Journal of Control, Automation and Electrical Systems (2021) 32:842–852
https://doi.org/10.1007/s40313-021-00714-z

Controlling and Synchronization of Chaotic Systems Via Takagi–Sugeno Fuzzy Adaptive Feedback Control Techniques

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Received: 4 July 2020 / Revised: 20 January 2021 / Accepted: 6 March 2021 / Published online: 2 April 2021

Abstract

In this research article, we have addressed the T-S (Takagi–Sugeno) fuzzy modeling and controlling and adaptive synchronization of chaotic systems. Based on the T-S fuzzy model, the fuzzy logic for controlling and synchronization for chaotic systems are designed via linear matrix inequality (LMI). We have illustrated the new chaotic Chen system. Lyapunov exponents and bifurcation diagrams of new Chen system are obtained to justify the chaos in system. Analytical and computational studies of new Chen systems with triangular fuzzy membership function have been performed by using LMI toolbox. Numerical simulation illustrates the controlling chaos as well as adaptive synchronization for the identical systems. Feedback gain matrices and Lyapunov positive definite matrix for the synchronization of identical new Chen systems are obtained.

Keywords Adaptive control · T-S fuzzy model · Linear matrix inequalities · Synchronization

1 Introduction

Chaos, bounded aperiodic in nature, is the inevitable phenomenon. It is highly sensitive to the initial conditions. Last several decades, it has become the attractive subject in the field of nonlinear dynamical systems due to its potential applications in various inter-disciplines activities such as mechanical engineering, chemical reaction, power converters, signal process, secure communication and biological system, etc. (Pecora and Carroll 1990; Chang et al. 2009).

Controlling chaos and synchronization of chaotic systems is the important application of chaos. Chaotic systems can be controlled and synchronized due to their potential application in sciences and technologies. These applications have been done through linear or nonlinear feedback control, sliding mode control, robust control, optimal control, adaptive control, fuzzy control, anti-synchronization using fuzzy logic constant controller, adaptive sliding mode control techniques, etc. (Pecora and Carroll 1990; Khan and Kumar 2018, 2016; Ahn 2011).

Zadeh's fuzzy logic theory made a revolution in recent research and development of sciences and technologies. Fuzzy logic has played the vital role in the control theory. It has given the new insight in reasoning (Zadeh 1988). T-S (Takagi–Sugeno) fuzzy model has mathematical simplicity. It has been widely accepted tools for designing and analysis of control systems (Tanaka and Wang 2001). Lian et al. (2001) have presented a synthesis approach for the synchronization of chaotic systems based on T-S fuzzy models. Complete Takagi–Sugeno (T-S) fuzzy logic has been addressed by many researchers and scientists (Tanaka and Wang 2001; Khan and Kumar 2019; Reddy and Samuel 2019; Saidi et al. 2019; Khan and Kumar 2016, 2017; Kumar 2020).

Fuzzy adaptive control technique is generally applied when the parameters are unknown or time varying (Kim 2005). It is classified into direct and indirect adaptive controllers. In direct fuzzy adaptive control, the parameters of the controller are constructed initially from human control knowledge, and then, iteratively it is adjusted to reduce the output error between master and slave systems. In indirect fuzzy adaptive control, parameters are constructed through human knowledge for the systems and then, it is adjusted iteratively to reduce the output error between master and slave systems (Kim 2005). Wang et al. (2003) has established the fuzzy LMI stabilization and synchronization of

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Chen system. Wang et al. (2004) has addressed the adaptive synchronization for Chen chaotic system with fully unknown parameters. Khan and Tyagi (2016) have proposed the analysis of a new 4-D hyper-chaotic system by using optimal and adaptive control. Khan and Singh (2016) has established the hybrid function projective synchronization of chaotic systems via adaptive control. Lin and Liao (2005), Kim (2005), Ting (2005) have addressed an adaptive robust observer-based synchronization of chaotic systems with time-delay. (Fradkov et al. 1919; Fradkov 2017, 2019a, b) have written a lot of research articles on adaptive control techniques.

In this article, we represent the controlling chaos and adaptive synchronization of chaotic systems based on T-S fuzzy modeling using feedback control techniques. We investigate the adaptive law to stabilize the error systems and stability analysis via Lyapunov approach of chaotic systems. Main contribution of this paper lies in four features. First, the chaotic systems are mainly redesigned by T-S fuzzy model. Second, fuzzy feedback control methodologies are used for synchronization of systems. Fuzzy triangular membership function with respect to one of state variables of fuzzy system is drawn. Third, Lyapunov exponent and bifurcation diagrams of new Chen systems are obtained to justify the chaos in the system. Fourth, numerical simulations are presented to verify the controlling and adaptive synchronization of systems. It yields the more robustness and efficient results. These studies can help the researchers and scientists of chaos control in the field secure communication. These establish the novelty of our research paper.

This article is organized as follows: Sect. 1 is introduction; Sect. 2 describes the system description fuzzy modeling of master–slave systems on the basis on adaptive control techniques; in Sect. 3, we describe stability analysis via Lyapunov approach; numerical simulations is used for verify the effectiveness of proposed adaptive synchronization of identical new Chen systems in Sect. 4; finally, conclusion is given in Sect. 5.

2 System Description of Fuzzy Modeling of Chaotic System

Consider a continuous-time nonlinear dynamical system as

$$\dot{x}(t) = Ax(t) + f(x(t)), \tag{1}$$

where $x(t) \in R^n$ is the state variable of the system, A is the $n \times n$ matrix of the system parameters and $f : R^n \longrightarrow R^n$ is the nonlinear part of the system. The system (1) is taken as the master or drive system. Its slave or response system is written as

$$\dot{y}(t) = By(t) + g(y(t)) + u(t), \tag{2}$$

where $y(t) \in R^n$ is the state variable of the slave system, B is the $n \times n$ matrix of the system parameters, $g : R^n \longrightarrow R^n$ is nonlinear part of the system and $u(t) \in R^n$ is the controller of the slave system. If $A = B$ and $f = g$, then $x(t)$ and $y(t)$ are known as the two identical chaotic systems. If $A \neq B$ or $f \neq g$, then $x(t)$ and $y(t)$ are said to be two nonidentical chaotic systems.

