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Generalized Bekenstein–Hawking system: logarithmic correction

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Abstract The present work is a generalization of the recent work [arXiv.1206.1420] on the modified Hawking temperature on the event horizon. Here the Hawking temperature is generalized by multiplying the modified Hawking temperature by a variable parameter α representing the ratio of the growth rate of the apparent horizon to that of event horizon. It is found that both the first and the generalized second law of thermodynamics are valid on the event horizon for any fluid distribution. Subsequently, the Bekenstein entropy is modified on the event horizon and the thermodynamical laws are examined. Finally, an interpretation of the parameters involved is presented.

1 Introduction

In black hole physics a semi-classical description shows that a black hole behaves as a black body emitting thermal radiation with temperature (known as the Hawking temperature) and entropy (known as the Bekenstein entropy) proportional to the surface gravity at the horizon and area of the horizon [1,2], respectively. Further, this Hawking temperature and the Bekenstein entropy are related to the mass of the black hole through the first law of thermodynamics [3]. Due to this relationship between the physical parameters (namely, entropy and temperature) and the geometry of the horizon, there is natural speculation about the relationship between black hole thermodynamics and the Einstein field equations. A first step in this direction was put forward by Jacobson [4], who derived the Einstein field equations from the first law of thermodynamics: $\delta Q = T\mathrm{d}S$ for all locally Rindler causal horizons with δQ and T as the energy flux and Unruh temperature measured by an accelerated observer just inside the horizon. Subsequently, Padmanabhan [5,6] on the other side was able to derive the first law of thermodynamics on the horizon starting from the Einstein equations for a general static spherically symmetric space-time.

This idea of equivalence between Einstein field equations and the thermodynamical laws has been extended in the context of cosmology. Usually, the universe bounded by the apparent horizon is assumed to be a thermodynamical system with Hawking temperature and the entropy as

$$T_A = \frac{1}{2\pi R_A},$$
$$S_A = \frac{\pi R_A^2}{G} \tag{1}$$

where R_A is the radius of the apparent horizon. It was shown that the first law of thermodynamics on the apparent horizon and the Friedmann equations are equivalent [7]. Subsequently, this equivalent idea was extended to higher dimensional space-time, namely gravity theory with a Gauss–Bonnet term and Lovelock gravity theory [7–10]. It is presumed that such an inherent relationship between the thermodynamics at the apparent horizon and the Einstein field equations may lead to some clue on the properties of dark energy.

Although the cosmological event horizon does not exist in the usual standard big bang cosmology, in the perspective of the recent observations [11–16], the universe is in an accelerating phase dominated by dark energy ($\omega_d < -1/3$) and the event horizon distinct from the apparent horizon. By defining the entropy and temperature on the event horizon similar to those for the apparent horizon (given above) Wang et al. [17] showed that both the first and the second law of thermodynamics break down on the cosmological event horizon. They justified it arguing that the first law is applicable to nearby states of local thermodynamic equilibrium, while the event horizon reflects the global features of space-time. As a result, the thermodynamical parameters on the non-equilibrium configuration of the event horizon may not be as simple as on the apparent horizon. Further, they speculated that the region bounded by the apparent horizon may be

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taken as the Bekenstein system, i.e., the Bekenstein entropy or mass bound, $S < 2\Pi R_E$, and the entropy or area bound, $S < A/4$, are satisfied in this region. Now due to universality of the Bekenstein bounds and as all gravitationally stable special regions with weak self-gravity should satisfy the above Bekenstein bounds, the corresponding thermodynamical system is termed a Bekenstein system. Further, due to the radius of the event horizon being larger than the apparent horizon, Wang et al. [17] termed the universe bounded by the event horizon a non-Bekenstein system.

