



H_∞ Model Reduction for 2-D Discrete Markovian Jump Systems

Khalid Badie¹ · Mohammed Alfidi¹ · Zakaria Chalh¹

Received: 19 June 2020 / Revised: 7 October 2020 / Accepted: 28 October 2020 / Published online: 29 November 2020
© Brazilian Society for Automatics–SBA 2020

Abstract

This paper is concerned with the problem of H_∞ model reduction for two-dimensional (2-D) discrete Markovian jump systems. The mathematical model of 2-D Markovian jump systems is described by the Fornasini–Marchesini (F–M) second model. Our attention is focused on the design of a 2-D reduced-order model, which ensures the model error system to be stochastically stable and has a prescribed H_∞ performance index. By using the Lyapunov functional approach and introducing some zero equations, a new condition for H_∞ performance analysis of model error system is developed. Based on this condition, the desired reduced-order model parameters can be obtained by solving a set of linear matrix inequalities. Two examples are presented to show the effectiveness of the proposed method.

Keywords 2-D systems · Markovian jump · Model reduction · H_∞ performance · Linear matrix inequalities

1 Introduction

In the past few decades, there have been remarkable advances in the analysis and synthesis of two-dimensional (2-D) systems. The key feature of 2-D systems is that the information is propagated along two independent directions. As is all known, many modern engineering fields, such as process control, transmission lines, image processing and signal filtering Du and Xie (2002), Kaczorek (1985), Lu (1992), have intrinsic 2-D characteristics, which motivates researchers to study the analysis and the design methods for 2-D systems. Accordingly, many problems on 2-D systems have been addressed; for example, stability and stabilization problems are studied in Badie et al. (2018), Badie et al. (2019), Tadepalli and Leite (2018), Yao et al. (2013), control problem is investigated Badie et al. (2020a), Duan and Xiang (2013), Fei et al. (2017), Luo et al. (2016) and filtering problem is discussed in Badie et al. (2020b), Li and Gao (2013), Liang et al. (2015), Xu et al. (2017).

On the other side, Markovian jump systems have attracted considerable interest in the control community in the last

two decades. Markovian jump system is a class of multi-model systems that are composed of a set of subsystems (modes), with mode-to-mode transition probabilities being governed by Markov chain Li et al. (2014), Liu et al. (2018), Zhang et al. (2018). As a special class of hybrid systems, Markovian jump systems have powerful modeling ability of many practical systems subject to abrupt variations in their structures caused by external causes, for instance component failures, sudden environmental changes and changing subsystem interconnections. Recently, numerous works have been published on Markovian jump systems Chen and Ma (2017), Huang et al. (2012), Saravanakumar et al. (2016), Zhu et al. (2020). However, the previously mentioned works only treat one-dimensional (1-D) systems. In relationship with 2-D systems, there are few published results which treat the problems of analysis and synthesis for 2-D Markovian jump systems; for example, the problem of stochastic stability analysis for 2-D Markovian jump systems described by the Roesser model with time-varying delays and uncertain transition probabilities was studied in Hien and Trinh (2016). In Zhai et al. (2016), the problem of asynchronous H_∞ filtering for 2-D Markovian jump systems described by the Roesser model with sensor failure was investigated. And in Hien and Trinh (2018), the problem of asynchronous observer-based control for 2-D Markovian jump systems was addressed.

In addition, it is well known that in many areas of engineering, physical systems of interest are generally represented

✉ Khalid Badie
Khalid.badie@usmba.ac.ma

¹ Engineering, Systems and Applications Laboratory, National School of Applied Sciences, Sidi Mohamed Ben Abdellah University, My Abdallah Avenue Km 5 Road Imouzzer, BP 72, Fez, Morocco

by high-order mathematical models. These high-order models lead to serious difficulties in analysis, synthesis and simulations of the systems concerned. Therefore, the H_∞ model reduction problem plays an important role in helping to improve our knowledge about the system concerned and to reduce the complexity of the control problems. Generally, it consists of designing a reduced-order model to minimize the H_∞ norm of the model error system under the assumption that the input noise signal is energy bounded. For 2-D systems, the problem of H_∞ model reduction has been investigated in some prior works; for example, the problem of H_∞ model reduction 2-D discrete time singular systems described by the Roesser model was studied in Xu et al. (2005). In Wu et al. (2006), the problem of H_∞ model reduction has been investigated for 2-D discrete state-delayed systems in the F–M second local state-space model. In Li et al. (2016), the problem of generalized H_∞ model reduction for 2-D systems described by the Roesser model and the F–M local state-space model, respectively, was studied. However, to the best of the authors' knowledge, there are not many works that address the H_∞ model reduction problem for 2-D Markovian jump systems in the literature. This motivates us for the present study.

In response to the discussion above, in this paper, we are interested in the problem of H_∞ model reduction for a class of 2-D discrete Markovian jump systems. Our attention is focused on the design of a 2-D reduced-order model, which approximates the original system well in a H_∞ norm sense. Based on a Lyapunov functional and some zero equations, the H_∞ analysis criterion is obtained. The analysis result is then used to the design of 2-D reduced-order model such that the resulting 2-D model error system is stochastically stable and has an H_∞ performance index. Two examples are presented to illustrate the effectiveness and merits of the proposed method.

Notation: The superscripts ' T ' and ' -1 ' stand for the transpose and the inverse of a matrix, respectively; \mathbb{R}^n denotes the n -dimensional Euclidean space; the null matrix and the identity matrix with appropriate dimensions are denoted by 0 and I , respectively. The notation $P > 0$ means that matrix P is real symmetric and positive definite; $\text{sym}(X)$ is the shorthand notation for $X + X^T$; $\text{diag} \dots$ denotes a block diagonal matrix; $\|\cdot\|$ denotes the Euclidean norm; the asterisk $*$ is used to denote term that is induced by symmetry. $\mathbb{E}\{x\}$ and $\mathbb{E}\{x|y\}$, respectively, mean the expectation of x and the expectation of x conditional on y .

2 Problem Formulation

The considered 2-D discrete systems in the F–M second model with Markov jump parameters can be described as follows:

$$\begin{aligned} x_{i+1,j+1} &= A_1(r_{i,j+1})x_{i,j+1} + A_2(r_{i+1,j})x_{i+1,j} \\ &\quad + B_1(r_{i,j+1})w_{i,j+1} + B_2(r_{i+1,j})w_{i+1,j}, \\ y_{i,j} &= C(r_{i,j})x_{i,j} + D(r_{i,j})w_{i,j}, \end{aligned} \quad (1)$$

where $x \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}^{n_y}$ is the measurement output vector, and $w \in \mathbb{R}^{n_w}$ is the disturbance input that belongs to $l_2\{[0, \infty), [0, \infty)\}$. The system matrices $A_1(r_{i,j+1})$, $A_2(r_{i+1,j})$, $B_1(r_{i,j+1})$, $B_2(r_{i+1,j})$, $C(r_{i,j})$ and $D(r_{i,j})$ are appropriately dimensioned. These matrices are functions of $r_{i,j}$, which takes values in a finite set $\mathcal{I} = \{1, \dots, N\}$, with transition probabilities:

$$\begin{aligned} \pi_{kl} &= \Pr\{r_{i+1,j+1} = l | r_{i,j+1} = k\} \\ &= \Pr\{r_{i+1,j+1} = l | r_{i+1,j} = k\}, \forall k, l \in \mathcal{I}, \end{aligned}$$

where $\pi_{kl} \geq 0$ and $\sum_{l=1}^N \pi_{kl} = 1$. To simplify the notation, when $r_{i,j} = k$, ($k \in \mathcal{I}$), the system matrices are denoted by: $A_1^{(k)}$, $A_2^{(k)}$, $B_1^{(k)}$, $B_2^{(k)}$, $C^{(k)}$ and $D^{(k)}$. Unless otherwise stated, the same simplification is also used to other matrices in the following.