A Takagi–Sugeno fuzzy model based on IF-THEN rules is described by a set of fuzzy implications. It is characterized as local relations of the system in the state space. T-S fuzzy master (drive) system (1) can be represented as

$$R^i : \begin{cases} \text{IF}\, s_1(t) \ \text{is in} \ M_{i1},\ s_2(t) \ \text{is in} \ M_{i2}, \\ \cdots s_n(t) \ \text{is in}\, M_{in}, \ \text{THEN} \\ \dot{x}(t) = A_i x(t) + A_{ui} x(t) + B_{ui}\phi(t), i = 1, 2, \ldots, n, \end{cases} \tag{3}$$

where $R^i (i = 1, 2 \ldots n)$ denotes the ith fuzzy rules. $s_1(t), s_2(t), \cdots s_n(t)$ are the premise variables which consist of state vectors of the system, $M_{ij} (j = 1, 2, \ldots, n)$ are fuzzy sets, $A_i \in R^{n\times n} (i = 1, 2, \ldots n)$ are constant matrices and (A_{ui}, B_{ui}) are unknown parameters matrices of master (drive) system, $\phi(t) \in R^m$ is the oscillated force or constant value in the chaotic dynamics systems. Using the fuzzifier, the output of the fuzzy master system is written as

$$\dot{x}(t) = \sum_{i=1}^{n} h_i(s(t))(A_i x(t) + A_{ui} x(t) + B_{ui}\phi(t)), \tag{4}$$

where

$$h_i(s(t)) = \frac{w_i(s(t))}{\sum_{i=1}^{r} w_i(s(t))},$$

$$w_i(s(t)) = \prod_{j=1}^{n} M_{ij}(s(t)),$$

$h_i(s(t))$ is denoted as the normalized weight of the IF-THEN rules which satisfies

$$0 \leq h_i(s(t)) \leq 1 \quad \text{and} \quad \sum_{i=1}^{n} h_i(s(t)) = 1.$$

Similarly, fuzzy slave (response) system can be written as:

$$R^i : \begin{cases} \text{IF}\, \hat{s}_1(t) \ \text{is in} \ M_{i1},\ \hat{s}_2(t) \ \text{is in} \ M_{i2}, \\ \cdots \hat{s}_n(t) \ \text{is in}\, M_{in}, \ \text{THEN} \\ \dot{y}(t) = A_i y(t) + \hat{A}_{ui} y(t) + \hat{B}_{ui}\phi(t) + u(t), i = 1, 2, \cdots, n, \end{cases} \tag{5}$$

where $\hat{s}(t) = (\hat{s_1}(t), \hat{s_2}(t) \ldots \hat{s_n}(t))^T \in R^n$ are the premise variables which consist of state vectors of the system,

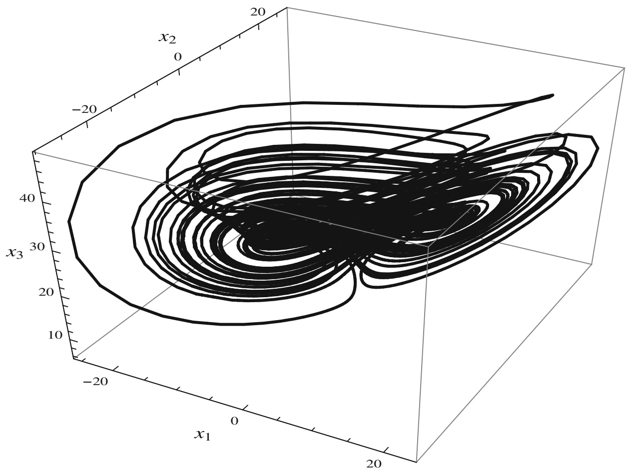


Fig. 1 Three-dimensional phase portrait of new chaotic Chen system (without controller)

$M_{ij}(j = 1, 2, \ldots n)$ are fuzzy sets. $u(t) \in R^n$ is the control input vector, $\hat{A}_{ui}$ and $\hat{B}_{ui}$ are the estimates of A_{ui} and B_{ui} of the master system, which is generated by an adaptive law. The total fuzzy response system can be rewritten as

$$\dot{y}(t) = \sum_{i=1}^{n} h_i(\hat{s}(t))(A_i y(t) + \hat{A}_{ui} y(t) + \hat{B}_{ui}\phi(t)) + u(t), \quad (6)$$

The error signal is written as

$$e(t) = y(t) - x(t). \quad (7)$$

The error dynamics of $e(t)$ is obtained as

$$\begin{aligned}\dot{e}(t) &= \dot{y}(t) - \dot{x}(t) \\ &= \sum_{i=1}^{n} h_i(\hat{s}(t))(A_i y(t) + \hat{A}_{ui} y(t) + \hat{B}_{ui}\phi(t)) \\ &\quad - \sum_{i=1}^{n} h_i(s(t))(A_i x(t) + A_{ui} x(t) \\ &\quad + B_{ui}\phi(t)) + u(t). \end{aligned} \quad (8)$$

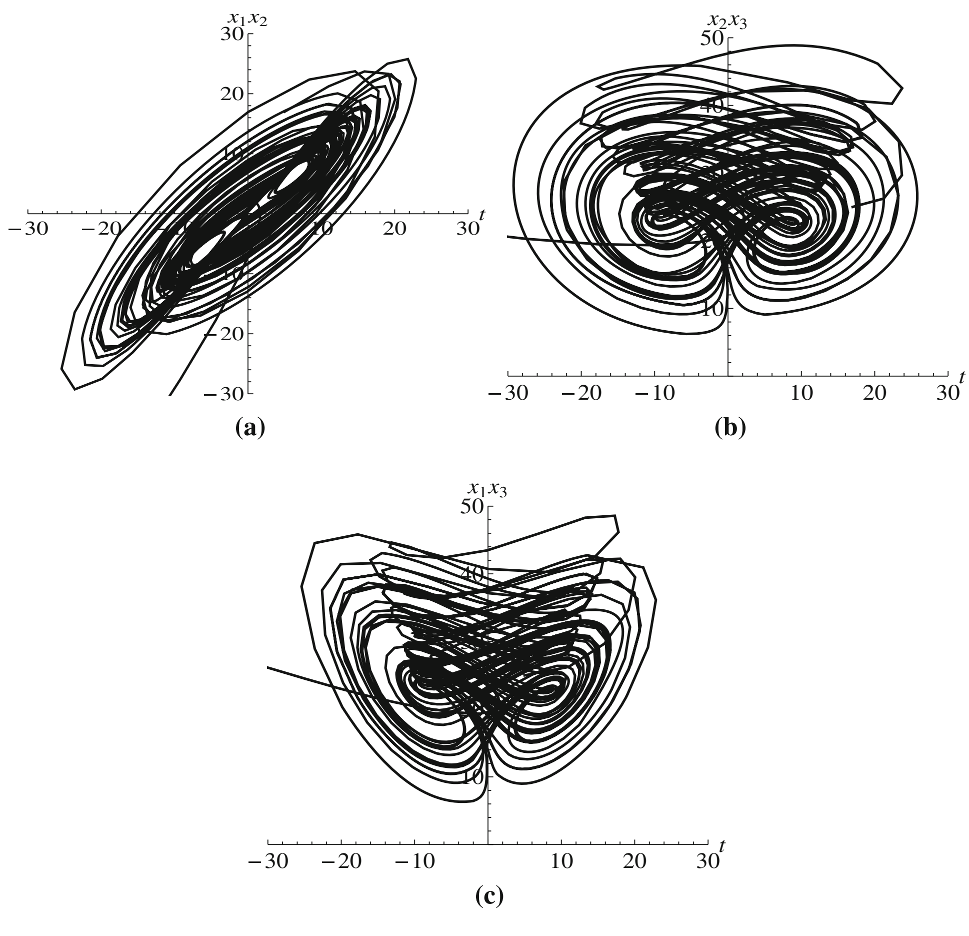


Fig. 2 Two-dimensional phase portrait of new chaotic Chen system (without controller)

