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STATISTICAL ANALYSIS OF LANGMUIR WAVES ASSOCIATED WITH TYPE III RADIO BURSTS: II. SIMULATION AND INTERPRETATION OF THE WAVE ENERGY DISTRIBUTIONS

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Abstract. We have modeled electrostatic Langmuir waves by an electric field, consisting of superposition of Gaussian wave packets with several probability distributions of amplitudes and with several Poisson distributions of wave packets. The outcome of the model is that the WIND satellite observations, especially in the low frequency domain (the WAVES experiment), do not allow to conclude whether the input wave amplitude distributions are closer to the log-normal than to the Pearson type I or uniform. The average number of wave packets in 1 s is found to be between 0.1 and 50. Therefore, there is a clear need to measure Langmuir wave energy distributions directly at the waveform level, not a posteriori in the spectral domain. This is planned to be implemented on the RPW (Radio and Plasma Wave Analyzer) instrument in the Solar Orbiter mission.

Key words: solar wind – physical processes: plasmas – Langmuir waves

1. ELECTRIC FIELD MODEL – SIMULATIONS

We modeled the electric field, $E(t)$, detected by the WIND satellite antennas (Bougeret et al. 1995) as a superposition of Gaussian wave packets (Figure 1):

$$E(t) = \sum_{i=1}^{N} E_i \, \mathrm{e}^{-\frac{1}{2}\left(\frac{t-t_{0i}}{\Delta t_i}\right)^2} \cos(2\pi f_i t + \varphi_i), \tag{1}$$

where E_i is the amplitude, t_{0i} is the time where maximum of i-th wave packet occurs, Δt_i determines the spread of t-values about t_{0i}, f_i is the frequency and φ_i is the phase of the wave packet. Number of wave packets in 1 s is modeled as a Poisson distribution, $\mathcal{P}(\lambda)$. If we consider the number N to be determined by Poisson law with a flux parameter λ [s^{-1}], the probability to have N wave packets within the time T_S is given by:

$$\mathcal{P}(\lambda) = \mathrm{e}^{\lambda T_S} \frac{(\lambda T_S)^N}{N!}, \tag{2}$$

where N is the number of wave packets observed during the sampling time T_S of the LFR instrument (Law Frequency Receiver) which measures the voltage power spectral density in V^2/Hz, i.e., the Fourier transform of the autocorrelation function of the voltage measured by the dipole antennas.

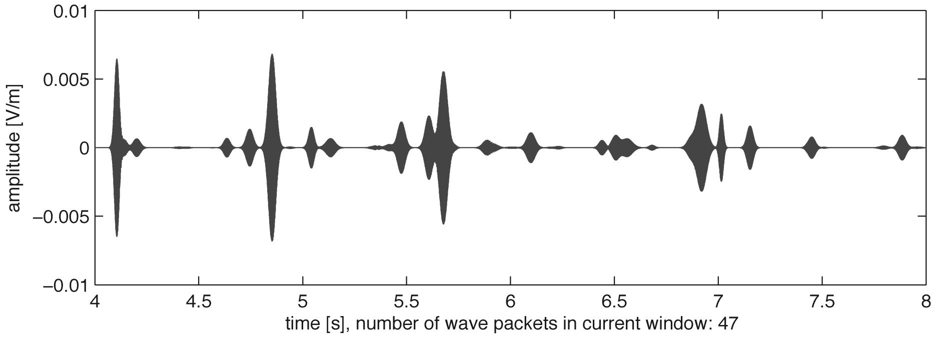

Fig. 1. An example of the wave packet simulation with $\log(A_i^2)$: $\mathcal{N}(\mu = -7, \sigma^2 = 1.7^2)$; $\mathcal{P}(\lambda = 10)$. A_i is the amplitude of wave packet, μ is the first moment – mean value and σ is the second moment – standard deviation of normal probability distribution.

