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Adaptive Observer-Based Decentralized Sliding Mode Control for Connected Systems in the Presence of Unknown Interconnection and Bounded Disturbance

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Abstract
In this manuscript, the control problem for a class of connected systems with unknown interconnections in the presence of distributed disturbance is studied. For this purpose, an adaptive observer-based decentralized sliding mode controller is proposed to provide the stability of the closed-loop system. In this approach, the connection matrix between subsystems is also assumed unknown, and therefore the dependency to the knowledge of connections is reduced. Due to the lack of availability of most states, a combination of the Luenberger observer and the sliding mode controller is proposed. The observer is to estimate the unmeasurable states, observing the disturbance effects, and thereby facilitating determination of the control law in an adaptive scheme. Furthermore, the effects of unknown interconnections between subsystems are estimated using adaptive laws, to find equivalent of interconnections among subsystems. To control and stabilize the overall system, a decentralized sliding mode controller is designed to stabilize the original system using the estimated disturbance and states. To analyze the stability, based on the proposed observer, direct method of Lyapunov is proposed together with the LMI technique. The stability of system and convergence of the estimation error of each subsystem to zero is guaranteed. The observer-based method is finally applied into two numerical examples to verify the significance of the proposed approach.

Keywords Connected systems · Decentralized observer · Adaptive sliding mode · Bounded disturbance · Unknown interconnection · Disturbance observer

1 Introduction

In recent years, complex systems, including large scale, multi-agent, and interconnected systems, have attracted the interests of many kinds of research. Connected systems are applicable in many practical systems such as mechanical systems (Zhang et al. 2018a, b, c), aerospace systems, wind turbines (Ke et al. 2019), urban traffic, computer networks, and economic systems. Lack of sufficient information about the connections between subsystems has made it difficult to design controller and observer for such connected systems (Zhang et al. 2018a, b; Xu et al. 2018; Bento et al. 2018; Babu et al. 2019).

In the past few decades, control of connected systems through decentralized controller has investigated by several scholars (Phat et al. 2015, 2016). Main reason to use decentralized controller in the connected systems is its independence from the information exchange between subsystems. This leads to simpler design of the controller to use only local information. The solution for addressing the problem of information of the connection between subsystems is one of the most important challenges in the decentralized design. The information drawback may sometimes cause instability of the overall system. To solve this problem, some cases have been considered in the design of decentralized controllers for connected systems. Yi and Zheng (2016) state that a decentralized proportional-integral control scheme has been suggested to control large scale systems.

In most practical problems, due to technical and/or economic limitations, subsystem state variables are not fully available. One solution to this problem is to use the reconstruction of the inaccessible state variables by an observer (Yang and Yin 2018, 2019; Yang et al. 2018). Since the pioneering work of Luenberger in which a primary observer design to estimate unavailable states was proposed, the observer design has been considered in many systems. There

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is usually a mismatch between the actual model and that of the estimated. This can be treated as an unknown input and may degrade the performance of the observer. The sliding mode observer can be a possible choice to estimate the states of the system in the presence of such uncertainties.

Due to complexity of interconnections, the dimensions of system, and the economic costs of the state estimation, most states of system are not available and it is necessary to be estimated by an observer. Sliding mode observer is used to estimate the state of the connected system because of its robustness in dealing with uncertainties together with gaining a routine implementation. Liu (2016) states that a sliding mode observer has been designed for a linear large scale system in the presence of disturbance, to estimate states in such a way that the convergence estimation error is minimized. In Mohamed et al. (2018), the states of a class of interconnected systems have been estimated in the presence of uncertainty through a variable structure observer. Mohamed et al. (2019) state that an observer has been designed with combination of sliding mode and adaptive technique to estimate state variables in the presence of unknown time-varying parameters. One of the most important applications of the observer is the use of the feedback control in sensorless structure. An observer-based control procedure consists of two basic structures:

1. Fully state feedback based on availability of all states.
2. A reduced-order observer to reconstruct most required states.

The concept of separating the control scheme in the latter is usually interpreted as the separation principle, which is often a suitable solution to solve linear control system problems For example, in three-phase power converters (Liu et al. 2016), a second-order sliding mode controller based on the extended state observer has been used. A controller, based on third-order sliding mode control, has been proposed in Rinaldi et al. (2017) for optimal load frequency control in power grid. Most of the researches use the powerful optimization tools of the linear matrix (LMI) to ensure and guarantee appropriate conditions for the closed-loop system stability (Zhang and Yang 2018; Wang et al. 2019; Zhang et al. 2019).

Extensive researches have been developed for observer-based decentralized control on the connected systems (Goel and Swarup 2017; Fateh and Fateh 2020; Ranjbar et al. 2020; and Yu 2020). Goel and Swarup (2017) state that a robust super-twisting sliding mode controller for a multi-input–multi-output system has been provided. The proposed controller gains are selected, using an adaptive estimation mechanism to tackle the gain overestimation problem. Fateh and Fateh (2020) state that a decentralized adaptive fuzzy control design (AFC) based on voltage control strategy is presented with an approximate fuzzy error compensator. Asymptotic tracking performance is adjusted by compensating for the approximate fuzzy errors adaptively. Yu (2020) states that an observer-based decentralized output tracking control problem has been presented for a class of interconnected systems. The stability is guaranteed using the linear matrix inequalities (LMI) approach. (Tong et al. 2015) presented a decentralized fuzzy-adaptive output feedback scheme based on the observer for unknown switching systems together with the dead zone. By designing a reduced-order observer to estimate unmeasurable stats, in Hua et al. (2015) a decentralized output feedback adaptive neural network tracking control has been proposed for stochastic connected systems. Koo et al. (2015) state that a sampled data observer has been designed such that each observer is solved as an optimization problem by the LMI approach. However, there is a lack of connection information. Furthermore, the unknown distributed disturbance still degrades the performance of control.

