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A Control Approach to Web Speed and Tension Regulation of Web Transport Systems Based on Dynamic Surface Control

Ly Tong Thi[1,2] · Cuong Nguyen Manh[2] · Tung Lam Nguyen[2]

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Abstract
This paper proposes an approach to deal with control problems of unmodeled components of the web transport system. It is commonly challenging to construct model-based controllers to guarantee tracking quality due to the unknown terms in the mathematical model. Hence, our main contribution is to design a robust controller for the uncertain model of the web transport system based on Dynamic Surface Control (DSC) technique to ensure high accuracy in the tracking process of the web speed and tension. The proposed controller is designed in the face of the mathematical model of the web transport system that is adversely affected by bounded uncertainties. The stability of the controlled system is proved using the Lyapunov standard. The simulation results show the validity and the effectiveness of the proposed control law when the system is lack of system model's information.

Keywords Roll to roll · Dynamic surface control · Lyapunov stability · Rewinding system

1 Introduction

Various applications related to web transport systems, for example, flexible films, paper, lighting, and solar cells, using roll-to-roll (R2R) frameworks are commonly winding up in the industry. The key point to guarantee the system output quality is to properly handle web moving speed and tension. In order to eliminate the disturbance in the entire procedure, the control law for R2R frameworks is crucial for high-resolution printing (Chang and Weng 2001)–(Glaoui et al. 2013). Based on dynamic modeling of the system depicted by the authors of (Choi 2010) and (Choi et al. 2009), several research works have been published to enhance the control performance with numerous control schemes, for instance, sliding mode control (SMC) in (Abjadi et al. 2009) and (Chen et al. 2004), back stepping technique (Choi et al. 2011) and (Bouchiba et al. 2011) to counter impacts of system uncertainties for nonlinear R2R web transport systems. Besides, the study of rotational roll inertia which directly influences the speed and the performance of the system are considered in (Kang and Lee 2007). The controller proposed by Lin el.al solves the problem arising from disturbance brought by friction and rotational inertia (Lin et al. 2001). An observer based on neural network is designed in (Mahfouf et al. 2005) to provide the roll load and torque information for a PID controller to regulate roll speed and gap. An interesting work on web handling system can be found in (Mathur and Messner 1998); the authors propose a control scheme for low-tension high-speed system where air entrainment can deteriorate the control performance. In (Thi et al. 2019), a flexible coupling problem is considered; the authors employ backstepping control to minimize flexible coupling effects on the rewinding system. In web rewinding system, information of roll inertia is crucial for control design; Manh et al. design a neural network combined with backstepping sliding mode control to cope with uncertain roll inertia (Manh et al. 2019). The paper results show good speed and tension tracking performance under unavailability of the unwinding and rewinding inertia.

In most cases, these solutions give positive results and properly handle the problems affecting the system. In this paper, we propose an approach that solves some of the limitations of conventional algorithms. The classic backstepping technique is known as a method of building the controller for nonlinear robot systems, but in some cases, the control law

✉ Tung Lam Nguyen
lam.nguyentung@hust.edu.vn

1 Hanoi University of Industry, Hanoi, Vietnam

2 Hanoi University of Science and Technology, Hanoi, Vietnam

using this technique tends to sensitive to noise (Mokhtari and Sicard 2013). In addition, the higher the order of the system, the more complex the calculation of the virtual control signal, and thus, it reduces the response speed of the system and leads to the phenomena “explosion of terms.” In fact, the roll-to-roll web transport system is affected by various nonlinear elements such as friction, torque, and hardware design features. Therefore, to have the capacity to meet the aimed control quality, the number of conditions of the system must be idealized when using backstepping control law. In that context, the SMC emerged as a strong candidate that could overcome this phenomenal because of its noise-eliminated ability. However, the “chattering” is a factor that significantly affects the quality of the system (Drakunov and Utkin 2015). Thus, we propose an approach using dynamic surface control to handle the above problems because it takes advantage of the two techniques and its low-pass filter is to generate the virtual control signal. Therefore, simultaneously increases the computation speed as well as the response speed of the system. Compared to other advanced control techniques such as (Wang et al. 2016; Wang and Pan 2019; Wang and Deng 2020), dynamic surface control has its own merit in term of controlling the web transport system. On the other hand, other approaches using adaptive/gain scheduling PI controllers are investigated to improve the system performance as in (Nishida, et al. 2013; Giannoccaro, et al. 2016; Raul and Pagilla 2016). In (Nishida, et al. 2013), particle swarm optimization (PSO) is employed to identify the system parameters, and thus, the performance is improved in the presence of noises affecting the system; another adaptive mechanism is presented in (Raul and Pagilla 2016). Besides, because control parameters play a vital role in PI controller, self-tuning parameters method is investigated in (Giannoccaro, et al. 2016). However, the web transport system commonly contains the nonlinear factors which the method based on transfer function could not consider in some cases.