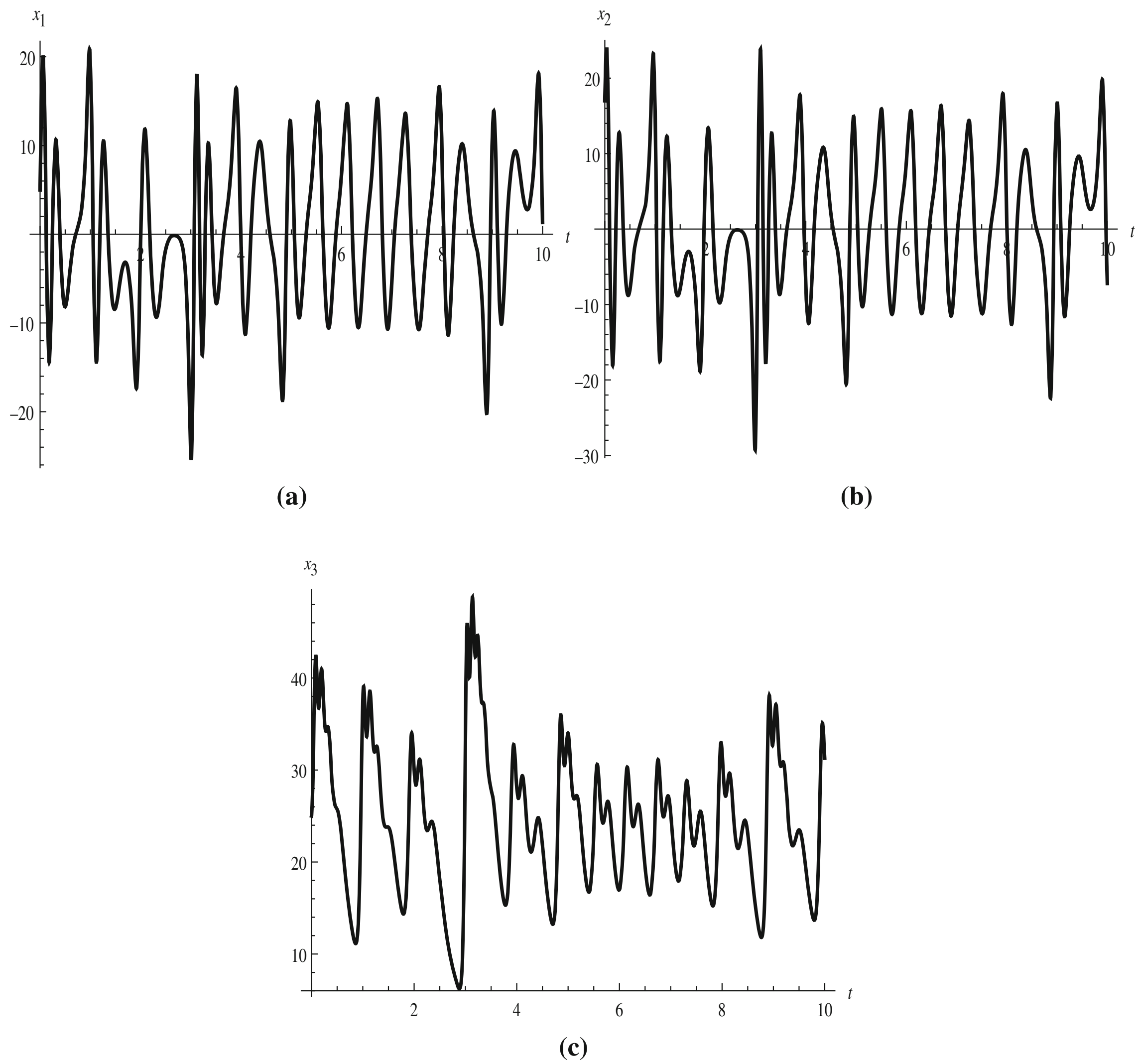


Fig. 3 Time series graphs of new chaotic Chen system (without controller)

It can be rewritten as

$$\begin{aligned}\dot{e}(t) = &\sum_{i=1}^{n} h_i(\hat{s}(t))(A_i e(t) + \hat{A}_{ui} e(t) + \tilde{A}_{ui} x(t) \\ &+ \tilde{B}_{ui}\phi(t)) + u(t) + \psi(t),\end{aligned} \tag{9}$$

where $\tilde{A}_{ui} = \hat{A}_{ui} - A_{ui}$, $\tilde{B}_{ui} = \hat{B}_{ui} - B_{ui}$ and

$$\psi(t) = \sum_{i=1}^{n}(h_i(s(t)) - h_i(\hat{s}(t)))(A_i x(t) + A_{ui} x(t) + B_i \phi(t)). \tag{10}$$

We design the adaptive control law to stabilize the error dynamics (8).

Control rule i:

$$R^i : \begin{cases} \text{IF } \hat{s}_1(t) \text{ is in } M_{i1},\ \hat{s}_2(t) \text{ is in } M_{i2}, \quad \cdots\ \hat{s}_n(t) \text{ is in } M_{in}, \text{ THEN} \\ u(t) = \hat{A}_{ui} e(t) + K_i e(t),\ i = 1, 2, \ldots, p, \end{cases} \tag{11}$$

where K_i are the feedback gain matrices.
The inferred control law is written as

$$u(t) = \sum_{i=1}^{n} h_i(\hat{s}(t))\hat{A}_{ui} e(t) + \sum_{i=1}^{n} h_i(\hat{s}(t)) K_i e(t). \tag{12}$$

We assume that the adaptive law is represented by

$$\begin{aligned}\dot{\hat{A}}_{ui} &= \sigma_{A_{ui}} h_i(\hat{s}(t)) e(t) x^T(t), \\ \dot{\hat{B}}_{ui} &= \sigma_{B_{ui}} h_i(\hat{s}(t)) e(t) \phi^T(t),\end{aligned} \tag{13}$$

