnetic brane backgrounds.

3 Thermodynamics of black holes in $AdS_7 \times S^4$ space

In this section, we discuss the magnetic solution associated with the near geometry $AdS_7 \times S^4$. According to [34,36], the corresponding metric takes the following form:

$$ds^2 = \frac{r}{L} \left(-f dt^2 + \sum_{i=1}^5 dx_i^2 \right) + \frac{L^2}{r^2} f^{-1} dr^2 + L^2 d\Omega_4^2, \quad (19)$$

where $d\Omega_4^2$ is the metric of a four-dimensional sphere with unit radius. As in the case of M2-branes, the metric function reads

$$f = 1 - \frac{m}{r^4} + \frac{r^2}{L^2}. \quad (20)$$

In M-theory, the eleven-dimensional spacetime Eq. (19) can be considered as the near horizon geometry of N coincident M5-branes. For this solution, the AdS radius L is related to the number N via the relation [34,37]

$$L^9 = N^3 \frac{\kappa_{11}^2}{2^7 \pi^5}. \quad (21)$$

The mass of the black hole can be computed using Eq. (20). The calculation gives the following expression:

$$M_7 = \frac{5 m \omega_5}{8 \pi \mathcal{G}_7} = \frac{5 r^4 \omega_5 (L^2 + r^2)}{16 \pi \mathcal{G}_7 L^2}. \quad (22)$$

It is found that the entropy is

$$S = \frac{A}{4 \mathcal{G}_7} = \frac{\omega_5 r^5}{4 \mathcal{G}_7}, \quad \mathcal{G}_7 = \frac{6 \mathcal{G}_{11}}{2 \pi \omega_5 L^7}. \quad (23)$$

Combining these expressions, one can write the mass in terms of the entropy S and N as follows:

$$M_7(S, N) = \frac{5 \left(2^{23/45} 3^{4/5} \pi^{2/15} N^{8/5} S^{4/5} + 96 \cdot 2^{2/45} \sqrt[5]{3} S^{6/5} \right)}{48 \pi^{23/45} N^{17/15}}. \quad (24)$$

The Hawking temperature can be obtained using the first law of thermodynamics, $dM = T dS + \mu dN$. Indeed, it is given by

$$T_7 = \frac{\partial M_7(S, N)}{\partial S} \Big|_N = \frac{2^{23/45} 3^{4/5} \pi^{2/15} N^{8/5} + 144 \cdot 2^{2/45} \sqrt[5]{3} S^{2/5}}{12 \pi^{23/45} N^{17/15} \sqrt[5]{S}}. \quad (25)$$

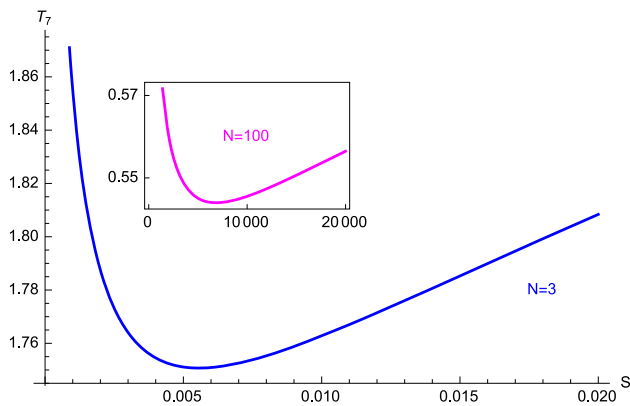


Fig. 6 The temperature as a function of the entropy, with $N = 3$ and $N = 100$

It is found, after calculations, that the chemical potential μ , conjugate to the number of M5-branes, reads

$$\mu_7 = \frac{\partial M_7(S, N)}{\partial N} \Big|_S = \frac{7 \cdot 2^{23/45} 3^{4/5} \pi^{2/15} N^{8/5} S^{4/5} - 1632 \cdot 2^{2/45} \sqrt[5]{3} S^{6/5}}{144 \pi^{23/45} N^{32/15}}. \quad (26)$$

Similarly, the Gibbs free energy can be computed. It is given by

$$G_7(T, N) = M_7 - T_7 S = \frac{2^{23/45} 3^{4/5} \pi^{2/15} N^{8/5} S^{4/5} - 96 \cdot 2^{2/45} \sqrt[5]{3} S^{6/5}}{48 \pi^{23/45} N^{17/15}}. \quad (27)$$

As in the previous case, the stability discussion can be done by varying the two variables S and N . We first deal with the phase transition. Indeed, it can be studied in terms of the monotony of the Hawking temperature using the entropy as a variable. This variation is plotted in Fig. 6.

