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Dynamics of interacting quintessence

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Abstract   In this paper, we investigate coupled quintessence with scaling potential assuming specific forms of the coupling as A namely, $\alpha \dot{\rho}_m$, $\beta \dot{\rho}_\phi$ and $\sigma (\dot{\rho}_m + \dot{\rho}_\phi)$, and present phase space analysis for three different interacting models. We focus on the attractor solutions that can give rise to late time acceleration with $\Omega_{\rm DE}/\Omega_{\rm DM}$ of order unity in order to alleviate the coincidence problem.

1 Introduction

A large number of cosmological observations [1–5] reveal that our Universe is experiencing an accelerated expansion at present and the transition from deceleration phase to acceleration phase took place in the recent past [3]. In the standard Einstein gravity, the late time cosmic acceleration is driven by an exotic energy component with huge negative pressure filling the Universe, known as 'dark energy' [6–9]. One of the simplest candidate of dark energy (DE) is the cosmological constant (CC) Λ. However, it is plagued with difficult theoretical issues such as fine tuning and cosmic coincidence problem [10–13]. This is important to explore whether dark energy is cosmological constant or it has dynamics. To this effect, a variety of dynamical dark energy models have been explored in the references [14–48]. The models of unstable dark matter with a non zero cosmological constant can also mimic such dynamics [49–53]. Alternatively, large scale modification of gravity has been used to obtain late time cosmic acceleration. At present late time cosmic acceleration is treated as an established phenomenon however its underly-

ing cause is still unknown. Within the framework of Einstein gravity and modified theories of gravity, numerous models can explain the said phenomenon.

Although ΛCDM model is consistent with present observations, yet there is no satisfactory argument for coincidence problem. Interaction of dark energy with dark matter is one novel approach that might address the mentioned problem. The interacting dark energy models have been recently proposed by several authors [54–59]. The interaction between dark energy and dark matter may enhance the dark matter, and also affect structure formation. The investigation of phase space analysis is the one conclusive test for dark energy models. Specifically, the attractor solutions are independent for a wide range of initial conditions. If the dark energy models have $\Omega_{\rm DE}/\Omega_{\rm DM}$ of the order 1 and an accelerated scaling attractor solution, then the coincidence problem can be alleviated. The non-interacting quintessence [60–62] and quintom [63–66] models show late time accelerated attractors, and possess $\Omega_{\rm DE}$ as 1, therefore, they do not provide an adequate solution for coincidence problem. In the literature, two forms of interactions have been discussed namely, local and non-local. Local forms of interactions are directly proportional to energy density whereas non-local forms are directly proportional to Hubble parameter H and energy density ρ. In this paper we consider local forms of interactions proportional to energy density. Some of the local forms have been discussed in references [67–71]. Note that some of the choices of interacting terms appeared implicitly in the literature [72]. There is also approach to discuss the interacting term without the assumption of a specific form of interacting term [73]. The plan of the work is organised as follows: In Sect. 2 we establish the interacting quintessence cosmological framework and construct an autonomous dynamical system which is worthy for phase space investigation. In Sect. 3 we discuss phase space analysis and find stationary points and their stability for three

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interacting quintessence cosmological models. Our results are presented in Sect. 4.

2 Quintessence cosmology

We consider two components first one is canonical scalar field (quintessence) as a source of dark energy in spatially flat Universe, and second one is matter (Baryonic+DM). The total energy density of the Universe is conserved, and the individual components of energy density may not be conserved. Thus, we are considering following conservation equations of energy density as:

$$
\begin{aligned}
&\dot{\rho}_{\text{tot}} + 3H(1 + w_{\text{tot}})\rho_{\text{tot}} = 0, \\
&\dot{\rho}_\phi + 3H(1 + w_\phi)\rho_\phi = -A, \\
&\dot{\rho}_m + 3H(1 + w_m)\rho_m = A,
\end{aligned}
\tag{1}
$$

where $\rho_{\text{tot}} = \rho_\phi + \rho_m$, $1 + w_\phi = \dot{\phi}^2/\rho_\phi$, $w_m = 0$ is the equation of state of matter, A is the interaction strength and H is the Hubble parameter which is given as

$$
H^2 = \frac{8\pi G}{3}\rho_{\text{tot}}
\tag{2}
$$

The sign of A gives information about the direction of flow of energy between two components. There are three cases:

Case I: If $A > 0$, in this case transfer of energy occurs from quintessence to dark matter. Consequently, quintessence losses self strength and gives dark matter.

Case II: If $A < 0$, under this condition dark matter losses its strength and there is energy transfer from dark matter to quintessence.

Case III: If $A = 0$, under this condition quintessence do not interact with dark matter, and no energy transfer at all between two components considered in the literature. Therefore, we are not considering this case.

Since, there is no fundamental theory of dark energy and dark matter interaction (interaction in dark sector) at present, therefore it is not possible to construct the functional form of interaction strength A from first principle. Different forms of interaction strength (linear and non-linear) have been considered by several authors [74–83]. Motivated from the left hand side of the energy conservation equation (1) it is natural that A should be the function of Hubble parameter and energy density that is

$$
A = A(H, \rho_m, \rho_\phi)
\tag{3}
$$

Here we consider three specific forms of interaction strength heuristically as:

$$
A = \alpha\dot{\rho}_m,
\tag{4}
$$

$$
A = \beta\dot{\rho}_\phi,
\tag{5}
$$

$$
A = \sigma(\dot{\rho}_m + \dot{\rho}_\phi)
\tag{6}
$$

Since, Hubble parameter has the dimension of inverse of time, and inverse of time is sitting in the rate change of energy density, see Eqs. (4)–(6), therefore we are not incorporating H separately in the functional form of A. The evolution equations in a flat Friedmann–Lemaitre–Robertson–Walker (FLRW) Universe can be written as:

$$
\begin{aligned}
H^2 &= \frac{\kappa^2}{3}(\rho_m + \rho_\phi) \\
2H\dot{H} &= \frac{\kappa^2}{3}(\dot{\rho}_m + \dot{\rho}_\phi)
\end{aligned}
\tag{7}
$$

where $\kappa^2 = 8\pi G$, $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ and $p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$. We introduce following dimensionless parameters

$$
X^2 = \frac{\kappa^2\dot{\phi}^2}{6H^2}; \quad Y^2 = \frac{\kappa^2 V}{3H^2}; \quad \lambda = -\frac{V'}{\kappa V}
\tag{8}
$$

to form an autonomous system of evolution equations (1) and (7) as:

$$
\begin{aligned}
\frac{dX}{dN} &= -3X + \sqrt{\frac{3}{2}}\lambda Y^2 - X\frac{\dot{H}}{H^2} \\
\frac{dY}{dN} &= -\sqrt{\frac{3}{2}}\lambda XY - Y\frac{\dot{H}}{H^2}
\end{aligned}
\tag{9}
$$

where $N = \ln a$. The total equation of state and field density parameter are given as:

$$
\begin{aligned}
W_{\text{tot}} &= -1 - \frac{2\dot{H}}{3H^2} \\
\Omega_\phi &= \frac{\kappa^2\rho_\phi}{3H^2} = X^2 + Y^2
\end{aligned}
\tag{10}
$$

The condition for acceleration is $W_{\text{tot}} < -\frac{1}{3}$.

