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RESEARCH ARTICLE - HYDROLOGY

A simplified approach for the evaluation of groundwater flow in stream–aquifer interaction

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Abstract

Stream–aquifer interaction process plays an important role in modulating flood wave propagation in a channel. The most elementary understanding of stream–aquifer interaction can be interpreted by the flux direction between a surface water body and the underlying aquifer. At the time of floods, stream stage rises, and the water gets infiltrated into the aquifer, and this process gets reversed at the time when the stream stage gets declined. Therefore, an integrated mechanism between the surface and subsurface flows is particularly important for models, where the response of the system is based on simultaneous interactions between these two major flow domains. In this study, numerical simulation of a flood wave has been demonstrated considering stream–aquifer interaction. The calibration has been executed on a hypothetical flood event accessed by routing a known stage hydrograph for a channel reach having a rectangular cross section which fully penetrates the adjoining aquifer given by Zitta and Wiggert (Water Resour Res 7:1341–1345, 1971). A simplified mathematical approach, based on Darcy's law, has been presented here for the solution of groundwater flow equations. The results obtained from the adopted procedure are also compared with the solution proposed by Zitta and Wiggert in 1971. The NSE and RMSE (m^3/s) estimate assessed for the simulated hydrographs using the proposed methodology with respect to the procedure adopted by Zitta and Wiggert (Water Resour Res 7:1341–1345, 1971) is 0.9983 and 0.8544, respectively. Therefore, the use of Simpson's (3/8)-rule is not suggestible due to its complicated calculation and its sensitivity, and it is better to use the proposed simplified approach for the evaluation of lateral flow.

Keywords Stream–aquifer interaction · Rectangular channel · Fully penetrating stream

Introduction

Surface water and groundwater both depend significantly on the streams as they control the existence, distribution, and quality of water. Groundwater and surface water both are intimately related units. Streams and aquifers are hydraulically connected with water passing between the adjoining aquifer and the stream channel, and vice versa during the passage of flood wave in the stream reach (Castro and Hornberger 1991; Bencala 1993). Hence, degradation or exhaustion of one will distress the other in terms of both quality and quantity of water. The stream–aquifer interaction proceeds on various temporal and spatial scales, and are complex in nature (Kalbus 2009). The study of stream–aquifer interaction is necessary, because the streams enforce the boundary condition on flow into the aquifer. Stream stage fluctuations may vary due to both anthropogenic (e.g., dams or reservoirs operation) or natural factors (e.g., floods). During the passage of a flood wave, the stream stage rises, and the water seeps to the adjoining aquifer, and it is termed as influent seepage. During the recession of the stream stage, the process becomes reversed, i.e., the direction of flow is from the aquifer to stream, and it termed as effluent seepage. The hydraulic gradient is the main factor which decides the nature of flow exchange, i.e., losing or gaining stream. In gaining stretches of a river, the groundwater table elevation is generally higher, in comparison

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with the prevailing stream stage, and vice versa for the losing stream reaches. A saturated zone or an unsaturated zone can act as a medium for connection or disconnection to the stream, when the water table is below the streambed. The level of groundwater table and stream stage can alter or cause temporal changes in magnitude and direction of flows due to an unusual precipitation event or due to the seasonal variations in the precipitation. The gaining or losing condition of a stream can be found out by observing the elevation of the groundwater table and stream water level in a nearby monitoring well. The complexities in stream–aquifer interaction process arise mainly due to the complex porosity of aquifers, catchment physiographical characteristics, surface water positioning and also due to the difference in the opinions of hydrologists and hydrogeologists regarding the selection of methods or models to investigate the interaction between them (Madlala 2015). Surplus water after a high precipitation and flood events may contribute to aquifer storage by percolation process. Stream–aquifer interaction study comprises the temporal and spatial piezometric head variation, flow rate, and bank storage due to stream-stage variation.

Investigators acknowledged the stream–aquifer interaction process in the early '50 s. Todd (1956) highlighted the relationship between flooding streams to groundwater flow. Several investigators (Todd 1956; Cooper and Rorabaugh 1963; Hornberger et al. 1970; Verma and Brutsaert 1970; Pinder and Sauer 1971; Gill 1978; Tung 1985; Yoon and Padmanabhan 1993; Swamee et al. 2000; Choudhary and Chahar 2007; Kim et al. 2001; Kim et al. 2008; Majumdar et al. 2013) provided solutions for stream–aquifer interaction problem assuming the channel section of the interacting reach with bank aquifer fully penetrates the aquifer. In addition, it is assumed that the river–aquifer interaction takes place perpendicular to the riverbank. This assumption implies that the flow is one-dimensional in the riverbank aquifer (Cooper and Rorabaugh 1963; Hornberger et al. 1970; Zitta and Wiggert 1971; Hall and Moench 1972; Hogarth et al. 1999).

In the present study, the lateral flow due to stream–aquifer interaction has been estimated based on the application of mass conservation equation applied to a cascade of subreaches in the bank-aquifer strip perpendicular to the stream. The appropriateness of the proposed method is verified with the hypothetical stage-hydrograph routing solution obtained by Zitta and Wiggert (1971).