The main objective of this manuscript is to provide closed-loop stability of a class of connected systems with unknown interconnections in the presence of bounded disturbance. To achieve this goal, a decentralized sliding mode controller, based on adaptive sliding mode observer, is proposed. In several researches, interconnections between subsystems are assumed available. However, in this manuscript, the interconnection matrix between the subsystems is assumed unknown and only a few information is available. Since sliding mode technique is robust against uncertainties, this technique is designed to estimate states of subsystems. The sliding term is used to reduce effect of the disturbance during the state estimation. Furthermore, adaptive terms are brought to assess an interconnection equivalent. Finally, a sliding surface is designed to control the connected system through estimated states. Simulations results are given to assess quality of the proposed decentralized sensorless control, in the presence of unknown interconnection, together with bounded unknown disturbances.

The rest of this paper is organized as follows:

Section 2 deals with the description of connected system and governing dynamics. These systems are exposed at bounded unknown disturbance, in addition to the lack of information about the interconnection between subsystems. Section 3 is devoted to design and implement a decentralized observer. In Sect. 4, decentralized sliding mode controller is designed based on an adaptive sliding mode observer which is designed by selecting an appropriate sliding term. Thereafter, the stability of the system is maintained, while the disturbance effects are effectively reduced. In Sect. 5, by considering appropriate Lyapunov function together with the LMI theorem, the controller and sliding term gains are chosen such that the stability of the closed-loop system is guaranteed. In Sect. 6, simulations are presented by MATLAB™ on two numerical examples of connected systems with sta-

ble/unstable subsystems. Finally, Sect. 7 closes the work by a conclusion.

It should be noted that in the following, observability and controllability of system are defined through pairs of $(A, C)$and (A, B) matrices. The operator $\|\cdot\|$ indicates the matrix or vector two norm, $(\cdot)^{\mathrm{T}}$ indicates the transpose operator. Meanwhile A^{+} is the right pseudoinverse of none-square matrix A.

2 Connected System Equation

A connected system with N subsystems together with unknown interconnections in the presence of bounded disturbance is considered as follows:

$$\begin{aligned}\dot{x}_i(t) &= A_i\, x_i(t) + B_i\, u_i(t) \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\, x_j(t) + B_i\, f_i(x_i(t), t), \\ y_i(t) &= C_i\, x_i(t), \quad i = 1, 2, \ldots, N\end{aligned} \tag{1}$$

where $x_i(t) \in \Re^{n_i}$ is the state variable $u_i(t) \in \Re^{m_i}$ is the control input and $y_i(t) \in \Re^{r_i}$ is the output of the ith subsystem and N indicates the number of connected subsystems. A_i, B_i, and C_i are the state $(n_i \times n_i)$, the input $(n_i \times m_i)$, and the output $(r_i \times n_i)$ matrices, respectively. Furthermore, A_{ij} is unknown interconnection matrix of $(n_i \times n_i)$ and f_i $(x_i(t), t) \in \Re^{n_i}$ is distributed disturbance, applied into the ith subsystem. It is also assumed that each subsystem is completely observable (Liu 2016) and controllable (Liu 2018). To estimate and control the closed-loop system, dependency of unknown interconnections in the output (available states) and control input of each subsystem (assuming 1) must be available.

Assumption 1 Due to observability and controllability of each subsystem, A_{ij} matrix can be rewritten as (Liu 2016):

$$A_{ij} = B_i\, H_{ij}\, C_j, \quad i = 1, 2, \ldots, N. \tag{2}$$

where the H_{ij} matrix for each subsystem is bounded in such a way that $\left\| H_{ij} \right\| < \beta_{ij}$ and β_{ij} is a known finite positive constant.

Assumption 2 Again and due to observability and controllability of each subsystem:

$$C_i\, B_i \neq 0, \quad i = 1, 2, \ldots, N. \tag{3}$$

B_i and C_i, are the input and the output matrices coefficients of the ith subsystem, respectively (Yan et al. 2017).

Assumption 3 The disturbance $f_i(x_i(t), t)$ in (1) is bounded by a certain amount of ρ_i as:

$$\| f_i(x_i(t), t) \| < \rho_i \tag{4}$$

Lemma 1 *The right pseudoinverse matrix of A, i.e.,* (A^{+}) *is defined according to* (5) (Liu 2016; Liu and Han 2016):

$$A^{+} = A^{\mathrm{T}}(A\, A^{\mathrm{T}})^{-1} \tag{5}$$

The operation A^{+} *is needed when none-square matrix A is of a full row rank* (Liu 2016).

3 Decentralized Disturbance Observer Design

A decentralized disturbance observer is proposed to estimate states of the connected system with unknown interconnections, in the presence of disturbance $f_i(x_i(t), t) \in \Re^{n_i}$ in (1) by assuming observability of pair (A_i, C_i). A combination of the Luenberger and sliding mode observer is proposed to estimate the disturbance and also effects of unknown interconnections via adaptive laws are estimated.

Let us define $e_i(t)$ as the estimation error between the states of system $(x_i(t))$ and that of the estimated by the observer $(\hat{x}_i(t))$ for each subsystem, *i.e.:*

$$e_i(t) \triangleq x_i(t) - \hat{x}_i(t) \tag{6}$$

A sliding surface $s_i(t)$ for each subsystem is selected to design the sliding mode observer, which is as follows:

$$\begin{aligned}s_i(t) &\triangleq y_i(t) - \hat{y}_i(t) = C_i\big(x_i(t) - \hat{x}_i(t)\big) \\ &= C_i\, e_i(t)\end{aligned} \tag{7}$$

Accordingly, the overall sliding surface vector is configured as follows:

$$S(t) \triangleq [s_1(t), s_2(t), \ldots, s_N(t)]^{\mathrm{T}} \tag{8}$$

As a result, to estimate states of the connected system with unknown interconnections (1) in the presence of disturbance, the following decentralized closed-loop disturbance observer (9) is proposed:

$$\begin{aligned}\dot{\hat{x}}_i(t) &= A_i\, \hat{x}_i(t) + B_i\, u_i(t) \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{N} \hat{A}_{ij}\, \hat{x}_j(t) + L_i\big(y_i(t) - \hat{y}_i(t)\big) \\ &\quad + B_i\, R_i(t) \\ \hat{y}_i(t) &= C_i \hat{x}_i(t)\end{aligned} \tag{9}$$

where $L_i \in \Re^{n_i \times r_i}$ is the observer gain, $\hat{A}_{ij}$ is an equivalent of the interconnection matrix and $R_i(t)$ is the sliding term to reduce the convergence error between the actual state and that of the estimated for each subsystem. Furthermore, effect of the disturbances during the estimation of states is considered in the sliding terms.