In the recent past there were published a series of works [18–23] investigating the validity of the generalized second law of thermodynamics of the universe bounded by the event horizon for Einstein gravity [18,19] and in other gravity theories [18–21] and for different fluid systems [18,19,22,23] (including dark energy [19,22,23]). In these works the validity of the first law of thermodynamics on the event horizon was assumed and it was possible to show the validity of the generalized second law of thermodynamics with some reasonable restrictions. However, the validity of the first law of thermodynamics on the event horizon was still followed by a question mark. Very recently, the author [24] was able to show that the first law of thermodynamics is satisfied on the event horizon with a modified Hawking temperature for two specific examples of single DE fluids. The present work is a further extension of it. Here, by generalizing the Hawking temperature, or modifying the Bekenstein entropy, it is possible to show that both the first and the generalized second law of thermodynamics (GSLT) are always satisfied on the event horizon. The paper is organized as follows: Sect. 2 deals with basic equations related to earlier works. The thermodynamical laws with generalized Hawking temperature and modified Bekenstein entropy are studied, respectively, in Sect. 3 and in Sect. 4. The interpretation of the parameters involved in generalized Hawking temperature and modified Bekenstein entropy has been analyzed in Sect. 5. Finally, a summary of the work and possible conclusions are presented in Sect. 6.

2 Basic equations and earlier works

The homogeneous and isotropic FRW model of the universe can locally be expressed by the metric

$$ds^2 = h_{ij}(x^i)dx^i dx^j + R^2 d\Omega_2^2 \tag{2}$$

where i, j can take values 0 and 1, and the two dimensional metric tensor h_{ij}, known as the normal metric, is given by

$$h_{ij} = \text{diag}(-1, a^2/1 - kr^2) \tag{3}$$

with x^i being the associated co-ordinates ($x^0 = t, x^1 = r$). $R = ar$ is the area radius and is considered as a scalar field in the normal 2D space. Another relevant scalar quantity on this normal space is

$$\chi(x) = h^{ij}(x)\partial_i R \partial_j R = 1 - (H^2 + k/a^2)R^2 \tag{4}$$

where $k = 0, \pm 1$ stands for flat, closed or open model of the universe. The Friedmann equations are

$$H^2 + k/a^2 = \frac{8\Pi G\rho}{3} \tag{5}$$

and

$$\dot{H} - k/a^2 = -4\Pi G(\rho + p) \tag{6}$$

where the energy density ρ and the thermodynamic pressure p of the matter distribution obey the conservation relation

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{7}$$

Usually, the apparent horizon is defined at the vanishing of the scalar i.e. $\chi(x) = 0$, which gives

$$R_A = \frac{1}{\sqrt{H^2 + k/a^2}}. \tag{8}$$

Now the surface gravity on the apparent horizon is defined as

$$\kappa_{A=-\frac{1}{2}} \frac{\partial \chi}{\partial R} \|_{R=R_A} = \frac{1}{R_A}. \tag{9}$$

So the usual Hawking temperature on the apparent horizon is given by (as in (1))

$$T_A = \frac{\|\kappa_A\|}{2\Pi} = \frac{1}{2\Pi R_A}. \tag{10}$$

It has been shown by Wang et al. [17] and others [7–9,25,26] that a universe bounded by the apparent horizon (with parameters given by (1)) is a thermodynamical system satisfying both the first and the second law of thermodynamics, not only in Einstein gravity but also in any other gravity theory and also for baryonic as well as for exotic matter.

On the other hand, the difficulty starts from the very definition of the event horizon. The infinite integral in the definition

$$R_E = a \int_t^\infty \frac{dt}{a} \tag{11}$$

converges only if $a \sim t^\alpha$ with $\alpha > 1$ i.e. the event horizon does not exist in the decelerating phase, it has only relevance in the present accelerating era. In the literature, the Hawking temperature on the event horizon is usually taken similar to the apparent horizon (replacing R_A by R_E) as (see (10))

$$T_E = \frac{1}{2\Pi R_E}. \tag{12}$$

This choice is also supported from the measurement of the temperature by a freely falling detector in a de Sitter background (where the two horizons coincide) using quantum field theory [27]. But unfortunately, with this choice of temperature and the entropy in the form of Bekenstein, i.e.,

$$T_E = \frac{1}{2\Pi R_E}, \qquad S_E = \frac{\Pi R_E^2}{G}, \tag{13}$$

the universe bounded by the event horizon is not a realistic thermodynamical system as both thermodynamical laws fail to hold there [17].