The main contributions of the paper can be highlighted as:

(1) The paper proposes a DSC controller which has not been constructed for an uncertain model of the roll-to-roll system. The design step considers all the uncertain elements of the system which can be measured inaccurately or unknown, but only the nominal values of these parameters are required. Dissimilar to (Manh et al. 2019), the system uncertainties containing all the variable parameters are considered. Therefore, this approach is more comprehensive than the previous work in (Manh et al. 2019) which only consider the torque as the uncertainties.

(2) The proposed approach has the outstanding characteristic in diminishing the amount of the computation, but still guarantees the system’s stability and simultaneously cope with uncertainties, while the approach in (Manh et al. 2019) sometimes struggles with the computational burden when the system is considerably affected by the unknown parts because of the neural network calculation.

(3) The control parameters are also analyzed and chosen with a limited range. Meanwhile, model-based controllers for the roll-to-roll system as in (Thi et al. 2019) and (Tran and Choi 2014) are heavily depended on the control parameter selection.

The paper is organized as follows. The uncertain model of single-pan roll-to-roll web transport system is presented in Sect. 2. Section 3 dedicates to developing a dynamic surface control for web speed and tension regulation; the control design is summarized in two propositions. The simulation results are given in Sect. 4. Finally, some concluding remarks are stated in Sect. 5.

2 Model of Roll-to-Roll Web Handling Systems

Figure 1 shows a single-span web control system that contains unwinder, rewinder, a loadcell subsystem with idle rollers and two dancer subsystems. Input torques τ_u and τ_r generated on unwinder and rewinder motors, respectively, operate the system, and we use these signals to control the web’s velocity and tension. The idle rollers keep the moving web around the load cell in a fixed angel, while two dancer subsystems on unwind and rewind sides deal with the slack during startup and shutdown process.

Assume that the web’s slippage and deformation do not occur, and the loadcell and dancers dynamic are totally ignored, and web property obeys Hook’s law. The nonlinear dynamic equations of the single-span roll-to-roll web control system are shown as (Choi et al. 2009):

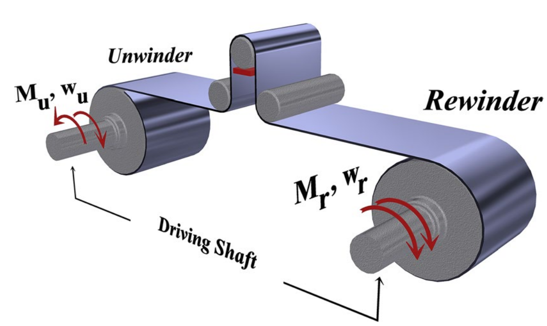

Fig. 1 Single-span roll-to-roll web system (Manh et al. 2019)

$$\begin{cases} \dot{\omega}_u = \dfrac{B_u}{J_u}\omega_u + \dfrac{R_u}{J_u}T - \dfrac{1}{J_u}\tau_u, \\ \dot{T} = -KR_u\omega_u - \dfrac{R_r}{L}T\omega_r + KR_r\omega_r, \\ \dot{\omega}_r = \dfrac{R_r}{J_r}T - \dfrac{B_r}{J_r}\omega_r + \dfrac{1}{J_r}\tau_r. \end{cases} \tag{1}$$

The relevance among total moment of inertia, operating radius, and the thickness of the web is shown in following equations:

$$R_u(t) = R_{u0} - \frac{\theta_u h}{2\pi};\ R_r(t) = R_{r0} + \frac{\theta_r h}{2\pi}$$

$$J_u = J_{u0} + \pi\rho\omega\frac{\left(R_u^4 - R_{u0}^4\right)}{2}$$

$$J_r = J_{r0} + \pi\rho\omega\frac{\left(R_r^4 - R_{r0}^4\right)}{2}$$

where J_{u0} is the total moment of inertia of the unwind roll and motor at start-up time, J_{r0} is the total moment of inertia of the rewind roll and motor at start-up time, J_u is the total moment of inertia of the unwind roll and motor, J_r is the total moment of inertia of the rewind roll and motor, T is the web tension, ω_u is the angular velocity of the unwind roll, θ_u is the rotational angular of the unwind roll, ω_r is the angular velocity of the rewind roll, θ_r is the rotational angular of the rewind roll, R_{r0} is the initial radius of the rewind roll, R_{u0} is the initial radius of the unwind roll, R_u is the operating radius of the unwind roll, R_r is the operating radius of the rewind roll, τ_u is the torque generated by the unwind motor, τ_r is the torque generated by the rewind motor, B_u is the coefficient of viscous friction of the rewind roll, B_r is the coefficient of viscous friction of the unwind roll, L is the web total length, K is the spring constant of web, h is the thickness of web, ρ is the density of web, w is the web width.

It is noted that input torques τ_u and τ_r are used as control input to the system. This in commonly applicable in practice since actuators such as electric motors working in torque control mode can generate torque as required.