3 Phase space analysis: stationary points and their stability

In this section we shall use an autonomous system [Eq. (9)], which is appropriate for obtaining stationary points and stability. The stationary points shall be obtained by equating the left hand side of Eq. (9) to zero. Their stability will be confirmed from the sign of the corresponding eigenvalues, which will be obtained numerically.

3.1 Interacting model I

In this model we consider the following specific form of interaction strength

$$
A = \alpha\dot{\rho}_m
\tag{11}
$$

Using Eq. (7) for this phenomenological form of interaction we have

$$
\frac{\dot{H}}{H^2} = -\frac{3}{2}\left[\frac{1 - Y^2 + X^2(1 - 2\alpha)}{1 - \alpha}\right]
\tag{12}
$$

By using $\Omega_\phi + \Omega_m = 1$ where $\Omega_m = 1 - X^2 - Y^2$. Thus, from Eq. (9) with Eq. (12) we have following form of autonomous system

$$\frac{dX}{dN} = -3X + \sqrt{\frac{3}{2}}\lambda Y^2 + \frac{3}{2}X\left[\frac{1 - Y^2 + X^2(1 - 2\alpha)}{1 - \alpha}\right]$$

$$\frac{dY}{dN} = -\sqrt{\frac{3}{2}}\lambda XY + \frac{3}{2}Y\left[\frac{1 - Y^2 + X^2(1 - 2\alpha)}{1 - \alpha}\right] \quad (13)$$

Using Eqs. (10) and (12), the total equation of state for this model can be written as

$$W_{\text{tot}} = -1 + \left[\frac{1 - Y^2 + X^2(1 - 2\alpha)}{1 - \alpha}\right] \equiv W_\phi \Omega_\phi \quad (14)$$

The critical points of autonomous system (13) could be obtained by setting $\frac{dX}{dN} = 0$ and $\frac{dY}{dN} = 0$ simultaneously. Thus, we have following stationary points:

1. $X = -1, Y = 0$
   In this case, the corresponding eigenvalues are

   $\mu_1 = 6 + 3/(-1 + \alpha) < 0$,   for $\alpha < 1$,
   $\mu_2 = 3 + \sqrt{3/2}\lambda < 0$,   for $\lambda < -\sqrt{6}$

   The eigenvalues of this point show the negativity for $\alpha < 1$ and $\lambda < -\sqrt{6}$. Therefore, it is stable point.

2. $X = 0, Y = 0$
   In this case, the corresponding eigenvalues are

   $\mu_1 = 3/(2 - 2\alpha) < 0$,   for $\alpha > 1$,
   $\mu_2 = -3 + 3/(2 - 2\alpha) < 0$,   for $1 < \alpha < 1/2$

   The negativity of the eigenvalues represents the stability. This point is stable for $\alpha > 1$.

3. $X = 1, Y = 0$
   In this case, the corresponding eigenvalues are

   $\mu_1 = 6 + 3/(-1 + \alpha) < 0$,   for $\alpha < 1$,
   $\mu_2 = 3 - \sqrt{3/2}\lambda < 0$,   for $\lambda < \sqrt{6}$

   This point is stable for $\alpha < 1$ and $\lambda < \sqrt{6}$.

4. $X = \frac{\lambda}{\sqrt{6}}, Y = \sqrt{1 - \frac{\lambda^2}{6}}$
   In this case, the corresponding eigenvalues are

   $\mu_1 = (-6 + \lambda^2)/2 < 0$,   for $\lambda < \sqrt{6}$,
   $\mu_2 = 3/(-1 + \alpha) + \lambda^2 < 0$,   for $\alpha < 1$,
   $\lambda < \sqrt{3/(-1 + \alpha)}$

   This point is stable under above given conditions.

5. $X = \frac{\sqrt{\frac{3}{2}}}{\lambda(1 - \alpha)}, Y = \frac{\sqrt{\frac{3}{2} - 3\alpha}}{\lambda(\alpha - 1)}$
   In this case, the corresponding eigenvalues are

$$\mu_1 = -\frac{3(\lambda^2(\alpha - 1)^2(2\alpha - 1) + \sqrt{\lambda^2(\alpha - 1)^3(2\alpha - 1)(24 + \lambda^2(7 + 2\alpha)(\alpha - 1))}}{4\lambda^2(\alpha - 1)^3} < 0, \quad \text{for } \alpha < -7/2, \lambda > 0,$$

$$\mu_2 = \frac{3(\lambda^2(\alpha - 1)^2(2\alpha - 1) + \sqrt{\lambda^2(\alpha - 1)^3(2\alpha - 1)(24 + \lambda^2(7 + 2\alpha)(\alpha - 1))}}{4\lambda^2(\alpha - 1)^3} < 0, \quad \text{for } \alpha < -7/2, \lambda > \sqrt{2/3}$$

This point has negative eigenvalues for $\alpha < -7/2$ and $\lambda > \sqrt{2/3}$. Therefore, it is stable point. We are interested in this point because it has both the parameters α and λ. We evolve the autonomous system (13) numerically for the parameter values $\alpha = -3.6$, $\lambda = 0.94$, and $\alpha = -4.6$, $\lambda = 0.94$, and the obtained results are shown in Fig. 1. For the chosen parameters the stable point behaves as an attractive node which is confirmed by the panels (a) and (b) of Fig. 1. The lower panels of Fig. 1 show the scaling behaviour that gives acceleration. In addition we also calculate basic cosmological observables W_{tot}, W_ϕ, Ω_ϕ and obtained as $-0.78, -1.06, 0.73$ and $-0.82, -1.36, 0.60$ corresponding to $\alpha = -3.6, \lambda = 0.94$, and $\alpha = -4.6, \lambda = 0.94$, respectively. This point is summarized in Table 1.