Governing equation

Though a realistic flood propagation study in a river reach requires the use of equations governing the flood wave propagation in two-dimensions (Ferris 1951; Verma and Brutsaert 1970; Pinder and Sauer 1971; Safavi et al. 2004; Spanoudaki et al. 2005) but considering the model simplicity and data availability limitations for two-dimensional modeling, it is proposed in this study to use one-dimensional governing equations for streamflow routing. However, the equations governing flood wave propagation in channels should consider the presence of lateral outflow and inflow in the channel or river reach to consider the river–aquifer interaction process in the study reach. It is assumed that this interaction process takes place perpendicular to the riverbanks at the interface of the stream and aquifer, and this assumption enables to modify only the continuity equation governing the flood propagation process, and not the governing momentum equation. Accordingly, continuity equation considering lateral inflow and outflow in the river reach is expressed as:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = -2q_{\mathrm{L}} \tag{1}$$

where, Q = channel flow $\left(\mathrm{m}^3/\mathrm{s}\right)$; A = channel cross-sectional area $\left(\mathrm{m}^2\right)$; x = distance in the direction of channel flow (m); t = time (s); q_{L} = lateral flow in the study reach per unit length on one side of the riverbank $\left(\mathrm{m}^3/\mathrm{s}/\mathrm{m}\right)$;

The momentum equation governing the one-dimensional flood propagation is expressed as:

$$\frac{1}{A}\frac{\partial Q}{\partial t} + \frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + g\frac{\partial y}{\partial x} - g\left(S_0 - S_{\mathrm{f}}\right) = 0 \tag{2}$$

The nonlinear Boussinesq equation which governs unsteady, one-dimensional lateral flow in the unconfined aquifer adjacent to the study river reach is expressed as (Aravin and Numerov 1965; Jacobs and Hunter 1950; Polubarinova-Kochina 1962):

$$\frac{\partial^2 h^2}{\partial z^2} = \frac{2S_{\mathrm{y}}}{K}\frac{\partial h}{\partial t} \tag{3}$$

where S_0 = slope of the channel bottom (dimensionless); S_{f} = friction slope(dimensionless); R = hydraulic radius (m); h = height from the datum to the phreatic surface (m); z = distance perpendicular to the path of the channel (m); S_{y} = specific yield (dimensionless); K = hydraulic conductivity (m/s).

Prior solution: Zitta and Wiggert solution (1971)

Consider a rectangular channel which fully penetrates the adjoining unconfined bank aquifer, i.e., channel banks have been taken as vertical (Pinder and Sauer 1971; Zitta and Wiggert 1971; Perkins and Koussis 1996; Hantush et al. 2002; Miracapillo and Morel-Seytoux 2014). For the solution of the stream–aquifer system provided by Zitta and Wiggert (1971), they used the explicit finite difference solution of the full SVE for channel routing and Simpson's rule for solving the Boussinesq equation for flow transfer into and from the aquifer. The cross section and plan view of a stream channel reach under investigation is shown in Fig. 1.

The lateral flow per side per unit length can be estimated using the following equation (Zitta and Wiggert 1971),

$$q_{\mathrm{L}} = -S_{\mathrm{y}} \int \frac{\partial h}{\partial t} \mathrm{d}z \tag{4}$$

where h = height from the datum to the phreatic surface (m); z = distance perpendicular to the path

$$y(t) = y_0 + A_0\left[1 - \cos\left(\pi t/T_c\right)\right] \tag{5}$$

of the channel (m); S_y = specific yield (dimensionless); q_L = lateral flow in the study reach per unit length on one side of the riverbank $\left(\mathrm{m}^3/\mathrm{s}/\mathrm{m}\right)$ $\sum_{i=1}^{n}(X_i - \overline{X})^2$;$t$ = time(s).

The hypothetical input stage hydrograph used in the study is expressed mathematically as

$$(i) \quad h(t) = y_0 \quad \text{at} \quad 0 \le z \le Z_{\max}, \quad t = 0 \tag{6}$$

where, $x = 0$ and $0 \le t \le 2T_c$.

For a rectangular channel that fully penetrates the aquifer, the input details as used by Zitta and Wiggert (1971) is described in Fig. 2 and the loop rating curve at the inlet section is shown in the inset of Fig. 2.

The initial and boundary conditions used for this routing problem, respectively, are given as

$$(ii) \quad h(t) = y_0 \quad \text{at} \quad z = Z_{\max}\ , \quad t > 0 \tag{7}$$

$$(iii) \quad kh(t) = y(t) \quad \text{at} \quad z = 0\ , \quad t > 0 \tag{8}$$

The channel and aquifer characteristics details are given in Table 1.