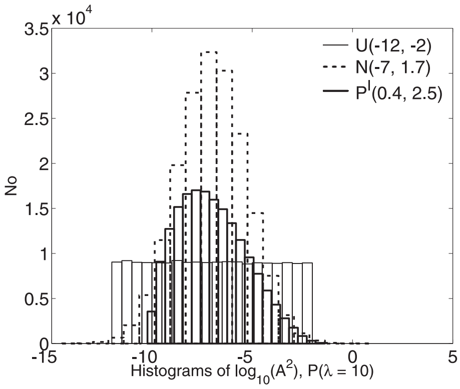

Fig. 2. Histograms of input wave packets with amplitudes $\log(A_i^2)$: light gray – $\mathcal{U}(a = -12, b = -2)$, black – $\mathcal{N}(\mu = -7, \sigma^2 = 1.7^2)$, very light gray – $\mathcal{P}^I(\mu = -0.5, \sigma^2 = 0.5^2, \beta_1 = 0.4, \beta_2 = 2.5)$; $\mathcal{P}(\lambda = 10)$.

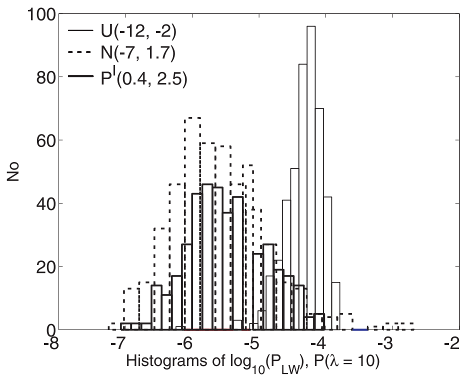

Fig. 3. Histograms of the Langmuir wave power, $\log(P_{LW})$, spectral domain. They correspond to the input wave packet amplitudes shown in Figure 2.

Histograms of input wave packet power logarithms with three different probability distributions of amplitudes $\log(A_i^2)$ are shown in Figure 2. Light gray represents histogram with uniform probability distribution, $\mathcal{U}(a = -12, b = -2)$ where a and b are parameters of the probability distribution; black histogram with normal distribution, $\mathcal{N}(\mu = -7, \sigma^2 = 1.7^2)$; and very light gray histogram with Pearson's type I probability distribution, $\mathcal{P}^I(\mu = -0.5, \sigma^2 = 0.5^2, \beta_1 = 0.4, \beta_2 = 2.5)$, where μ and σ are the first two parameters, the mean and standard deviations, and β_1 and β_2 (Eq. 4) are the second two parameters of the distribution. The flux parameter in the Poisson law, $\mathcal{P}(\lambda)$, is 10. The corresponding histograms of the Langmuir wave power logarithms, $\log(P_{LW})$, in the spectral domain are shown in Figure 3.

2. PEARSON'S SYSTEM OF DISTRIBUTIONS – β PLANE

Pearson (1895) defined a distribution system by the following equation for the probability density function $p(x)$:

$$-\frac{p'(x)}{p(x)} = \frac{b_0 + b_1 x}{c_0 + c_1 x + c_2 x^2}, \tag{3}$$

where b_0, b_1, c_0, c_1 and c_2 are real parameters. The form of the solutions of this differential equation depends on the parameter values, resulting in several distribution types. The classification of distributions in the Pearson system is entirely determined by the two moment ratios, square of skewness, β_1, and kurtosis, β_2:

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}, \quad \beta_2 = \frac{\mu_4}{\mu_2^2}. \tag{4}$$

We conclude that in the case $\lambda T_S \gg 1$ it is not possible to recover the initial distribution of the electric field. Whatever is this distribution, at the end we obtain a Gaussian distribution of V^2 (see Figures 2 and 3). The Gaussian parameters are related to the electric field ones (E_i and Δt_i) and to the flux at λ. Contrary, in the case $\lambda T_S \ll 1$, the distribution of V^2 should be comparable to the distribution of $\Delta t_i E_i^2$. In both cases, this does not explain the log-normal distributions predicted by Robinson's Stochastic Growth Theory (SGT) (Robinson 1992), some other process(es) must be responsible.