Recently, the surface gravity on the event horizon has been defined similar to that on the apparent horizon (see (9)) by [24]

$$\kappa_E = -\frac{1}{2}\frac{\partial \chi}{\partial R}|_{R=R_E} = \frac{R_E}{R_A^2}, \tag{14}$$

and as a result the modified Hawking temperature on the event horizon becomes

$$T_E^m = \frac{\|\kappa_E\|}{2\Pi} = \frac{R_E}{2\Pi R_A^2}, \tag{15}$$

which for the flat FRW model (i.e. $k = 0$) becomes

$$T_E = \frac{H^2 R_E}{2\Pi}. \tag{16}$$

As the two horizons are related by the inequality

$$R_A < R_E, \tag{17}$$

we always have

$$T_A < T_E. \tag{18}$$

Using this modified Hawking temperature the author [24] has been able to show the validity of the first law of thermodynamics on the event horizon for two specific single fluid DE models.

3 Generalized Hawking temperature and thermodynamical laws

In this section, to proceed searching for a general prescription, we start with a generalization of the modified Hawking temperature in the form

$$T_E^g = \frac{\alpha R_E}{2\Pi R_A^2} \tag{19}$$

where the dimensionless parameter α is to be determined so that $\alpha = 1$ on the apparent horizon.

The amount of energy flux across a horizon within the time interval $\mathrm{d}t$ is [7,28]

$$-\,\mathrm{d}E_h = 4\Pi R_h^2 T_{ab}k^a k^b \mathrm{d}t, \tag{20}$$

with k^a a null vector. So for the event horizon we get

$$-\,\mathrm{d}E = 4\Pi R_E^3 H(\rho + p)\mathrm{d}t. \tag{21}$$

Now using the Einstein field Eq. (6) and the definition of the apparent horizon (i.e. (8)), the above expression for the energy flux simplifies to

$$-\,\mathrm{d}E = \left(\frac{R_E}{R_A}\right)^3 \frac{\dot{R}_A}{G}\mathrm{d}t. \tag{22}$$

From the Bekenstein entropy–area relation (see (13)) we have

$$T_E \mathrm{d}S_E = \alpha \left(\frac{R_E}{R_A}\right)^2 \frac{\dot{R}_E}{G}\mathrm{d}t. \tag{23}$$

Hence for the validity of the first law of thermodynamics i.e.

$$-\,\mathrm{d}E = \mathrm{d}Q = T_E \mathrm{d}S_E, \tag{24}$$

we have

$$\alpha = \frac{\dot{R}_A/R_A}{\dot{R}_E/R_E}. \tag{25}$$

Thus reciprocal of α gives the relative growth rate of the radius of the event horizon to the apparent horizon.

For the generalized second law of thermodynamics, we start with the Gibbs law [17,29] to find the entropy variation of the bounded fluid distribution:

$$T_f \mathrm{d}S_f = \mathrm{d}E + p\mathrm{d}V, \tag{26}$$

where T_f and S_f are the temperature and entropy of the given fluid distribution, respectively, $V = 4\Pi R_E^3/3$ and $E = \rho V$. The above equation explicitly takes the form

$$T_f \mathrm{d}S_f = 4\Pi R_E^2(\rho + p)(\dot{R}_E - HR_E)\mathrm{d}t. \tag{27}$$