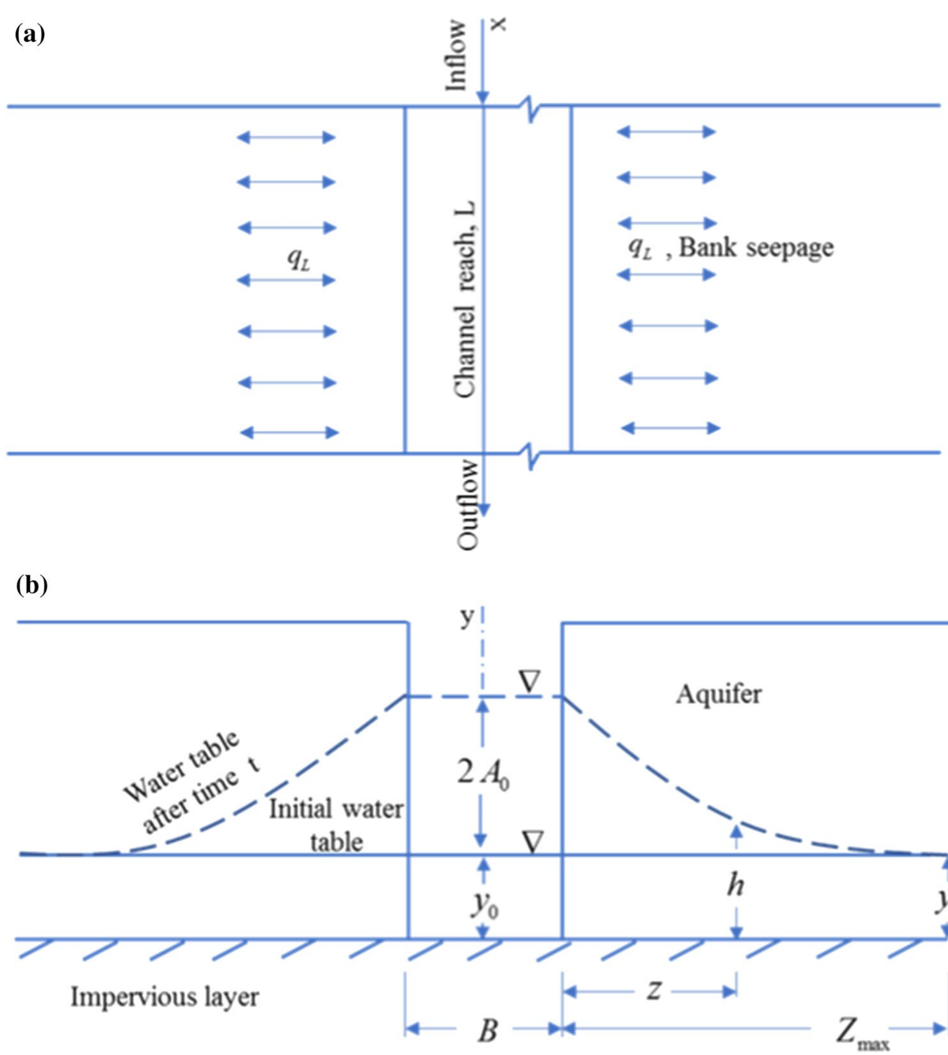

Fig. 1 Plan (**a**) and section (**b**) views of the considered stream and aquifer interaction system (from Zitta and Wiggert 1971)

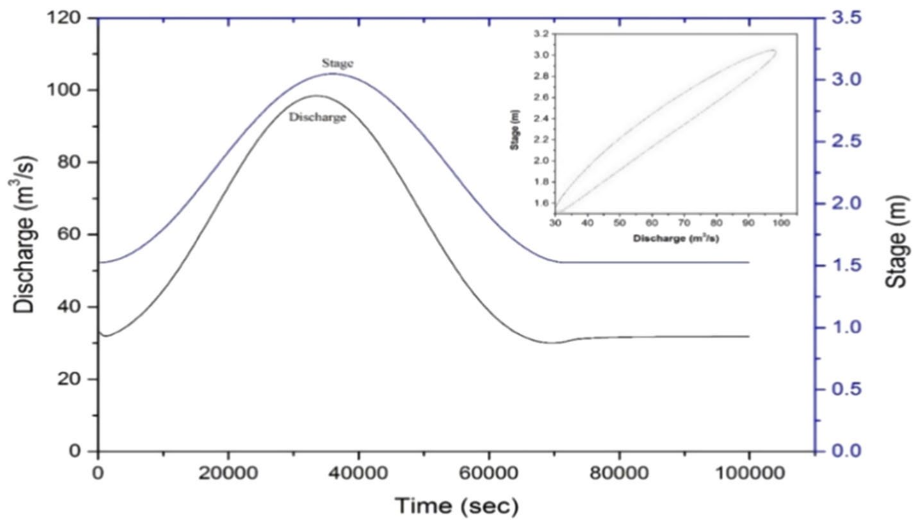

Fig. 2 Input stage and corresponding discharge hydrograph

Table 1 Numerical data

Channel characteristics	Aquifer characteristics
$y_0 = 1.524$m	Hydraulic conductivity $(K) = 0.000945$ m/s
Channel width = 30.48 m	Specific yield $(S_y) = 0.16$
Length = 16 km	$Z_{max} = 300$ m
Bed slope = 0.000189	*Finite difference routing parameters*
Manning's $n = 0.025$	$\Delta x = 2000$ m
$T_c = 36000$ s	$\Delta t = 300$ s

Solution procedure for solving groundwater flow equations using Simpson's (3/8)-rule

To evaluate the lateral flow using Simpson's (3/8)-rule, the following procedure has been adopted. The estimation of lateral flow has been shown for a sub-reach of the channel.

(1) Solve Eqs. (1) and (2) using explicit finite difference solution of the full SVE considering $q_L = 0$ and subsequently find out the river stage and discharge for all sub-sections.

(2) Discretize the width of the aquifer z into n equal (a multiple of 3) sub-reaches as $\Delta z = \frac{z_{max}}{n}$.

(3) Initially, consider the hydraulic head and the river stage are at the same level.

(4) For the present time $(t + \Delta t)$, consider the hydraulic depth to be equal to the average flow depth at the interface of the river.

(5) Using Eq. (3), obtain the values of hydraulic heads at all the nodal points of the aquifer using an explicit finite difference scheme.

(6) From Eq. (4), consider $f(z) = \frac{\partial h}{\partial t} = \frac{h(t+\Delta t, z) - h(t,z)}{\Delta t} \frac{n!}{r!(n-r)!}$ where z has been divided into 'n' equal sub-intervals from $i = 0, 1, 2 \ldots n$. So, it can be said that

$$f(z_i) = \frac{h(t + \Delta t, z_i) - h(t, z_i)}{\Delta t}, \quad i = 0, 1, 2 \ldots n \tag{9}$$

Evaluate the values of $f(z_i)$ for all the values from $0, 1, 2 \ldots n$.

(7) Apply the Simpson's (3/8)-rule to find out the value of the integral in Eq. (4) which can be defined as (Matthews 2004),

$$\int_{z_0}^{z_{max}} f(z)\,dz = \frac{3\Delta z}{8}\left[f(z_0) + 3\sum_{i=1,4,7,\ldots}^{n-2} f(z_i) + 3\sum_{i=2,5,8,\ldots}^{n-1} f(z_i) + 3\sum_{i=3,6,9,\ldots}^{n-3} f(z_i) + f(z_n)\right] \tag{10}$$

(8) Multiply the value of the integral obtained in step (7) with the specific yield $\left(S_y\right)$ to find the lateral flow as given in Eq. (4).
(9) Again solve Eqs. (1) and (2) using the value of q_L obtained from step (8), and find out the river stage and discharge for all sub-sections considering lateral flow.
(10) Repeat the steps (1) to (9) for each time interval while routing the complete inflow hydrograph in the channel.