We can clearly see that the Hawking temperature is not a monotonic function. It involves a minimum at the point

$$S_{7,1} = \frac{\sqrt[3]{\pi} N^4}{6912 \cdot 2^{5/6} \sqrt{3}}, \quad (28)$$

associated with the temperature $T_{7,\min} = \frac{2 \cdot 2^{5/18} \sqrt{3}}{\pi^{4/9} \sqrt[3]{N}}$. It is observed that for the minimal temperature no black hole can survive. Otherwise, two branches appear. Indeed, the first branch, associated with small entropy values, is thermodynamically unstable. The second one, corresponding to the large entropy values, is considered as a thermodynamically stable branch.

It follows from the Gibbs free energy, given in Eq. (27), that the Hawking–Page phase transition occurs where the corresponding phase-transition temperature obeys

$$T_{HP_7} = \frac{2 \cdot 2^{5/18} \sqrt{3}}{\pi^{4/9} \sqrt[3]{N}}. \quad (29)$$

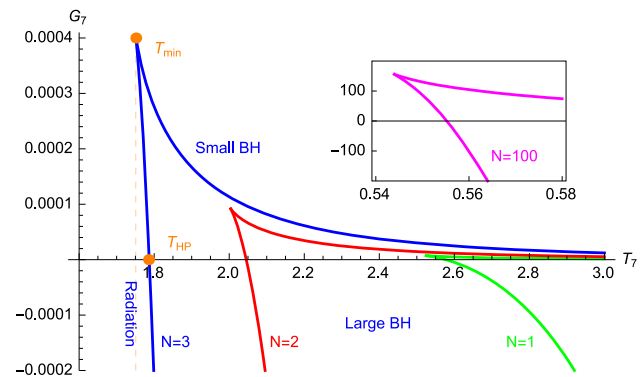


Fig. 7 The Gibbs free energy as a function of the temperature, for $N = 1, 2, 3, 100$

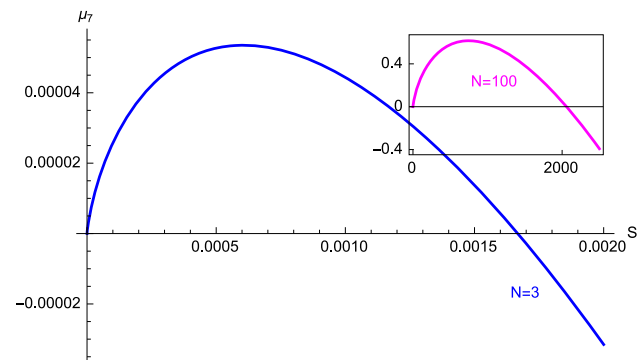


Fig. 8 The chemical potential μ as a function of the entropy, with $N = 3, 100$

This quantity is larger than the temperature $T_{7,\min} = T|_{S=S_{7,1}}$. At the Hawking–Page transition, the corresponding entropy is given by

$$S_{7,2} = \frac{\sqrt[3]{\frac{\pi}{2}} N^4}{6144}. \quad (30)$$

Figure 7 illustrates the Gibbs free energy with respect to the Hawking temperature T for some fixed values of N .

For a fixed N , it follows that the down branch Gibbs free energy changes its sign at the point $S = S_{7,2}$, corresponding to the Hawking–Page transition point.

To study the phase transition, we vary the chemical potential in terms of the entropy. In Fig. 8, we plot such a variation by fixing the number of M5-branes in M-theory.