Also using the first law (i.e. (24)) we have from (21)

$$T_E \mathrm{d}S_E = 4\Pi R_E^3 H(\rho + p)\mathrm{d}t. \tag{28}$$

Now for equilibrium distribution, we assume $T_f = T_E^g$, i.e., the inside matter has the same temperature as the bounding surface and we obtain

$$T_E \mathrm{d}S_T = 4\Pi R_E^2(\rho + p)\dot{R}_E \mathrm{d}t, \tag{29}$$

with $S_T = S_E + S_f$, the total entropy of the universal system. Again using the Einstein field Eq. (6), the conservation relation (7), and (8) we have on simplification

$$T_E^g \frac{\mathrm{d}S_T}{\mathrm{d}t} = \left(\frac{R_E}{R_A}\right)^2 \frac{\dot{R}_A \dot{R}_E}{GHR_A}. \tag{30}$$

Now using the generalized Hawking temperature (19) the time variation of the total entropy becomes

$$\frac{\mathrm{d}S_T}{\mathrm{d}t} = \frac{2\Pi}{GH}\dot{R_E}^2, \tag{31}$$

which is positive definite for an expanding universe and hence the generalized second law of thermodynamics always holds on the event horizon.

4 Modified Bekenstein entropy and thermodynamical laws

In the previous section we have generalized the Hawking temperature, keeping the Bekenstein entropy–area relation unchanged and we are able to show the validity of both the first law of thermodynamics and GSLT on the event horizon, irrespective of any fluid distribution and we may term this universe bounded by the event horizon a generalized Bekenstein system. However, it is possible to have two other modifications of the entropy and temperature on the event horizon as follows.

(a) $S_E^{(m)} = \beta S_E^{(B)},\ T_E = T_E^{(m)}$, and

(b) $S_E^{(m)} = \delta S_E^{(B)},\ T_E = \frac{1}{\delta} T_E^{(m)}$.

We shall now examine the validity of the thermodynamical laws for these choices.

(a) $S_E^{(m)} = \beta S_E^{(B)}$ and $T_E = T_E^{(m)}$.

Here $S_E^{(m)}$ and $S_E^{(B)}$ are, respectively, the modified entropy and the usual Bekenstein entropy on the event horizon, $T_E^{(m)}$ is the modified Hawking temperature on the event horizon (given by (15) or (16)) and β is a parameter having value unity on the apparent horizon. Then as before from the validity of the Clausius relation β can be determined as

$$\beta = \frac{2}{R_E^2} \int R_E^2 \frac{\mathrm{d}R_A}{R_A}. \tag{32}$$

Thus for this choice of β the above modified entropy and modified Hawking temperature satisfy the first law of thermodynamics on the event horizon. Now we shall examine the validity of the generalized second law of thermodynamics (GSLT) on the event horizon for this choice of entropy and temperature on the horizon. Proceeding as before (assuming that the temperature of the inside fluid is the same as the modified Hawking temperature for thermodynamical equilibrium), we have

$$\frac{\mathrm{d}S_T}{\mathrm{d}t} = \frac{2\Pi}{GH}\left(\frac{R_E}{R_A}\right)\dot{R}_A\dot{R}_E. \tag{33}$$

Thus validity of GSLT depends on the evolution of the two horizons (apparent and event)—if both horizons increase or decrease simultaneously the GSLT is always satisfied. However, as long as the weak energy condition (WEC) is satisfied $\dot{R}_A > 0$ and $\dot{R}_E > 0$ if $R_E > R_A$ and GSLT is satisfied. But if WEC is violated then $\dot{R}_A < 0$ and GSLT will be satisfied only if $\dot{R}_E < 0$ i.e. $R_E < R_A$, which may be possible only in the phantom era. Hence for this choice of entropy and temperature on the event horizon GSLT is always satisfied as long as WEC is satisfied and when WEC is violated then GSLT will be valid if $R_E < R_A$. Further, it should be noted that if we choose the temperature on the event horizon as the generalized Hawking temperature (i.e. T_E^g) then β turns out to be unity i.e. we get back the previous generalized Bekenstein system (in Sect. 3).