4 The Decentralized Observer-Based Control Design

To stabilize the closed-loop connected system (1), assuming the controllability of the pair (A_i, B_i) for ith subsystem, with $(i = 1, 2, \ldots, N)$, a sliding mode controller is proposed, based on the disturbance observer in (9). To design the controller, the sliding surface $\delta_i(t)$ is considered as follows:

$$\delta_i(t) \triangleq y_i(t) - y_{di}(t) \tag{10}$$

which $\delta_i(t) \in \Re^{m_i}$ is the sliding surface, $y_i(t)$ and $y_{di}(t)$ are the actual and desired output (reference input) of the ith subsystem, respectively. The overall sliding surface $\delta(t)$ for the connected system (1) is then defined as:

$$\delta(t) \triangleq [\delta_1(t), \delta_2(t), \ldots, \delta_N(t)]^{\mathrm{T}} \tag{11}$$

According to the output equation in (1), i.e.,$y_i(t) = C_i\, x_i(t)$, where the connected system stabilization goal is $y_{di}(t) = 0$, then the sliding surface (10) can be rewritten as:

$$\delta_i(t) = y_i(t) = C_i\, x_i(t) \tag{12}$$

on which the derivative of the sliding surface $\delta_i(t)$ is yielded as:

$$\dot{\delta}_i(t) = \dot{y}_i(t) = C_i\, \dot{x}_i(t) \tag{13}$$

Finally, the control law to stabilize the system (1) is proposed as follows:

$$u_i(t) = -\begin{bmatrix} k_i\, \hat{x}_i(t) + \sum\limits_{\substack{j=1 \\ j\neq i}}^{N} k_{ij}\, \hat{y}_j(t) \\ + \sum\limits_{\substack{j=1 \\ j\neq i}}^{N} k_j \left(y_j(t) - \hat{y}_j(t)\right) + G_i(t) \end{bmatrix} \tag{14}$$

Where $k_i \in \Re^{m_i \times n_i}$, $k_{ij} \in \Re^{m_i \times r_i}$ and $k_j \in \Re^{m_i \times r_i}$ are controller gains and $G_i(t)$ is the sliding term of the sliding mode controller, to reduce or even eliminate the effects of the disturbances. Now, we have:

$$\begin{aligned} y_j(t) - \hat{y}_j(t) &= C_j(x_j(t) - \hat{x}_j(t)) \\ &= C_j\, e_j(t) \end{aligned} \tag{15}$$

From (15) and replacing the control law in (14) into the connected system Eq. (1), we have:

$$\begin{aligned} \dot{x}_i(t) &= A_i\, x_i(t) \\ &\quad - B_i \begin{bmatrix} k_i\, \hat{x}_i(t) + \sum\limits_{\substack{j=1 \\ j\neq i}}^{N} k_{ij}\, \hat{y}_j(t) \\ + \sum\limits_{\substack{j=1 \\ j\neq i}}^{N} k_j\, C_j\, e_j(t) + G_i(t) \end{bmatrix} \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\, x_j(t) + B_i\, f_i(x_i(t), t) \end{aligned} \tag{16}$$

By adding and subtracting the term $\left(\sum\limits_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\, \hat{x}_j(t)\right)$ from the term $\sum\limits_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\, x_j(t)$, we have:

$$\begin{aligned} &\sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\, x_j(t) \\ &= \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\, x_j(t) - \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\, \hat{x}_j(t) + \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\, \hat{x}_j(t) \\ &= \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\, e_j(t) + \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\, \hat{x}_j(t) \end{aligned} \tag{17}$$

The same action in (17) for the term $B_i\ k_i\, \hat{x}_i(t)$ also yields:

$$\begin{aligned} B_i\ k_i\, \hat{x}_i(t) &= B_i\ k_i\, \hat{x}_i(t) - B_i\ k_i\, x_i(t) \\ &\quad + B_i\ k_i\, x_i(t) = -B_i\ k_i\, e_i(t) + B_i\ k_i\, x_i(t) \end{aligned} \tag{18}$$

Replacing these terms into (17) and also into their main equations, i.e., (16) achieves:

$$\begin{aligned} \dot{x}_i(t) &= (A_i - B_i\ k_i\ )\, x_i(t) + B_i\ k_i\, e_i(t) \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\, e_j(t) + \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\, \hat{x}_j(t) \\ &\quad - \sum_{\substack{j=1 \\ j\neq i}}^{N} B_i\, k_{ij}\, \hat{y}_j(t) - \sum_{\substack{j=1 \\ j\neq i}}^{N} B_i\, k_j\, C_j\, e_j(t) \end{aligned}$$

$$- B_i\, G_i(t) + B_i\, f_i(x_i(t), t) \tag{19}$$

The time derivative of the convergence error $e_i(t)$, using the connected system (1) together with the observer Eq. (9), is obtained as:

$$\begin{aligned}\dot{e}_i(t) &= (A_i - L_i\, C_i)\, e_i(t) \\ &+ \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\, x_j(t) - \sum_{\substack{j=1 \\ j\neq i}}^{N} \hat{A}_{ij}\, \hat{x}_j(t) \\ &- B_i\, R_i(t) + B_i\, f_i(x_i(t), t)\end{aligned} \tag{20}$$