We see that the chemical potential is positive when we consider small values of S . However, it changes to become negative for large values of S . The chemical potential changes its sign at

$$S_{7,3} = \frac{49 \sqrt{\frac{7}{17}} \sqrt[3]{\frac{\pi}{2}} N^4}{1775616}. \quad (31)$$

As in the case of M2-branes, one has

$$S_{7,3} < S_{7,2} < S_{7,1}. \quad (32)$$

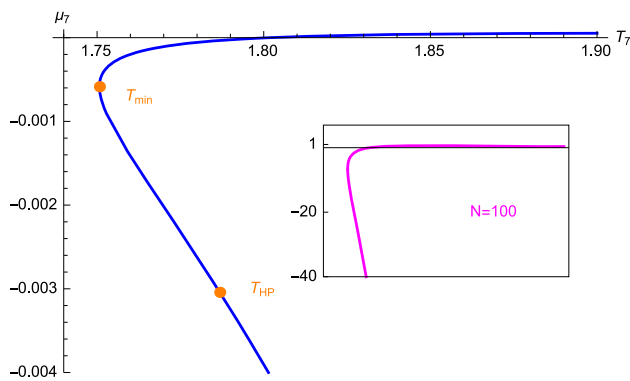


Fig. 9 The chemical potential μ as a function of the temperature T_7 , with $N = 3, 100$

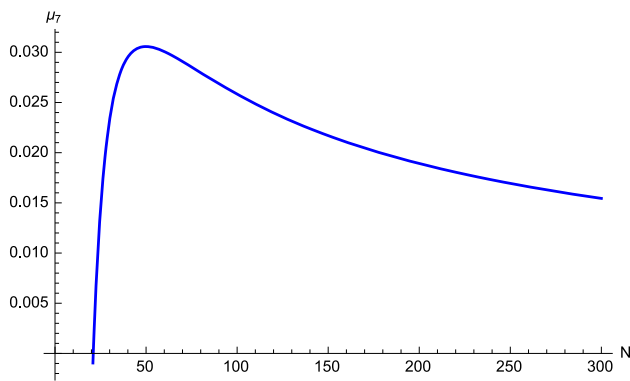


Fig. 10 The chemical potential μ as a function of N ; we have set $S_7 = 4$

As we will see, the vanishing of the chemical potential appears in the unstable branch. Similar behaviors appeared in the case of M2-branes. This implies that the vanishing of the chemical potential does not make any sense from the point of view of dual conformal field theory. This point deserves deeper study. We hope to come back to this point in the future.

To see the effect of the temperature, Fig. 9 presents the chemical potential as a function of the temperature T_7 for fixed values of N . It is noted that one has similar behaviors appearing in the case of M2-branes.

Figure 10 shows the chemical potential as a function of N in the case of a fixed entropy.

We can observe that the chemical potential μ presents a maximum at

$$N = 16 \left(\frac{17}{7} \right)^{5/8} \sqrt[4]{3} \sqrt[12]{\frac{2}{\pi}} \sqrt[4]{S}, \quad \text{namely} \\ S_{7,4} = \frac{49 \sqrt{\frac{7}{17}} \sqrt[3]{\frac{\pi}{2}} N^4}{56819712}. \quad (33)$$

In the following section, we will study the thermodynamical geometry of the M2- and M5-branes black holes in the extended phase space to reconsider the stability problem.

4 Geothermodynamics of AdS black holes in M-theory

In this section, we investigate the geothermodynamics AdS black holes in $AdS_{p+2} \times S^{11-p-2}$. This study concerns singular limits of certain thermodynamical quantities including the heat capacity. This quantity is the relevant one in the study of the stability of such black hole solutions.