(b) $S_E^{(m)} = \delta S_E^{(B)},\ T_E = \frac{1}{\delta} T_E^{(m)}$.

As before the parameter δ should be unity on the apparent horizon to match with the Bekenstein system. Again for the validity of the first law of thermodynamics (i.e. the Clausius relation) δ turns out to be R_A^2/R_E^2 and as a result the entropy on the event horizon becomes constant (equal to that at the apparent horizon). So this choice of entropy–temperature is not of much physical interest.

5 Interpretation of the parameters α and β

(I) α-parameter.

In this section we shall try to find some implications of the factor α. Note that α can be termed the ratio of the expansion rate of the two horizons. If we compare the expansion rate of the expanding matter with that for both horizons, we have

$$\frac{\dot{R}_E}{R_E} - H = -\frac{1}{R_E},\ \frac{\dot{R}_A}{R_A} - H = \left(\frac{3\omega+1}{2}\right)H \tag{34}$$

where the matter in the universe is chosen as a barotropic fluid with equation of state $p = \omega\rho$. As an event horizon exists only for the accelerating phase we have $\omega < -\frac{1}{3}$. Hence both the horizons expand slower than comoving. So the expansion rate of both the horizons coincide (i.e. $\alpha = 1$) when

$$HR_E = -\frac{2}{(3\omega+1)}. \tag{35}$$

Before proceeding, we present a comparative characterization of the two horizons in Table 1.

We shall now try to estimate the parameter α for some known fluid systems.

(a) Perfect fluid with constant equation of state $\omega(< -\frac{1}{3})$.

For a flat FRW model, the cosmological solution is

$$a(t) = a_0 t^{\left[\frac{2}{3(\omega+1)}\right]},$$

i.e.

$$H(t) = \frac{2}{3(\omega+1)t} \tag{36}$$

and

$$R_E(t) = -\frac{3(1+\omega)}{(1+3\omega)}t. \tag{37}$$

Hence

$$HR_E = -\frac{2}{(1+3\omega)} \tag{38}$$

Table 1 A comparative study of the horizons

Horizon	Location or definition	Causal character	Velocity	Acceleration
Apparent horizon	$R_A = \frac{1}{\sqrt{H^2+\frac{\kappa}{a^2}}}$	Time like if $-1 < \omega < \frac{1}{3}$, Null if $\omega = -1 or \frac{1}{3}$, Space like if $\omega < -1 or \omega > \frac{1}{3}$.	$4\Pi H R_A^2(\rho + p) = \frac{3}{2}(1 + \frac{p}{\rho})$	$\frac{9H}{2}(1 + \frac{p}{\rho})(\frac{p}{\rho} - \frac{\dot{p}}{\dot{\rho}})$
Event horizon	$R_E = a\int_t^{\infty} \frac{dt'}{a(t')}$	Null	$HR_E - 1$	$-H(1 + qHR_E)$

i.e. (33) is identically satisfied for all $\omega(< -\frac{1}{3})$. So for a perfect fluid with constant equation of state $(< -\frac{1}{3})$ we always have $\alpha = 1$ and hence the expansion rates of the two horizons are identical throughout the evolution. Thus the universe bounded by the event horizon with modified/generalized Hawking temperature (given by (15)/(19)) is a Bekenstein system and it supports the results in [24].

(b) Interacting holographic dark energy fluid.

We shall now study an interacting holographic dark energy (HDE) model that consists of dark matter in the form of dust (of energy density ρ_m) and HDE in the form of a perfect fluid: $p_d = \omega_d \rho_d$. The interaction between them is chosen as $3b^2 H(\rho_m + \rho_d)$ with b^2 the coupling constant. If R_E is taken as the I.R. cutoff, then the radius of the event horizon (R_E) and the equation of state parameter ω_d are given by [30]

$$R_E = \frac{c}{\sqrt{\Omega_d} H} \tag{39}$$

and

$$\omega_d = -\frac{1}{3} - \frac{2\sqrt{\Omega_d}}{3c} - \frac{b^2}{\Omega_d} \tag{40}$$