The difference between the actual amount of the interconnection matrix A_{ij} and its approximation $\hat{A}_{ij}$ is also defined by $\tilde{A}_{ij}$ as:

$$\tilde{A}_{ij} = A_{ij} - \hat{A}_{ij} \tag{21}$$

From expansion $\sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\, x_j(t) - \sum_{\substack{j=1 \\ j\neq i}}^{N} \hat{A}_{ij}\, \hat{x}_j(t)$ together with using (21), we have:

$$\begin{aligned}&\sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\, x_j(t) - \sum_{\substack{j=1 \\ j\neq i}}^{N} \hat{A}_{ij}\, \hat{x}_j(t) \\ &= \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\, x_j(t) - \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\, \hat{x}_j(t) \\ &+ \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\, \hat{x}_j(t) - \sum_{\substack{j=1 \\ j\neq i}}^{N} \hat{A}_{ij}\, \hat{x}_j(t) \\ &= \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\, e_j(t) + \sum_{\substack{j=1 \\ j\neq i}}^{N} \tilde{A}_{ij}\, \hat{x}_j(t)\end{aligned} \tag{22}$$

Finally, with replacing (22) into (20), the time derivative of $e_i(_t)$ is obtained as:

$$\begin{aligned}\dot{e}_i(t) &= (A_i - L_i\, C_i)\, e_i(t) \\ &+ \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\, e_j(t) + \sum_{\substack{j=1 \\ j\neq i}}^{N} \tilde{A}_{ij}\, \hat{x}_j(t) \\ &- B_i\, R_i(t) + B_i\, f_i(x_i(t), t)\end{aligned} \tag{23}$$

In the following, will be shown that the proposed closed-loop control provides stability of the original overall system (1).

5 The Stability Analysis

In the following theorem, it will be proved that the proposed observer-based decentralized controller guarantees the stability of the closed-loop system. That is, the estimation error of the connected system in the presence of unknown interconnections and disturbance in (1) will be converging.

Theorem 1 *Consider the connected system with unknown interconnections in* (1) *concerning the observability and controllability assumptions for each subsystem. The control law* $u_i(t)$ *in* (14) *stabilizes the overall system in* (1) *in the presence of unknown disturbance and interconnection, through the Lyapunov candidate* (24), *when states of each subsystems are estimated.*

It should be noted that for $i = 1, 2, \ldots, N$, $R_i(t)$ is the observer sliding term, L_i is the gain of observer, and $G_i(_t)$ is the sliding term of controller. $\hat{A}_{ij}$ is an equivalent of the estimation of unknown connected matrix, k_j, k_i and k_{ij} with $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, N$, $i \neq j$ are controller gains.

Proof The Lyapunov candidate $V(Z(t))$ is considered as:

$$V(Z(t)) = \sum_{i=1}^{N} \begin{pmatrix} \delta_i^{\mathrm{T}}(t)\ \delta_i(t) + s_i^{\mathrm{T}}(t)\, s_i(t) \\ +\mathrm{trace}\left(\sum_{\substack{j=1 \\ j\neq i}}^{N} (\tilde{A}_{ij}^{T} \tilde{A}_{ij}) \right) \end{pmatrix} \tag{24}$$

With $Z_i^{\mathrm{T}}(t) = [\,\delta_{\mathrm{i}}^{\mathrm{T}}(t)\ s_i^{\mathrm{T}}(t)\,]$, $i = 1, 2, \ldots, N$. It is seen shown that the Lyapunov function $V(Z(t))$ is always positive ($V(Z(t))>0$). The first time derivative of $V(Z(t))$ is as follows:

$$\begin{aligned}&\dot{V}(Z(t)) \\ &= \sum_{i=1}^{N} \begin{pmatrix} \dot{\delta}_i^{\mathrm{T}}(t)\ \delta_i(t) + \delta_i^{\mathrm{T}}(t)\, \dot{\delta}_i(t) + \dot{s}_i^{\mathrm{T}}(t)\, s_i(t) \\ +s_i^{T}(t)\, \dot{s}_i(t) + 2\mathrm{trace}\left(\sum_{\substack{j=1 \\ j\neq i}}^{N} (\dot{\tilde{A}}_{ij}^{T} \tilde{A}_{ij}) \right) \end{pmatrix}\end{aligned} \tag{25}$$

where $\dot{\tilde{A}}_{ij} = \dot{A}_{ij} - \dot{\hat{A}}_{ij}$. Since the actual interconnection dynamic $\dot{A}_{ij} = 0$, which means $\dot{\tilde{A}}_{ij} = -\dot{\hat{A}}_{ij}$. Therefore, (25) becomes:

$$\dot{V}(Z(t)) = \sum_{i=1}^{N} \begin{pmatrix} \dot{\delta}_i^T(t)\,\delta_i(t) + \delta_i^{\mathrm{T}}(t)\,\dot{\delta}_i(t) \\ +\dot{s}_i^{\mathrm{T}}(t)\,s_i(t) + s_i^T(t)\,\dot{s}_i(t) \\ -2\mathrm{trace}\left(\sum_{\substack{j=1 \\ j\neq i}}^{N} (\dot{\hat{A}}_{ij}^{\mathrm{T}}\tilde{A}_{ij})\right) \end{pmatrix} \tag{26}$$