3.2 Interacting model II

This model is specified by a coupling of the form

$$A = \beta \dot{\rho}_\phi \quad (15)$$

For this coupling term we have

$$\frac{\dot{H}}{H^2} = -\frac{3}{2}\left[1 - Y^2 + \frac{X^2(1 - \beta)}{1 + \beta}\right] \quad (16)$$

$$\frac{dX}{dN} = -3X + \sqrt{\frac{3}{2}}\lambda Y^2 + \frac{3}{2}X\left[1 - Y^2 + \frac{X^2(1 - \beta)}{1 + \beta}\right]$$

$$\frac{dY}{dN} = -\sqrt{\frac{3}{2}}\lambda XY + \frac{3}{2}Y\left[1 - Y^2 + \frac{X^2(1 - \beta)}{1 + \beta}\right] \quad (17)$$

Using Eqs. (10) and (16), the total equation of state for this model can be written as

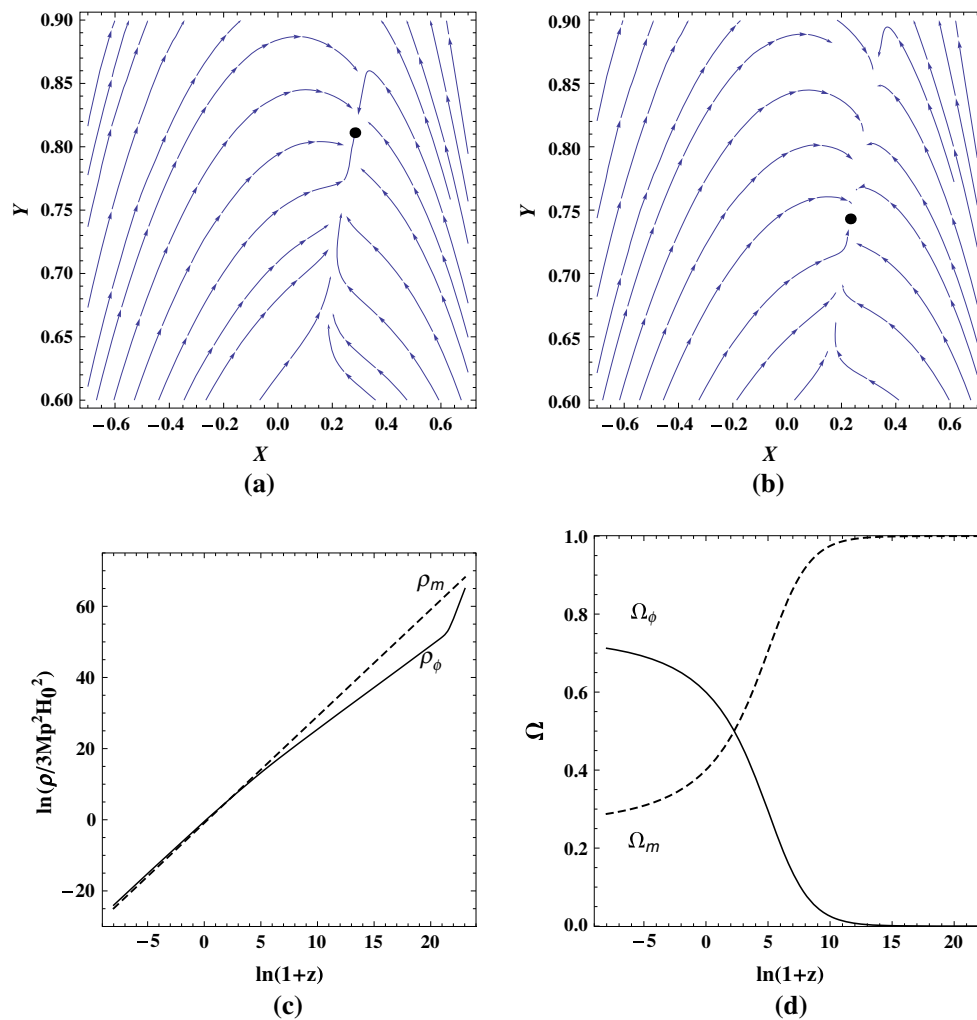$$W_{\text{tot}} = -1 + \left[1 - Y^2 + \frac{X^2(1 - \beta)}{1 + \beta}\right] \equiv W_\phi \Omega_\phi \quad (18)$$

Fig. 1 This figure shows the phase portrait, evolution of energy density and density parameter for model I for the stable fixed point 5 which is an attractive node. The panel (**a**) corresponds to $\alpha = -3.6$ and $\lambda = 0.94$, for these values of the parameters we obtain $\Omega_\phi = 0.73$, $W_{tot} = -0.78$, $W_\phi = -1.06$ and an accelerating attractor solution. The panel (**b**) corre-sponds to $\alpha = -4.6$ and $\lambda = 0.94$, correspondingly we get $\Omega_\phi = 0.60$, $W_{tot} = -0.82$, $W_\phi = -1.36$ and an accelerated attractor solution. In both the panels, black dots represent attractor stable point. The panels (**c**) and (**d**) have same values of the parameters as panel (**a**), and show scaling behaviours that provide an accelerated expansion

This interacting model has following stationary points:

1. $X = 0, Y = 0$
   In this case, the corresponding eigenvalues are

   $$\mu_1 = -3/2, \quad \mu_2 = 3/2$$

   This is unstable point because one of the eigenvalue is positive.

2. $X = \frac{\sqrt{-1-\beta}}{\sqrt{-1+\beta}}, \quad Y = 0$
   In this case, the corresponding eigenvalues are

   $$\mu_1 = 3, \quad \mu_2 = 3 - \frac{\sqrt{-3(1+\beta)/2}\,\lambda}{\sqrt{-1+\beta}} < 0,$$
   $$\text{for } \sqrt{-3(1+\beta)/2}\,\lambda > 3(-1+\beta)$$

It is also unstable point because its one eigenvalue is positive.