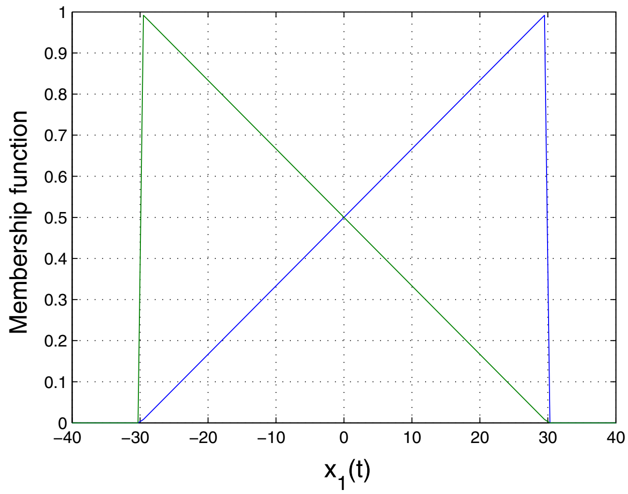


Fig. 4 Membership function of $x_1(t)$

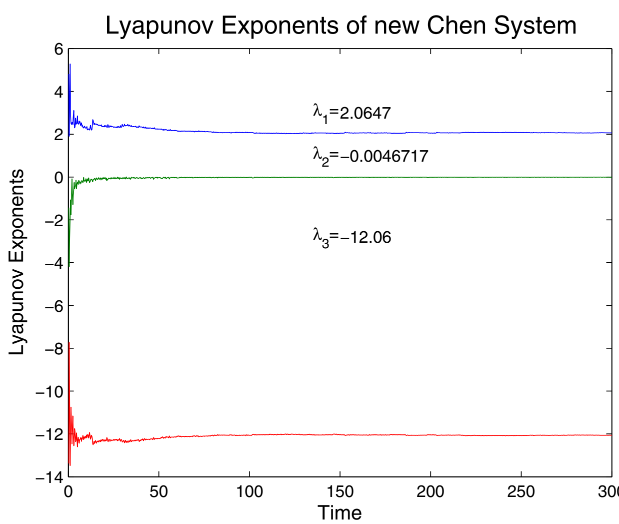


Fig. 5 Lyapunov exponent of new chaotic Chen System

where $\sigma_{A_{ui}}$ and $\sigma_{B_{ui}}$ are the constant adaptation gains. With (9) and (10) , the synchronized error dynamics by control law is obtained as

$$\dot{e}(t) = \sum_{i=1}^{n} h_i(\hat{s}(t))(A_i - K_i)e(t) + \hat{A}_{ui}x(t) + \hat{B}_{ui}\phi(t) + \psi(t), \tag{14}$$

Let us consider the following robust performance

$$\int_0^{tf} e^T(t)\tau e(t)dt \le \chi_0 + \eta^2 \int_0^{tf} \psi^T(t)\tau\psi(t)\mathrm{d}t, \tag{15}$$

where tf is the terminal time; in tf, the letter f is index; τ is a positive definite matrix, χ_0 is a value related to the system initial conditions and η is a prescribed attenuation level.

We define the Lyapunov function as

$$V(e(t), \tilde{A}_{ui}, \tilde{B}_{ui}) = e^T(t)Pe(t) + \sum_{i=1}^{n} tr\left(\frac{\tilde{A}_{ui}^T P\tilde{A}_{ui}}{\sigma_{A_{ui}}}\right) + \sum_{i=1}^{n} tr\left(\frac{\tilde{A}_{ui}^T P\tilde{A}_{ui}}{\sigma_{A_{ui}}}\right), \tag{16}$$

where P is a positive definite matrix and $tr(A)$ represents the trace of A. The time derivative of V along the trajectory of (13) and (14) is written as

$$\begin{aligned}
\dot{V}(e(t)) &= \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) \\
&\quad + \sum_{i=1}^{n} tr\left(\frac{\dot{\tilde{A}}_{ui}^T P\tilde{A}_{ui}}{\sigma_{A_{ui}}} + \frac{\tilde{A}_{ui}^T P\dot{\tilde{A}}_{ui}}{\sigma_{A_{ui}}}\right) \\
&\quad + \sum_{i=1}^{n} tr\left(\frac{\dot{\tilde{B}}_{ui}^T P\tilde{B}_{ui}}{\sigma_{B_{ui}}} + \frac{\tilde{B}_{ui}^T P\dot{\tilde{B}}_{ui}}{\sigma_{B_{ui}}}\right). \qquad (17) \\
&= \sum_{i=1}^{n} h_i\hat{s}(t)(A_i - K_i)^T P(A_i - K_i)e(t) \\
&\quad + 2e^T(t)P\sum_{i=1}^{n} h_i(\hat{s}(t))\tilde{A}_{ui}x(t) \\
&\quad + 2e^T(t)P\sum_{i=1}^{n} h_i(\hat{s}(t))\tilde{B}_{ui}\phi(t) + e^T(t)P\psi(t) \\
&\quad + \psi^T(t)Pe(t) - \sum_{i=1}^{n} 2tr\left(\frac{\dot{\tilde{A}}_{ui}^T P\dot{\tilde{A}}_{ui}}{\sigma_{A_{ui}}}\right) \\
&\quad - \sum_{i=1}^{n} 2tr\left(\frac{\dot{\tilde{B}}_{ui}^T P\dot{\tilde{B}}_{ui}}{\sigma_{B_{ui}}}\right).
\end{aligned}$$

After some algebraic manipulations, we have

$$\begin{aligned}
\dot{V} &\le \sum_{i=1}^{n} h_i(\hat{s}(t))e(t)((A_i - K_i)^T P + P(A_i - K_i) \\
&\quad + \tau + \frac{1}{\eta^2}PP)e(t) - e^T(t)\tau e(t) + \eta^2\psi^T\psi \\
&\quad + 2tr\left(\sum_{i=1}^{n} h_i(\hat{s}(t))\tilde{A}_{ui}Pe(t)x^T(t) - \sum_{i=1}^{n}\frac{\tilde{A}_{ui}^T P\dot{\tilde{A}}_{ui}}{\sigma_{A_{ui}}}\right) \\
&\quad + 2tr\left(\sum_{i=1}^{n} h_i(\hat{s}(t))\tilde{B}_{ui}Pe(t)\phi^T(t) - \sum_{i=1}^{n}\frac{\tilde{B}_{ui}^T P\dot{\tilde{B}}_{ui}}{\sigma_{B_{ui}}}\right).
\end{aligned}$$

If there exists a symmetric and positive definite matrix P such that the following linear matrix inequalities (17) hold,