where $\Omega_d = \rho_d/(3H^2)$ is the density parameter for the dark energy and the dimensionless parameter 'c' carries the uncertainties of the theory and is assumed to be constant. In this case (33) is modified as

$$HR_E = -\frac{2}{(1 + 3\omega_t)} \tag{41}$$

with

$$\omega_t = \frac{p_d}{(\rho_m + \rho_d)} = \omega_d \Omega_d. \tag{42}$$

We shall now examine whether for this model relation (39) is satisfied or not. Using (37), (38), and (40) in (39) we obtain a cubic equation in $x (= \sqrt{\Omega_d})$,

$$2x^3 + cx^2 - 2x - (1 - b^2)c = 0. \tag{43}$$

This cubic equation has a positive root (x_p) if $b^2 < 1$ (the other two roots are either both negative or a pair of complex conjugates). In the Table 2 we present the value of x for different choices of b and c within the observational bounds:

Table 2 Value of x for different values of c and b^2 from (43)

c	b^2	Ω_d	x
0.7	0.92	0.73	0.85
0.8	0.84	0.73	0.85
0.82	0.91	0.70	0.84
0.76	0.8	0.76	0.87

The table shows that for interacting DE fluids it is possible to have identical expansion rates (i.e. $\alpha = 1$) for the two horizons within the observational limit of Ω_d and c.

(II) β-parameter.

To interpret the parameter β we consider thermal fluctuations of the apparent horizon so that the area changes by an infinitesimal amount, i.e., $A_a^{(m)} = A_a + \epsilon$; then the entropy and temperature of the modified apparent horizon can be written as (from the choice (a)) $S_a^{(m)} = \beta S_a^B$, $T_a = T_a^{(m)}$.

Now the modified radius of the apparent horizon is related to the original radius as (in the first approximation)

$$R_a^{(m)} = R_a + \frac{\epsilon\prime}{2\Pi R_a}, \qquad \epsilon\prime = \frac{\epsilon}{4}, \tag{44}$$

and β approximates

$$\beta = 1 - \frac{\epsilon\prime}{\Pi R_a^2} + \frac{2\epsilon\prime}{\Pi R_a^2} ln R_a. \tag{45}$$

Hence we have

$$S_a^{(m)} = S_a^{(B)} + \frac{2\epsilon\prime}{G} ln R_a \tag{46}$$

and

$$T_a = \frac{1}{2\Pi R_a} + \frac{\epsilon\prime}{2\Pi R_a^3}. \tag{47}$$

Thus there is a logarithmic correction to the Bekenstein entropy, and the Hawking temperature is corrected by a term proportional to R_a^{-3} due to this thermal fluctuation. However, if we consider the infinitesimal change in the radius of the apparent horizon due to the thermal fluctuation, i.e. $R_a^{(m)} = R_a + \epsilon$, then the modified entropy and temperature on the horizon become

$$S_a^{(m)} = S_a^{(B)} + \frac{4\Pi\epsilon R_a}{G}, \quad T_a = T_a^H + \frac{\epsilon}{2\Pi R_a^2}, \tag{48}$$

i.e., the correction to the Bekenstein entropy is proportional to the radius of the horizon, and that of the temperature is proportional to the inverse square of the radius.