Proposed approach for studying flow movement in the riverbank aquifer system

The assumptions considered in the development of river routing using explicit method considering stream–aquifer interaction are as follows:

(1) The study river reach is characterized by a prismatic section.
(2) The considered unconfined bank river aquifer located on either side adjacent to the stream is symmetrical in form and assumed to be characterized by the same aquifer properties.
(3) The river channel fully penetrates the aquifer.
(4) The flow in the stream is one-dimensional, and one-dimensional flow perpendicular to the river face prevails in the riverbank aquifer.
(5) The initial water level in the aquifer is the same as the water level in the stream prior to the arrival of a flood wave in the stream channel.
(6) During the progress of the stream–aquifer interaction process, no rain is recorded, and this assumption avoids the aquifer storage variation due to recharging.

The bi-symmetrical stream–aquifer system studied herein is shown in Fig. 3a, b with sectional detail of the

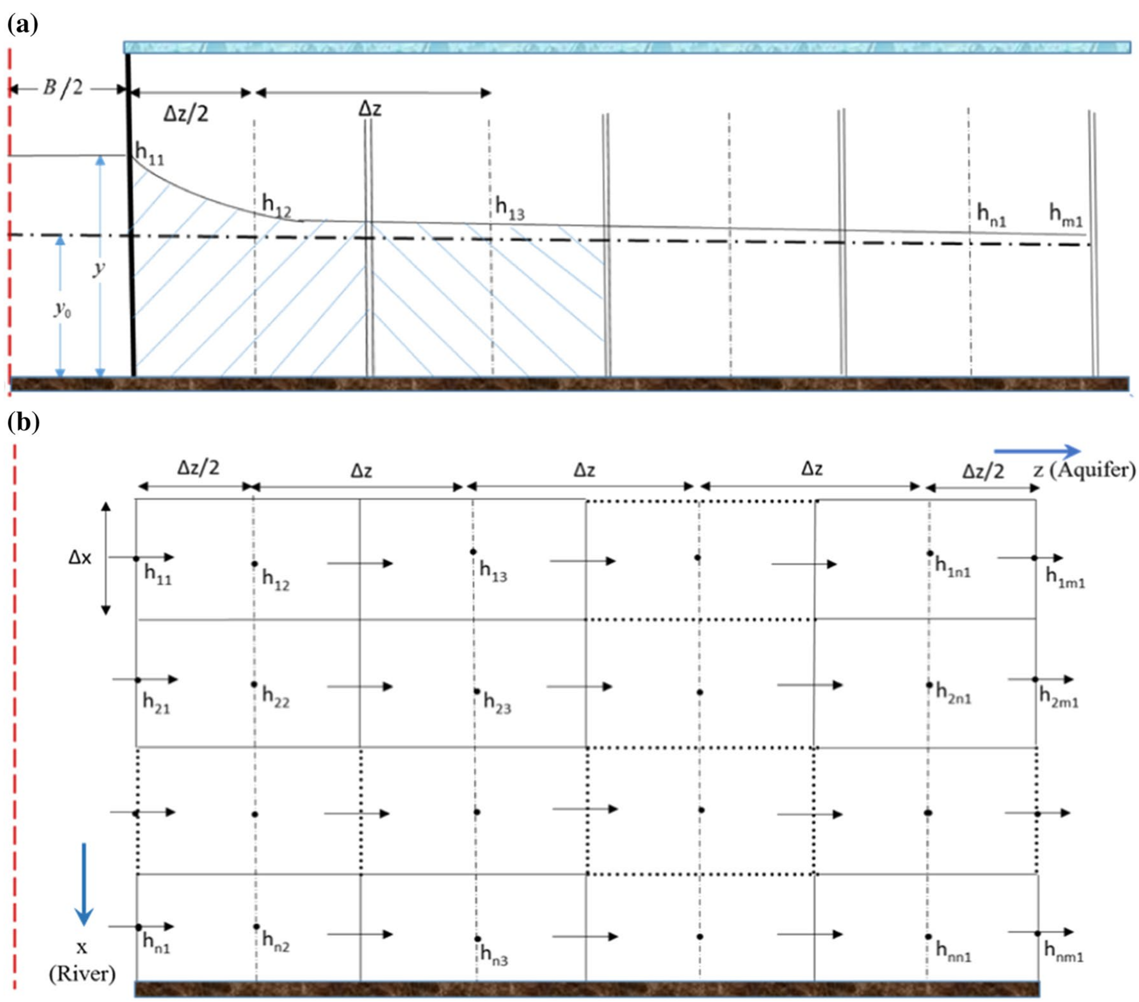

Fig. 3 **a** Cross section of the stream–aquifer system **b** Plan view of stream–aquifer system

stream–aquifer system and its representation in plan form with the stream reach discretized in n sub-reaches for enabling the application of stream routing using the explicit solution of full SVE.

Determination of the first grid size

For the development of the stream–aquifer interaction model, the aquifer is discretized into seamlessly connected rectangular grids to form an equal size grid network except for two narrow strips, one adjacent to the river face and the other at the far end of the aquifer boundary parallel to the river face. The size of the grid is decided in such a manner that the end section of the first grid located immediately adjacent to the stream is at a distance of greater than 0.75 times of the maximum possible saturated thickness perpendicular to the stream face formed due to the passage of flood hydrograph. The restriction on the size of the narrow strips is imposed to satisfy the applicability of the Dupuit's assumption which states that for a small inclinations of the line of seepage the streamlines can be taken as horizontal (hence the equipotential lines approach the vertical), as advocated by Bear (1972).