$$((A_i - K_i)^T P + P(A_i - K_i) + \tau + \frac{1}{\eta^2}PP) < 0, \tag{18}$$

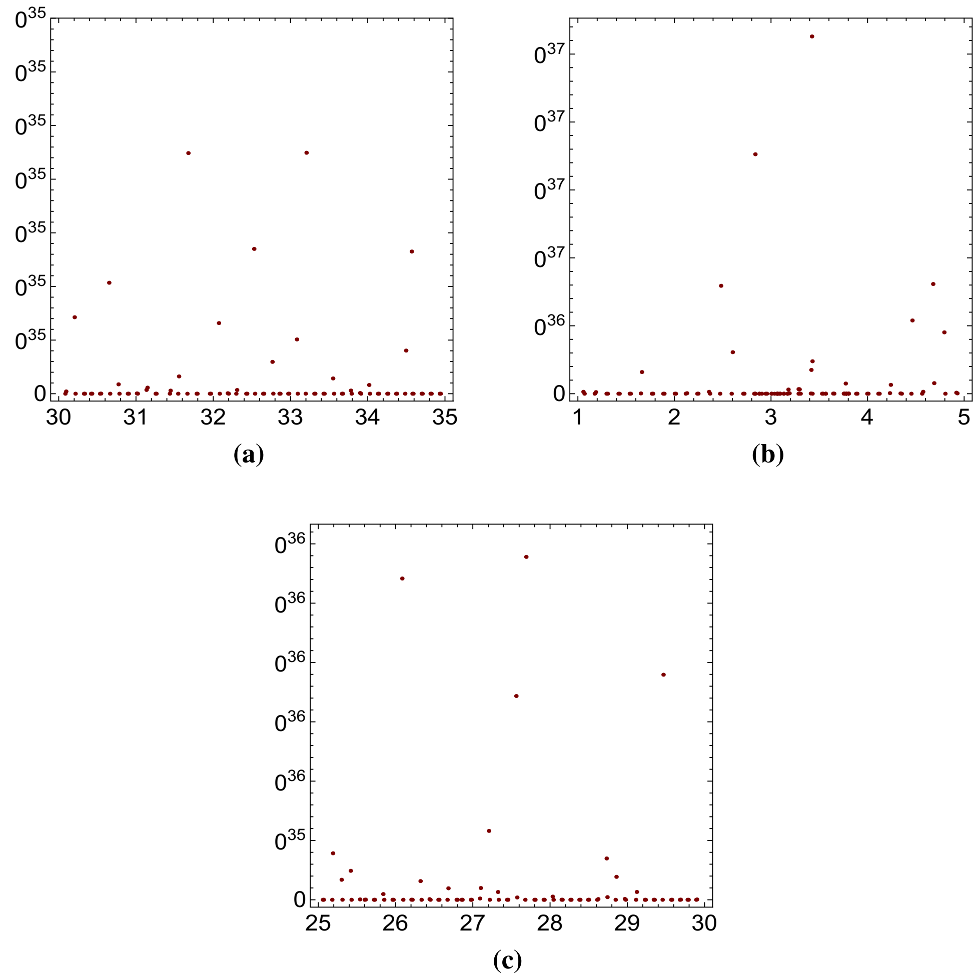


Fig. 6 Bifurcation of new chaotic Chen system with respect to the parameters α, β, and γ, respectively

we obtain that

$$\dot{V} \leq e^T(t)\tau e(t) + \eta^2 \psi^T \psi. \tag{19}$$

Integrating on both sides of (19) from $t = 0$ to $t = tf$ yields

$$\begin{aligned} &V(e(tf), \tilde{A}_{ui}(tf), \tilde{B}_{ui}(tf)) - V(e(0), \tilde{A}_{ui}(0), \tilde{B}_{ui}(0)) \\ &\quad \leq -\int_0^{tf} e^T(t)\tau e(t)dt + \eta^2 \int_0^{tf} \psi^T(t)\tau\psi(t)\mathrm{d}t, \end{aligned} \tag{20}$$

which implies

$$\begin{aligned} &\int_0^{tf} e^T(t)\tau e(t)\mathrm{d}t \\ &\quad \leq V(e(0), \tilde{A}_{ui}(0), \tilde{B}_{ui}(0)) + \eta^2 \int_0^{tf} \psi^T(t)\tau\psi(t)\mathrm{d}t. \end{aligned} \tag{21}$$

Letting $\chi(0) = V(e(0), \tilde{A}_{ui}(0), \tilde{B}_{ui}(0))$, then the robust performance (15) is guaranteed for $i = 1, 2 \ldots n$. Using Equation (12) and (18), $\dot{V} < 0$ if $\psi(t) = 0$. Therefore, the total synchronization system (14) is asymptotically stable, i.e., the two fuzzy chaotic system (3) and (5) can be synchronized. Equation (18) can be rewritten in Schur complement form as

$$\begin{bmatrix} A_i^T P + P A_i - X_i^T - X_i & P \\ P & -\eta^2 I \end{bmatrix} < 0, \quad i = 1, 2, \ldots n. \tag{22}$$

We determine the feedback gain matrices K_i from the fact that $K_i = P^{-1} X_i$ for $i = 1, 2, \ldots n$.

The trajectories of $e(t)$, $\tilde{A}_{ui}$ and $\tilde{B}_{ui}$ are bounded for all $t > 0$, $e(t)$ and $x(t) \in L_\infty$ space. This means that $y(t) \in L_\infty$ space. Therefore, all signals in (13) and (14) are uniformly bounded. $\psi \in L_2$ space and V is bounded.

$$\lim_{t\to\infty} V(e(t), \tilde{A}_{ui}, \tilde{B}_{ui}) = V_\infty < \infty. \tag{23}$$

Using (20) and (23), we get

$$\int_0^{tf} e^T(t)\tau e(t)dt \leq V_0 - V_\infty + \eta^2 \int_0^{tf} \psi^T(t)\tau\psi(t)\mathrm{d}t, \tag{24}$$

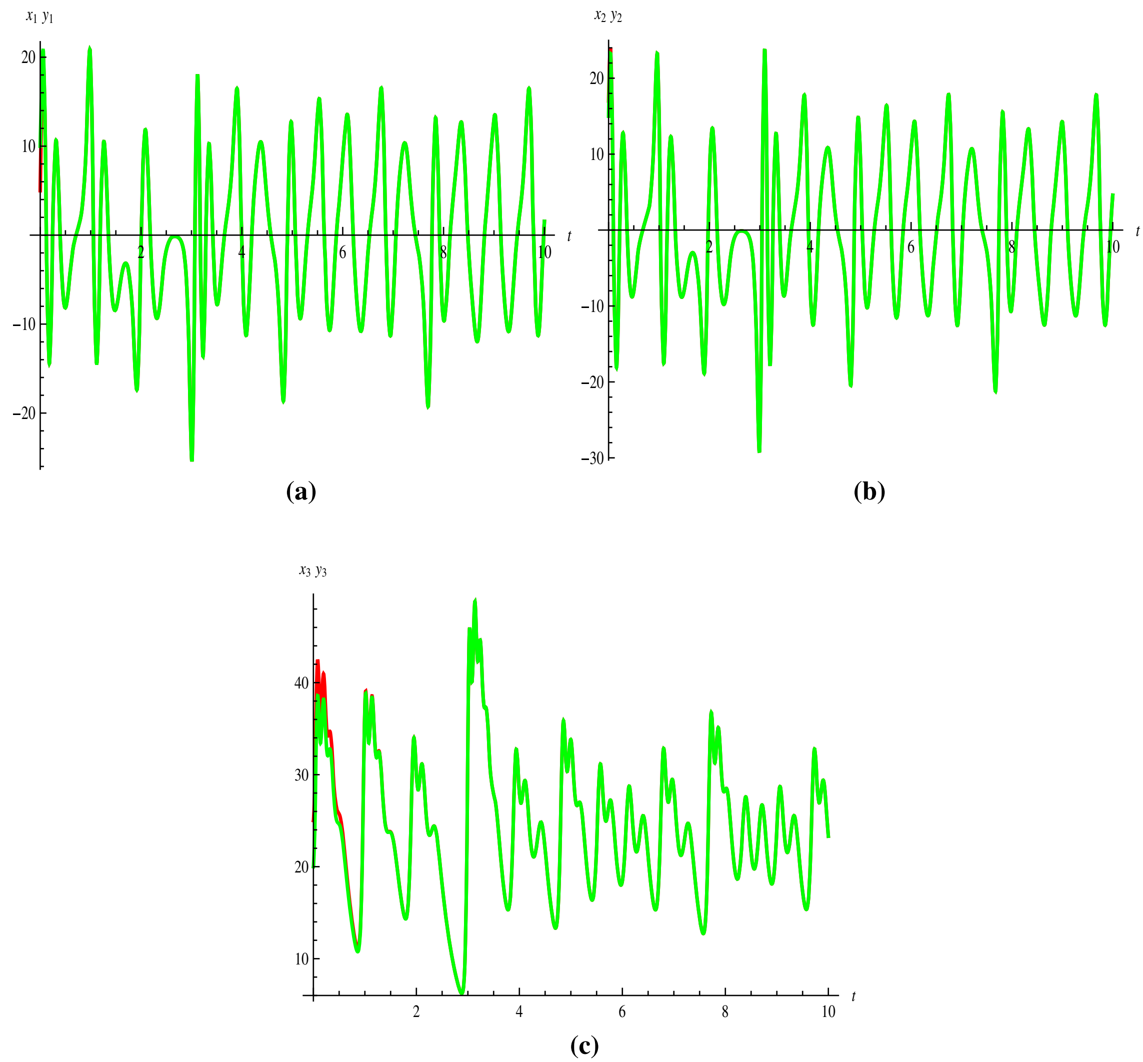


Fig. 7 Stabilization of state of trajectory of 3D new chaotic Chen system in **a** x_1, y_1 **b** x_2, y_2 and **c** x_3, y_3 , respectively (with controller)