To elaborate this discussion, the number of branes N should be fixed to consider a canonical ensemble. For a fixed N , the heat capacity for the M2- and M5-branes are given, respectively, by

$$C_{N,4} = T_4 \left(\frac{\partial S}{\partial T_4} \right)_N = \left(\frac{1}{\frac{8}{9} \sqrt[3]{\frac{2}{\pi}} N + S} - \frac{1}{2S} \right)^{-1}, \quad (34)$$

$$C_{N,7} = T_7 \left(\frac{\partial S}{\partial T_7} \right)_N \\ = \frac{720S^{7/5} + 5 \cdot 2^{7/15} 3^{3/5} \pi^{2/15} N^{8/5} S}{144S^{2/5} - 2^{7/15} 3^{3/5} \pi^{2/15} N^{8/5}}. \quad (35)$$

These equations contain many interesting thermodynamical properties. Indeed, the heat capacity involves a divergence at the point of $S_{i,1i \in \{4,7\}}$. For a fixed N , this point can be identified with the point corresponding to the minimal Hawking temperature. In the case $S < S_{i,1i \in \{4,7\}}$, the heat capacity is negative, showing the thermodynamical instability. However, it becomes positive in the region defined by $S > S_{i,1i \in \{4,7\}}$. These behaviors of $C_{N,i \in \{4,7\}}$ as a function of S can be illustrated in Fig. 11.

To show the singularity of the corresponding heat capacity, the thermodynamical geometry of such black hole solutions should be discussed including the thermodynamical curvature. To compute such a quantity, one can use several metrics. However, we can explore the Quevedo metric, which reads [38–41]

$$g^Q = (ST + N\mu) \begin{pmatrix} M_{SS} & 0 \\ 0 & M_{NN} \end{pmatrix}, \quad (36)$$

where M_{ij} stands for $\partial^2 M / \partial x^i \partial x^j$, and $x^1 = S$, $x^2 = N$. The scalar curvature of this metric can be computed in a direct way. For M2- and M5-branes, respectively, the calculation gives the following expressions:

$$R_4^Q = \frac{A_4}{B_4} \quad (37)$$

where one has

$$A_4 = 55296 \sqrt[9]{2} \pi^{14/9} N^{7/3} \left(-8192N^3 - 729\pi S^3 \right. \\ \left. + 9792 \sqrt[3]{2} \pi^{2/3} N S^2 + 2112 \cdot 2^{2/3} \sqrt[3]{\pi} N^2 S \right), \quad (38)$$

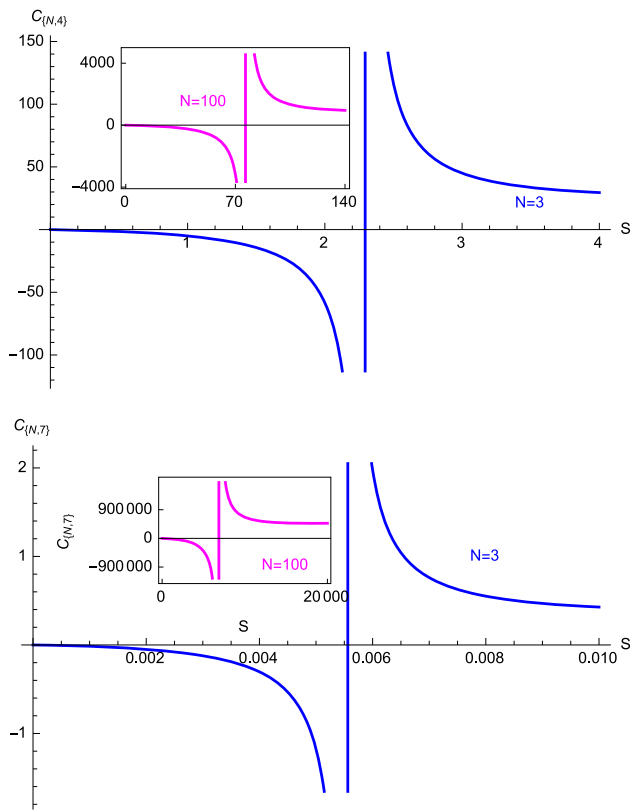


Fig. 11 The heat capacity in the case with a fixed $N = 3$ and 100 as a function of entropy S for the two backgrounds