3. $X = \frac{(3+\lambda^2)(1+\beta)-\delta}{2\sqrt{6}\,\lambda}$,
   $Y = \frac{\sqrt{(1-\beta^2)(6+9/\lambda^2)-(1+\beta^2)\lambda^2+\delta(1+\beta-3/\lambda^2+3\beta/\lambda^2)}}{2\sqrt{3}}$
   In this case, the corresponding eigenvalues are

   $$\mu_1 = \frac{1}{8}\Big(3(-5+\lambda^2) + 3\beta(3+\lambda^2) - 3\delta$$
   $$-\sqrt{\Big(\frac{2}{\lambda^2}(216 - 63\lambda^2 + \lambda^6 + \beta^2(-24+\lambda^2)}$$
   $$\times (3+\lambda^2)^2 - 72\delta - 3\lambda^2\delta - \lambda^4\delta$$
   $$+ \beta(2\lambda^6 + 72\delta - \lambda^4(18+\delta) + 3\lambda^2(30+7\delta))))\Big)$$

   $$\mu_2 = \frac{1}{8}\Big(3(-5+\lambda^2) + 3\beta(3+\lambda^2) - 3\delta$$

Table 1 We present the stable points 5, 3 and 3 for interacting models I, II and III respectively, these stable points have two parameters, namely, α, λ and β, λ and σ, λ. We also introduce the expressions of Ω_ϕ, W_{tot} and the condition of acceleration in terms of the parameters.

Two numerical choices of the parameters are shown for each model. The symbols δ and η are used for shorting the expressions and given by Eqs. (19) and (24) respectively

Model	X	Y	Stable for	Ω_ϕ	W_{tot}	$W_\phi = \frac{W_{\text{tot}}}{\Omega_\phi}$	Acceleration
I	$\frac{\sqrt{3/2}}{\lambda(1-\alpha)}$	$\frac{\sqrt{3/2-3\alpha}}{\lambda(\alpha-1)}$	$\alpha < -7/2, \lambda > \sqrt{2/3}$	$\frac{3}{\lambda^2(1-\alpha)}$	$\frac{\alpha}{1-\alpha}$		For $\alpha < -1/2$
			$\alpha = -3.6, \lambda = 0.94$	0.73	−0.78	−1.06	Yes
			$\alpha = -4.6, \lambda = 0.94$	0.60	−0.82	−1.36	Yes
II	$\frac{1}{2\sqrt{6}\,\lambda}$ $[(3+\lambda^2)$ $(1+\beta)-\delta]$	$\frac{1}{2\sqrt{3}}\sqrt{[(1-\beta^2)}$ $(6+9/\lambda^2)-(1$ $+\beta^2)\lambda^2+\delta(1+\beta$ $-3/\lambda^2+3\beta/\lambda^2)]$	For all values of β and λ provided that $\lambda \neq 0$	$\frac{(3+\lambda^2)(1+\beta)-\delta}{2\lambda^2}$	$\frac{\lambda^2-3+\beta(3+\lambda^2)-\delta}{6}$		$\lambda^2+\beta(3+\lambda^2)$ $-\delta < 1$
			$\beta = 0.7, \lambda = 1.3$	0.76	−0.57	−0.75	Yes
			$\beta = 0.3, \lambda = 1.3$	0.83	−0.53	−0.64	Yes
III	$\frac{9-(1-\sigma)^2\lambda^4+\eta}{2\sqrt{6}\lambda(3-(1-\sigma)\lambda^2)}$	$\frac{\sqrt{9+6\lambda^2-(1-\sigma)^2\lambda^4+\eta}}{2\sqrt{3}\lambda}$	$\sigma < 0.2 \neq$ $-1, \lambda \leq$ $\frac{\sqrt{6(1+\sigma)}}{1-\sigma}$	$\frac{9-(1-\sigma)^2\lambda^4+\eta}{6\lambda^2-2(1-\sigma)\lambda^4}$	$\frac{1}{18-6(1-\sigma)\lambda^2}[(1$ $-\sigma)\lambda^2(6-(1$ $-\sigma)\lambda^2)+\eta-9]$		$27-3(1-\sigma)\lambda^2$ $[4-(1-\sigma)\lambda^2]$ $-3\eta < 0$
			$\sigma = -0.3, \lambda = 1.57$	0.65	−0.46	−0.71	Yes
			$\sigma = 0.1, \lambda = 2.85$	0.45	0.22	0.49	No

$$+ \sqrt{\Big(\frac{2}{\lambda^2}(216 - 63\lambda^2 + \lambda^6 + \beta^2(-24 + \lambda^2)}$$
$$\times (3 + \lambda^2)^2 - 72\delta - 3\lambda^2\delta - \lambda^4\delta$$
$$+ \beta(2\lambda^6 + 72\delta - \lambda^4(18 + \delta) + 3\lambda^2(30 + 7\delta)))\Big) \Big),$$

where

$$\delta = \sqrt{(1 + \beta)((-3 + \lambda^2)^2 + \beta(3 + \lambda^2)^2)} \tag{19}$$

Above eigenvalues are negative for $-\infty \leq \beta \leq \infty$ and $-\infty \leq \lambda \leq \infty$ (but $\lambda \neq 0$).

The negativity of the eigenvalues exhibits the stability. This point is stable for all the values of β and λ provided that $\lambda \neq 0$. We elaborate the autonomous system (17) numerically for the parameter choices $\beta = 0.7$, $\lambda = 1.3$, and $\beta = 0.3$, $\lambda = 1.3$, and obtain W_{tot}, W_ϕ and Ω_ϕ as -0.57, -0.75, 0.76 and -0.53, -0.64, 0.83, respectively. The phase space trajectories of this stable point are shown in panels (a) and (b) of Fig. 2, and the point behaves as an attractive focus. The lower panels of Fig. 2 exhibit the scaling behaviour that gives late time acceleration. The results of the stable point are abbreviated in Table 1.

3.3 Interacting model III

In this model we consider the coupling form as a linear combination of $\dot{\rho}_m$ and $\dot{\rho}_\phi$ as

$$A = \sigma(\dot{\rho}_m + \dot{\rho}_\phi) \tag{20}$$

For this interaction form we have

$$\frac{\dot{H}}{H^2} = -\frac{3}{2}\left[\frac{1 + X^2 - Y^2}{1 - \sigma}\right] \tag{21}$$

$$\frac{dX}{dN} = -3X + \sqrt{\frac{3}{2}}\lambda Y^2 + \frac{3}{2}X\left[\frac{1 + X^2 - Y^2}{1 - \sigma}\right] \tag{22}$$

$$\frac{dY}{dN} = -\sqrt{\frac{3}{2}}\lambda XY + \frac{3}{2}Y\left[\frac{1 + X^2 - Y^2}{1 - \sigma}\right]$$

The total equation of state for this model can be obtained by using Eqs. (10) and (21) as

$$W_{\text{tot}} = -1 + \left[\frac{1 + X^2 - Y^2}{1 - \sigma}\right] \equiv W_\phi \Omega_\phi \tag{23}$$

This model has following stationary points:

1. $X = 0, Y = 0$
   In this case, the corresponding eigenvalues are

   $$\mu_1 = -\frac{3}{2(-1 + \sigma)} < 0, \quad \text{for } \sigma > 1$$
   $$\mu_2 = \frac{3 - 6\sigma}{2(-1 + \sigma)} < 0, \quad \text{for } \sigma > 1/2$$

   This point is stable for $\sigma > 1$.