Therefore, in the present case, the aquifer has been discretized in the direction perpendicular to the channel flow with a grid size of $\Delta z = 6\,m$ and the first observation point has been taken at the distance half of the selected grid size (i.e., $\Delta z/2$) following the Dupuit's assumptions (Bear 1972). The

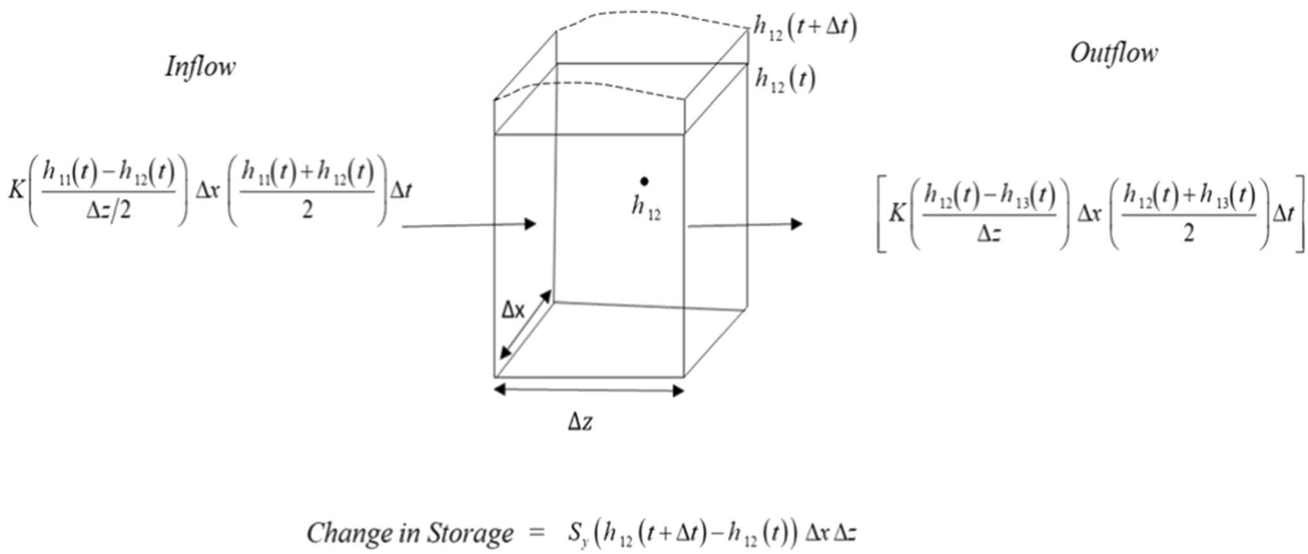

Fig. 4 Mass balance in h_{12} grid

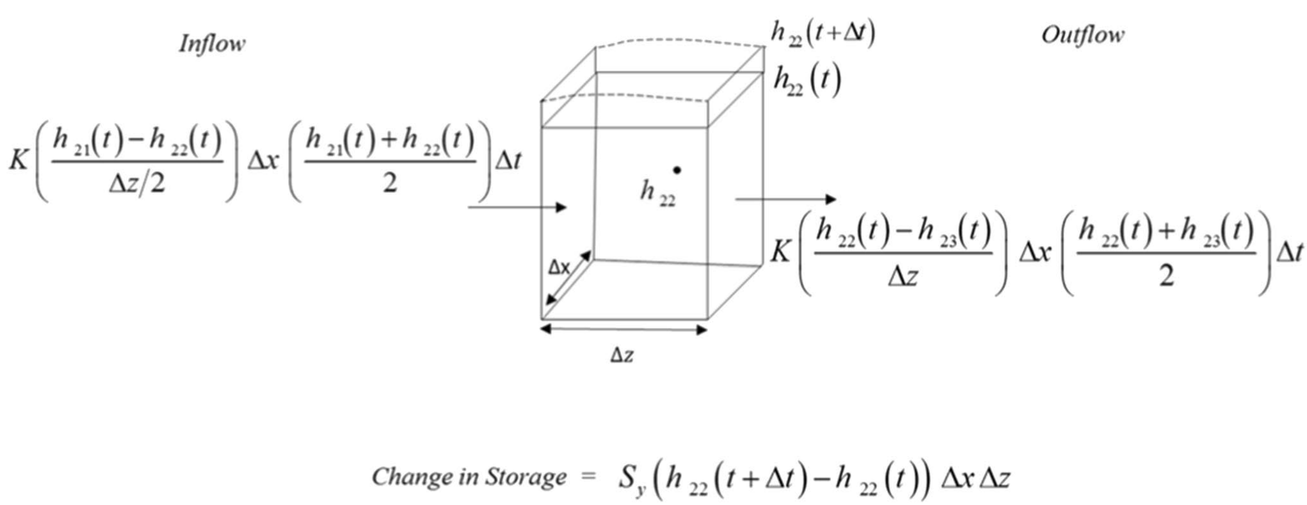

Fig. 5 Mass balance in h_{22} grid

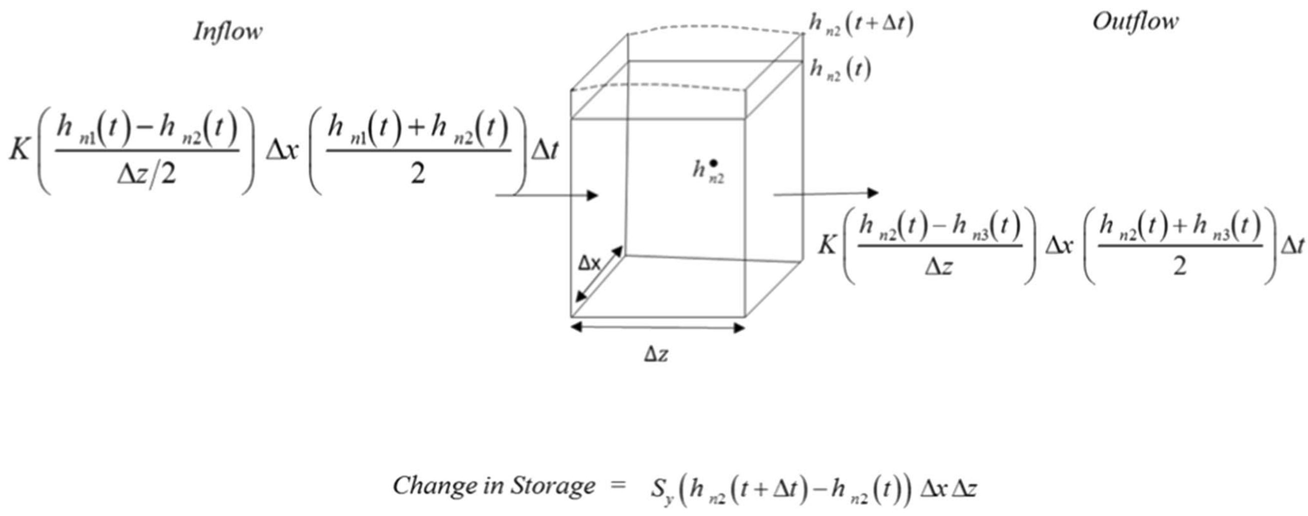

Fig. 6 Mass balance in h_{n2} grid

locations of all observation points are marked, as shown in Fig. 3b. For each of the considered grid, the mass balance equation in the respective flow direction is expressed as

$$I - O = \Delta S/\Delta t \tag{11}$$

where I = rate of inflow volume $(\mathrm{m}^3/\mathrm{s})$; O = the rate of outflow volume $(\mathrm{m}^3/\mathrm{s})$; $\Delta S/\Delta t$ = the rate of change of storage $(\mathrm{m}^3/\mathrm{s})$; S = aquifer storage volume (m^3); t = time (s).

In the present study, the explicit finite difference equations for the mass balance approach have been solved for each grid in the flow direction of the aquifer to find out the value of the water table level for each grid.

Application details of the mass balance equation for a typical grid of the considered narrow strips located on the boundaries of the main aquifer grid network along with the flow directions in the considered grids are shown in Figs. 4, 5, 6.

Using Eq. (11), the value of $h_{12}(t + \Delta t)$ can be obtained for the time step $t + \Delta t$ considering the values at time t are known.

The flow in/out to the aquifer computed for the grid h_{12} at the time $t + \Delta t$ in the finite difference form has been written as:

$$q_{\mathrm{L},1}(t+\Delta t) = K \times \frac{\left(y_{M,1}(t+\Delta t) - h_{12}(t+\Delta t)\right)}{(\Delta z/2)} \times \frac{\left(y_{M,1}(t+\Delta t) + h_{12}(t+\Delta t)\right)}{2} \tag{12}$$

where $y_{\mathrm{M},1}(t + \Delta t)$ is the depth at mid-section for the first sub-reach along the channel length for the time step $t + \Delta t$.