3. RESULTS AND CONCLUSIONS

1. We have shown that for 36 events – intense locally formed Langmuir waves associated with type III radio bursts measured by the WIND spacecraft – the probability distributions of the power logarithm belong to the three types of the Pearson probability distributions: type I, type IV and type VI (Figure 4, black dots). The goodness of the fits test (e.g., χ^2) shows that the Pearson probability distributions fit the data better than the Gaussian ones for all of the considered events. This is in contradiction with the Stochastic Growth Theory (Robinson 1992) which assumes log-normal distributions for the wave energy, see Vidojević et al. (2012).

2. We have modeled Langmuir waves by the electric field, $E(t)$, consisting of Gaussian wave packets with several distributions of amplitudes, $\log(A^2)$, and with several Poisson distributions of the number of wave packets in 1 s, $\mathcal{P}(\lambda)$.

3. The outcome of these simulations is that the $\beta_1 - \beta_2$ plane of the WIND observations can be covered by a combination of the following assumptions: (a) from WIND observations it is not possible to conclude whether the input wave

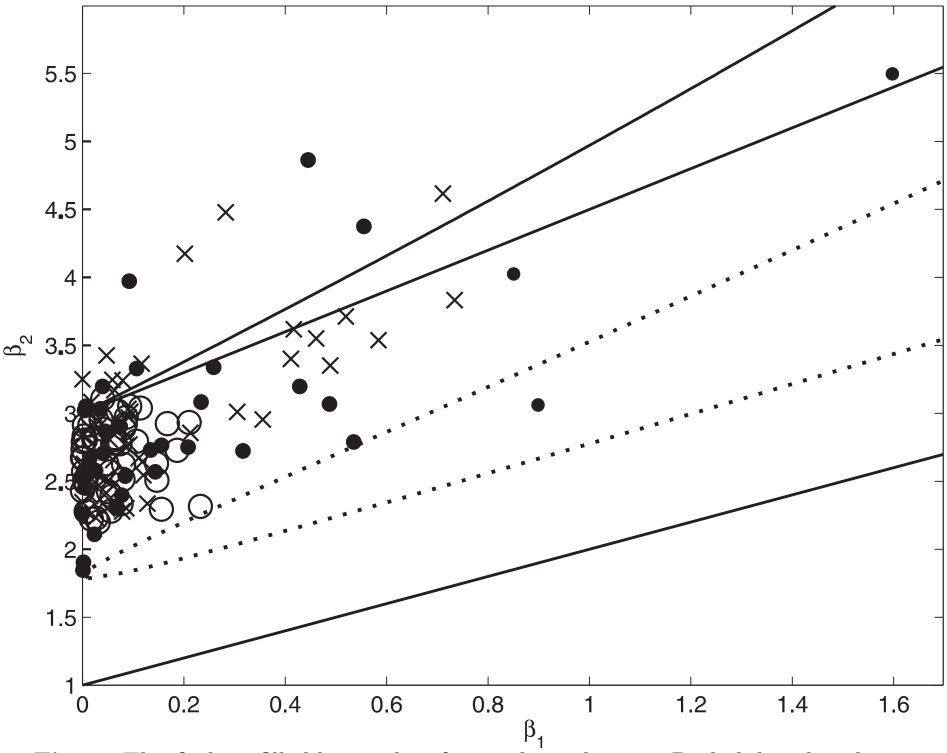

Fig. 4. The β plane filled by results of several simulations. Probability distribution of wave packet amplitudes, $\log(A_i^2)$: $\mathcal{P}^I(\mu, \sigma^2, \beta_1, \beta_2)$. Symbols: circles – $\mathcal{P}^I(\mu = -7, \sigma^2 = 1.7^2, \beta_1 = 0.4, \beta_2 = 2.5)$; $\times$ – $\mathcal{P}^I(\mu = -0.5, \sigma^2 = 0.5^2, \beta_1 = 0.4, \beta_2 = 2.5)$, black dots – 36 WIND events.

amplitude distributions are closer to the log-normal than to the Pearson type I or uniform; (b) the average number of wave packets in 1 s is between 0.1 and 50.

4. Therefore, there is a clear need to measure Langmuir wave energy distributions directly at the waveform level and not *a posteriori* in the spectral domain. This is what is planned to be implemented on the RPW (Radio and Plasma Waves) instrument on the Solar Orbiter, a new space mission.

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