Replacing equations $\dot{\delta}_i(t) = C_i\,\dot{x}_i(t)$(13) and the time derivative (7), i.e.,$\dot{s}_i(t) = C_i\,\dot{e}_i$, into (26) results:

$$\begin{aligned} \dot{V}(Z(t)) &= \sum_{i=1}^{N} \left(\begin{array}{l} \left(C_i \left(\begin{array}{l} (A_i - B_i\,k_i)x_i(t) + B_i\,k_i\,e_i(t) \\ +\sum_{\substack{j=1 \\ j\neq i}}^{N} (A_{ij} - B_i k_{ij}\,C_j)\,\hat{x}_j(t) \\ +\sum_{\substack{j=1 \\ j\neq i}}^{N} (A_{ij} - B_i\,k_j\,C_j)\,e_j(t) \\ -B_i\,G_i(t) + B_i\,f_i(x_i(t),t) \end{array} \right) \right)^{\mathrm{T}} \delta_i(t) \\ +\delta_i^{\mathrm{T}}(t) \left(C_i \left(\begin{array}{l} (A_i - B_i\,k_i)x_i(t) + B_i\,k_i\,e_i(t) \\ +\sum_{\substack{j=1 \\ j\neq i}}^{N} (A_{ij} - B_i k_{ij}\,C_j)\,\hat{x}_j(t) \\ +\sum_{\substack{j=1 \\ j\neq i}}^{N} (A_{ij} - B_i\,k_j\,C_j)\,e_j(t) \\ -B_i\,G_i(t) + B_i\,f_i(x_i(t),t) \end{array} \right) \right) \\ +\left(C_i \left(\begin{array}{l} (A_i - L_i\,C_i)\,e_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\,e_j(t) \\ +\sum_{\substack{j=1 \\ j\neq i}}^{N} \tilde{A}_{ij}\,\hat{x}_j(t) - B_i\,R_i(t) \\ +B_i\,f_i(x_i(t),t) \end{array} \right) \right)^{\mathrm{T}} s_i(t) \\ +s_i^{\mathrm{T}}(t) \left(C_i \left(\begin{array}{l} (A_i - L_i\,C_i)\,e_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\,e_j(t) \\ +\sum_{\substack{j=1 \\ j\neq i}}^{N} \tilde{A}_{ij}\,\hat{x}_j(t) - B_i\,R_i(t) \\ +B_i\,f_i(x_i(t),t) \end{array} \right) \right) \\ -2\mathrm{trace}\left(\sum_{\substack{j=1 \\ j\neq i}}^{N} (\dot{\hat{A}}_{ij}^{T}\tilde{A}_{ij})\right) \end{array} \right) \end{aligned} \tag{27}$$

Now, we split the function into three by:

$$\dot{V}(Z(t)) = \dot{V}_1(Z(t)) + \dot{V}_2(Z(t)) + \dot{V}_3(Z(t)) \tag{28}$$

where they are defined as in the following. The first term, i.e., $\dot{V}_1(Z(t))$ is defined as:

$$\begin{aligned} \dot{V}_1(Z(t)) &= \sum_{i=1}^{N} \left(\left(C_i(A_i - B_i\,k_i)x_i(t) + B_i\,k_i\,e_i(t) \right)^{\mathrm{T}} \delta_i(t) \right) \\ &+ \delta_i^{\mathrm{T}}(t) \left(C_i \left((A_i - B_i\,k_i)x_i(t) + B_i\,k_i\,e_i(t) \right) \right) \\ &+ \left(C_i\,((A_i - L_i\,C_i)\,e_i(t)) \right)^{\mathrm{T}} s_i(t) \\ &+ s_i^{\mathrm{T}}(t)\,(C_i\,((A_i - L_i\,C_i)\,e_i(t))) \end{aligned} \tag{29}$$

The second term, i.e., $\dot{V}_2(Z(t))$ can be defined as:

$$\begin{aligned} \dot{V}_2(Z(t)) &= \sum_{i=1}^{N} \left(\begin{array}{l} \left(C_i \left(\begin{array}{l} \sum_{\substack{j=1 \\ j\neq i}}^{N} (A_{ij} - B_i\,k_{ij}\,c_j)\,\hat{x}_j(t) \\ +\sum_{\substack{j=1 \\ j\neq i}}^{N} (A_{ij} - B_i\,k_j\,C_j)\,e_j(t) \\ -B_i\,G_i(t) + B_i\,f_i(x_i(t),t) \end{array} \right) \right)^{\mathrm{T}} \delta_i(t) \\ +\delta_i^{\mathrm{T}}(t) \left(C_i \left(\begin{array}{l} \sum_{\substack{j=1 \\ j\neq i}}^{N} (A_{ij} - B_i\,k_{ij}\,c_j)\,\hat{x}_j(t) \\ +\sum_{\substack{j=1 \\ j\neq i}}^{N} (A_{ij} - B_i\,k_j\,C_j)\,e_j(t) \\ -B_i\,G_i(t) + B_i\,f_i(x_i(t),t) \end{array} \right) \right) \\ +\left(C_i \left(\begin{array}{l} \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\,e_j(t) \\ -B_i\,R_i(t) + B_i\,f_i(x_i(t),t) \end{array} \right) \right)^{\mathrm{T}} s_i(t) \\ +s_i^{\mathrm{T}}(t) \left(C_i \left(\begin{array}{l} \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}\,e_j(t) \\ -B_i\,R_i(t) + B_i\,f_i(x_i(t),t) \end{array} \right) \right) \end{array} \right) \end{aligned} \tag{30}$$

and lastly:

$$\dot{V}_3(Z(t)) = \sum_{i=1}^{N} \begin{pmatrix} \left(C_i \left(\sum_{\substack{j=1 \\ j\neq i}}^{N} \tilde{A}_{ij}\, \hat{x}_j(t) \right) \right)^{\mathrm{T}} s_i(t) \\ +s_i^T(t) \left(C_i \left(\sum_{\substack{j=1 \\ j\neq i}}^{N} \tilde{A}_{ij}\, \hat{x}_j(t) \right) \right) \\ -2\mathrm{trace} \left(\sum_{\substack{j=1 \\ j\neq i}}^{N} (\dot{\hat{A}}_{ij}^T \tilde{A}_{ij}) \right) \end{pmatrix} \tag{31}$$

In the following, with the proper selection of gains and parameters, it will be shown that $\dot{V}_1(Z(t)) < 0$, $\dot{V}_2(Z(t)) < 0$ and $\dot{V}_3(Z(t)) = 0$. Therefore, the Lyapunov function $V(Z(t))$ will be a suitable candidate to guarantee the stability of the closed-loop system. In the following, each step is described.