The positive value of $q_{\mathrm{L},1}(t + \Delta t)$ in Eq. (12) represents the influent stream (losing stream), whereas the negative value represents the effluent stream (gaining stream) (Sophocleous 2002). Following the same procedure, the values of water table levels in the direction of the aquifer and the flow in/out to the aquifer for all the channel sub-sections can be obtained.

In this study, the explicit finite difference scheme has been used for the solution of the flow equations. The procedure of solving this system of equations follows two steps: in the first step, the open channel flow equations are solved for steady flow conditions. Then in the second step, the groundwater equations are solved for steady flow by using the calculated stream elevations. Further, the open channel flow equations must now be solved once again, because the groundwater inflow term q_{L} has been modified by the new head distribution in the aquifer.

To proceed with the solution of the transient problem, the procedure indicated above is repeated for each time interval Δt.

The volume of bank storage can be calculated from the equation given in Eq. (13) (Chen and Chen 2003);

$$V = \int_0^t q_{\mathrm{L}}\,\mathrm{d}t \tag{13}$$

Results and discussion

The simulations have been performed here using FORTRAN-77 code that solves the river–aquifer interaction flow

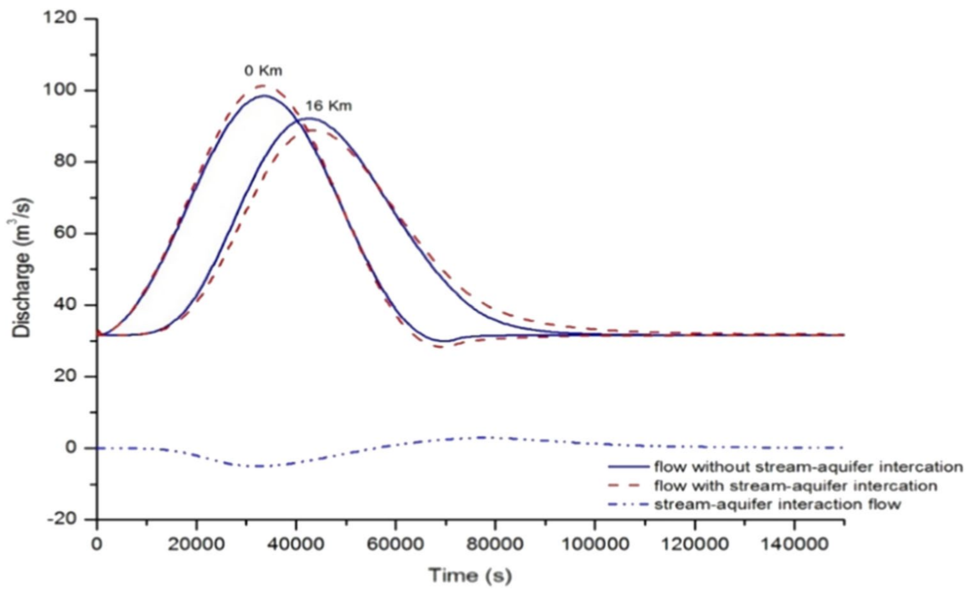

Fig. 7 Effect of bank seepage in a 16 km reach using explicit method

problem formulated herein using the related governing flow equations discussed. Channel routing has been performed using an explicit finite difference solution of full SVE in both the methodologies. For analyzing the modification of flood wave due to interactive bank flow, the given stage hydrograph has been routed through the channel reach with and without the consideration of interactive flow. The channel has been considered as impervious in the first cycle of routing and the interactive flow then has been considered in the second cycle of routing. In the first cycle of routing, which consists of two iterative routings with the first iterative routing used for the determination of unrefined discharge and the second one used for estimating the refined discharge using the refined routing parameters estimated using the unrefined routed discharge of the first iteration. Prior to the second cycle of routing, the river–aquifer interaction flow is taken into account for determining outflow.