A problem of uncertain parameters always exists in the mathematical model, and thus, model-based controllers cannot completely ensure the overall system's stability. In roll-to-roll system's model (1), B_u, B_r, R_u, R_r, J_u, and J_r are commonly uncertain values or cannot be measured accurately. Hence, these parameters are nominal values in modeled system. Assume that B_u, B_r, R_u, R_r, J_u, and J_r are the actual values of the system's parameter, in which

$$\begin{cases} B_u = \hat{B}_u + \tilde{B}_u \\ B_r = \hat{B}_r + \tilde{B}_r \\ R_u = \hat{R}_u + \tilde{R}_u \\ R_r = \hat{R}_r + \tilde{R}_r \\ J_u = \hat{J}_u + \tilde{J}_u \\ J_r = \hat{J}_r + \tilde{J}_r \end{cases} \tag{2}$$

$\hat{B}_u$, $\hat{B}_r$, $\hat{R}_u, \hat{R}_r, \hat{J}_u$, and $\hat{J}_r$ are nominal values and these values are used to design the controllers. Additionally, $\tilde{B}_u$, $\tilde{B}_r$, $\tilde{R}_u, \tilde{R}_r, \tilde{J}_u$, and $\tilde{J}_r$ are the errors between the actual values and the nominal ones. Then, the system model (1) becomes:

$$\begin{cases} \dot{\omega}_u = \dfrac{\hat{B}_u + \tilde{B}_u}{\hat{J}_u + \tilde{J}_u}\omega_u + \dfrac{\hat{R}_u + \tilde{R}_u}{\hat{J}_u + \tilde{J}_u}T - \dfrac{1}{\hat{J}_u + \tilde{J}_u}\tau_u, \\ \dot{T} = -K(\hat{R}_u + \tilde{R}_u)\omega_u - \dfrac{\hat{R}_r + \tilde{R}_r}{L}T\omega_r + K(\hat{R}_r + \tilde{R})_r\omega_r, \\ \dot{\omega}_r = \dfrac{\hat{R}_r + \tilde{R}_r}{\hat{J}_r + \tilde{J}_r}T - \dfrac{\hat{B}_r + \tilde{B}_r}{\hat{J}_r + \tilde{J}_r}\omega_r + \dfrac{1}{\hat{J}_r + \tilde{J}_r}\tau_r. \end{cases} \tag{3}$$

To facilitate the controller design, (3) is rewritten as

$$\begin{cases} \hat{J}_u\dot{\omega}_u = \hat{B}_u\omega_u + \hat{R}_uT - \tau_u + (-\tilde{J}_r\dot{\omega}_u + \tilde{B}_u\omega_u + \tilde{R}_uT), \\ \dot{T} = -K\hat{R}_u\omega_u - \dfrac{\hat{R}_r}{L}T\omega_r + K\hat{R}_r\omega_r + (-K\tilde{R}_u\omega_u - \dfrac{\tilde{R}_r}{L}T\omega_r + K\tilde{R}_r\omega_r), \\ \hat{J}_r\dot{\omega}_r = \hat{R}_rT - \hat{B}_r\omega_r + \tau_r + (-\tilde{J}_r\dot{\omega}_r + \tilde{R}_rT - \tilde{B}_r\omega_r). \end{cases} \tag{4}$$

The uncertainties of each equation of motion are defined as

$$\begin{cases} \Delta\xi_u = -\tilde{J}_r\dot{\omega}_u + \tilde{B}_u\omega_u + \tilde{R}_uT, \\ \Delta\xi_T = -K\tilde{R}_u\omega_u - \dfrac{\tilde{R}_r}{L}T\omega_r + K\tilde{R}_r\omega_r, \\ \Delta\xi_r = -\tilde{J}_r\dot{\omega}_r + \tilde{R}_rT - \tilde{B}_r\omega_r. \end{cases} \tag{5}$$

Then, we have the nominal model of the roll-to-roll system as follows

$$\begin{cases} \hat{J}_u\dot{\omega}_u = \hat{B}_u\omega_u + \hat{R}_uT - \tau_u + \Delta\xi_u, \\ \dot{T} = -K\hat{R}_u\omega_u - \dfrac{\hat{R}_r}{L}T\omega_r + K\hat{R}_r\omega_r + \Delta\xi_T, \\ \hat{J}_r\dot{\omega}_r = \hat{R}_rT - \hat{B}_r\omega_r + \tau_r + \Delta\xi_r. \end{cases} \tag{6}$$

The system model (6) describes the mathematical model, in which the uncertainties are also considered. $\Delta\xi_u$, $\Delta\xi_T$, and $\Delta\xi_r$ are unknown bounded elements and the proposed controller in the next section will cope with this problem and simultaneously ensure the system's tracking quality.

3 Dynamic Surface Control For Roll-to-Roll Systems

Dynamic Surface Control (DSC) is constructed based on integrator backstepping (IB) technique and multiple sliding mode (MSS) method. So that, this controller inherits all the advantages of both the above mechanisms. Beside, with an important addition in the design: the low-pass filter, this controller also brought significant effect in diminishing error in calculating and minimizing the amount of computation by avoiding the "explosion of terms" phenomenon.

The controller aims to keep web tension and web speed at desired values; web speed is controlled through tracking angular velocity at references. This section will express the DSC controller formula for the roll-to-toll system; then, we will demonstrate the stability of system with that controller.