Step 1 [Eqs. (34)–(37)]: Proving the stability of the controller based on the proposed observer, with the LMI matrix inequality theorem (Lemma 2).
Step 2 [Eqs. (38)–(41)]: Selecting control parameters to estimate and eliminate the effects of disturbances with the sliding mode technique.
Step 3 [Eqs. (42)–(44)]: Calculating the equivalent of the interconnections among subsystems ($\hat{A}_{ij}$) with adaptive rules.

Lemma 2 *The LMI matrix inequality* (Lien 2004):

$$\begin{bmatrix} Q(y) + \rho I & S(y) \\ S^T(y) & R(y) \end{bmatrix} < 0 \tag{32}$$

is equivalent to:

$$\begin{aligned} &R(y) < 0, \\ &Q(y) - S(y)\, R^{-1}(y)\, S^{\mathrm{T}}(y) < -\rho I \end{aligned} \tag{33}$$

where $Q(y) = Q^{\mathrm{T}}(y)$, $R(y) = R^{\mathrm{T}}(y)$ *and* $S(y)$ *is dependent on y. If the right pseudoinverse of the matrix* C_i *is denoted by* C_i^+ (*As in Lemma 1*; Liu 2016), *from* (7) *and* (12) *is yielded as*:

$$\begin{aligned} e_i(t) &= C_i^+\, S_i(t) \\ x_i(t) &= C_i^+\, \delta_i(t) \end{aligned} \tag{34}$$

By replacing(34) *into* (29), $\dot{V}_1(Z(t))$ *is achieved as*:

$$\dot{V}_1(Z(t)) = \sum_{i=1}^{N} \begin{pmatrix} \delta_i^{\mathrm{T}}(t) \begin{pmatrix} \left(C_i(A_i - B_i\, k_i)C_i^+\right)^{\mathrm{T}} \\ +C_i(A_i - B_i\, k_i)C_i^+ \end{pmatrix} \delta_i(t) \\ +s_i^{\mathrm{T}}(t)\, (C_i^{+^i}\, k_i^{\mathrm{T}}\, B_i^{\mathrm{T}}\, C_i^{\mathrm{T}})\, \delta_i(t) \\ +\delta_i^{\mathrm{T}}(C_i\, B_i\, k_i\, C_i^+)\, s_i \\ +s_i^{\mathrm{T}} \begin{pmatrix} \left(C_i(A_i - L_i\, C_i)\, C_i^+\right)^{\mathrm{T}} \\ +C_i(A_i - L_i\, C_i)\, C_i^+ \end{pmatrix} s_i \end{pmatrix} \tag{35}$$

From (35) *and definition of new variable* $\sum_i = \begin{bmatrix} \sum_{11_i} & \sum_{12_i} \\ \sum_{21_i} & \sum_{22_i} \end{bmatrix}$, $\dot{V}_1(Z(t))$ *can be written as:*

$$\dot{V}_1(Z(t)) = \sum_{i=1}^{N} Z_i^{\mathrm{T}}(t) \sum_i Z_i(t) \tag{36}$$

where each element of $\sum_i$ *in* (35) *for* $i = 1, 2, \ldots, N$ *is defined as*:

$$\begin{aligned} \sum_{11_i} &= \left(C_i(A_i - B_i\, k_i)C_i^+\right)^{\mathrm{T}} \\ &\quad + \left(C_i(A_i - B_i\, k_i)C_i^+\right), \\ \sum_{12_i} &= C_i\, B_i\, k_i\, C_i^+, \\ \sum_{21_i} &= \left(C_i\, B_i\, k_i\, C_i^+\right)^{\mathrm{T}}, \\ \sum_{22_i} &= \left(C_i(A_i - L_i\, C_i)\, C_i^+\right)^{\mathrm{T}} \\ &\quad + C_i(A_i - L_i\, C_i)\, C_i^+ \end{aligned} \tag{37}$$

Aiding proper selection of k_i *and* L_i, *according to Lemma 2, i.e.,*. LMI inequality in (33) *leads to an answer, which provides* $\dot{V}_1(Z(t)) < 0$.

For the second term, $\dot{V}_2(Z(t))$, from the norm product properties of matrices $\|AB\| \leq \|A\|\, \|B\|$ and replacing (2) into (30) we have:

$$\dot{V}_2(Z(t))$$
$$\leq \sum_{i=1}^{N} \left(\begin{array}{l} \left\| \delta_i^T(t)\, C_i\, B_i \right\| \\ \left(\left(\begin{array}{l} \sum\limits_{\substack{j=1 \\ j\neq i}}^{N} (\beta_{ij} - \left\| k_{ij} \right\|) \left\| \hat{y}_j(t) \right\| \\ + \sum\limits_{\substack{j=1 \\ j\neq i}}^{N} \left(\begin{array}{r} (\beta_{ij} - \left\| k_j \right\|) \left\| y_j(t) - \hat{y}_j(t) \right\| \\ -G_i(t) + \rho_i \end{array} \right) \end{array} \right) \right)^{\mathrm{T}} \\ + \left(\left(\begin{array}{l} \sum\limits_{\substack{j=1 \\ j\neq i}}^{N} (\beta_{ij} - \left\| k_{ij} \right\|) \left\| \hat{y}_j(t) \right\| \\ + \sum\limits_{\substack{j=1 \\ j\neq i}}^{N} \left(\begin{array}{r} (\beta_{ij} - \left\| k_j \right\|) \left\| y_j(t) - \hat{y}_j(t) \right\| \\ -G_i(t) + \rho_i \end{array} \right) \end{array} \right) \right) \\ \left\| \left(\delta_i^T(t)\, C_i\, B_i \right)^{\mathrm{T}} \right\| \\ + \left\| s_i^T(t)\, C_i\, B_i \right\| \\ \left(\sum\limits_{\substack{j=1 \\ j\neq i}}^{N} \beta_{ij} \left\| y_j(t) - \hat{y}_j(t) \right\| - R_i(t) \right) \\ + \left(\sum\limits_{\substack{j=1 \\ j\neq i}}^{N} \beta_{ij} \left\| y_j(t) - \hat{y}_j(t) \right\| - R_i(t) \right)^{\mathrm{T}} \\ \left\| (s_i^{\mathrm{T}}(t)\, C_i\, B_i)^{\mathrm{T}} \right\| \end{array} \right) \tag{38}$$