The boundary condition of the state vector is supposed to be:

$$\lim_{s \rightarrow \infty} \mathbb{E} \left\{ \sum_{i=0}^s (\|x_{0,i}\|^2 + \|x_{i,0}\|^2) \right\} < \infty. \quad (2)$$

In this paper, we are concerned in approximating the original 2-D system (1) by a 2-D reduced-order model, and its desired structure is considered to be:

$$\begin{aligned} \tilde{x}_{i+1,j+1} &= \tilde{A}_1(r_{i,j+1})\tilde{x}_{i,j+1} + \tilde{A}_2(r_{i+1,j})\tilde{x}_{i+1,j} \\ &\quad + \tilde{B}_1(r_{i,j+1})w_{i,j+1} + \tilde{B}_2(r_{i+1,j})w_{i+1,j}, \\ \tilde{y}_{i,j} &= \tilde{C}(r_{i,j})\tilde{x}_{i,j} + \tilde{D}(r_{i,j})w_{i,j}, \end{aligned} \quad (3)$$

where $\tilde{x} \in \mathbb{R}^{\tilde{n}}$ is the state vector of the reduced-order model with $\tilde{n} < n$. $\tilde{A}_1(r_{i,j+1})$, $\tilde{A}_2(r_{i+1,j})$, $\tilde{B}_1(r_{i,j+1})$, $\tilde{B}_2(r_{i+1,j})$, $\tilde{C}(r_{i,j})$ and $\tilde{D}(r_{i,j})$ are appropriately dimensioned matrices to be determined. From (3), it is seen that the considered reduced-order model is assumed to jump synchronously with the modes in system (1).

Defining:

$$\bar{x}_{i,j} = \begin{bmatrix} x_{i,j} \\ \tilde{x}_{i,j} \end{bmatrix}, \quad e_{i,j} = y_{i,j} - \tilde{y}_{i,j},$$

and augmenting the 2-D system (1) to include the states of the reduced-order model (3), we can obtain the following model error system:

$$\begin{aligned} \bar{x}_{i+1,j+1} &= \bar{A}_1(r_{i,j+1})\bar{x}_{i,j+1} + \bar{A}_2(r_{i+1,j})\bar{x}_{i+1,j} \\ &\quad + \bar{B}_1(r_{i,j+1})w_{i,j+1} + \bar{B}_2(r_{i+1,j})w_{i+1,j}, \\ e_{i,j} &= \bar{C}(r_{i,j})\bar{x}_{i,j} + \bar{D}(r_{i,j})w_{i,j}, \end{aligned} \quad (4)$$

where

$$\begin{aligned} \bar{A}_1(r_{i,j+1}) &= \begin{bmatrix} A_1(r_{i,j+1}) & 0 \\ 0 & \tilde{A}_1(r_{i,j+1}) \end{bmatrix}, \\ \bar{A}_2(r_{i,j+1}) &= \begin{bmatrix} A_2(r_{i,j+1}) & 0 \\ 0 & \tilde{A}_2(r_{i,j+1}) \end{bmatrix}, \\ \bar{B}_1(r_{i,j+1}) &= \begin{bmatrix} B_1(r_{i,j+1}) \\ \tilde{B}_1(r_{i,j+1}) \end{bmatrix}, \quad \bar{B}_2(r_{i,j+1}) = \begin{bmatrix} B_2(r_{i,j+1}) \\ \tilde{B}_2(r_{i,j+1}) \end{bmatrix}, \\ \bar{C}(r_{i,j}) &= [C(r_{i,j}) \quad -\tilde{C}(r_{i,j})], \\ \bar{D}(r_{i,j}) &= D(r_{i,j}) - \tilde{D}(r_{i,j}). \end{aligned}$$

Definition 1 System (3) with $w_{i,j} = 0$ is said to be stochastically stable if for any initial conditions satisfying (2), the following condition holds

$$\mathbb{E} \left\{ \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (||\bar{x}_{i,j+1}||^2 + ||\bar{x}_{i+1,j}||^2) \right\} < \infty.$$

Definition 2 Given a scalar $\gamma > 0$, the model error system in (3) is said to be stochastically stable with an H_∞ disturbance attenuation level γ , if it is stochastically stable with $w_{i,j} = 0$, and satisfies:

$$||\hat{e}||_{\mathbb{E}2} < \gamma ||\hat{w}||_2, \quad (5)$$

for all nonzero $w_{i,j} \in l_2\{[0, \infty), [0, \infty)\}$ and under zero initial conditions, where

$$\begin{aligned} ||\hat{e}||_{\mathbb{E}2} &= \sqrt{\mathbb{E} \left\{ \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (||e_{i,j+1}||^2 + ||e_{i+1,j}||^2) \right\}}, \\ ||\hat{w}||_2 &= \sqrt{\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (||w_{i,j+1}||^2 + ||w_{i+1,j}||^2)}. \end{aligned}$$

Therefore, the objective of this paper is to design a 2-D reduced-order model in the form of (3), such that the model

error system in (4) is stochastically stable with an H_∞ disturbance attenuation level γ . Note that throughout this paper, we assume that the original 2-D system (1) is stochastically stable, as commonly used in the literature Zhang et al. (2009).

3 Main Results

3.1 H_∞ Performance Analysis

Theorem 1 The model error system in (4) is stochastically stable with an H_∞ disturbance attenuation level $\gamma > 0$ if there exist matrices $R_1^{(k)} > 0$, $R_2^{(k)} > 0$, $X^{(k)}$ and $Y^{(k)}$, $k \in \mathcal{I}$, such that the following LMIs hold for $k \in \mathcal{I}$:

$$\begin{bmatrix} \Xi^{(k)} & \sqrt{\pi_{k1}}(\Gamma^{(k,1)})^T & \sqrt{\pi_{k2}}(\Gamma^{(k,2)})^T & \dots & \sqrt{\pi_{kN}}(\Gamma^{(k,N)})^T & (\Upsilon^{(k)})^T \\ * & -R^{(1)} & 0 & 0 & 0 & 0 \\ * & * & -R^{(2)} & 0 & 0 & 0 \\ & & & \ddots & & \\ * & * & * & * & 0 & 0 \\ * & * & * & * & -R^{(N)} & 0 \\ * & * & * & * & * & -I_{2n_y} \end{bmatrix} < 0, \quad (6)$$

where

$$\begin{aligned} \Xi^{(k)} &= \begin{bmatrix} -\text{sym}(Z^{(k)}) & 0.5(Z^{(k)})^T + R^{(k)} & 0 \\ * & -2R^{(k)} & 0 \\ * & * & -\gamma^2 I_{2n_w} \end{bmatrix}, \\ \Gamma^{(k,l)} &= [Z^{(l)} \mathcal{A}^{(k)} \quad 0 \quad Z^{(l)} \mathcal{B}^{(k)}], \quad \Upsilon^{(k)} = [C^{(k)} \quad 0 \quad \mathcal{D}^{(k)}], \\ R^{(k)} &= \begin{bmatrix} R_1^{(k)} & 0 \\ 0 & R_2^{(k)} \end{bmatrix}, \quad Z^{(k)} = \begin{bmatrix} X^{(k)} & 0 \\ 0 & Y^{(k)} \end{bmatrix}, \\ \mathcal{A}^{(k)} &= \begin{bmatrix} \bar{A}_1^{(k)} & \bar{A}_2^{(k)} \\ \bar{A}_1^{(k)} & \bar{A}_2^{(k)} \end{bmatrix}, \\ \mathcal{B}^{(k)} &= \begin{bmatrix} \bar{B}_1^{(k)} & \bar{B}_2^{(k)} \\ \bar{B}_1^{(k)} & \bar{B}_2^{(k)} \end{bmatrix}, \quad C^{(k)} = \begin{bmatrix} \bar{C}^{(k)} & 0 \\ 0 & \bar{C}^{(k)} \end{bmatrix}, \\ \mathcal{D}^{(k)} &= \begin{bmatrix} \bar{D}^{(k)} & 0 \\ 0 & \bar{D}^{(k)} \end{bmatrix}. \end{aligned}$$

Proof Define the following Lyapunov functional for the model error system (4):

$$V(i, j) = V_1(i, j+1) + V_2(i+1, j), \quad (7)$$

with

$$\begin{aligned} V_1(i, j+1) &= \bar{x}_{i,j+1}^T (X(r_{i,j+1}))^T P_1(r_{i,j+1}) \\ &\quad (X(r_{i,j+1})) \bar{x}_{i,j+1}, \\ V_2(i+1, j) &= \bar{x}_{i+1,j}^T (Y(r_{i+1,j}))^T P_2(r_{i+1,j}) \\ &\quad (Y(r_{i+1,j})) \bar{x}_{i+1,j}. \end{aligned}$$

where $P_1(r_{i,j}) > 0$ and $P_2(r_{i,j}) > 0$ are Lyapunov matrices. $X(r_{i,j})$ and $Y(r_{i,j})$ are any appropriate matrices.