2. $X = \sqrt{1 - 2\sigma}, Y = 0$
   In this case, the corresponding eigenvalues are

   $$\mu_1 = 6 + \frac{3}{-1 + \sigma} < 0, \quad \text{for } \sigma < 1/2$$
   $$\mu_2 = 3 - \sqrt{3/2 - 3\sigma}\,\lambda < 0, \quad \text{for } \sqrt{3/2 - 3\sigma}\,\lambda > 3$$
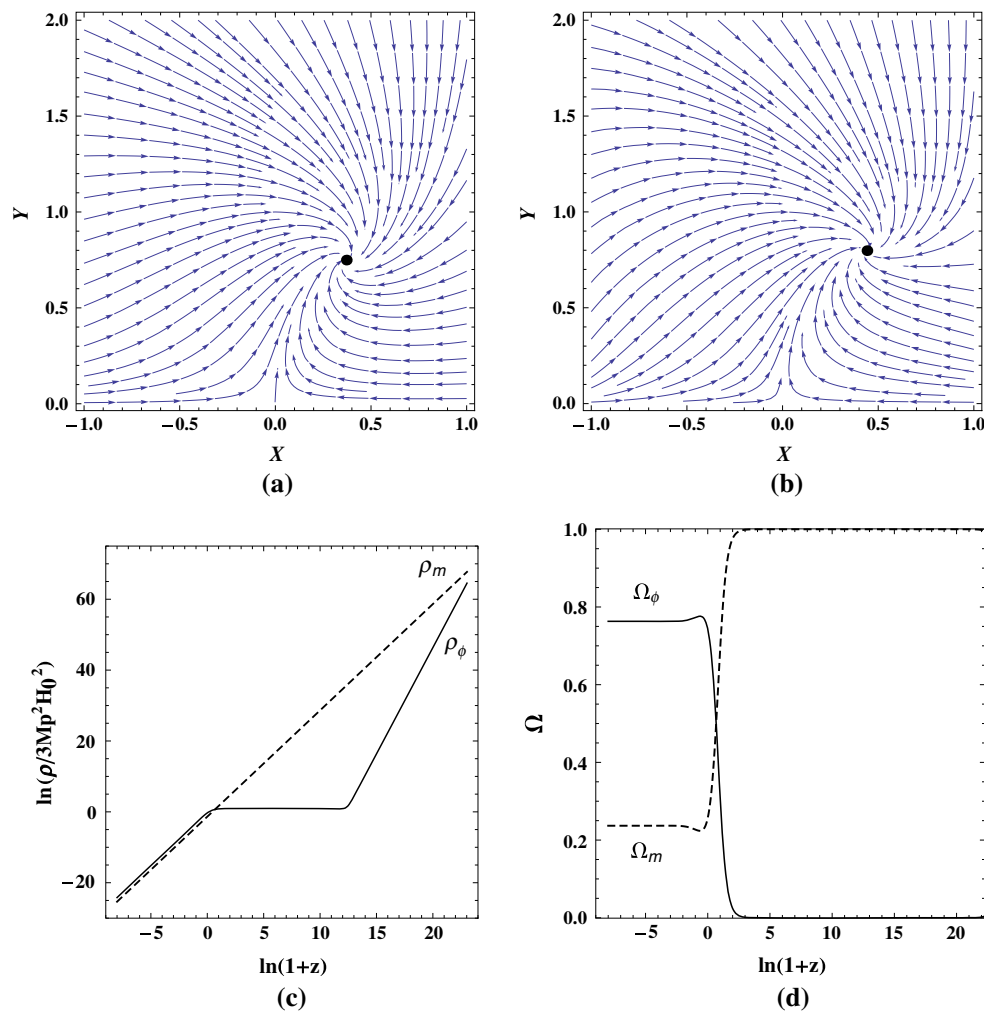
(a)

(b)

(c)

(d)

Fig. 2 The figure represents phase space trajectories, evolution of energy density and density parameter for interacting model II for the stable fixed point 3. The panel (**a**) corresponds to $\beta = 0.7$ and $\lambda = 1.3$, for these values of the parameters we obtain $\Omega_\phi = 0.76$, $W_{tot} = -0.57$, $W_\phi = -0.75$ and an accelerating attractor solution. The panel (**b**) corresponds to $\beta = 0.3$ and $\lambda = 1.3$, correspondingly we get $\Omega_\phi = 0.83$,

$W_{tot} = -0.53$, $W_\phi = -0.64$ and an accelerated attractor solution. In both the panels, black dots designate attractor stable point, and the stable point behaves as an attractive focus under the chosen parameters. The panels (**c**) and (**d**) are plotted for the same values of the parameters as panel (**a**), and show scaling behaviours that gives an accelerated expansion

This point is also stable for the above given conditions.

3. $X = \frac{9 - (-1+\sigma)^2\lambda^4 + \eta}{2\sqrt{6}\lambda(3 + (-1+\sigma)\lambda^2)}$, $Y = \frac{\sqrt{9 + 6\lambda^2 - (-1+\sigma)^2\lambda^4 + \eta}}{2\sqrt{3}\lambda}$

In this case, the corresponding eigenvalues are

$$\mu_1 = \frac{1}{8}\left(-3 + \frac{12}{-1+\sigma} - 3(-1+\sigma)\lambda^2\right.$$
$$+ \frac{3\eta}{3 + (-1+\sigma)\lambda^2} - \sqrt{2}\left(\frac{9(-7 + \sigma(2 + 13\sigma))}{(-1+\sigma)^2}\right.$$
$$- \frac{216}{(-1+\sigma)\lambda^2} + 6\sigma\lambda^2 + (-1+\sigma)^2\lambda^4 - \eta$$
$$- \frac{24\eta}{(-1+\sigma)\lambda^2} - \frac{30\eta}{(-1+\sigma)(3 + (-1+\sigma)\lambda^2)}$$
$$\left.\left.+ \frac{18\sigma\eta}{(-1+\sigma)(3 + (-1+\sigma)\lambda^2)}\right)\right)$$