where $V_0 = V(e(0), \tilde{A}_{ui}(0), \tilde{B}_{ui}(0))$. $\phi(t)$, $\psi(t)$, $\hat{A}_{ui}$, $\hat{B}_{ui}$, $\hat{x}(t)$, $e(t)$ belong to L_∞ space.

Using equation (14), $\dot{e}(t) \in L_\infty \Rightarrow e(t) \in L_\infty$

Thus,

$$\lim_{t\to\infty} e(t) = 0. \tag{25}$$

That is, master and slave systems are in synchronization. It means $e(t) \to 0$ when $t \to \infty$.

3 Stability Conditions Via Lyapunov Approach

Proposition Duan and Yu (2013): The continuous-time linear system in the form of

$$\dot{x}(t) = Ax(t), \tag{26}$$

with $A \in R^{n\times n}$ is Hurwitz stable if and only if there exists a matrix $P \in R^{n\times n}$, such that

$$\begin{bmatrix} P > 0 \\ A^T P + PA < 0. \end{bmatrix} \tag{27}$$

Lemma 1 *(Tanaka and Wang 2001; Duan and Yu 2013): Equilibrium of the synchronization error system (18) is asymptotically stable if the following condition holds*

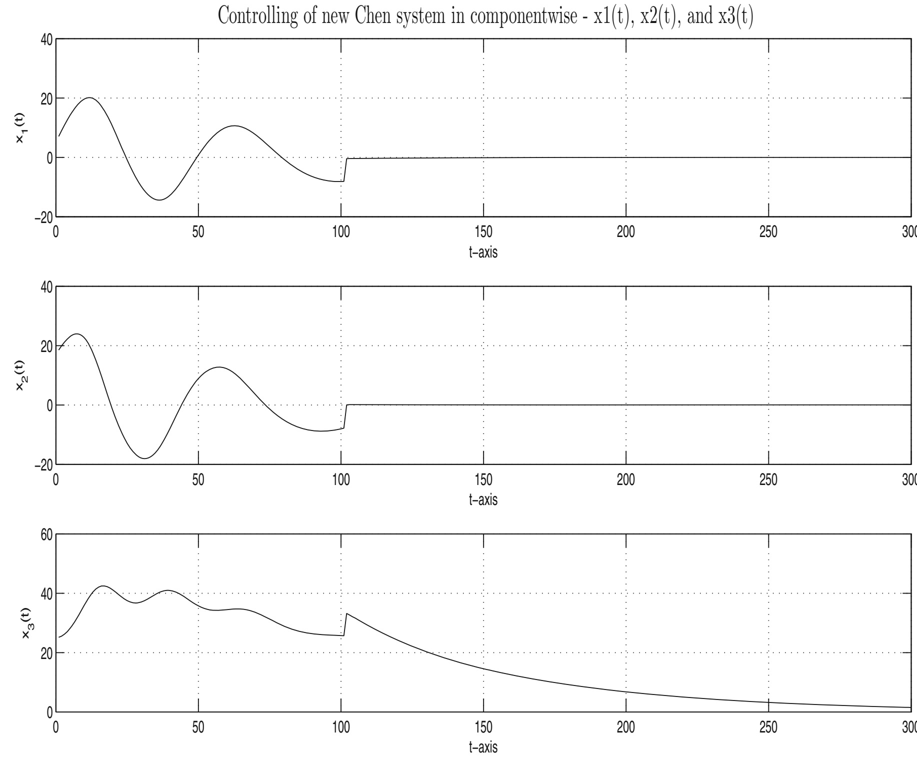
$$G_i^T P + PG_i < 0, \quad i = 1, 2, \ldots r. \tag{28}$$

where $G_i = A_i - K_i$ and P is a common matrix.

4 Numerical Simulation

Consider the new Chen system (Cermak and Nechvatal 2019) as

Fig. 8 Controlling of drive new chaotic Chen System in component-wise activated at $t = 100$



$$\begin{cases} \dot{x}_1 = \alpha(x_2 - x_1), \\ \dot{x}_2 = \gamma x_1 - \alpha x_1 - x_1 x_3 + \gamma x_2, \\ \dot{x}_3 = x_1 x_2 - \beta x_3, \end{cases}$$

where $\alpha,\ \beta,\ \gamma\ >\ 0$ are assumed as constant parameters. For simplicity, we assume that $x_1(t)$ is observable. Because "$x_1(t)$ contains all nonlinear terms" in the 3D new Chen system. It is also assumed that $x_1(t) \in [x_{min},\ \ x_{max}] = [-k,\ \ k]$ with $k = 30$.

The fuzzy master (drive) system of new Chen attractor is written as

$$\text{Rule } R^i\colon \begin{cases} \text{IF } x_1(t) \text{ is in } M_i, \text{THEN} \\ \dot{x}(t) = A_i x(t) + A_{ui} x(t) + B_{ui}\phi(t), i = 1, 2, \end{cases}$$

where $x(t) = (x_1, x_2, x_3)^T$, $A_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -k \\ 0 & k & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & k \\ 0 & -k & 0 \end{bmatrix}$, $A_{u1} = A_{u2} = \begin{bmatrix} -\alpha & \alpha & 0 \\ (\gamma - \alpha) & \gamma & 0 \\ 0 & 0 & -\beta \end{bmatrix}$.