Using (8), (9) and adding the right-hand sides of (10a)–(10d) into \mathcal{J} yields:

$$\mathcal{J} = \xi_{i,j}^T \left\{ \Xi^{(k)} + (\Upsilon^{(k)})^T \Upsilon^{(k)} + \left(\sum_{l=1}^N \pi_{kl} (\Gamma^{(k)})^T (Z^{(l)})^T P^{(l)} Z^{(l)} \Gamma^{(k)} \right) \right\} \xi_{i,j},$$

Taking the difference of the Lyapunov functional along the trajectories of 2-D model error system (4) yields:

$$\Delta V(i, j) = \Delta V_1(i, j) + \Delta V_2(i, j),$$

where

$$\begin{aligned} \Delta V_1(i, j) &= \mathbb{E}\{V_1(i+1, j+1)|r_{i,j+1}=k\} - V_1(i, j+1) \\ &= \bar{x}_{i+1,j+1}^T \left(\sum_{l=1}^N \pi_{kl} (X^{(l)})^T P_1^{(l)} X^{(l)} \right) \bar{x}_{i+1,j+1} \\ &\quad - \bar{x}_{i,j+1}^T (X^{(k)})^T P_1^{(k)} X^{(k)} \bar{x}_{i,j+1}, \end{aligned} \quad (8)$$

and

$$\begin{aligned} \Delta V_2(i, j) &= \mathbb{E}\{V_2(i+1, j+1)|r_{i+1,j}=k\} - V_2(i+1, j) \\ &= \bar{x}_{i+1,j+1}^T \left(\sum_{l=1}^N \pi_{kl} (Y^{(l)})^T P_2^{(l)} Y^{(l)} \right) \bar{x}_{i+1,j+1} \\ &\quad - \bar{x}_{i+1,j}^T (Y^{(k)})^T P_2^{(k)} Y^{(k)} \bar{x}_{i+1,j}. \end{aligned} \quad (9)$$

Then, based on the Lyapunov functional (7), it is known that the 2-D model error system is stochastically stable with an H_∞ performance γ for all nonzero $w_{i,j} \in l_2\{[0, \infty), [0, \infty)\}$ and under zero initial conditions, if the following inequality holds,

$$\mathcal{J} = \Delta V(i, j) + \|\hat{e}\|_{\mathbb{E}_2}^2 - \gamma^2 \|\hat{w}\|_2^2 < 0,$$

where $\|\hat{e}\|_{\mathbb{E}_2}$ and $\|\hat{w}\|_2$ are defined in Definition 2.

According to the Lyapunov functional (7), the following zero equations hold:

$$0 = 2\bar{x}_{i,j+1}^T (X^{(k)})^T P_1^{(k)} \left\{ X^{(k)} \bar{x}_{i,j+1} - (P_1^{(k)})^{-1} P_1^{(k)} X^{(k)} \bar{x}_{i,j+1} \right\}, \quad (10a)$$

$$0 = 2\bar{x}_{i,j+1}^T (X^{(k)})^T \left\{ (P_1^{(k)})^{-1} (P_1^{(k)}) \bar{x}_{i,j+1} - \bar{x}_{i,j+1} \right\}, \quad (10b)$$

$$0 = 2\bar{x}_{i+1,j}^T (Y^{(k)})^T P_2^{(k)} \left\{ Y^{(k)} \bar{x}_{i+1,j} - (P_2^{(k)})^{-1} P_2^{(k)} Y^{(k)} \bar{x}_{i+1,j} \right\}, \quad (10c)$$

$$0 = 2\bar{x}_{i+1,j}^T (Y^{(k)})^T \left\{ (P_2^{(k)})^{-1} P_2^{(k)} \bar{x}_{i+1,j} - \bar{x}_{i+1,j} \right\}. \quad (10d)$$

where

$$\begin{aligned} \xi_{i,j} &= \begin{bmatrix} \bar{x}_{i,j+1}^T \\ \bar{x}_{i+1,j}^T \\ P_1(r_{i,j+1}) X(r_{i,j+1}) \bar{x}_{i,j+1} \\ P_2(r_{i+1,j}) Y(r_{i+1,j}) \bar{x}_{i+1,j} \end{bmatrix}, \\ P^{(k)} &= \begin{bmatrix} P_1^{(k)} & 0 \\ 0 & P_2^{(k)} \end{bmatrix}, \quad Z^{(k)} = \begin{bmatrix} X^{(k)} & 0 \\ 0 & Y^{(k)} \end{bmatrix}. \end{aligned}$$

In addition, consider the change of variables as: $R_1^{(k)} = (P_1^{(k)})^{-1}$, $R_2^{(k)} = (P_2^{(k)})^{-1}$ and define: $R^{(k)} = \text{diag}\{R_1^{(k)}, R_2^{(k)}\}$. Then, according to the Schur complement lemma Boyd et al. (1994), LMI (6) guarantees $\mathcal{J} < 0$, and this completes the proof. \square

Remark 1 From Theorem 1, it can be seen that the LMI condition (6) no longer contain product terms between the positive definite matrices $(R_1^{(k)}, R_2^{(k)})$ and model error system matrices. This is made possible by the use of a new structure of the Lyapunov functional (7) and the introduction of some zero Eqs. (10a)–(10d). This decoupling feature provides an additional degrees of freedom to the LMI condition and facilitate the design task. On the other hand, the introduction of the matrices $X^{(k)}$ and $Y^{(k)}$, ($k \in \mathcal{I}$), leads to increase the computational complexity. Therefore, it is the future topic to make the proper tradeoff between the conservatism of the results and computational complexity.

3.2 H_∞ Reduced-Order Model Design

Theorem 2 Consider the 2-D discrete system (1), and let $\gamma > 0$ be a given scalar, there exists a 2-D reduced-order model in the form of (3) such that the error system in (4) is stochastically stable with an H_∞ performance level γ if there exist matrices $\bar{R}_{11}^{(k)} > 0$, $\bar{R}_{13}^{(k)} > 0$, $\bar{R}_{21}^{(k)} > 0$, $\bar{R}_{23}^{(k)} > 0$, $\bar{X}_1^{(k)}$, $\bar{X}_2^{(k)}$, $\bar{Y}_1^{(k)}$, $\bar{Y}_2^{(k)}$, $W^{(k)}$, $\hat{A}_1^{(k)}$, $\hat{A}_2^{(k)}$, $\hat{B}_1^{(k)}$, $\hat{B}_2^{(k)}$, $\hat{C}^{(k)}$ and $\hat{D}^{(k)}$, $k \in \mathcal{I}$, such that the following LMIs hold for $k \in \mathcal{I}$:

$$\begin{bmatrix} \bar{\mathcal{E}}^{(k)} & \sqrt{\pi_{k1}}(\Theta^{(1)})^T & \sqrt{\pi_{k2}}(\Theta^{(2)})^T & \dots & \sqrt{\pi_N}(\Theta^N)^T & (\Lambda^{(k)})^T \\ * & -\bar{R}^{(1)} & 0 & 0 & 0 & 0 \\ * & * & -\bar{R}^{(2)} & 0 & 0 & 0 \\ * & * & * & \ddots & 0 & 0 \\ * & * & * & * & -\bar{R}^{(N)} & 0 \\ * & * & * & * & * & -I_{2n_y} \end{bmatrix} < 0, \quad (11)$$

$$\bar{R}_1^{(k)} = \begin{bmatrix} \bar{R}_{11}^{(k)} & \bar{R}_{12}^{(k)} \\ * & \bar{R}_{13}^{(k)} \end{bmatrix} > 0, \quad \bar{R}_2^{(k)} = \begin{bmatrix} \bar{R}_{21}^{(k)} & \bar{R}_{22}^{(k)} \\ * & \bar{R}_{23}^{(k)} \end{bmatrix} > 0, \quad (12)$$

where

$$\bar{\mathcal{E}}^{(k)} = \begin{bmatrix} -\text{sym}(\bar{X}^{(k)}) & 0 & 0.5(\bar{X}^{(k)})^T + \bar{R}_1^{(k)} & 0 & 0 \\ * & -\text{sym}(\bar{Y}^{(k)}) & 0 & 0.5(\bar{Y}^{(k)})^T + \bar{R}_2^{(k)} & 0 \\ * & * & -2\bar{R}_1^{(k)} & 0 & 0 \\ * & * & * & -2\bar{R}_2^{(k)} & 0 \\ * & * & * & * & -\gamma^2 I_{2n_w} \end{bmatrix},$$