$$\mu_2 = \frac{1}{8}\left(-3 + \frac{12}{-1+\sigma} - 3(-1+\sigma)\lambda^2\right.$$
$$+ \frac{3\eta}{3 + (-1+\sigma)\lambda^2} + \sqrt{2}\left(\frac{9(-7 + \sigma(2 + 13\sigma))}{(-1+\sigma)^2}\right.$$
$$- \frac{216}{(-1+\sigma)\lambda^2} + 6\sigma\lambda^2 + (-1+\sigma)^2\lambda^4 - \eta$$
$$- \frac{24\eta}{(-1+\sigma)\lambda^2} - \frac{30\eta}{(-1+\sigma)(3 + (-1+\sigma)\lambda^2)}$$
$$\left.\left.+ \frac{18\sigma\eta}{(-1+\sigma)(3 + (-1+\sigma)\lambda^2)}\right)\right)$$

where

$$\eta = \sqrt{(3 + (-1+\sigma)\lambda^2)^2(9 - 6(1+\sigma)\lambda^2 + (-1+\sigma)^2\lambda^4)}$$
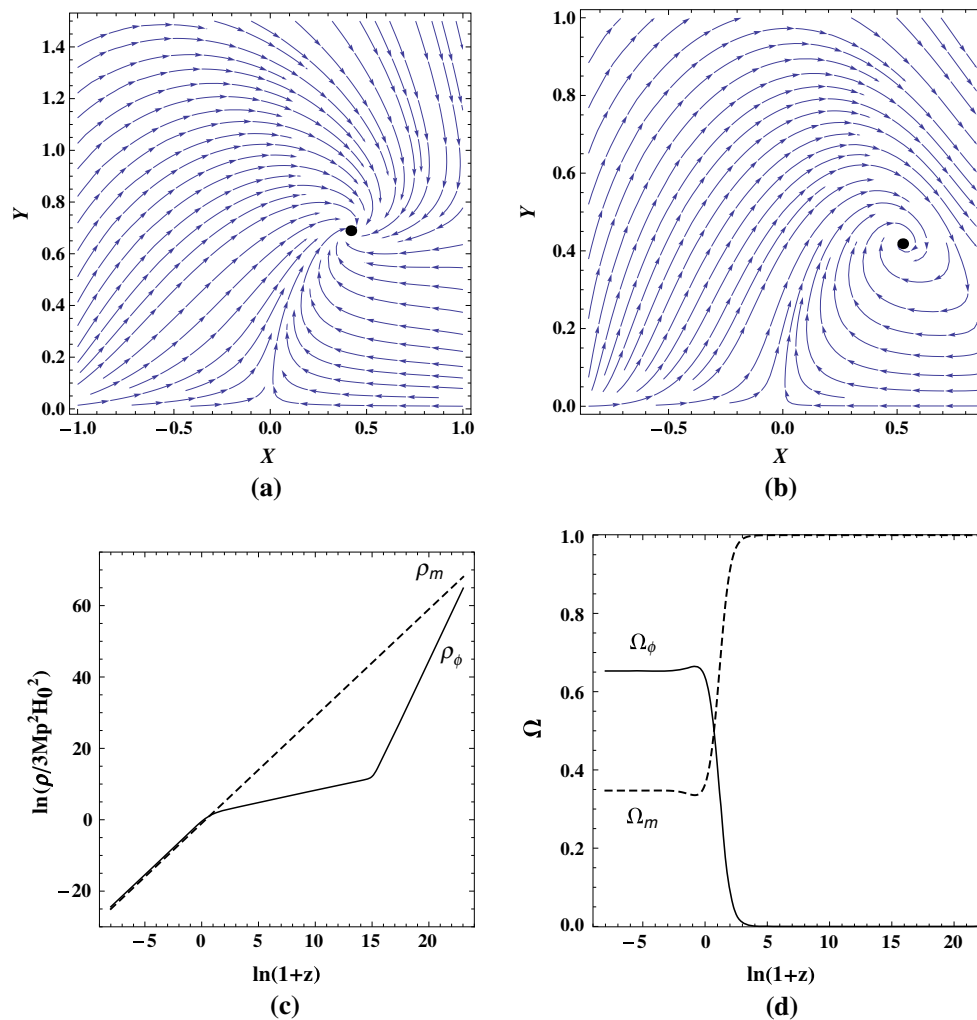$$\tag{24}$$

(a)

(b)

(c)

(d)

Fig. 3 The figure displays phase space trajectories, evolution of energy density and density parameter for interacting model III for the stable fixed point 3. The panel (**a**) corresponds to $\sigma = -0.3$ and $\lambda = 1.57$, for these values of the parameters we get $\Omega_\phi = 0.65$, $W_{\text{tot}} = -0.46$, $W_\phi = -0.71$ and an accelerating attractor solution. The panel (**b**) corresponds to $\sigma = 0.1$ and $\lambda = 2.85$, and correspondingly we obtain $\Omega_\phi = 0.45$, $W_{\text{tot}} = 0.22$, $W_\phi = 0.49$, since total equation of state is positive therefore, attracting solution but not accelerating. In both the panels, black dots represent attractor stable point, and the stable point acts as an attractive focus under the chosen parameters. For panels (**c**) and (**d**), we use same values of the parameters as panel (**a**). Both panels are showing attractor behaviour that corresponds to scaling solutions

Above eigenvalues are negative for $\sigma < 0.2$ provided that $\sigma \neq -1$ and $\lambda \leq \sqrt{\frac{6(1+\sigma)}{(-1+\sigma)^2}}$.

This point has both the parameters σ, λ and shows the stability for the choices of the parameters $\sigma < 0.2$ ($\sigma \neq -1$) and $\lambda \leq \sqrt{\frac{6(1+\sigma)}{(-1+\sigma)^2}}$. We numerically elaborate the autonomous system (22) for the parameter choices $\sigma = -0.3$, $\lambda = 1.57$, and $\sigma = 0.1$, $\lambda = 2.85$, and obtain W_{tot}, W_ϕ, Ω_ϕ as -0.46, -0.71, 0.65 and 0.22, 0.49, 0.45, respectively. We do not find accelerating solution corresponding to $\sigma = 0.1$ as it has positive equation of state. Also, we check numerically for all positive values of σ ($0 < \sigma < 0.2$) and do not find accelerating phase. The results of this stable point are concised in Table 1. The phase portraits of the point are displayed in panels (a) and (b) of Fig. 3, and the point acts as an attractive focus. The lower

panels of Fig. 3 exhibit the scaling behaviour that provides late time acceleration.