Firstly, define the error of the web tension that is considered for design controller step as follows:

$$\varsigma_T = T_d - T \tag{7}$$

where T_d is desired web tension. Taking derivative of ς_T, we obtain:

$$\begin{aligned}\dot{\varsigma}_T &= \dot{T}_d - \dot{T} \\ &= \dot{T}_d + K\hat{R}_u\omega_u + \frac{\hat{R}_r}{L}T\omega_r - K\hat{R}_r\omega_r + \Delta\overline{\xi}_T\end{aligned} \tag{8}$$

where $-\Delta\xi_T = \Delta\overline{\xi}_T$ and $\left|\Delta\overline{\xi}_T\right| \le \delta$. The conventional idea is using virtual control signal generated through backstepping technique in order to drive $T \to T_d$. In this scenario, a unwinder's speed $\overline{\omega}_u$ is considered as the virtual control signal. However, in this research, instead of using directly virtual signal, we use the value that is filtered signal by using a low-pass filter that was defined as:

$$\begin{cases}\tau\dot{\omega}_{ud} + \omega_{ud} = \overline{\omega}_u \\ \overline{\omega}(0) = \omega_{ud}(0)\end{cases} \tag{9}$$

where ω_{ud} is denoted as the reference speed of the unwinding roll and τ is the time constant. This addition step helps to significantly decrease the amount of computation from calculating the virtual control's derivative that may generate "explosion of terms" phenomenon.

Consider the relationship between web tension and other system parameters specified in Eq. (6), if the uncertain value $\Delta\overline{\xi}_T$ is ignored and the unwider's speed has the form:

$$\overline{\omega}_u = \frac{1}{K\hat{R}_u}\left(-\frac{\hat{R}_r}{L}T\omega_r + K\hat{R}_r\omega_r - \dot{T}_d - k_1\varsigma_T\right) \tag{10}$$

where k_1 is a positive gain. Propose the Lyapunov candidate function for error signal of web tension as follows:

$$V_T = \frac{1}{2}\varsigma_T^2 \tag{11}$$

Taking time derivative of (11) and using (8), we obtain:

$$\dot{V}_T = \varsigma_T\left(\dot{T}_d + K\hat{R}_u\omega_u + \frac{\hat{R}_r}{L}T\omega_r - K\hat{R}_r\omega_r + \Delta\overline{\xi}_T\right) \tag{12}$$

With the virtual control signal (10) and assuming that $\overline{\omega}_u = \omega_u$, (12) becomes:

$$\dot{V}_T = -k_1\varsigma_T^2 + \Delta\overline{\xi}_T \tag{13}$$

Ignoring $\Delta\overline{\xi}_T$, we have $\dot{V}_T = -k_1\varsigma_T^2 \le 0$ and this satisfies the Lyapunov stability standard. However, with the existence of $\Delta\overline{\xi}_T$, the virtual control signal in (10) cannot ensure the system's stability. Therefore, the following virtual control is proposed

$$\overline{\omega}_u = \frac{1}{K\hat{R}_u}\left(-\frac{\hat{R}_r}{L}T\omega_r + K\hat{R}_r\omega_r - \dot{T}_d - k_1\varsigma_T - \varsigma_T\frac{\delta^2}{2\varepsilon}\right) \tag{14}$$

Next, the unwinder speed's tracking error is denoted as

$$\varsigma_u = \omega_u - \omega_{ud} \tag{15}$$

Taking derivative of (15), we have:

$$\begin{aligned}\dot{\varsigma}_u &= \dot{\omega}_u - \dot{\omega}_{ud} \\ &= \frac{\hat{B}_u}{\hat{J}_u}\omega_u + \frac{\hat{R}_u}{\hat{J}_u}T - \frac{1}{\hat{J}_u}\tau_u + \Delta\overline{\xi}_u - \dot{\omega}_{ud}\end{aligned} \tag{16}$$

in which $\Delta\overline{\xi}_u = \frac{\Delta\xi_u}{\hat{J}_u}$ and $\left|\Delta\overline{\xi}_u\right| \le \beta_u$. With this definition, the first control signal τ_u is proposed:

$$\tau_u = \hat{J}_u\left(\frac{\hat{B}_u}{\hat{J}_u}\omega_u + \frac{\hat{R}_u}{\hat{J}_u}T - \dot{\omega}_{ud} + k_2\varsigma_u + k_3 sgn(\varsigma_u)\right) \tag{17}$$

$\dot{\omega}_{ud}$ is calculated from the low-pass filter (9) by $\dot{\omega}_{ud} = \frac{\overline{\omega}_u - \omega_{ud}}{\tau}$. Thus, (17) is rewritten as:

$$\tau_u = \hat{J}_u\left(\frac{\hat{B}_u}{\hat{J}_u}\omega_u + \frac{\hat{R}_u}{\hat{J}_u}T - \frac{\overline{\omega}_u - \omega_{ud}}{\tau} + k_2\varsigma_u + k_3 sgn(\varsigma_u)\right) \tag{18}$$

with k_2 and k_3 being positive values. This control signal is designed to guarantee that ω_u approach ω_{ud} or $\varsigma_u \to 0$.