The comparison between the stream discharges for these simulations at the upstream and downstream with and without considering stream–aquifer interaction indicates the effect of stream–aquifer interaction flow on the flood wave has been shown in Fig. 7. Here, the upstream discharge considering stream–aquifer interaction is more than without considering interaction. It is due to the steeping of the energy grade line (Zitta and Wiggert 1971).

The comparison between the downstream discharge hydrograph obtained by the proposed method and the Zitta and Wiggert Solution, 1971 is shown in Fig. 8.

The NSE and RMSE (m^3/s) estimate assessed for the simulated hydrographs using the proposed methodology with respect to the procedure adopted by Zitta and Wiggert

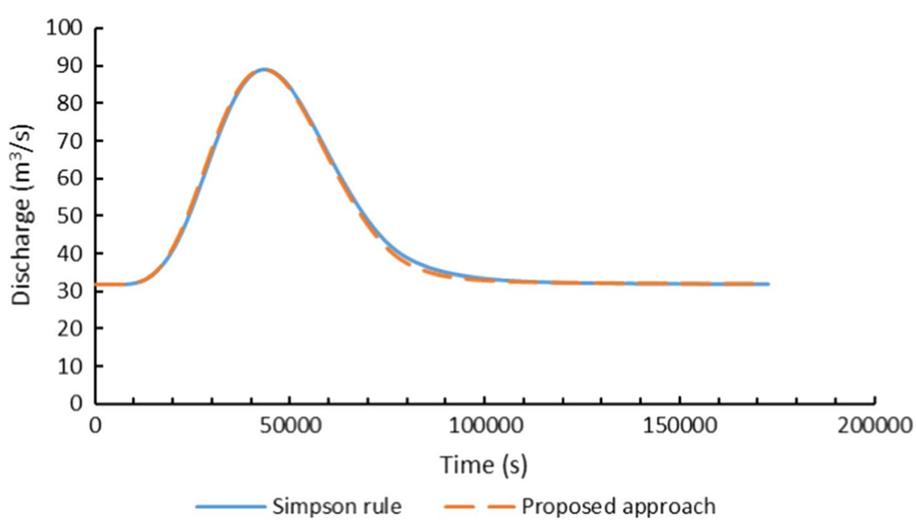

Fig. 8 Comparison of downstream discharge obtained by the proposed method with the Simpson's rule