$$B_4 = 5 \left(15 \cdot 2^{2/3} \sqrt[3]{\pi} S - 16N \right)^2 \left(9 \sqrt[3]{\pi} S - 8 \sqrt[3]{2} N \right)^2 \times \left(8 \sqrt[3]{2} N + 3 \sqrt[3]{\pi} S \right)^3, \quad (39)$$

and

$$R_7^Q = \frac{A_7}{B_7 C_7}. \quad (40)$$

The quantities A , B , and C are given in this case by

$$A_7 = 414720 \cdot 2^{2/45} \pi^{52/45} N^{58/15} \times \left(361361 \cdot 2^{7/15} \sqrt[5]{3} \pi^{8/15} N^{32/5} + 82685568 \cdot 3^{3/5} \pi^{2/5} N^{24/5} S^{2/5} + 388512000 \cdot 2^{8/15} \pi^{4/15} N^{16/5} S^{4/5} + 6806372352 \cdot 15 \sqrt[2]{{23}^{2/5}} \pi^{2/15} N^{8/5} S^{6/5} + 5474746368 \cdot 2^{3/5} 3^{4/5} S^{8/5} \right), \quad (41)$$

$$B_7 = S^{6/5} \left(-133 \cdot 3^{3/5} \pi^{4/15} N^{16/5} + 61680 \cdot 2^{8/15} \pi^{2/15} N^{8/5} S^{2/5} + 104448 \cdot 15 \sqrt[2]{{23}^{2/5}} S^{4/5} \right)^2, \quad (42)$$

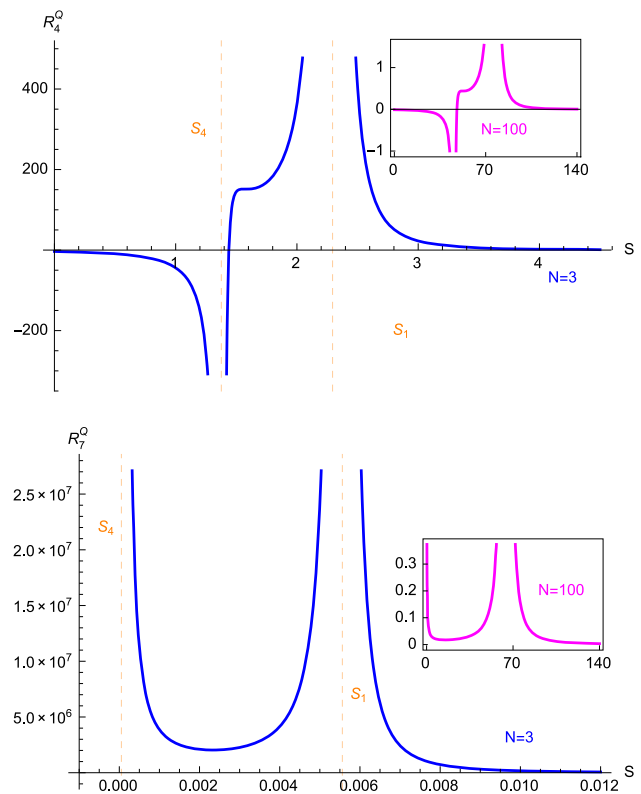


Fig. 12 The scalar curvature vs. entropy for the Quevedo metric case with $N = 3, 100$ for different backgrounds

$$C_7 = \left(-19 \cdot 3^{3/5} \pi^{4/15} N^{16/5} + 1320 \cdot 2^{8/15} \pi^{2/15} N^{8/5} S^{2/5} + 2304 \cdot 15 \sqrt[2]{{23}^{2/5}} S^{4/5} \right)^2. \quad (43)$$

In Fig. 12, we plot the scalar curvature of the Quevedo metric as a function of the entropy for M2- and M5-branes.

It follows from Fig. 12 that one has two divergent points located at $S_{i,4i \in \{4,7\}}$ and $S_{i,1i \in \{4,7\}}$, respectively. The first one coincides with the divergent point of C_N . However, the second one is associated with the maximum of the chemical potential considered as a function of N . It is worth noting that this result is in good agreement with the recent study reported in [42–44], saying that the divergences of scalar curvature for the Quevedo metric corresponds to the divergence or zero for the heat capacity. It has been suggested that these results might be explored to understand the link between the phase transition and the thermodynamical curvature.