Then, the error of the rewinder speed is defined as:

$$\varsigma_r = \omega_r - \omega_{rd} \tag{19}$$

where ω_{rd} is the reference speed of the rewinding roll. Subsequently, we have its derivative

$$\dot{\varsigma}_r = \dot{\omega}_r - \dot{\omega}_{rd} = \frac{\hat{R}_r}{\hat{J}_r}T - \frac{\hat{B}_r}{\hat{J}_r}\omega_r + \frac{1}{\hat{J}_r}\tau_r + \Delta\bar{\xi}_r - \dot{\omega}_{rd} \tag{20}$$

where $\Delta\bar{\xi}_r = \frac{\Delta\xi_r}{\hat{J}_r}$ and $\left|\Delta\bar{\xi}_r\right| \leq \beta_r$. The remaining control signal is proposed as follows

$$\tau_r = -\hat{J}_r(\frac{\hat{R}_r}{\hat{J}_r}T - \frac{\hat{B}_r}{\hat{J}_r}\omega_r - \dot{\omega}_{rd} + k_4\varsigma_r + k_5\text{sgn}(\varsigma_r)) \tag{21}$$

with k_4 and k_5 being positive values. Finally, we define the filtered error

$$\zeta = \omega_{ud} - \bar{\omega}_u \tag{22}$$

and its derivative

$$\dot{\zeta} = \dot{\omega}_{ud} - \dot{\bar{\omega}}_u = -\frac{\zeta}{\tau} + \frac{\mathrm{d}}{\mathrm{d}t}\left(-\frac{1}{K\hat{R}_u}\left(-\frac{\hat{R}_r}{L}T\omega_r + K\hat{R}_r\omega_r - \dot{T}_d - k_1\varsigma_T - \varsigma_T\frac{\delta^2}{2\varepsilon}\right)\right) \tag{23}$$

Let $\gamma = \frac{d}{dt}\left(-\frac{1}{K\hat{R}_u}(-\frac{\hat{R}_r}{L}T\omega_r + K\hat{R}_r\omega_r - \dot{T}_d - k_1\varsigma_T - \varsigma_T\frac{\delta^2}{2\varepsilon})\right)$ with the bounded condition $\gamma \leq M$, $M > 0$, (23) is rewritten as

$$\dot{\zeta} = -\frac{\zeta}{\tau} + \gamma \tag{24}$$

Remark From Eq. (17) and (21), it can be seen that the control inputs τ_u and τ_r comprise of available information of the system, namely web tension and speed, that can be recorded using tension and speed sensors. Moreover, the developed controller is constructed to cope with system parameters variation, and thus, only nominal values are required.

Theorem Consider an uncertain model of the roll-to-roll system (6), with the virtual controller in (14) followed by the low-pass filter (9) and the control signal (18), (21), if the filter time constant τ is chosen as

$$\frac{1}{\tau} = 1 + \frac{M^2}{2\varepsilon} + k_0 \tag{25}$$

where k_0 is a positive gain; the overall system is asymptotically stable.

Proof The web tension error (8) is rewritten as follows:

$$\dot{\varsigma}_T = \dot{T}_d + K\hat{R}_u\left[(\omega_u - \omega_{ud}) + (\omega_{ud} - \bar{\omega}_u) + \bar{\omega}_u\right] + \frac{\hat{R}_r}{L}T\omega_r - K\hat{R}_r\omega_r + \Delta\bar{\xi}_T \tag{26}$$

Substituting (14), (15), and (22) into (26), we obtain:

$$\dot{\varsigma}_T = K\hat{R}_u(\varsigma_u + \zeta) - k_1\varsigma_T - \varsigma_T\frac{\delta^2}{2\varepsilon} + \Delta\bar{\xi}_T \tag{27}$$

Using the control signal (18) and (21), (16) and (20) turn into:

$$\begin{cases} \dot{\varsigma}_u = -k_2\varsigma_u - k_3\text{sgn}(\varsigma_u) + \Delta\bar{\xi}_u \\ \dot{\varsigma}_r = -k_4\varsigma_r - k_5\text{sgn}(\varsigma_r) + \Delta\bar{\xi}_r \end{cases} \tag{28}$$

Next, a Lyapunov candidate function is chosen as

$$V = \frac{1}{2}\varsigma_T^2 + \frac{1}{2}\varsigma_u^2 + \frac{1}{2}\varsigma_r^2 + \frac{1}{2}\zeta^2 \tag{29}$$

Taking derivative of (29), using (24), (27), (28), and (29) results in

$$\begin{aligned} \dot{V} &= \varsigma_T\left(K\hat{R}_u(\zeta_u + \zeta) - k_1\varsigma_T - \varsigma_T\frac{\delta^2}{2\varepsilon} + \Delta\bar{\xi}_T\right) \\ &+ \varsigma_u\left(-k_2\varsigma_u - k_3\text{sgn}(\varsigma_u) + \Delta\bar{\xi}_u\right) \\ &+ \varsigma_r\left(-k_4\varsigma_r - k_3\text{sgn}(\varsigma_r) + \Delta\bar{\xi}_r\right) + \zeta\left(-\frac{\zeta}{\tau} + \gamma\right) \end{aligned} \tag{30}$$