By defining the sliding term of the observer $R_i(t)$ as:

$$R_i(t) = \mu_i \frac{(s_i^{\mathrm{T}}(t)\, C_i\ B_i)^{\mathrm{T}}}{\left\| s_i^T(t)\, C_i\ B_i \right\|} \left(\sum_{\substack{j=1 \\ j\neq i}}^{N} \beta_{ij} \left\| y_j(t) - \hat{y}_j(t) \right\| + \rho_i \right), \quad \mu_i > 1 \tag{39}$$

and similarly, the sliding term of the controller $G_i(t)$ as:

$$G_i(t) = \xi_i \frac{(\partial_i^{\mathrm{T}}(t)\ C_i\ B_i)^{\mathrm{T}}}{\left\| \partial_i^{\mathrm{T}}(t)\, C_i\ B_i \right\|} \rho_i, \quad \xi_i > 1 \tag{40}$$

together with the Assumption 2 that $A_{ij} = B_i H_{ij} C_j$, assuming $\left\| H_{ij} \right\| < \beta_{ij}$, to become $\dot{V}_2(Z(t))$ negative, gains k_j and k_{ij} must be determined such that:

$$\left\| k_{ij} \right\| = \left\| k_j \right\| \geq \beta_{ij} \tag{41}$$

From the vector properties, for two vectors of a and b we know (Ioannou and Sun 2012) that:

$$a^{\mathrm{T}} b = \mathrm{trace}(b\, a^{\mathrm{T}}) \tag{42}$$

From Eq. (31) and property (42), $\dot{V}_3(Z(t))$ is obtained as:

$$\dot{V}_3(Z(t)) = \sum_{i=1}^{N} \left(\begin{array}{l} 2trace \sum\limits_{\substack{j=1 \\ j\neq i}}^{N} \hat{x}_j(t)\, s_i^T(t)\, C_i\, \tilde{A}_{ij} \\ -2trace(\sum\limits_{\substack{j=1 \\ j\neq i}}^{N} (\dot{\hat{A}}_{ij}^T \tilde{A}_{ij})) \end{array} \right)$$
$$= \sum_{i=1}^{N} \left(2trace \left(\sum_{\substack{j=1 \\ j\neq i}}^{N} \left(\hat{x}_j(t)\, s_i^T(t)\, C_i - \dot{\hat{A}}_{ij}^T \right) \tilde{A}_{ij} \right) \right) \tag{43}$$

To achieve $\dot{V}_3(Z_i(t)) = 0$, the following adaptive laws (44) must be met.

$$\dot{\hat{A}}_{ij}^T = \left(\hat{x}_j(t)\, s_i^T(t)\, C_i \right) \tag{44}$$

where the matrix $\hat{A}_{ij}$ is an equivalent of the interconnection matrix. Since $\dot{V}_1(Z(t)) < 0$, based on the LMI; $\dot{V}_2(Z(t)) < 0$ by selecting suitable sliding terms of the observer (39) and of the controller (40) and finally $\dot{V}_3(Z(t)) = 0$ based on the adaptive law in (44), $\dot{V}(Z(t)) < 0$.

Since the Lyapunov function $V(Z(t))$ in (24) is always positive and its derivative ($\dot{V}(Z(t))$ according to (28) $\dot{V}(Z(t)) = \dot{V}_1((Z(t)) + \dot{V}_2(Z(t)) + \dot{V}_3(Z(t))$ is always negative, the controller (14) is a stabilizing controller for the disturbance observer in (9). □

Remark 1 Equation (37) is Bilinear Matrix Inequality (BMI) due to the product of gains and Lyapunov matrices. Typically, there is not any approach to get the global solution BMI optimization problems. In this manuscript, a V–K algorithm (Rodrigues and How 2003) is used to by fixing a set of parameters constant in each iteration divide BMI into two LMIs.

In the following, the proposed procedure is presented as an algorithm to provide closed-loop stability of the connected system in (1).

Algorithm 1 Controller design procedure, aiding the disturbance observer with unknown interconnections

Goal: Stabilization of the closed-loop system based on the disturbance observer to estimate and reduce the disturbance effect in the presence of unknown connection.

***Given**: Observability and controllability of each subsystem, assuming there is unknown interconnections and bounded disturbance.*

Step 1 Perform the controllability and observability tests of pairs (A_i, B_i) and (A_i, C_i) for $i = 1, 2, \ldots, N$, respectively, for each subsystem.

Step 2 Solve the LMI inequality (37) and determine gains of the observer in (9) and the controller in (14) (based on Lemma 2 and Remark 1).

Step 3 Define the observer sliding surface $s_i(t) \in \Re^{m_i}$ as in (7) and the sliding term $R_i(t)$ in (39).

Step 4 Use the adaptation law (44) to assess the equivalent of the unknown interconnections matrix $\hat{A}_{ij}$.

Step 5 Determine the controller sliding surface $\delta_i(t) \in \Re^{m_i}$ as in (10) and the sliding term $G_i(t)$ in (40).

Step 6 Determine the controller gains k_j and k_{ij} in (41) from (2) such that $\|H_{ij}\| < \beta_{ij}$.

Based on the proposed plan in Algorithm 1, an overall block diagram of the connected system with two subsystems in the presence of disturbances $f_i(x_i(t), t)$ and unknown interconnections A_{ij} is shown in Fig. 1.

As seen in Fig. 1, u_i and y_i are control input and output signals of each subsystem (as the only available state of each subsystem), x_i and $\hat{x}_i$ are the actual and the estimated state of each subsystem, respectively. Figure 1 shows the decentralized schematic of the proposed technique.