5 Conclusion

In this paper, we have investigated the thermodynamics and thermodynamical geometry of AdS black holes from M2- and M5-branes in M-theory. Concretely, we have considered AdS

black holes in $AdS_{p+2} \times S^{11-p-2}$, where $p = 2, 5$ by viewing the number of M2- and M5-branes as a thermodynamical variable. First, we have discussed the corresponding phase transition by computing the relevant quantities. For M2- and M5-branes, we have computed the chemical potential and discussed the corresponding stabilities. Then we have studied the thermodynamical geometry associated with such AdS black holes. More precisely, we have computed the scalar curvatures from the Quevedo metric. The calculations show similar thermodynamical properties appearing in the phase-transition program. This present work, concerning M-theory, may support the relation between the phase transition and divergence of thermodynamical curvature studied in type IIB superstring theory.

This work poses a question concerning a nine-dimensional AdS black holes associated with D7-branes on AdS_9 -space. In fact, it may be possible to consider a geometry of the form

$$AdS_9 \times S^1 \times T^2,$$

inspired by the recent work on black holes in F-theory [45]. This may support the results concerning the link between the phase transition and the thermodynamical curvature in superstring and related theories.

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References

1. S. Hawking, D.N. Page, Thermodynamics of black holes in Anti-de Sitter space. *Commun. Math. Phys.* **83**, 577 (1987)
2. E. Witten, Anti-de Sitter space, thermal phase transition, and confinement in gauge theories. *Adv. Theor. Math. Phys.* **2**, 505–532 (1998)
3. A. Chamblin, R. Emparan, C. Johnson, R. Myers, Charged AdS black holes and catastrophic holography. *Phys. Rev. D* **60**, 064018 (1999)
4. A. Chamblin, R. Emparan, C. Johnson, R. Myers, Holography, thermodynamics, and fluctuations of charged AdS black holes. *Phys. Rev. D* **60**, 104026 (1999)
5. M. Cvetič, G.W. Gibbons, D. Kubiznak, C.N. Pope, Black hole enthalpy and an entropy inequality for the thermodynamic volume. *Phys. Rev. D* **84**, 024037 (2011). [arXiv:1012.2888](https://arxiv.org/abs/1012.2888) [hep-th]
6. B.P. Dolan, D. Kastor, D. Kubiznak, R.B. Mann, J. Traschen, Thermodynamic volumes and isoperimetric inequalities for de Sitter black holes. [arXiv:1301.5926](https://arxiv.org/abs/1301.5926) [hep-th]
7. D. Kastor, S. Ray, J. Traschen, Enthalpy and the mechanics of AdS black holes. *Class. Quant. Grav.* **261**, 95011 (2009). [arXiv:0904.2765](https://arxiv.org/abs/0904.2765)
8. B.P. Dolan, Vacuum energy and the latent heat of AdS-Kerr black holes. *Phys. Rev. D* **90**(8), 084002 (2014). [arXiv:1407.4037](https://arxiv.org/abs/1407.4037) [gr-qc]
9. C.V. Johnson, Holographic heat engines. *Class. Quant. Grav.* **31**, 205002 (2014). [arXiv:1404.5982](https://arxiv.org/abs/1404.5982) [hep-th]
10. N. Altamirano, D. Kubiznak, R.B. Mann, Z. Sherkatghanad, Thermodynamics of rotating black holes and black rings: phase transitions and thermodynamic volume. *Galaxies* **2**, 89 (2014). [arXiv:1401.2586](https://arxiv.org/abs/1401.2586) [hep-th]
11. A. Belhaj, M. Chabab, H. El Moumni, M.B. Sedra, On thermodynamics of AdS black holes in arbitrary dimensions. *Chin. Phys. Lett.* **29**(10), 100401 (2012)
12. A. Belhaj, M. Chabab, H. El Moumni, L. Medari, M.B. Sedra, The thermodynamical behaviors of Kerr–Newman AdS black holes. *Chin. Phys. Lett.* **30**, 090402 (2013)
