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Comment

A note on "Electron self-energy in logarithmic electrodynamics" by P. Gaete and J. Helayël-Neto

Dmitry M. Gitman[1,2,3,a], **Anatoly E. Shabad**[2,3,b]

[1] Instituto de Física, Universidade de São Paulo, Caixa Postal 66318, São Paulo, SP CEP 05508-090, Brazil
[2] P. N. Lebedev Physics Institute, Leninsky Prospekt 53, Moscow 117924, Russia
[3] P. Tomsk State University, Lenin Prospekt 36, Tomsk 634050, Russia

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Abstract We propose an identification of the free parameter in the model of nonlinear electrodynamics proposed in Gaete and Helayël-Neto (Eur Phys J C 74:2816, 2014) by equating the second term in the power expansion of its Lagrangian with that in the expansion of the Heiseberg–Euler Lagrangian. The resulting value of the field-energy of a point-like charge makes 0.988 of the electron mass, if the charge is that of the electron.

Recently Gaete and Helayël-Neto [1] proposed a nonlinear Lagrangian for what they called the logarithmic electrodynamics, which, in the special case of presence, in a certain Lorentz frame, of only electric field E, has the form

$$L = -\beta^2 \ln\left[1 - \frac{E^2}{2\beta^2}\right], \tag{1}$$

where β is a nonlinear coupling constant. For small fields one has $L = \frac{1}{2}E^2$, in correspondence with the classical electrodynamics. The same as in the Born–Infeld theory this Lagrangian has a singularity at a certain value of the electric field, namely $E = 2^{1/2}\beta$, which makes the maximum value of the electric field produced by a point-like charge Q

$$E(r) = \frac{4\pi\beta^2}{Q}\left[\left(r^4 + \frac{2Q^2}{(4\pi\beta)^2}\right)^{1/2} - r^2\right] \tag{2}$$

in the origin of coordinates $r = 0$, where the charge is placed. The fineness of the maximum electric field results in convergence of the integral for the electrostatic field-energy produced by the charge, again the same as in the Born–Infeld model. It is calculated in [1] as the space integral over the Noether energy density

$$\Theta^{00} = \frac{E^2}{1 - \frac{E^2}{2\beta^2}} + \beta^2 \ln\left(1 - \frac{E^2}{2\beta^2}\right)$$

taken on the field (2) to be

$$U = 4\pi \int_0^\infty \Theta^{00} r^2 dr = 0.391\sqrt{e^3\beta}. \tag{3}$$

To fix the model, the authors of [1] have proposed, when setting the point charge equal to that of the electron, $Q = e$, to equalize the maximum field $E = 2^{1/2}\beta$ with the characteristic QED value m^2/e, where m is the electron mass. This led them to the value of the field-energy $U = 4\pi \int_0^{1/m} \Theta^{00} r^2 dr = 8.67 \times 10^{-4} m$ that makes a small part of the electron mass. In contrast to this suggestion, ours is to equalize the Lagrangian (1) and the Heisenberg–Euler Lagrangian, taken on the electric field alone, within the accuracy to the second power in the field strength squared [2,3] (similarly to what we [4] and, previously, other authors (e.g., [5]) acted in different models following [6]) $\frac{E^2}{2} + \frac{E^4}{8\beta^2} = \frac{E^2}{2} + \frac{E^4}{8}\frac{e^4}{45\pi^2 m^4}$. This supplies the value

$$\beta = \frac{m^2}{e}\frac{3\pi\sqrt{5}}{e} \tag{4}$$

to the nonlinear coupling constant β in (1) and the maximum electric field $3\pi\sqrt{10}/e = 98$ times the characteristic value m^2/e. With the value (4) and taking into account that $e^2/4\pi = 1/137$, the field energy of the point charge (3) becomes impressively close to the electron mass:

$$U = 0.988m. \tag{5}$$

If one imagined that the whole mass of the electron might have been of purely electrostatic origin, $U = m$, such

[a] e-mail: gitman@dfn.if.usp.br
[b] e-mail: anshabad@yahoo.com; shabad@lpi.ru

assumption would imply via (3) and (4) the equality

$$0.391\sqrt{e3\pi\sqrt{5}} = 1,$$

from which the value $1/130$ would follow for $e^2/4\pi$. Certainly, other interactions, in which electron is involved, must also contribute into its field-mass. The first of them all is the contribution of the static magnetic dipole field of the electron. Equation (5) leaves yet the spare room of 1.2 % for such extra contributions.

Note that an inclusion of a heavier charged lepton, say the μ-meson, contribution into the Heisenberg–Euler Lagrangian as a lepton-antilepton loop, would give rise to the negligible correction $-\frac{1}{2}\left(\frac{m}{m_\mu}\right)^4 \sim -3 \times 10^{-8}$ % to β (4). This means that the self-energy of the point particle charged as the electron remains practically the same as (5). So, if this particle is a muon, only a small portion of its mass may be attributed to its electromagnetic energy, in contrast to the electron.

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