To facilitate the proof of system stability, (30) is rewritten as

$$\begin{aligned} V &= K\hat{R}_u(\varsigma_u\varsigma_T + \zeta\varsigma_T) - k_1\varsigma_T^2 - \varsigma_T^2\frac{\delta^2}{2\varepsilon} + \varsigma_T\Delta\bar{\xi}_T \\ &+ (-k_2\varsigma_u^2 - \varsigma_u k_3\text{sgn}(\varsigma_u) + \varsigma_u\Delta\bar{\xi}_u) \\ &+ (-k_4\varsigma_r^2 - \varsigma_r k_5\text{sgn}(\varsigma_r) + \varsigma_r\Delta\bar{\xi}_r) - \frac{\zeta^2}{\tau} + \zeta\gamma \end{aligned} \tag{31}$$

From Young's inequality, it is straightforward to show that:

$$\begin{cases} \varsigma_u\varsigma_T \leq \dfrac{\varsigma_u^2 + \varsigma_T^2}{2}, \\ \zeta\varsigma_T \leq \dfrac{\zeta^2 + \varsigma_T^2}{2}, \\ \varsigma_T\Delta\bar{\xi}_T \leq \left|\varsigma_T\right|\delta \leq \dfrac{\varsigma_T^2\delta^2}{2\varepsilon} + \dfrac{\varepsilon}{2}, \\ \zeta\gamma \leq \dfrac{\varepsilon}{2} + \dfrac{\zeta^2\gamma^2}{2\varepsilon}. \end{cases} \tag{32}$$

Then, (32) becomes

$$\begin{aligned} \dot{V} &\leq K\hat{R}_u\left(\frac{\varsigma_u^2 + \varsigma_T^2}{2} + \frac{\zeta^2 + \varsigma_T^2}{2}\right) \\ &+ \varepsilon - \frac{\zeta^2}{\tau} + \frac{\zeta^2\gamma^2}{2\varepsilon} + (-k_1\varsigma_T^2 - k_2\varsigma_u^2 - \varsigma_u k_3\text{sgn}(\varsigma_u) \\ &+ \varsigma_u\Delta\bar{\xi}_u - k_4\varsigma_r^2 - \varsigma_r k_5\text{sgn}(\varsigma_r) + \varsigma_r\Delta\bar{\xi}_r) \end{aligned} \tag{33}$$

Accompanied with the assumptions that the uncertainties are bounded, (33) is transformed into

$$\dot{V} \leq K\hat{R}_u\left(\frac{2\varsigma_T^2 + \varsigma_u^2 + \zeta^2}{2}\right) + \varepsilon - \frac{\zeta^2}{\tau} + \frac{\zeta^2\gamma^2}{2\varepsilon} + (|\varsigma_u|(-k_3 + \beta_u) + |\varsigma_r|(-k_5 + \beta_r)) + (-k_1\varsigma_T^2 - k_2\varsigma_u^2 - k_4\varsigma_r^2) \tag{34}$$

By properly choosing $k_3 \geq \beta_u$ and $k_5 \geq \beta_r$, we obtain

$$\dot{V} \leq K\hat{R}_u(\frac{2\varsigma_T^2 + \varsigma_u^2 + \zeta^2}{2}) + \varepsilon - \frac{\zeta^2}{\tau} + \frac{\zeta^2\gamma^2}{2\varepsilon} + (-k_1\varsigma_T^2 - k_2\varsigma_u^2 - k_4\varsigma_r^2) \tag{35}$$

Then, the remaining control parameters are chosen as $k_1 = K\hat{R}_u + k_0$, $k_2 = \frac{1}{2}K\hat{R}_u + k_0$, $k_4 = \frac{1}{2}K\hat{R}_u + k_0$; (35) is rewritten as

$$\begin{aligned}\dot{V} &\leq K\hat{R}_u\left(\frac{2\varsigma_T^2 + \varsigma_u^2 + \zeta^2}{2}\right) + \varepsilon - \frac{\zeta^2}{\tau} + \frac{\zeta^2\gamma^2}{2\varepsilon} \\ &+ (-(K\hat{R}_u + k_0)\varsigma_T^2 - \left(\frac{1}{2}K\hat{R}_u + k_0\right)\varsigma_u^2 - \left(\frac{1}{2}K\hat{R}_u + k_0\right)\varsigma_r^2) \\ &\leq -k_0(\varsigma_T^2 + \varsigma_u^2 + \varsigma_r^2) + \varepsilon - \frac{\zeta^2}{\tau} + \frac{\zeta^2\gamma^2}{2\varepsilon}\end{aligned} \tag{36}$$

With the time filter constant selected according to (25), (36) becomes

$$\dot{V} \leq -k_0(\varsigma_T^2 + \varsigma_u^2 + \varsigma_r^2 + \zeta^2) + \varepsilon - \left(1 - \frac{\gamma^2}{M^2}\right)\frac{M^2\zeta^2}{2\varepsilon} \tag{37}$$

With bounded condition $\gamma \leq M$, and $M > 0$, finally we have:

$$\dot{V} \leq -2k_0V + \varepsilon \tag{38}$$

where ε is an arbitrary small positive constant, the system error is ultimately bounded with an arbitrary small region. That leads to the stability of the system with arbitrarily bounded error, and the theorem is proved.