The corresponding membership functions fuzzy sets are $M_1(x_1(t)) = \frac{1}{2}\left(1 + \frac{(x_1(t))}{k}\right)$, $M_2(x_1(t)) = \frac{1}{2}\left(1 - \frac{(x_1(t))}{k}\right)$. We take $k = 30$. Slave new Chen system is written as:

$$\begin{cases} \dot{x}_1 = \alpha(x_2 - x_1) + u_1, \\ \dot{x}_2 = \gamma x_1 - \alpha x_1 - x_1 x_3 + \gamma x_2 + u_2, \\ \dot{x}_3 = x_1 x_2 - \beta x_3 + u_3, \end{cases}$$

The fuzzy response system of new Chen systems is written as

$$\text{Rule } R^i\colon \begin{cases} \text{IF } y_1(t) \text{ is in } M_i, \text{THEN} \\ \dot{y}(t) = A_i y(t) + A_{ui} y(t) + u(t), i = 1, 2. \end{cases}$$

where $\hat{A}_{u1} = \begin{bmatrix} -\hat{\alpha}_{11} & \hat{\alpha}_{12} & 0 \\ (\hat{\gamma}_{11} - \hat{\alpha}_{13}) & \hat{\gamma}_{12} & 0 \\ 0 & 0 & \hat{\beta}_1 \end{bmatrix}$, $\hat{A}_{u2} = \begin{bmatrix} -\hat{\alpha}_{21} & \hat{\alpha}_{22} & 0 \\ (\hat{\gamma}_{21} - \hat{\alpha}_{23}) & \hat{\gamma}_{22} & 0 \\ 0 & 0 & \hat{\beta}_2 \end{bmatrix}$

Here, $u(t) = (u_1, u_2, u_3)$. $(\hat{\alpha}_{11}, \hat{\alpha}_{12}, \hat{\alpha}_{13}, \hat{\alpha}_{21}, \hat{\alpha}_{22}, \hat{\alpha}_{23},)$, $(\hat{\beta}_1, \hat{\beta}_2)$ and $\hat{\gamma}_{11}, \hat{\gamma}_{12}, \hat{\gamma}_{21}, \hat{\gamma}_{22}$ are the estimated unknown parameters of α, β and γ, respectively.

Simulation results for the new chaotic Chen system: Take the initial conditions of master and slave new chaotic Chen systems, $x(0) = (5, 17, 25)$ and $y(0) = (10, 15, 20)$. Figure 1 represents the three-dimensional phase portraits of system. Figure 2a–c represents the two-dimensional phase portraits of system in x_1x_2, x_2x_3 and x_3x_1 , respectively. Figure 3a–c represents the time-series graphs of system. Fuzzy membership function of $x_1(t)$ of the system is shown in Fig. 4. We have computed the Lyapunov exponents of system. We have $\lambda_1 = 2.0647$, $\lambda_2 = -0.0046717$ and $\lambda_3 = 12.06$. On calculating the Lyapunov exponents, we observe that out of these three Lyapunov exponent values, one is positive, one is negative, and one of these tends to zero. It verified the chaos

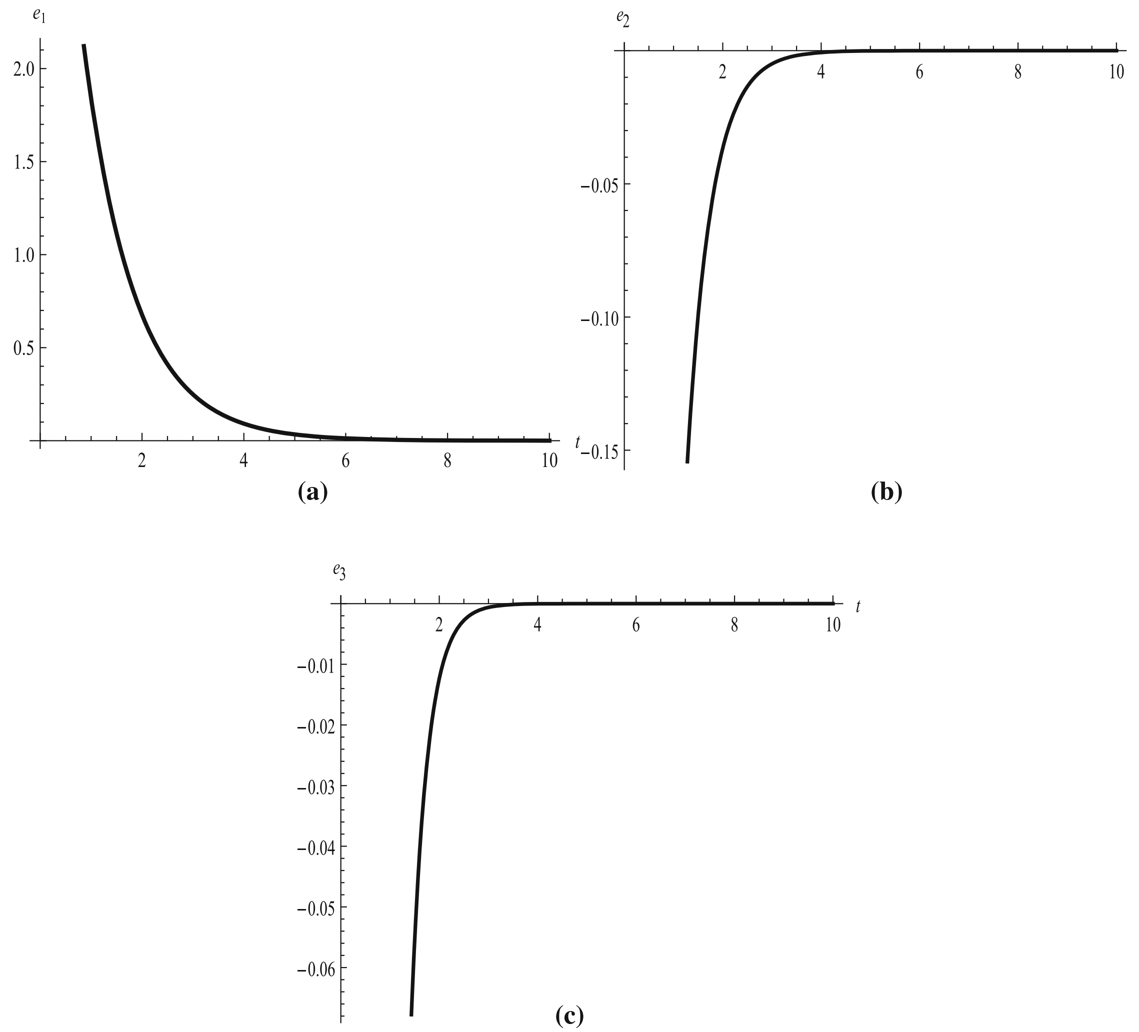


Fig. 9 Synchronization of error dynamics of new chaotic Chen systems

in the new Chen system. It is shown in Fig. (5). Figure (6)a–c shows the bifurcation diagrams with respect to the parameters α, β and γ, respectively. The parameters range of the chaotic system is taken as $\alpha = 30 - 35$, $\beta = 1 - 5$ and $\gamma = 25 - 30$. These show the strange points. The unknown parameters of $\hat{\alpha}_{11}$, $\hat{\alpha}_{12}$, $\hat{\alpha}_{21}$, $\hat{\alpha}_{22}$, $\hat{\alpha}_{13}$, $\hat{\alpha}_{23}$, $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\gamma}_{11}$, $\hat{\gamma}_{12}$, $\hat{\gamma}_{21}$, $\hat{\gamma}_{22}$ with zero initial conditions are adjusted by an adaptive law (13), where the adaption gains $\sigma_{A_{u1}} = 1, \sigma_{A_{u2}} = 3.5$ are used. The state of slave trajectories of the system tracks that of master system (a) x_1, y_1 (b) x_2, y_2, (c) x_3, y_3 which is shown in Fig. 7a–c. Figure 8 shows the controlling of new chaotic Chen system in component-wise. These are activated at $t = 100$. At the initial conditions, $e(0) = (5, -2, -5)$, Fig. 9a–c represents the synchronization of error dynamics for identical systems. Figure 10a–d shows the estimation parameters of (a)$\hat{\alpha}_{11}$, $\hat{\alpha}_{12}$, $\hat{\alpha}_{21}$, $\hat{\alpha}_{22}$, $\hat{\alpha}_{13}$ and $\hat{\alpha}_{23}$ (b) $\hat{\beta}_1$ and $\hat{\beta}_2$ and (c) $\hat{\gamma}_{11}$, $\hat{\gamma}_{12}$, $\hat{\gamma}_{21}$, and $\hat{\gamma}_{22}$ tend to the actual parameters values of α, β and γ, respectively. That is,