and

$$\bar{R}^{(k)} = \begin{bmatrix} \bar{R}_1^{(k)} & 0 \\ 0 & \bar{R}_2^{(k)} \end{bmatrix}, \quad \bar{X}^{(k)} = \begin{bmatrix} \bar{X}_1^{(k)} & E W^{(k)} \\ \bar{X}_2^{(k)} & W^{(k)} \end{bmatrix},$$

$$\bar{Y}^{(k)} = \begin{bmatrix} \bar{Y}_1^{(k)} & E W^{(k)} \\ \bar{Y}_2^{(k)} & W^{(k)} \end{bmatrix},$$

$$\Theta^{(k)} = \begin{bmatrix} \Theta_1^{(k)} & \Theta_2^{(k)} & 0 & 0 & \Theta_3^{(k)} \\ \Theta_1^{(k)} & \Theta_2^{(k)} & 0 & 0 & \Theta_3^{(k)} \end{bmatrix},$$

$$\Theta_1^{(k)} = \begin{bmatrix} A_1^{(k)}(\bar{X}_1^{(k)})^T & A_1^{(k)}(\bar{X}_2^{(k)})^T \\ \hat{A}_1^{(k)} E^T & \hat{A}_1^{(k)} \end{bmatrix},$$

$$\Theta_2^{(k)} = \begin{bmatrix} A_2^{(k)}(\bar{Y}_1^{(k)})^T & A_2^{(k)}(\bar{Y}_2^{(k)})^T \\ \hat{A}_2^{(k)} E^T & \hat{A}_2^{(k)} \end{bmatrix}, \quad \Theta_3^{(k)} = \begin{bmatrix} B_1^{(k)} & B_2^{(k)} \\ \hat{B}_1^{(k)} & \hat{B}_2^{(k)} \end{bmatrix},$$

$$\Lambda^{(k)} = \begin{bmatrix} A_1^{(k)} & A_2^{(k)} & 0 & 0 & \Lambda_3^{(k)} \end{bmatrix},$$

$$\Lambda_1^{(k)} = \begin{bmatrix} C^{(k)}(\bar{X}_1^{(k)})^T - \hat{C}^{(k)} E^T & C^{(k)}(\bar{X}_2^{(k)})^T - \hat{C}^{(k)} \\ 0 & 0 \end{bmatrix},$$

$$\Lambda_2^{(k)} = \begin{bmatrix} 0 & 0 \\ C^{(k)}(\bar{Y}_1^{(k)})^T - \hat{C}^{(k)} E^T & C^{(k)}(\bar{Y}_2^{(k)})^T - \hat{C}^{(k)} \end{bmatrix},$$

$$\Lambda_3^{(k)} = \begin{bmatrix} D^{(k)} - \hat{D}^{(k)} & 0 \\ 0 & D^{(k)} - \hat{D}^{(k)} \end{bmatrix},$$

$$E = [I_{nr} \ 0_{n_r, n-n_r}]^T.$$

Moreover, the parameters of the desired 2-D reduced-order model (3) are given by:

$$\begin{bmatrix} \hat{A}_1^{(k)} & \hat{B}_1^{(k)} \\ \hat{A}_2^{(k)} & \hat{B}_2^{(k)} \\ \hat{C}^{(k)} & \hat{D}^{(k)} \end{bmatrix} = \begin{bmatrix} \hat{A}_1^{(k)} & \hat{B}_1^{(k)} \\ \hat{A}_2^{(k)} & \hat{B}_2^{(k)} \\ \hat{C}^{(k)} & \hat{D}^{(k)} \end{bmatrix} \begin{bmatrix} (W^{(k)})^{-T} & 0 \\ 0 & I \end{bmatrix}, \quad k \in \mathcal{I}. \quad (13)$$

Proof According to Theorem 1, the error system (4) is stochastically stable and satisfies the H_∞ performance specification (5), if there exist symmetric positive definite matrices $R_1^{(k)}$, $R_2^{(k)}$ and matrices $X^{(k)}$, $Y^{(k)}$, $k \in \mathcal{I}$, satisfying the condition (6).

Defining the following matrix:

$$\mathcal{T} = \text{diag}\{(Z^{(k)})^{-1}, (Z^{(k)})^{-1}, I_{2n_w}, (Z^{(1)})^{-1}, (Z^{(2)})^{-1}, \dots, (Z^{(N)})^{-1}, I_{2n_y}\},$$

by pre- and post-multiply in (6) by \mathcal{T} and \mathcal{T}^T , respectively, and considering the following change of variables:

$$\bar{R}_1^{(k)} = (X^{(k)})^{-1} R_1^{(k)} (X^{(k)})^{-T},$$

$$\bar{R}_2^{(k)} = (Y^{(k)})^{-1} R_2^{(k)} (Y^{(k)})^{-T},$$

$$\bar{X}^{(k)} = (X^{(k)})^{-1}, \quad \bar{Y}^{(k)} = (Y^{(k)})^{-1},$$

we obtain:

$$\begin{bmatrix} \bar{\mathcal{E}}^{(k)} & \sqrt{\pi_{k1}}(\bar{\Gamma}^{(k)})^T & \sqrt{\pi_{k2}}(\bar{\Gamma}^{(k)})^T & \dots & \sqrt{\pi_N}(\bar{\Gamma}^{(k)})^T & (\bar{\Upsilon}^{(k)})^T \\ * & -\bar{R}^{(1)} & 0 & 0 & 0 & 0 \\ * & * & -\bar{R}^{(2)} & 0 & 0 & 0 \\ * & * & * & \ddots & 0 & 0 \\ * & * & * & * & -\bar{R}^{(N)} & 0 \\ * & * & * & * & * & -I_{2n_y} \end{bmatrix} < 0, \quad (14)$$

Table 1 Comparison of minimum values of H_∞ performance γ_{\min} (Example 1)

\tilde{n}	γ_{\min}
3	0.5310
2	0.9544
1	0.9971

where

$$\begin{aligned}\bar{\mathcal{E}}^{(k)} &= \begin{bmatrix} -\text{sym}(\bar{Z}^{(k)}) & 0.5(\bar{Z}^{(k)})^T + \bar{R}^{(k)} & 0 \\ * & -2\bar{R}^{(k)} & 0 \\ * & * & -\gamma^2 I_{2n_w} \end{bmatrix}, \\ \bar{F}^{(k)} &= [\mathcal{A}^{(k)}(\bar{Z}^{(k)})^T \ 0 \ \mathcal{B}^{(k)}], \\ \mathcal{Y}^{(k,l)} &= [\mathcal{C}^{(k)}(\bar{Z}^{(k)})^T \ 0 \ \mathcal{D}^{(k)}], \\ \bar{R}^{(k)} &= \begin{bmatrix} \bar{R}_1^{(k)} & 0 \\ 0 & \bar{R}_2^{(k)} \end{bmatrix}, \quad Z^{(k)} = \begin{bmatrix} \bar{X}^{(k)} & 0 \\ 0 & \bar{Y}^{(k)} \end{bmatrix}, \\ \mathcal{A}^{(k)} &= \begin{bmatrix} \bar{A}_1^{(k)} & \bar{A}_2^{(k)} \\ \bar{A}_1^{(k)} & \bar{A}_2^{(k)} \end{bmatrix}, \\ \mathcal{B}^{(k)} &= \begin{bmatrix} \bar{B}_1^{(k)} & \bar{B}_2^{(k)} \\ \bar{B}_1^{(k)} & \bar{B}_2^{(k)} \end{bmatrix}, \quad \mathcal{C}^{(k)} = \begin{bmatrix} \bar{C}^{(k)} & 0 \\ 0 & \bar{C}^{(k)} \end{bmatrix}, \\ \mathcal{D}^{(k)} &= \begin{bmatrix} \bar{D}^{(k)} & 0 \\ 0 & \bar{D}^{(k)} \end{bmatrix}.\end{aligned}$$

Next, we partition the matrices $\bar{R}_1^{(k)}$, $\bar{R}_2^{(k)}$, $\bar{X}^{(k)}$ and $\bar{Y}^{(k)}$ as:

$$\begin{aligned}\bar{R}_1^{(k)} &= \begin{bmatrix} \bar{R}_{11}^{(k)} & \bar{R}_{12}^{(k)} \\ * & \bar{R}_{13}^{(k)} \end{bmatrix}, \quad \bar{R}_2^{(k)} = \begin{bmatrix} \bar{R}_{21}^{(k)} & \bar{R}_{22}^{(k)} \\ * & \bar{R}_{23}^{(k)} \end{bmatrix}, \\ \bar{X}^{(k)} &= \begin{bmatrix} \bar{X}_1^{(k)} & E_h W^{(k)} \\ \bar{X}_2^{(k)} & W^{(k)} \end{bmatrix}, \quad \bar{Y}^{(k)} = \begin{bmatrix} \bar{Y}_1^{(k)} & E_h W^{(k)} \\ \bar{Y}_2^{(k)} & W^{(k)} \end{bmatrix}.\end{aligned}$$