4 Simulation Results

In this section, the performance of the roll-to-roll system with the proposed controller will be examined through a set of numerical simulations. To illustrate the efficiency of the proposed controller, the system is considered in two cases and the simulation results of the DSC controller are compared to those of the backstepping controller (Thi et al. 2019). With the used model of the web transport system, model-based controllers are represented by backstepping technique as in (Thi et al. 2019) given the significant performance in guaranteeing the tracking control quality in acceptable range. However, the phenomena “explosion of terms” when the system is affected by unknown factors are not considered. The simulation scenario would show the results in comparison with the backstepping controller in case of model-based controller to show the superior of the proposed method.

In the simulation scenarios, parameter variations are considered as of 10% of the nominal value. The parameters of the roll-to-roll system are given in Table 1.

Case 1: The two parameters J_u and J_r are considered as the uncertainties.

Practically, in the roll-to-roll web transport system, the roll inertias are commonly measured inaccurately because they significantly depend on both various mechanical factors and system’s coefficients. The control laws only have the information about nominal values, and thus, the proposed controller based on DSC technique can show the superior performance to the one of the conventional model-based controllers designed in (Thi et al. 2019) as shown in Fig. 2, Fig. 3, and Fig. 4.

The web tension’s performance is shown in Fig. 2 with the remarkably different control quality of the two controllers. A model-based approach of the backstepping technique gives a result which contains a steady error. Moreover, at the time approximately 10 s and 17 s, a phenomenon of overshoot occurs and causes the maximum error. Meanwhile, with the robust characteristic in coping with uncertain elements, DSC still ensures overall stability of the system.

Besides, Fig. 3 shows similar results with the outstanding performance of DSC controller in guaranteeing the rewinder speed’s tracking quality. Additionally, tracking errors are given in Fig. 4. In general, in this case, when the torques are uncertain, DSC gives superior results and clearly shows the efficiency of the proposed method.

Case 2: B_u, B_r, R_u, R_r, J_u, and J_r are the uncertainties with the change in the desired rewinder speed.

To verify the validation of the proposed method, all the uncertain components leading to an uncertain model in (6) are considered. Moreover, the rewinder’s reference speed is changed in each period.

Table 1 Parameters

System’s nominal parameters
$R_{u0} = 0.04\text{m}$, $R_{r0} = 0.015\text{m}$, $J_{u0} = J_{r0} = 1\text{kg/m/s}$, $\hat{B}_u = \hat{B}_r = 0.00002533\text{kgms/rad}$, $h = 0.00002\text{m}$; $L = 1\text{m}$, $w = 0.3\text{m}$, $K = 200\text{kg/m}$,
Control parameters
$k_0 = 5$, $k_3 = 10$, $k_4 = 10$