$$\lim_{t \to \infty} e(t) = 0.$$

The feedback gain matrices and Lyapunov positive definite matrix of system are simulated with the parameter values $\alpha = 35$, $\beta = 3$ and $\gamma = 28$. Estimated parameters for system are shown in Fig. 10a–d. To examine the validity of stability conditions for chaotic system, we use the MATLAB LMI toolbox to obtain the feedback gain matrices, K_1 and K_2 and Lyapunov positive definite matrices for $\eta = 0.001$ as:

$$K_1 = 1.0e^{-12} \begin{bmatrix} 0.1600 & 0.1570 & 0.1514 \\ 0.1570 & 0.3446 & 0.0221 \\ 0.1514 & 0.0221 & 0.2364 \end{bmatrix},$$

$$K_2 = 1.0e^{-10} \begin{bmatrix} 0.0607 & 0.0191 & -0.0208 \\ 0.1016 & 0.0325 & -0.0378 \\ 0.0281 & 0.0085 & -0.0074 \end{bmatrix}.$$

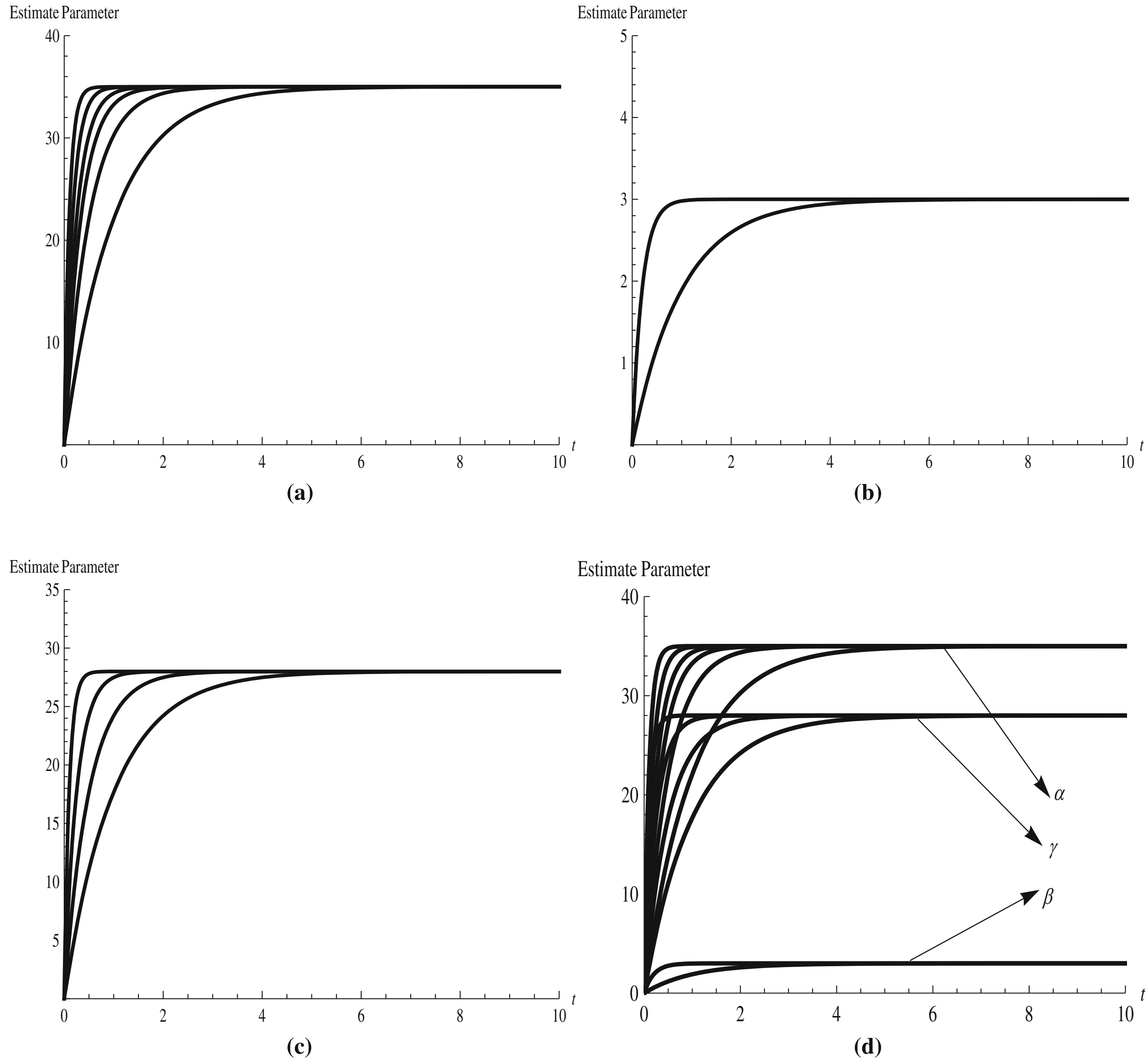


Fig. 10 Estimated parameters for the new chaotic Chen system **a** $\hat{\alpha}_{11}, \hat{\alpha}_{12}, \hat{\alpha}_{21}, \hat{\alpha}_{22}, \hat{\alpha}_{13}$ and $\hat{\alpha}_{23}$ **b** $\hat{\beta}_1$ and $\hat{\beta}_2$, **c** $\hat{\gamma}_{11}, \hat{\gamma}_{12}, \hat{\gamma}_{21}$, and $\hat{\gamma}_{22}$ and **d** α, β, γ

$$P = \begin{bmatrix} 0.2361 & -0.3377 & 0.3024 \\ -0.3377 & 1.0915 & -0.0591 \\ 0.3024 & -0.0591 & 0.7564 \end{bmatrix}.$$

5 Conclusions

In this research article, we have investigated the T-S fuzzy model based on controlling and adaptive synchronization of chaotic systems. These methodologies provide the new insight in control systems using LMI techniques. T-S fuzzy modeling with the help of LMI technique is an effective and fruitful results for controlling and synchronization chaotic systems. New Chen system illustrates the effectiveness of the proposed approach. Lyapunov exponents and bifurcation diagrams for new Chen system are calculated. In numerical solution, the feedback gain matrices and Lyapunov positive definite matrix have been obtained. It confirms the stability of fuzzy control systems and satisfies the linear matrix inequalities (LMIs). These results verify the efficiency of the feedback gains and T-S fuzzy control theory application to the synchronization for two identical new Chen systems.

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