By substituting the partitioned matrices above into (14), and defining the following change of variables for $k \in \mathcal{I}$:

$$\begin{bmatrix} \hat{A}_1^{(k)} & \hat{B}_1^{(k)} \\ \hat{A}_2^{(k)} & \hat{B}_2^{(k)} \\ \hat{C}^{(k)} & \hat{D}^{(k)} \end{bmatrix} = \begin{bmatrix} \tilde{A}_1^{(k)} & \tilde{B}_1^{(k)} \\ \tilde{A}_2^{(k)} & \tilde{B}_2^{(k)} \\ \tilde{C}^{(k)} & \tilde{D}^{(k)} \end{bmatrix} \begin{bmatrix} (W^{(k)})^T & 0 \\ 0 & I \end{bmatrix}, \quad k \in \mathcal{I}.$$

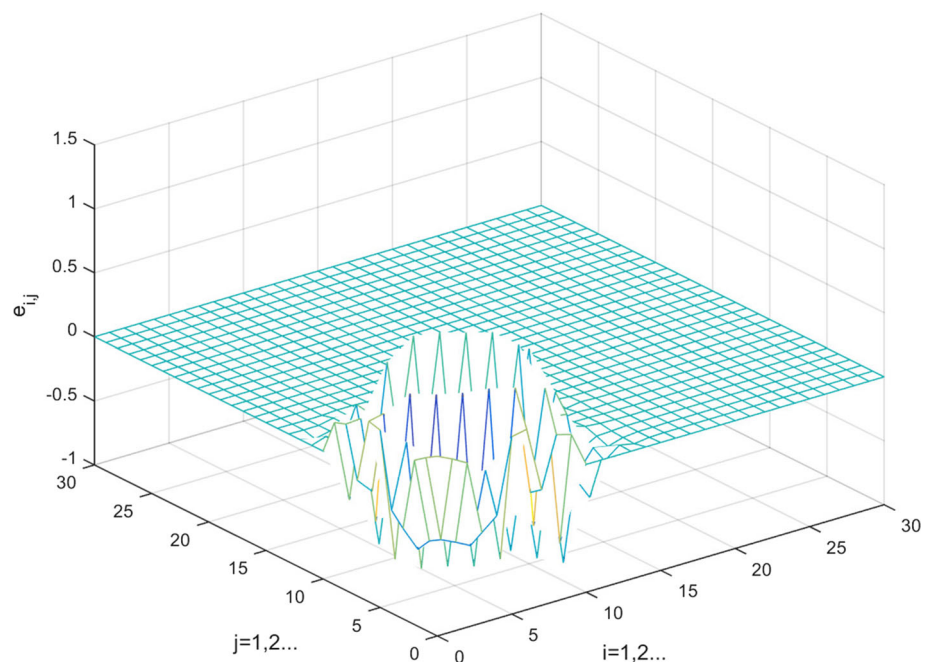
Thus, the LMI conditions (11), can be obtained readily. This completes the proof of Theorem 2. \square

Remark 2 For any solutions of the LMI condition (11), a corresponding 2-D reduced-order model of the form (3) can be obtained from the relation (13). In the other hand, by solving the following convex optimization problem:

$$\min \delta \text{ subject to (11) with } \delta \triangleq \gamma^2$$

we can obtain a 2-D reduced-order model such that the H_∞ performance index γ of the resulting error model system is minimized. In this case, those reduced-order model parameters (13) are said to be the optimal parameters of the 2-D reduced-order model (3).

Remark 3 The importance of the results presented in the present section relates to the following two aspects: (i) the

Fig. 1 Error signal $e_{i,j}$ 

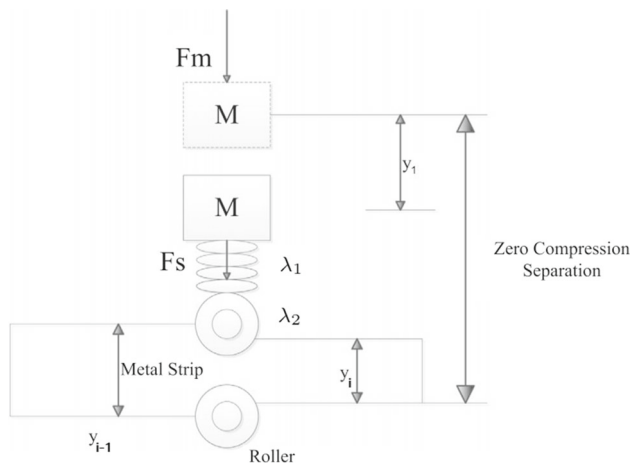


Fig. 2 Metal rolling process

study of the H_∞ model reduction problem, and as mentioned in the Introduction section, the H_∞ model reduction problem plays a key role in helping to improve our knowledge about the system concerned, and to reduce the complexity of the control problems. (ii) the study of the 2-D discrete Markovian jump systems, which have shown a powerful ability to represent plenty of real systems. In the next section, we will consider a numerical example and an industrial example to further clarify the importance of our results.

4 Illustrative Examples

Example 1 Consider a 2-D Markovian jump system in the form of (1), with two operation modes and $n = 4$. The parameters are given as follows:

The first mode:

$$\begin{bmatrix} A_1^{(1)} & B_1^{(1)} \\ A_2^{(1)} & B_2^{(1)} \\ C^{(1)} & D^{(1)} \end{bmatrix}$$

Table 2 Metal rolling process parameters

Parameters	Definition
p	The differentiation operator d/dt
$y_i(t)$	The i th actual roll-gap thickness
M	The force developed by the motor
λ_1	The stiffness of the adjusting mechanism spring
λ_2	The hardness of the metal strip
λ	The composite stiffness of the metal strip and the roll mechanism

Table 3 Comparison of minimum values of H_∞ performance γ_{\min} (Example 2)

\tilde{n}	γ_{\min}
4	0.0003
3	0.0131
2	2.2579
1	2.6605

$$= \begin{bmatrix} 0 & -0.45 & 0 & 0 & 0.4 \\ 0 & 0 & -0.55 & 0 & 0.8 \\ 0 & 0 & 0 & -0.3 & 0.1 \\ -0.2 & 0 & 0 & 0 & 0 \\ 0 & -0.6 & 0 & -0.4 & 0.2 \\ 0 & 0 & -0.5 & 0 & 0 \\ -0.1 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & -0.3 & -0.1 \\ 0.1 & 0.1 & 0.8 & -0.5 & 1 \end{bmatrix}.$$

The second model:

$$\begin{bmatrix} A_1^{(2)} & B_1^{(2)} \\ A_2^{(2)} & B_2^{(2)} \\ C^{(2)} & D^{(2)} \end{bmatrix} = \begin{bmatrix} 0 & -0.25 & 0 & 0 & 0.8 \\ 0 & 0 & -0.25 & 0 & -0.5 \\ 0 & 0 & 0 & -0.2 & 0.7 \\ -0.2 & 0 & 0 & 0.2 & 1 \\ 0 & -0.5 & 0 & -0.5 & 0.6 \\ 0 & -0.2 & 0 & -0.2 & 0 \\ 0.2 & 0 & 0 & 0.2 & 0.5 \\ 0 & 0 & 0 & 0.2 & -0.5 \\ 0.6 & 0 & -0.4 & 0.2 & 1 \end{bmatrix}.$$

The transition probability matrix is given by:

$$\pi \triangleq \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}.$$

The main purpose here is to find a 2-D reduced-order model (3) such that the resulting model error system is stochastically stable with an H_∞ disturbance attenuation level γ . By applying Theorem 2, the different minimum val-

ues of γ for different reduced-order \tilde{n} are calculated and listed in Table 1. In addition, the corresponding 2-D reduced-order models for $\tilde{n}=\{1,2,3\}$ are given as follows:

Third-order reduced model:

$$\begin{bmatrix} \tilde{A}_1^{(1)} & \tilde{B}_1^{(1)} \\ \tilde{A}_2^{(1)} & \tilde{B}_2^{(1)} \\ \tilde{C}^{(1)} & \tilde{D}^{(1)} \end{bmatrix} = \begin{bmatrix} 0.0328 & -0.4976 & -0.0216 & 0.4195 \\ -0.1304 & 0.0070 & -0.5160 & 0.8327 \\ 0.1442 & 0.0049 & 0.0305 & 0.1044 \\ 0.0007 & -0.6569 & 0.0223 & 0.2530 \\ 0.0005 & 0.0151 & -0.4113 & -0.0881 \\ -0.1114 & -0.0015 & -0.0368 & 0.5598 \\ 0.0853 & 0.0965 & 0.8162 & 1.0000 \end{bmatrix},$$

$$\begin{bmatrix} \tilde{A}_1^{(2)} & \tilde{B}_1^{(2)} \\ \tilde{A}_2^{(2)} & \tilde{B}_2^{(2)} \\ \tilde{C}^{(2)} & \tilde{D}^{(2)} \end{bmatrix} = \begin{bmatrix} 0.0354 & -0.2228 & -0.0131 & 0.4368 \\ 0.3021 & 0.1558 & -0.1842 & 0.1005 \\ 0.0546 & -0.0066 & -0.0142 & -0.0518 \\ 0.0259 & -0.2633 & 0.0169 & 0.8489 \\ -0.0123 & -0.0925 & 0.0057 & 0.0180 \\ 0.2544 & -0.0087 & 0.0029 & 0.8365 \\ -0.4839 & 0.7100 & -8.3753 & 1.0000 \end{bmatrix}.$$

Second-order reduced model:

$$\begin{bmatrix} \tilde{A}_1^{(1)} & \tilde{B}_1^{(1)} \\ \tilde{A}_2^{(1)} & \tilde{B}_2^{(1)} \\ \tilde{C}^{(1)} & \tilde{D}^{(1)} \end{bmatrix} = \begin{bmatrix} 0.2782 & -0.6988 & 0.4608 \\ 0.1865 & -0.1066 & 0.8494 \\ 0.1022 & -0.8988 & 0.2081 \\ 0.0825 & -0.1245 & -0.0011 \\ 0.2678 & 0.1535 & 1.0000 \end{bmatrix},$$

$$\begin{bmatrix} \tilde{A}_1^{(2)} & \tilde{B}_1^{(2)} \\ \tilde{A}_2^{(2)} & \tilde{B}_2^{(2)} \\ \tilde{C}^{(2)} & \tilde{D}^{(2)} \end{bmatrix} = \begin{bmatrix} 0.0515 & -0.0855 & 0.2538 \\ 0.1298 & -0.0043 & 0.2528 \\ 0.3057 & -0.6698 & 0.5445 \\ 0.1004 & -0.2233 & 0.2854 \\ 0.6190 & 0.6143 & 1.0000 \end{bmatrix}.$$

First-order reduced model:

$$\begin{bmatrix} \tilde{A}_1^{(1)} & \tilde{B}_1^{(1)} \\ \tilde{A}_2^{(1)} & \tilde{B}_2^{(1)} \\ \tilde{C}^{(1)} & \tilde{D}^{(1)} \end{bmatrix} = \begin{bmatrix} 0.0570 & 0.3265 \\ -0.0813 & 0.2338 \\ 0.3570 & 1.0000 \end{bmatrix},$$

$$\begin{bmatrix} \tilde{A}_1^{(2)} & \tilde{B}_1^{(2)} \\ \tilde{A}_2^{(2)} & \tilde{B}_2^{(2)} \\ \tilde{C}^{(2)} & \tilde{D}^{(2)} \end{bmatrix} = \begin{bmatrix} -0.0010 & 0.4513 \\ -0.3074 & 0.7324 \\ 1.1898 & 1.0000 \end{bmatrix}.$$

According to the results listed in Table 1, it can be seen clearly that the achieved minimum γ_{\min} is related to the reduced-order \tilde{n} , such that the best value of the H_{∞} performance is obtained when \tilde{n} is large. However, when \tilde{n} increases the complexity of the 2-D reduced-order model increases. Therefore, \tilde{n} should be chosen appropriately by considering a compromise between the conservatism of the results and the complexity of the 2-D reduced-order model.

In the following, we shall illustrate the effectiveness of the designed 2-D reduced-order model by using simulation

results. Our simulation is based on the obtained 2-D reduced-order model with $\tilde{n} = 1$. To this end, let the initial conditions be:

$$x_{i,0} = x_{0,i} = \begin{cases} [0.4 \ 0.15 \ -0.2 \ 1.2]^T & 0 \leq i < 10, \\ [0 \ 0 \ 0 \ 0]^T, & \text{otherwise.} \end{cases}$$

and disturbance input $w(i, j)$ as:

$$w_{i,j} = \begin{cases} \sin(i+j) & 0 \leq i < 10, \quad 0 \leq j < 10, \\ 0, & \text{otherwise.} \end{cases}$$

Figure 1 shows the signal $e_{i,j}$. It can be clearly observed from the simulation curve that the signal $e_{i,j}$ of the model error system converge to zero. In addition, under the zero initial conditions, we obtain: $\|\hat{e}_{i,j}\|_{\mathbb{E}2} = 7.1125$ and $\|\hat{w}_{i,j}\|_2 = 9.9767$, which yields $\frac{\|\hat{e}_{i,j}\|_{\mathbb{E}2}}{\|\hat{w}_{i,j}\|_2} = 0.7504$ (below the minimum H_{∞} performance $\gamma_{\min} = 0.9971$), which confirm that the designed 2-D reduced-order model is efficient.

Example 2 In this example, we apply the proposed model reduction method to a metal rolling process Foda and Agathoklis (1992), Yamada et al. (1999), Yang et al. (2020) depicted in Fig. 2. This process can be described by the following equation:

$$y_i(t) = \frac{\lambda}{\lambda + Mp^2} \left\{ (1 + \frac{Mp^2}{\lambda_1})y_{i-1}(t) - \frac{1}{\lambda_2}F_m \right\}. \quad (15)$$

The definition of parameters in the metal rolling process is given in Table 2.

As in Ren et al. (2017), by replacing differentiation with backward difference and choosing the sampling period T_1 , Eq. (15) can be expressed in the following form:

$$y_i(t+T_1) = a_1 y_i(t) + a_2 y_i(t-T_1) + a_3 y_{i-1}(t+T_1) + a_4 y_{i-1}(t) + a_5 y_{i-1}(t-T_1) + b F_m, \quad (16)$$

where

$$a_1 = \frac{2M}{\lambda T_1^2 + M}, \quad a_2 = \frac{-M}{\lambda T_1^2 + M},$$

$$a_3 = \frac{\lambda}{\lambda T_1^2 + M} (T_1^2 + \frac{M}{\lambda}), \quad a_4 = \frac{-2\lambda M}{\lambda_1(\lambda T_1^2 + M)},$$

$$a_5 = \frac{\lambda M}{\lambda_1(\lambda T_1^2 + M)},$$

$$b = \frac{-\lambda T_1^2}{\lambda_2(\lambda T_1^2 + M)}, \quad \lambda = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}.$$

Define

$$x_{i,j} \triangleq \begin{bmatrix} y_{i-1}((j+1)T_1) & y_{i-1}(jT_1) & y_i(jT_1) \\ y_i((j-1)T_1) & y_{i-1}((j-1)T_1) & \end{bmatrix}^T,$$

$$y_{i,j} \triangleq y_i(jT_1), \quad u_{i,j} \triangleq F_m.$$

Then, it is easy to verify that Eq. (16) can be converted into the following F–M second model:

$$x_{i+1,j+1} = A_{o1}x_{i,j+1} + A_{o2}x_{i+1,j} + B_{o1}u_{i,j+1} + B_{o2}u_{i+1,j},$$

$$y_{i,j} = C_o x_{i,j},$$

with

$$A_{o1} = \begin{bmatrix} a_3 & a_4 & a_1 & a_2 & a_5 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_{o2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ a_3 & a_4 & a_1 & a_2 & a_5 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix},$$

$$B_{o1} = \begin{bmatrix} b \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, B_{o2} = \begin{bmatrix} 0 \\ 0 \\ b \\ 0 \\ 0 \end{bmatrix}, C_o = [0 \ 0 \ 1 \ 0 \ 0].$$

Set $\lambda_1 = 1800$, $\lambda_2 = 1800$, $T_1 = 0.8$ and $M = 100$, and similar to Ren et al. (2017), a state-feedback controller will be designed to stabilize this system. But, in the present work, it is assumed that the controller parameter $u_{i,j} = K(r_{i,j})x_{i,j}$ has a Markov jumping feature, with $r_{i,j} \in \{1, 2\}$ and

$$K^{(1)} = [1300 \ 0 \ 0 \ 0 \ 0],$$

$$K^{(2)} = [1200 \ 0 \ 0 \ 0 \ 0],$$

In addition, consider there exists a disturbance $w(i, j)$. Then, the closed-loop system is modeled in the form (1) with two operation modes:

The first mode:

$$\left[\begin{array}{c|c} A_1^{(1)} & B_1^{(1)} \\ \hline A_2^{(1)} & B_2^{(1)} \\ \hline C^{(1)} & D^{(1)} \end{array} \right] = \left[\begin{array}{ccccc|c} 0.3846 & -0.1479 & 0.2959 & -0.1479 & 0.0740 & -0.9000 \\ 0 & 0 & 1.0000 & 0 & 0 & 0.2000 \\ 0 & 0 & 0 & 0 & 0 & 0.5000 \\ 0 & 0 & 0 & 0 & 0 & 0.1000 \\ 0 & 0 & 0 & 0 & 0 & -0.4500 \\ \hline 0 & 0 & 0 & 0 & 0 & -0.1200 \\ 0 & 0 & 0 & 0 & 0 & 0.1000 \\ 0.3846 & -0.1479 & 0.2959 & -0.1479 & 0.0740 & -0.8000 \\ 0 & 0 & 1.0000 & 0 & 0 & -0.1700 \\ 0 & 1.0000 & 0 & 0 & 0 & 0.3000 \\ \hline 0 & 0 & 1.0000 & 0 & 0 & 1.0000 \end{array} \right].$$

The second mode:

$$\left[\begin{array}{c|c} A_1^{(2)} & B_1^{(2)} \\ \hline A_2^{(2)} & B_2^{(2)} \\ \hline C^{(2)} & D^{(2)} \end{array} \right] = \left[\begin{array}{ccccc|c} 0.4320 & -0.1479 & 0.2959 & -0.1479 & 0.0740 & -0.7000 \\ 0 & 0 & 1.0000 & 0 & 0 & 0.3000 \\ 0 & 0 & 0 & 0 & 0 & 0.1000 \\ 0 & 0 & 0 & 0 & 0 & 0.2000 \\ 0 & 0 & 0 & 0 & 0 & -0.2000 \\ \hline 0 & 0 & 0 & 0 & 0 & -0.1000 \\ 0 & 0 & 0 & 0 & 0 & 0.2000 \\ 0.4320 & -0.1479 & 0.2959 & -0.1479 & 0.0740 & -0.9000 \\ 0 & 0 & 1.0000 & 0 & 0 & -0.7000 \\ 0 & 1.0000 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1.0000 & 0 & 0 & 1.0000 \end{array} \right].$$

Fig. 3 Mean value of signal error $e_{i,j}$ by using Monte Carlo simulation (100 runs)

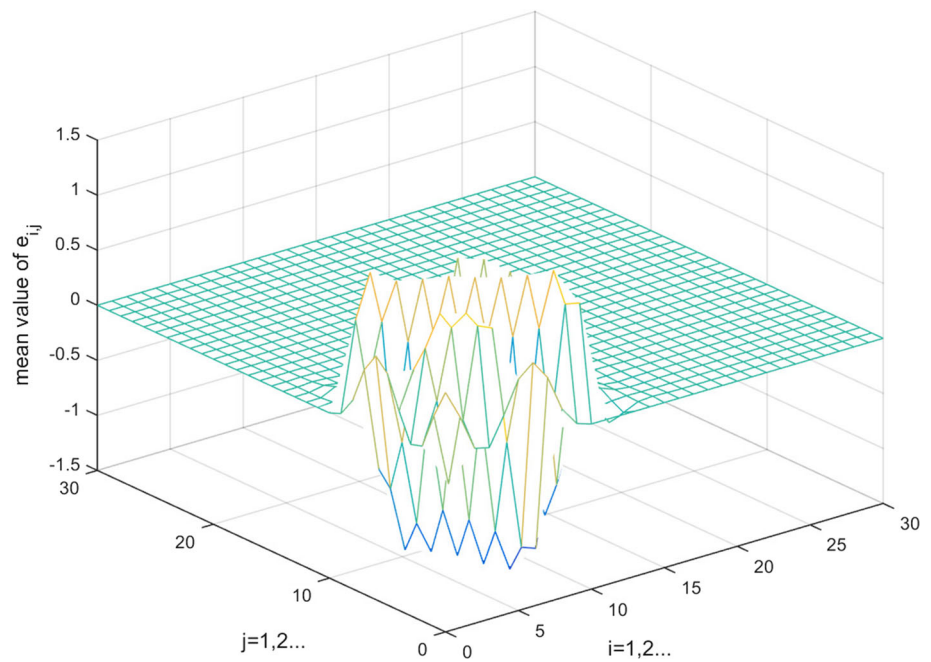
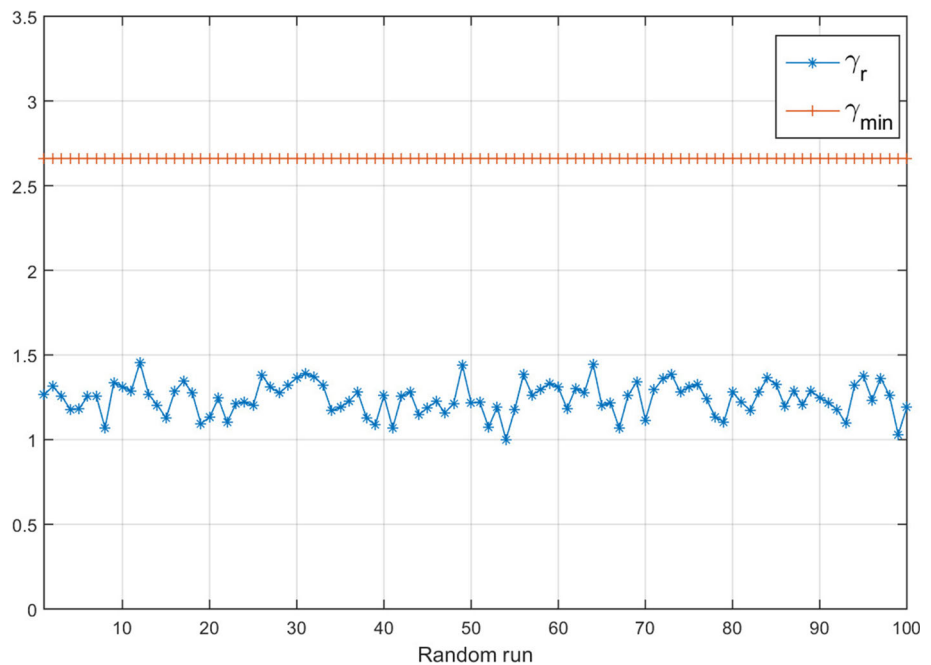


Fig. 4 Comparisons between γ_{\min} and the ratio γ_r



The transition probability matrix is assumed as:

$$\pi \triangleq \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}.$$

Next, we will apply the proposed method to design the reduced-order models. By applying Theorem 2 for $\tilde{n} = \{1, 2, 3, 4\}$, the different minimum values of γ are calculated and listed in Table 3. For $\tilde{n} = 1$, the corresponding 2-D reduced-order model is given as follows:

$$\begin{bmatrix} \tilde{A}_1^{(1)} & \tilde{B}_1^{(1)} \\ \tilde{A}_2^{(1)} & \tilde{B}_2^{(1)} \\ \tilde{C}^{(1)} & \tilde{D}^{(1)} \end{bmatrix} = \begin{bmatrix} 0.0582 & -0.6013 \\ -0.0057 & 0.0462 \\ -0.9747 & 0.5510 \end{bmatrix},$$

$$\begin{bmatrix} \tilde{A}_1^{(2)} & \tilde{B}_1^{(2)} \\ \tilde{A}_2^{(2)} & \tilde{B}_2^{(2)} \\ \tilde{C}^{(2)} & \tilde{D}^{(2)} \end{bmatrix} = \begin{bmatrix} 0.0667 & -0.8862 \\ -0.0019 & 0.2574 \\ -0.3960 & 0.8407 \end{bmatrix}.$$

In order to show the effectiveness of the designed 2-D reduced-order model with $\tilde{n} = 1$, we perform a Monte Carlo simulation (with 100 random runs) in MATLAB, for the

model error system, where the initial conditions are chosen as:

$$x_{i,0} = x_{0,i} = \begin{cases} [0.3 \ 0.5 \ 0.24 \ 0.7 \ 0.9]^T & 0 \leq i < 10, \\ [0 \ 0 \ 0 \ 0 \ 0]^T, & \text{otherwise.} \end{cases}$$

and disturbance input $w(i, j)$ as:

$$w_{i,j} = \begin{cases} \sin(i + j) & 0 \leq i < 10, \ 0 \leq j < 10, \\ 0, & \text{otherwise.} \end{cases}$$

Figure 3 shows the mean value of signal error $e_{i,j}$ for the Monte Carlo simulation. In addition, under the zero initial conditions, we calculate the ratio $\gamma_r = \frac{\|\hat{e}_{i,j}\|_{\mathbb{R}^2}}{\|\hat{w}_{i,j}\|_2}$ in 100 random runs. As shown in Fig. 4, all ratio values are smaller than the minimum value $\gamma_{\min} = 2.6605$. The average ratio is 1.2438, the minimal one is 1.0009, and the maximal one is 1.4561.