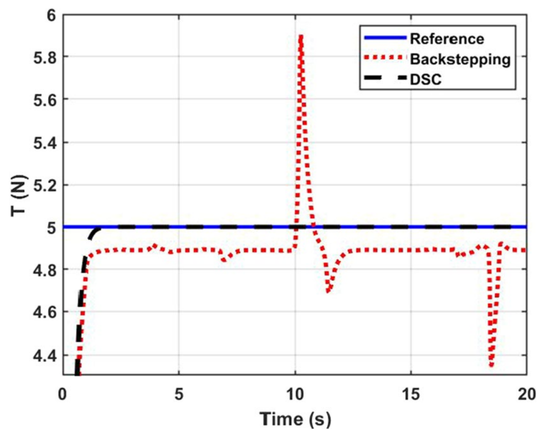

Fig. 2 Web tension when torques are unknown

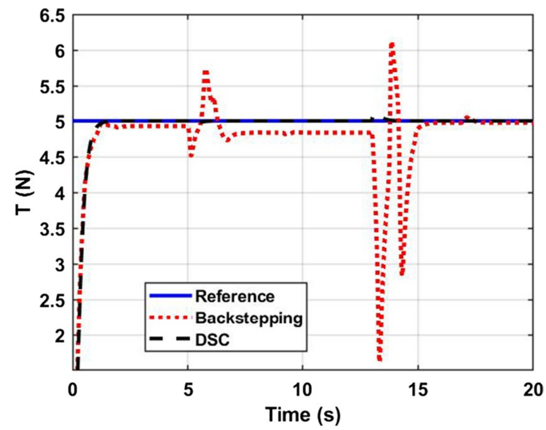

Fig. 5 Web speed in case 2

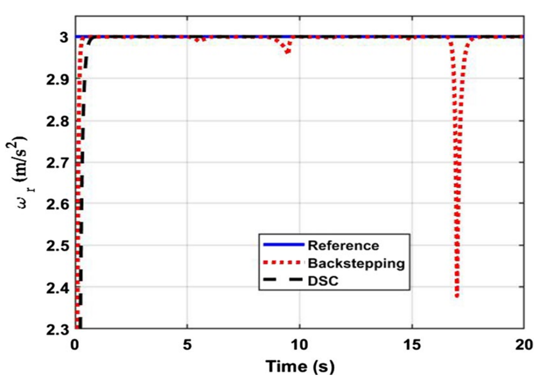

Fig. 3 Rewinder speed when torques are unknown

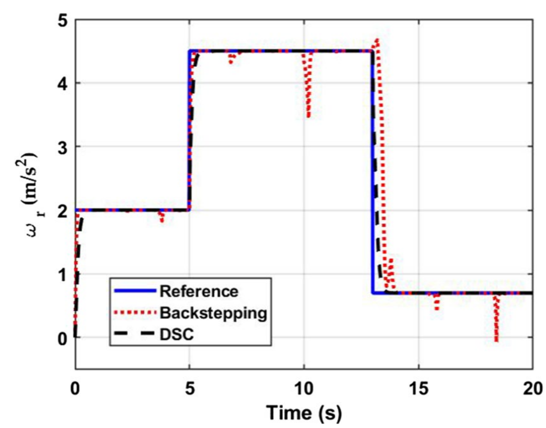

Fig. 6 Rewinder speed in case 1

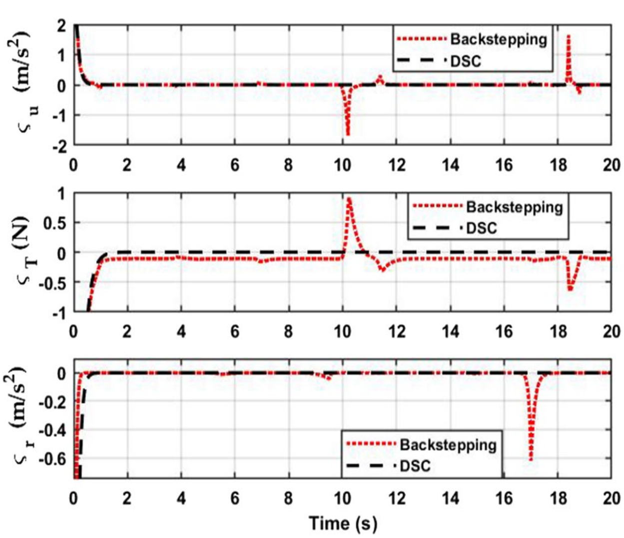

Fig. 4 Tracking errors when torques are unknown

There is a fluctuation, as well as the maximum error dramatically increases in the backstepping's performance of the web tension as shown in Fig. 5, while the DSC controller ensures the tracking quality. In Fig. 6, when the reference value of the rewinder speed is changed at the time 5 s and 13 s, the proposed controller shows the robust characteristic in keeping the system's stability during the time. The sudden deviation in the backstepping's performance can make the system operates un-smoothly with significant errors as shown in Fig. 7.

In general, it can be seen from the figures, it is obvious that the performance of the DSC controller shows better quality in comparison with the backstepping method. The results also reveal that the proposed controller is able to cope with the uncertain system model which is difficult to accurately calculate in practical.

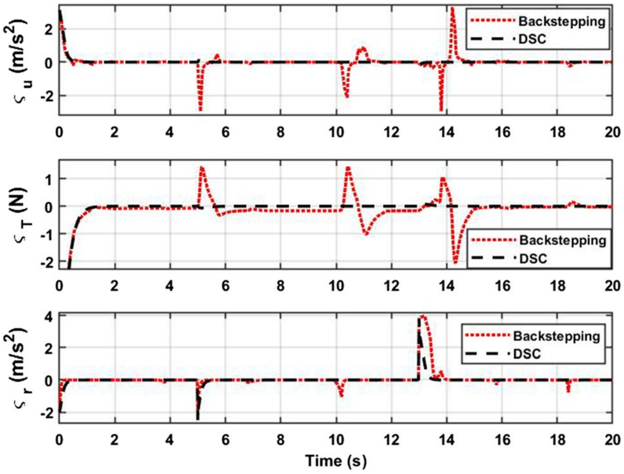

Fig. 7 Web tension response with changed reference

5 Conclusions

Web speed and tension control is a pivot problem in web transport systems, especially in presence of uncertain parameters. The paper proposes a control design based on dynamic surface control as an alternative way to model-based controllers. The paper also introduces comprehensive numerical simulations to demonstrate the effectiveness of the control design in the comparison with the backstepping technique. Dynamic surface control-based controller provides high tracking performance which is superior to that of backstepping controller even when the system is adversely affected by the uncertainties. In real-time systems, the limitation of the method is to determine appropriate time constants for the filter, which is bounded by the sampling frequency. Therefore, the processing speed of the hardware significantly takes an impact on the system performance. In the future, our work will target to consider nonlinear elasticity in modeling the web handling system and its control problem. The future work of the research aims at developing a multi-roll system model with unknown roll inertia.

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