13. A. Belhaj, M. Chabab, H. El Moumni, K. Masmar, M.B. Sedra, Critical behaviors of 3D black holes with a scalar hair. *Int. J. Geom. Methods Mod. Phys.* **12**(02), 1550017 (2014). [arXiv:1306.2518](https://arxiv.org/abs/1306.2518) [hep-th]
14. A. Belhaj, M. Chabab, H. El Moumni, K. Masmar, M.B. Sedra, Maxwell's equal-area law for Gauss–Bonnet–Anti-de Sitter black holes. *Eur. Phys. J. C* **75**(2), 71 (2015). [arXiv:1412.2162](https://arxiv.org/abs/1412.2162) [hep-th]
15. D. Kubiznak, R.B. Mann, P–V criticality of charged AdS black holes. *J. High Energy Phys.* **1207**, 033 (2012)
16. A. Belhaj, M. Chabab, H. El Moumni, K. Masmar, M.B. Sedra, A. Segui, On heat properties of AdS black holes in higher dimensions. *JHEP* **1505**, 149 (2015). [arXiv:1503.07308](https://arxiv.org/abs/1503.07308) [hep-th]
17. A. Belhaj, M. Chabab, H. EL Moumni, K. Masmar, M.B. Sedra, Ehrenfest scheme of higher dimensional topological AdS black holes in the third order Lovelock–Born–Infeld gravity. *Int. J. Geom. Methods Mod. Phys.* **12**, 1550115 (2015). [arXiv:1405.3306](https://arxiv.org/abs/1405.3306) [hep-th]
18. J.X. Zhao, M.S. Ma, L.C. Zhang, H.H. Zhao, R. Zhao, The equal area law of asymptotically AdS black holes in extended phase space. *Astrophys. Space Sci.* **352**, 763 (2014)
19. J.L. Zhang, R.G. Cai, H. Yu, Phase transition and thermodynamical geometry for Schwarzschild AdS black hole in $AdS_5 \times S^5$ spacetime. *JHEP* **1502**, 143 (2015). [arXiv:1409.5305](https://arxiv.org/abs/1409.5305) [hep-th]
20. J.L. Zhang, R.G. Cai, H. Yu, Phase transition and thermodynamical geometry of Reissner–Nordstrom–AdS black holes in extended phase space. *Phys. Rev. D* **91**(4), 044028 (2015). [arXiv:1502.01428](https://arxiv.org/abs/1502.01428) [hep-th]
21. B.P. Dolan, Bose condensation and branes. *JHEP* **1410**, 179 (2014). [arXiv:1406.7267](https://arxiv.org/abs/1406.7267) [hep-th]
22. J.M. Maldacena, The large N limit of superconformal field theories and supergravity. *Int. J. Theor. Phys.* **38**, 1113 (1999) [*Adv. Theor. Math. Phys.* **2**, 231 (1998)]. [arXiv:hep-th/9711200](https://arxiv.org/abs/hep-th/9711200)
23. E. Witten, Anti-de Sitter space and holography. *Adv. Theor. Math. Phys.* **2**, 253 (1998). [arXiv:hep-th/9802150](https://arxiv.org/abs/hep-th/9802150)
24. S.S. Gubser, I.R. Klebanov, A.M. Polyakov, Gauge theory correlators from noncritical string theory. *Phys. Lett. B* **428**, 105 (1998). [arXiv:hep-th/9802109](https://arxiv.org/abs/hep-th/9802109)
25. O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri, Y. Oz, Large N field theories, string theory and gravity. *Phys. Rep.* **323**, 183 (2000). [arXiv:hep-th/9905111](https://arxiv.org/abs/hep-th/9905111)
26. S. Katmadas, A. Tomasiello, AdS4 black holes from M-theory. [arXiv:1509.00474](https://arxiv.org/abs/1509.00474) [hep-th]
27. Y. Lozano, N.T. Macpherson, J. Montero, A $\mathcal{N} = 2$ solution in M-theory with purely magnetic flux. [arXiv:1507.02660](https://arxiv.org/abs/1507.02660) [hep-th]
28. N. Halmagyi, M. Petrini, A. Zaffaroni, BPS black holes in AdS_4 from M-theory. *JHEP* **1308**, 124 (2013). [arXiv:1305.0730](https://arxiv.org/abs/1305.0730) [hep-th]
29. I.R. Klebanov, S.S. Pufu, T. Tesileanu, Membranes with topological charge and AdS_4/CFT_3 correspondence. *Phys. Rev. D* **81**, 125011 (2010). [arXiv:1004.0413](https://arxiv.org/abs/1004.0413) [hep-th]
30. E. Witten, Solutions of four-dimensional field theories via M theory. *Nucl. Phys. B* **500**, 3 (1997). [arXiv:hep-th/9703166](https://arxiv.org/abs/hep-th/9703166)
31. E.S. Fradkin, A.A. Tseytlin, Quantum properties of higher dimensional and dimensionally reduced supersymmetric theories. *Nucl. Phys. B* **227**, 252 (1983)