4 Conclusions

We studied interaction of quintessence with dark matter in spatially flat Universe. In the absence of fundamental theory of specific interaction in the dark sector, the choice of interaction strength in the conservation of energy equations was phenomenological and heuristic. In this paper we considered three phenomenological interacting quintessence cosmological models as $\alpha \dot{\rho}_m$, $\beta \dot{\rho}_\phi$, and $\sigma(\dot{\rho}_m + \dot{\rho}_\phi)$. Our primary object was to inspect whether there exist late time accelerated scaling attractor having $\Omega_{\text{DE}}/\Omega_{\text{DM}}$ of the order one.

We studied dynamical behaviour and phase space analysis of the models under consideration. We focussed on the stable points which could give rise to scaling attractors. In all the models we obtained fundamental cosmological observables like Ω_ϕ, W_{tot}, W_ϕ corresponding to two numerical choices of the parameters. For the interacting model I we found that the fixed point 5 is stable for $\alpha < -7/2$ and $\lambda > \sqrt{2/3}$. The phase space trajectories, evolution of energy density and density parameter for different numerical choices of the parameters are shown in Fig. 1. The fixed point 5, in this case, corresponds to accelerated scaling attractor with $\Omega_{\text{DE}}/\Omega_{\text{DM}} = O(1)$. The point 3 of interacting model II shows stability for all values of β and λ provided that $\lambda \neq 0$. The phase portraits, evolution of energy density and density parameter for different numerical values of the parameters are displayed in Fig. 2. Clearly, this is scaling solution with the required property. For interacting model III we noticed that the fixed point 3 is stable for $\sigma < 0.2 \neq -1$ and $\lambda \leq \sqrt{6(1+\sigma)}/(1-\sigma)$. Figure 3 shows the phase space trajectories, evolution of energy density and density parameter for different values of the parameters. Our analysis shows that accelerating attractor, in this case, exists only for negative values of σ. The lower panels of Fig. 3 shows accelerated attractor solution for $\sigma = -0.3$ and $\lambda = 1.57$. For all the models, we obtained late time accelerated scaling attractor having $\Omega_{\text{DE}}/\Omega_{\text{DM}} = O(1)$. Therefore all the models considered in this paper are viable to solve the coincidence problem.

References

1. P.A.R. Ade et al. (Planck Collaboration), Planck 2013 results. XVI. Cosmological parameters. Astron. Astrophys. **571**, A16 (2014). arXiv:1303.5076 [astro-ph.CO]
2. S. Perlmutter et al., Measurements of omega and lambda from 42 high-redshift supernovae. Astrophys. J. **517**, 565 (1999)
3. A.G. Riess et al. (Supernova Search Team Collaboration), Observational evidence from supernovae for an accelerating Universe and a cosmological constant. Astron. J. **116**, 1009 (1998)
4. D.N. Spergel et al., Astrophys. J. Suppl. **148**, 175 (2003). arXiv:astro-ph/0302209
5. E. Komatsu et al., Seven-year Wilkinson microwave anisotropy probe (WMAP) observations: cosmological interpretation. ApJS **192**, 18 (2011)
6. E.J. Copeland, M. Sami, S. Tsujikawa, Dynamics of dark energy. Int. J. Mod. Phys. D **15**, 1753 (2006)
7. V. Sahni, A.A. Starobinsky, The case for a positive cosmological lambda-term. Int. J. Mod. Phys. D **9**, 373 (2000)
8. M. Sami, A primer on problems and prospects of dark energy. Curr. Sci. **97**, 887 (2009). arXiv:0904.3445
9. M. Sami, R. Myrzakulov, Late time cosmic acceleration: ABCD of dark energy and modified theories of gravity. arXiv:1309.4188 [hep-th]
10. S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989)
11. S.M. Carroll, Living Rev. Rel. **4**, 1 (2001). arXiv:astro-ph/0004075
12. P.J.E. Peebles, B. Ratra, Rev. Mod. Phys. **75**, 559 (2003). arXiv:astro-ph/0207347
13. T. Padmanabhan, Phys. Rep. **380**, 235 (2003). arXiv:hep-th/0212290
14. B. Ratra, P.J.E. Peebels, Phys. Rev. D. **37**, 3406 (1988)
15. R.R. Caldwell, M. Kamionkowski, N.N. Weinberg, Phys. Rev. Lett. **91**, 071301 (2003). arXiv:astro-ph/0302506
16. M.R. Setare, Eur. Phys. J. C **50**, 991 (2007)
17. A. Ali, R. Gannouji, M. Sami, Modified gravity a 'la Galileon: late time cosmic acceleration and observational constraints. Phys. Rev. D **82**, 103015 (2010)
18. R. Gannouji, M. Sami, Galileon gravity and its relevance to late time cosmic acceleration. Phys. Rev. D **82**, 024011 (2010)
19. M. Sami, M. Shahalam, M. Skugoreva, A. Toporensky, Phys. Rev. D **86**, 103532 (2012). arXiv:1207.6691
20. R. Myrzakulov, M. Shahalam, J. Cosmol. Astropart. Phys. **10**, 047 (2013). arXiv:1303.0194
21. R. Myrzakulov, M. Shahalam, Light mass Galileon and late time acceleration of the Universe. Gen. Relativ. Gravit. **47**, 81 (2015)
22. M. Shahalam, S. Sami, A. Agarwal, *Om* diagnostic applied to scalar field models and slowing down of cosmic acceleration. Mon. Not. R. Astron. Soc. **448**, 2948–2959 (2015). arXiv:1501.04047
23. M.M. Verma, S.D. Pathak, The BICEP2 data and a single Higgs-like interacting tachyonic field. Int. J. Mod. Phys. D **23**, 1450075 (2014). arXiv:1312.1175
24. C. Wetterich, Nucl. Phys. B **302**, 668 (1988)
25. A.R. Liddle, R.J. Scherrer, Phys. Rev. D **59**, 023509 (1998)
26. I. Zlatev, L.M. Wang, P.J. Steinhardt, Phys. Rev. Lett. **82**, 896 (1999)
27. Z.K. Guo, N. Ohta, Y.Z. Zhang, Mod. Phys. Lett. A **22**, 883 (2007)
28. R.R. Caldwell, Phys. Lett. B **545**, 23 (2002)
29. S. Nojiri, S.D. Odintsov, Phys. Lett. B **562**, 147 (2003)
30. V.K. Onemli, R.P. Woodard, Phys. Rev. D **70**, 107301 (2004). arXiv:gr-qc/0406098
31. E. N. Saridakis, Theoretical limits on the equation-of-state parameter of phantom cosmology. Phys. Lett. B **676**, 7–11 (2009)
32. B. Boisseau, G. Esposito-Farese, D. Polarski, A.A. Starobinsky, Phys. Rev. Lett. **85**, 2236 (2000)
33. S. Nojiri, S.D. Odintsov, M. Sasaki, Phys. Rev. D **71**, 123509 (2005)
34. M.Z. Li, B. Feng, X.M. Zhang, JCAP **0512**, 002 (2005)
35. S. Nojiri, S.D. Odintsov, Phys. Rev. D **72**, 023003 (2005)
36. S. Sur, S. Das, JCAP **0901**, 007 (2009)
37. K. Bamba, C.Q. Geng, S. Nojiri, S.D. Odintsov, Crossing of the phantom divide in modified gravity. Phys. Rev. D **79**, 083014 (2009)
38. C. Armendariz-Picon, V.F. Mukhanov, P.J. Steinhardt, Phys. Rev. D **63**, 103510 (2001). arXiv:astro-ph/0006373
39. B. Feng, X.L. Wang, X.M. Zhang, Phys. Lett. B **607**, 35 (2005)
40. Z.K. Guo et al., Phys. Lett. B **608**, 177 (2005)
41. M.-Z. Li, B. Feng, X.-M. Zhang, JCAP **0512**, 002 (2005)