5 Conclusions

In this paper, the problem of H_∞ model reduction for a class of 2-D discrete Markovian jump systems has been investigated. Based on an appropriate Lyapunov functional and by introducing some zero equations, a new sufficient condition has been developed to ensure the stochastic stability of the model error system as well as a prescribed H_∞ performance requirement. Then, a reduced-order design methodology has been presented and formulated in terms of LMIs which can be solved by LMI toolbox. The effectiveness of the proposed method has been illustrated by two examples.

Acknowledgements The authors would like to thank the editor and anonymous reviewers for their many helpful comments and suggestions to improve the quality of this paper.

References

- Badie, K., Alfidi, M., & Chalh, Z. (2018). New relaxed stability conditions for uncertain two-dimensional discrete systems. *Journal of Control, Automation and Electrical Systems*, 29(6), 661–669.
- Badie, K., Alfidi, M., & Chalh, Z. (2019). Exponential stability analysis for 2D discrete switched systems with state delays. *Optimal Control Applications and Methods*, 40(6), 1088–1103.
- Badie, K., Alfidi, M., & Chalh, Z. (2020). Further results on H_∞ filtering for uncertain 2-D discrete systems. *Multidim Systems and Signal Process.*, <https://doi.org/10.1007/s11045-020-00715-2>.
- Badie, K., Alfidi, M., Tadeo, F., & Chalh, Z. (2020a). Robust H_∞ controller design for uncertain 2D continuous systems with interval time-varying delays. *International Journal of Systems Science*, 51(3), 440–460.
- Boyd, S., El Ghaoui, L., Feron, E., & Balakrishnan, V. (1994). *Linear matrix inequalities in system and control theory*. Philadelphia: SIAM.
- Chen, M., & Ma, Y. (2017). Non-fragile fuzzy dissipative control for singular markovian jump systems with time-varying delay. *Journal of Control, Automation and Electrical Systems*, 28(6), 715–726.
- Duan, Z., & Xiang, Z. (2013). State feedback H_∞ control for discrete 2D switched systems. *Journal of the Franklin Institute*, 350(6), 1513–1530.
- Du, C., & Xie, L. (2002). *H_∞ control and filtering of two-dimensional systems*. Berlin: Springer.
- Fei, Z., Shi, S., Zhao, C., & Wu, L. (2017). Asynchronous control for 2-D switched systems with mode-dependent average dwell time. *Automatica*, 79, 198–206.
- Foda, S., & Agathoklis, P. (1992). Control of the metal rolling process: A multidimensional system approach. *Journal of the Franklin Institute*, 329(2), 317–332.
- Hien, L. V., & Trinh, H. (2016). Stability analysis of two-dimensional Markovian jump state-delayed systems in the Roesser model with uncertain transition probabilities. *Information Sciences*, 367, 403–417.
- Hien, L. V., & Trinh, H. (2018). Observer-based control of 2-D Markov jump systems. *IEEE Transactions on Circuits and Systems-II: Express Briefs*, 64(11), 1322–1326.
- Huang, H., Feng, G., & Chen, X. (2012). Stability and stabilization of Markovian jump systems with time delay via new Lyapunov functionals. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 59(10), 2413–2421.
- Kaczorek, T. (1985). *Two-dimensional linear systems*. Berlin: Springer.
- Liang, J., Huang, T., Hayat, T., & Alsaadi, F. (2015). H_∞ filtering for two-dimensional systems with mixed time delays, randomly occurring saturations and nonlinearities. *International Journal of General Systems*, 44(2), 226–239.
- Li, X., & Gao, H. (2013). Robust finite frequency H_∞ filtering for uncertain 2-D systems: The FM model case. *Automatica*, 49(8), 2446–2452.
- Li, X., Lam, J., & Cheung, K. C. (2016). Generalized H_∞ model reduction for stable two-dimensional discrete systems. *Multidimensional Systems and Signal Processing*, 27(2), 359–382.
- Li, X., Lam, J., Gao, H., & Li, P. (2014). Improved results on H_∞ model reduction for Markovian jump systems with partly known transition probabilities. *Systems & Control Letters*, 70, 109–117.
- Liu, X., Zhai, D., He, D. K., & Chang, X. H. (2018). Simultaneous fault detection and control for continuous-time Markovian jump systems with partially unknown transition probabilities. *Applied Mathematics and Computation*, 337, 469–486.
- Lu, W. S. (1992). *Two-dimensional digital filters*. New York: CRC Press.
- Luo, Y., Wang, Z., Liang, J., Wei, G., & Alsaadi, F. E. (2016). H_∞ control for 2-D fuzzy systems with interval time-varying delays and missing measurements. *IEEE Transactions on Cybernetics*, 47(2), 365–377.
- Ren, Y., Ding, D. W., & Li, Q. (2017). Finite-frequency fault detection for two-dimensional Fornasini–Marchesini dynamical systems. *International Journal of Systems Science*, 48(12), 2610–2621.
- Saravanakumar, R., Ali, M. S., Ahn, C. K., Karimi, H. R., & Shi, P. (2016). Stability of Markovian jump generalized neural networks with interval time-varying delays. *IEEE Transactions on Neural Networks and Learning Systems*, 28(8), 1840–1850.
- Tadepalli, S. K., & Leite, V. J. (2018). Robust stabilization of uncertain 2-D discrete delayed systems. *Journal of Control, Automation and Electrical Systems*, 29(3), 280–291.
- Wu, L., Shi, P., Gao, H., & Wang, C. (2006). H_∞ mode reduction for two-dimensional discrete state-delayed systems. *IEE Proceedings-Vision, Image and Signal Processing*, 153(6), 769–784.
- Xu, Z., Su, H., Lu, R., & Wu, Z. G. (2017). Non-fragile dissipative filtering for 2-D switched systems. *Journal of the Franklin Institute*, 354(14), 6234–6246.

- Xu, H., Zou, Y., Xu, S., Lam, J., & Wang, Q. (2005). H_∞ model reduction of 2-D singular Roesser models. *Multidimensional Systems and Signal Processing*, 16(3), 285–304.
- Yamada, M., Xu, L., & Saito, O. (1999). 2D model-following servo system. *Multidimensional Systems and Signal Processing*, 10(1), 71–91.
- Yang, R., Li, L., & Su, X. (2020). Finite-region dissipative dynamic output feedback control for 2-D FM systems with missing measurements. *Information Sciences*, 514, 1–14.
- Yao, J., Wang, W., & Zou, Y. (2013). The delay-range-dependent robust stability analysis for 2-D state-delayed systems with uncertainty. *Multidimensional Systems and Signal Processing*, 24(1), 87–103.
- Zhai, D., Lu, A. Y., Dong, J., & Zhang, Q. L. (2016). Asynchronous H_∞ filtering for 2D discrete Markovian jump systems with sensor failure. *Applied Mathematics and Computation*, 289, 60–79.
- Zhang, L., Boukas, E. K., & Shi, P. (2009). H_∞ model reduction for discrete-time Markov jump linear systems with partially known transition probabilities. *International Journal of Control*, 82(2), 343–351.
- Zhang, Q., Li, J., & Song, Z. (2018). Sliding mode control for discrete-time descriptor Markovian jump systems with two Markov chains. *Optimization Letters*, 12(6), 1199–1213.
- Zhu, Y., Song, X., Wang, M., & Lu, J. (2020). Finite-time asynchronous H_∞ filtering design of Markovian jump systems with randomly occurred Quantization. *International Journal of Control, Automation and Systems*, 18(2), 450–461.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.