32. M.B. Green, P. Vanhove, D instantons, strings and M theory. *Phys. Lett. B* **408**, 122 (1997). [arXiv:hep-th/9704145](#)
33. J.G. Russo, A.A. Tseytlin, One loop four graviton amplitude in eleven-dimensional supergravity. *Nucl. Phys. B* **508**, 245 (1997). [arXiv:hep-th/9707134](#)
34. S.S. Gubser, I.R. Klebanov, A.A. Tseytlin, Coupling constant dependence in the thermodynamics of $\mathcal{N} = 4$ supersymmetric Yang–Mills theory. *Nucl. Phys. B* **534**, 202 (1998). [arXiv:hep-th/9805156](#)
35. R. Kallosh, A. Rajaraman, Vacua of M theory and string theory. *Phys. Rev. D* **58**, 125003 (1998). [arXiv:hep-th/9805041](#)
36. M.J. Duff, H. Lu, C.N. Pope, The black branes of M theory. *Phys. Lett. B* **382**, 73 (1996). [arXiv:hep-th/9604052](#)
37. I.R. Klebanov, World volume approach to absorption by nondilatonic branes. *Nucl. Phys. B* **496**, 231 (1997). [arXiv:hep-th/9702076](#)
38. H. Quevedo, Geometrothermodynamics. *J. Math. Phys.* **48**, 013506 (2007). [arXiv:physics/0604164](#)
39. H. Quevedo, Geometrothermodynamics of black holes. *Gen. Relativ. Grav.* **40**, 971 (2008). [arXiv:0704.3102](#)
40. H. Quevedo, A. Sanchez, S. Taj, A. Vazquez, Phase transitions in geometrothermodynamics. *Gen. Relativ. Grav.* **43**, 1153 (2011). [arXiv:1010.5599](#)
41. A. Bravetti, D. Momeni, R. Myrzakulov, H. Quevedo, Geometrothermodynamics of higher dimensional black holes. *Gen. Relativ. Grav.* **45**, 1603 (2013). [arXiv:1211.7134](#)
42. S.A.H. Mansoori, B. Mirza, Correspondence of phase transition points and singularities of thermodynamic geometry of black holes. *Eur. Phys. J. C* **74**, 2681 (2014). [arXiv:1308.1543](#) [gr-qc]
43. S.H. Hendi, Thermodynamic properties of asymptotically Reissner–Nordstrom black holes. *Ann. Phys.* **346**, 42 (2014). [arXiv:1405.6996](#) [gr-qc]
44. J. Suresh, R. Tharanath, N. Varghese, V.C. Kuriakose, The thermodynamics and thermodynamic geometry of the Park black hole. *Eur. Phys. J. C* **74**, 2819 (2014). [arXiv:1403.4710](#) [gr-qc]
45. B. Haghighat, S. Murthy, C. Vafa, S. Vandoren, F-theory, spinning black holes and multi-string branches. [arXiv:1509.00455](#)