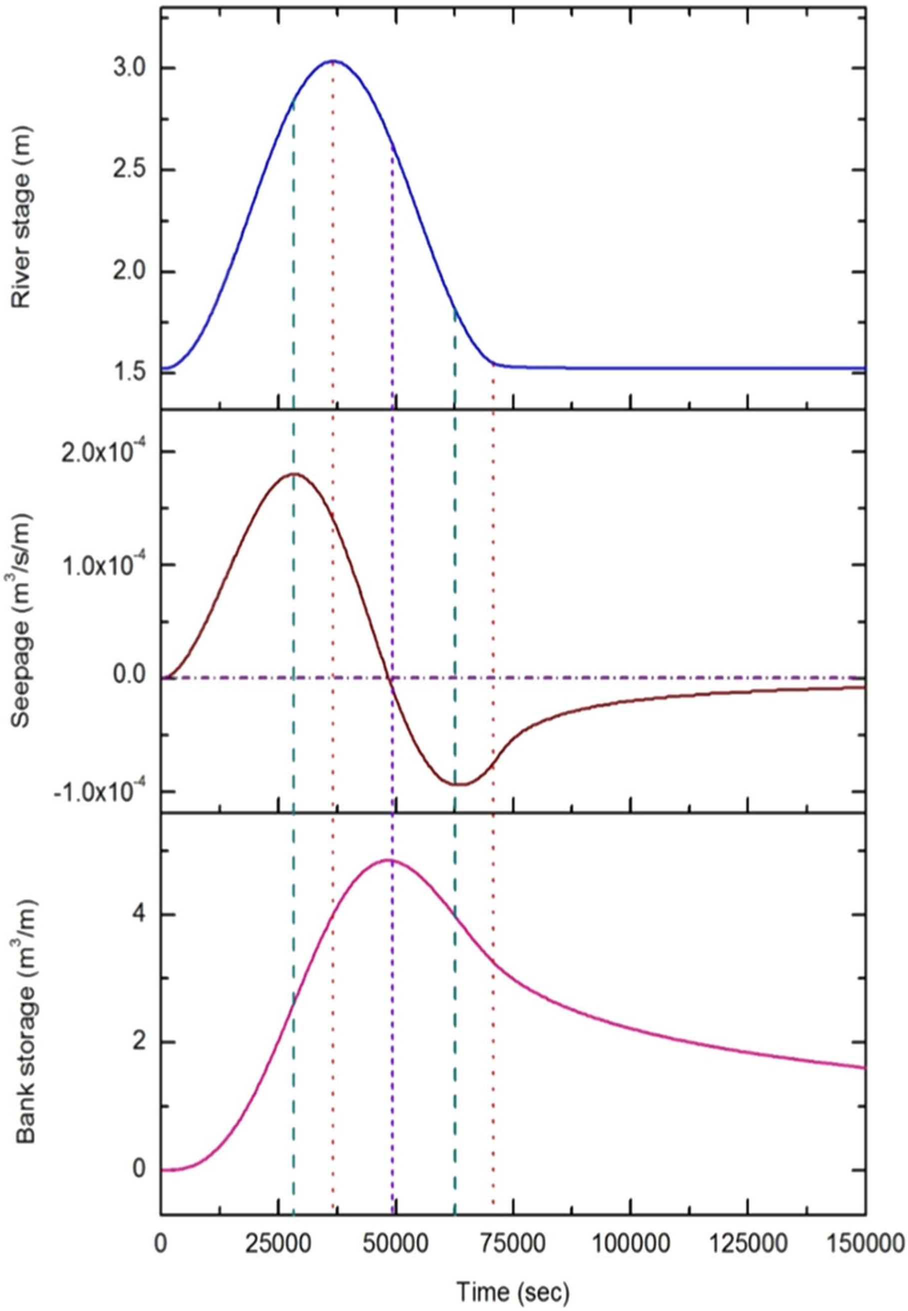

Fig. 9 Bank storage and bank seepage hydrographs simulated at the outlet of the first sub-reach using the VPMS method for the case study of Zitta and Wiggert (1971)

(1971) are 0.9983 and 0.8544, respectively. Therefore, the proposed method can be adopted for the simulation of hydrographs considering the stream–aquifer interaction.

In Fig. 9, the lateral flow for the first sub-section of the considered channel reach along with the corresponding average flow depth and the bank storage hydrographs are shown. The positive value of the bank seepage $\left(q_{\mathrm{L}}\right)$ represents the flow towards the aquifer during the flood period, and the negative value of the bank seepage represents the return flow towards the river. The bank seepage becomes maximum at the time prior to the maximum river stage during the flood period. At the later stage of a flood hydrograph, the return flow to the river starts at a lower rate, but it remains continuous for a long time after the flood period. The bank storage becomes maximum at the time where the bank seepage becomes zero. After that, bank storage will reduce at a lower rate.

Figure 10a, b show a set of groundwater profiles for upstream and downstream of the given stream which indicate that the most of the groundwater storage occurs at relatively short distances from the stream. Hence, the proposed methodology for lateral flow estimation using Darcy's law is more simple and less prone to the errors in the comparison of the Simpson's (3/8)-rule as it requires only two values of

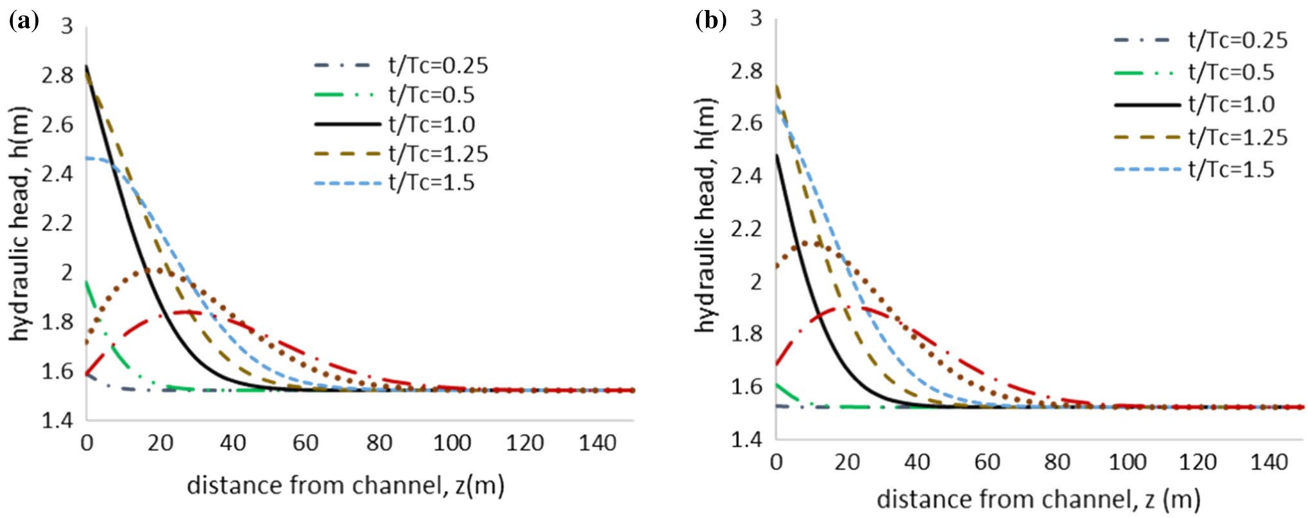

Fig. 10 Groundwater profile at 2 km below the upstream (**a**) and at 16 km away from downstream (**b**)

hydraulic heads for the lateral flow estimation; one at the stream–aquifer interface and the other at just adjacent to that for each sub-section of the reach at a time.

Conclusions

The proposed methodology reproduces similar outflow hydrograph in comparison to the Zitta and Wiggert (1971) approach. For the estimation of lateral flow using a given approach, only two values of phreatic surfaces are required, one at the interface of the stream and aquifer, which is actually the stream stage and other at just adjacent to the stream in the direction of the aquifer at the first observation point. On the other hand, for the implementation of Simpson's (3/8)-rule, the subdivision of the grids should always be a multiple of 3 for those the values of phreatic surfaces are required for the evaluation of bank seepage. On the other hand, for the implementation of Simpson's (3/8)-rule, the subdivision of the grids should always be a multiple of 3. Therefore, the sensitivity of the method got increased because of the involvement of hydraulic heads at each grid point. Therefore, the use of Simpson's (3/8)-rule is not suggestible due to its complicated calculation and its sensitivity, and it is better to use this simplified approach for the evaluation of lateral flow.

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Compliance with ethical standards

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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