6 Summary and concluding remarks

In this work we have studied thermodynamical laws on the event horizon for the following three choices of entropy and temperature on the event horizon:

(1) $S_E = S_E^{(B)}, T_E = T_E^{(g)}$,
(2) $S_E = \beta S_E^{(B)}, T_E = T_E^{(m)}$,
(3) $S_E = \delta S_E^{(B)}, T_E = \frac{1}{\delta} T_E^{(m)}$

where $S_E^{(B)}$ and $T_E^{(m)}$ are, respectively, the usual Bekenstein entropy and modified Hawking temperature (given in (15) or (16)) and the parameters α, β, and δ are evaluated using the Clausius relation. It is found that the parameters take value unity on the apparent horizon so that all the three choices reduce to the Bekenstein–Hawking system on the apparent horizon. However, for the third choice the entropy on the event horizon turns out to be constant (equal to that on the apparent horizon) and hence it is not of much physical interest. So we have not discussed it further. On the other hand, both thermodynamical laws hold on the event horizon unconditionally for any fluid distribution for the first choice, while for the second choice of entropy and temperature on the event horizon we must have $R_E < R_A$ in the phantom era for the validity of the GSLT. Hence we call a universe bounded by the event horizon (for the above two choices of entropy and temperature) a generalized Bekenstein–Hawking thermodynamical system. Also some interpretations of the parameters α and β have been presented in Sect. 5. Lastly, if in the present model of the universal thermodynamics there is thermal equilibrium with CMB photons then the temperature of the horizon must coincide with the CMB temperature ($\simeq 2.73K$) today [31,32]. Now restoring the dimension, the temperature of the event horizon (see (19)) can be written (in Kelvin) as [32]

$$T_E^{(g)} = \frac{(1+q)H^3 R_E^3}{HR_E - 1} \frac{1}{2\pi R_E} \left(\frac{\hbar c}{k_B}\right) \tag{49}$$

where $q = -(1+\dot{H}/H^2)$ is the usual deceleration parameter; $\hbar = 1.05 \times 10^{-27}$erg s, $c = 3 \times 10^{10}$cm/s, and $k_B = 1.38 \times 10^{-16}$erg/K are, respectively, the Planck constant, the speed of light, and the Boltzmann constant.

In particular, if we choose the cosmic fluid as holographic dark energy, then from (39) we get

$$HR_E = c/\sqrt{\Omega_d}$$

and we have

$$T_E^{(g)} = \frac{(1+q)c^3}{\Omega_d(c - \sqrt{\Omega_d})} \frac{0.23}{2\pi R_E} K. \tag{50}$$

Now using the observed values of c, ω_d, q, and choosing R_E appropriately, it is always possible to match $T_E^{(g)}$ with the temperature of the CMB photons. Finally, the conclusions are point-wise presented below.

I. The universe bounded by the event horizon (generalized Bekenstein–Hawking system) is a realistic thermodynamical system where both thermodynamical laws hold for any matter system within it.
II. In deriving the thermodynamical laws we have used the second Friedmann Eq. (6) and the energy conservation relation (7). On the other hand, assuming the first law of thermodynamics it is possible to derive the Einstein field equations. So we may conclude that the first law of thermodynamics and the Einstein field equations are equivalent (i.e. one can be derived from the other) on the event horizon irrespective of any fluid distribution.
III. The generalized Bekenstein–Hawking system i.e. a universe bounded by the event horizon supports the recent observations, i.e., the results of the present work are compatible (qualitatively) with the present observed data.
IV. If due to some thermal fluctuations the apparent horizon is modified so that its area changes infinitesimally then up to the first order of approximation the Bekenstein entropy is corrected by a logarithmic term and the correction to the Hawking temperature is proportional to the inverse cube of the radius of the apparent horizon.
V. The horizon temperature can be in thermal equilibrium with CMB photons by an appropriate choice of the parameters involved.

For future work one may consider the following issues.

(i) Do we have validity of the thermodynamical laws on the event horizon for other gravity theories?
(ii) Is this generalized Hawking temperature or the modified Bekenstein valid for other horizons (if it exists) of the universal thermodynamical system?
(iii) Further, what are the physical and geometrical implications of the parameters α and β?
(iv) Is the present generalized Bekenstein–Hawking system, i.e., $S_E = S_E^{(B)}, T_E = T_E^{(g)}$ or $S_E = \beta S_E^{(B)}, T_E = T_E^{(m)}$ or some other modified version on the event horizon physically more realistic?

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