42. B. Feng, M. Li, Y.-S. Piao, X. Zhang, Phys. Lett. B **634**, 101 (2006)
43. M.R. Setare, Phys. Lett. B **641**, 130 (2006)
44. W. Zhao, Y. Zhang, Phys. Rev. D **73**, 123509 (2006)
45. M.R. Setare, E.N. Saridakis, Phys. Lett. B **671**, 331 (2009)
46. S. Ray, M.Yu. Khlopov, P.P. Ghosh, U. Mukhopadhyay, Int. J. Theor. Phys. **50**, 939 (2011). arXiv:0711.0686 [gr-qc]
47. I.G. Dymnikova, M.Yu. Khlopov, Mod. Phys. Lett. A **15**, 2305 (2000). arXiv:astro-ph/0102094
48. I.G. Dymnikova, M.Yu. Khlopov, Eur. Phys. J. C **20**, 139 (2001)
49. A.G. Doroshkevich, M.Yu. Khlopov, Sov. J. Nucl. Phys. **39**, 551 (1984)
50. A.G. Doroshkevich, M.Yu. Khlopov, Mon. Not. R. Astron. Soc. **211**, 279 (1984)
51. A.G. Doroshkevich, M.Yu. Khlopov, Sov. Astron. Lett. **11**, 236 (1985)
52. A.G. Doroshkevich, A.A. Klypin, M.Yu. Khlopov, Sov. Astron. **32**, 127 (1988)
53. A.G. Doroshkevich, M.Yu. Khlopov, A.A. Klypin, Mon. Not. R. Astron. Soc. **239**, 923 (1989)
54. L. Amendola, Phys. Rev. D **62**, 043511 (2000)
55. W. Zimdahl, D. Pavon, Phys. Lett. B **521**, 133 (2011)
56. L.P. Chimento, Phys. Rev. D **81**, 043525 (2010). arXiv:0911.5687
57. J.H. He, B. Wang, P. Zhang, Phys. Rev. D **80**, 063530 (2009). arXiv:0906.0677
58. M.M. Verma, S.D. Pathak, A tachyonic scalar field with mutually interacting components. Int. J. Theor. Phys. **51**, 2370–2379 (2012)
59. M.M. Verma, S.D. Pathak, Astrophys. Space Sci. **344**, 505 (2013)
60. P.G. Ferreira, M. Joyce, Phys. Rev. Lett. **79**, 4740 (1997)
61. Y.G. Gong, A. Wang, Y.Z. Zhang, Phys. Lett. B **636**, 286 (2006)
62. E.J. Copeland, A.R. Liddle, D. Wands, Phys. Rev. D **57**, 4686 (1998)
63. H. Wei, S.N. Zhang, Phys. Rev. D **76**, 063005 (2007)
64. M.R. Setare, E.N. Saridakis, JCAP **0809**, 026 (2008)
65. M.R. Setare, E.N. Saridakis, Phys. Lett. B **668**, 177 (2008)
66. M.R. Setare, E.N. Saridakis, Quintom cosmology with general potentials. Int. J. Mod. Phys. D **18**, 549–557 (2009)
67. C.G. Böhmer, G. Caldera-Cabral, R. Lazkoz, R. Maartens, Phys. Rev. D **78**, 023505 (2008)
68. R. Cen, Astrophys. J. **546**, L77 (2001). arXiv:astro-ph/0005206
69. M. Oguri, K. Takahashi, H. Ohno, K. Kotake, Astrophys. J. **597**, 645 (2003)
70. K.A. Malik, D. Wands, C. Ungarelli, Phys. Rev. D **67**, 063516 (2003)
71. H. Ziaeepour, Phys. Rev. D **69**, 063512 (2004)
72. M. Szydlowski, T. Stachowiak, R. Wojtak, Phys. Rev. D **73**, 063516 (2006)
73. M. Szydlowski, Phys. Lett. B **632**, 1–5 (2006)
74. B. Wang, Y.G. Gong, E. Abdalla, Phys. Lett. B **624**, 141 (2005)
75. B. Gumjudpai, T. Naskar, M. Sami, S. Tsujikawa, JCAP **0506**, 007 (2005). arXiv:hep-th/0502191
76. S.D. Campo, R. Herrera, D. Pavon, *H(z)* diagnostics on the nature of dark energy. Int. J. Mod. Phys. D **20**, 561 (2011)
77. H. Wei, R.G. Cai, Phys. Rev. D **71**, 043504 (2005). arXiv:hep-th/0412045
78. H. Wei, S.N. Zhang, Phys. Lett. B **644**, 7 (2007). arXiv:astro-ph/0609597
79. X. Chen, Y. Gong, Phys. Lett. B **675**, 9–13 (2009). arXiv:0811.1698
80. X. Chen, Y. Gonga, E.N. Saridakis, JCAP **04**, 001 (2009). arXiv:0812.1117
81. C.G. Bohmer, G. Caldera-Cabral, R. Lazkoz, R. Maartens, PRD **78**, 023505 (2008). arXiv:0801.1565
82. C.G. Bohmer, N. Tamanini, M. Wright, arXiv:1501.06540
83. C.G. Bohmer, N. Tamanini, M. Wright, arXiv:1502.04030