6 Numerical Simulation

In this section, two cases are simulated to verify significance of the proposed method. In the first example (Liu 2016), a connected system with two stable subsystems is considered. Regardless of the stability of each subsystem, the overall connected system is still unstable. In the second example, both subsystems (regardless of interconnection between subsystems) are unstable.

Case 1: An interconnected system, consists of two subsystems, is presented when bounded disturbances are applied to each subsystem (Liu 2016). First subsystem is described by:

$$\begin{cases} \dot{x}_1(t) = \begin{bmatrix} -1 & 5 \\ 0 & -4 \end{bmatrix} x_1(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u_1(t) \\ \qquad + \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} x_2(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} f_1(x_i, t), \\ y_1(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_1(t) \end{cases} \tag{45}$$

where, the second subsystem is denoted as:

$$\begin{cases} \dot{x}_2(t) = \begin{bmatrix} -1 & 3 \\ 0 & -3 \end{bmatrix} x_2(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_2(t) \\ \qquad + \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} x_1(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} f_2(x_i, t), \\ y_2(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_2(t) \end{cases} \tag{46}$$

together with the following disturbances (Liu 2016), applied to each subsystem:

$$\begin{aligned} f_1(x_i, t) &= 0.15 \sin(x_{12}) \\ &\quad + 0.04[sign(\sin(2t)) + 1] \\ f_2(x_i, t) &= 0.2 \sin(x_{21}) - 0.1 \cos(x_{22}) \\ &\quad + 0.04[sign(\sin(2t) + 1] \end{aligned} \tag{47}$$

where:

$$x_1(t) = \begin{bmatrix} x_{11}(t) \\ x_{12}(t) \end{bmatrix}, \quad x_2(t) = \begin{bmatrix} x_{21}(t) \\ x_{22}(t) \end{bmatrix} \tag{48}$$

Initial conditions are set as $\begin{bmatrix} x_{11}(0) & x_{12}(0) \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} -1 & 1 \end{bmatrix}^{\mathrm{T}}$ and $\begin{bmatrix} x_{21}(0) & x_{22}(0) \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 1 & -1 \end{bmatrix}^{\mathrm{T}}$. Eigenvalues of each subsystem are found as $\lambda = \{-1, -4\}$ and $\lambda = \{-1, -3\}$. Meanwhile, modes of the connected system (1) with the interconnection matrix are equal to $\lambda = \{1.42,\ -4.72,\ -2.85 \pm 1.94\}$. These imply that the overall system, due to existence of interconnections between the subsystems, is still unstable (at least one positive eigenvalue is in the right hand side of the s-plane). A governing graph of system (1) with two subsystems of (45) and (46) is shown in Fig. 2

The decentralized controller design procedure is as in the following:

First step; the assumptions validity: Observability and controllability of both subsystems by investigating pairs of (A_1, C_1) and (A_2, C_2), and also $(A_1, B_1)$and (A_2, B_2) are concluded. From C_i and B_i themselves, the second assumption $C_i B_i \neq 0$ is also met.
Second step; the answer possibility: To design the observer (9) and controller (14), the LMI inequality in (37) provides an answer, using the observer gains $L_1 = \begin{bmatrix} 5.9449 & 3.8755 \end{bmatrix}^T$ and $L_2 = \begin{bmatrix} 5.3596 & 3.4558 \end{bmatrix}^T$, and the controller gains $k_1 = \begin{bmatrix} 0.7004 & 1.4514 \end{bmatrix}$ and $k_2 = \begin{bmatrix} 0.7180 & 1.2707 \end{bmatrix}$(according to Remark 1).
Third step; the observer design: To design an observer, sliding surfaces are defined as follows:

$$\begin{aligned} s_1(t) &\triangleq y_1(t) - \hat{y}_1(t), \\ s_2(t) &\triangleq y_2(t) - \hat{y}_2(t) \end{aligned} \tag{49}$$

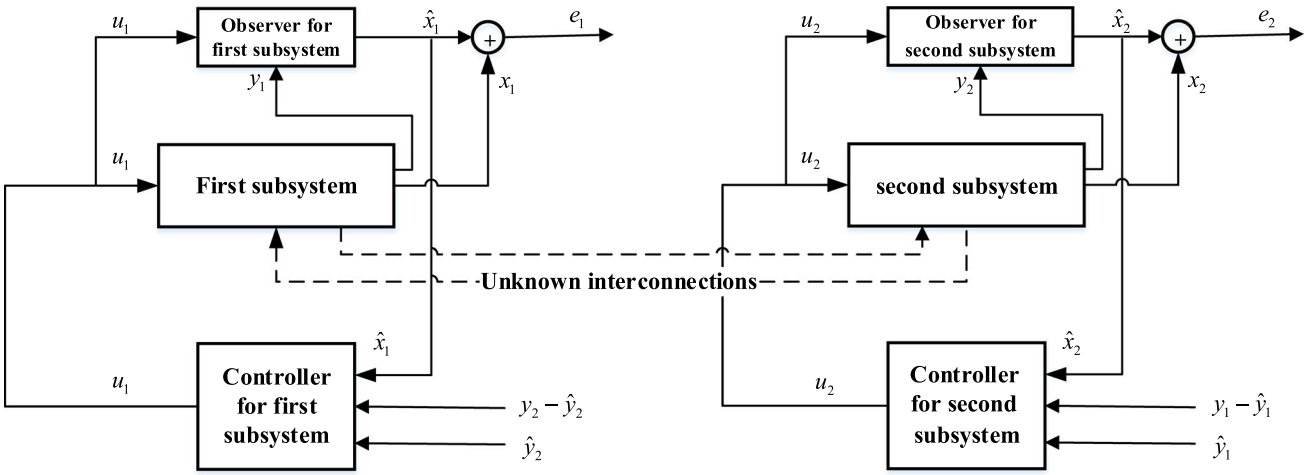


Fig. 1 A schematic diagram of the proposed decentralized adaptive sliding mode observer-based control

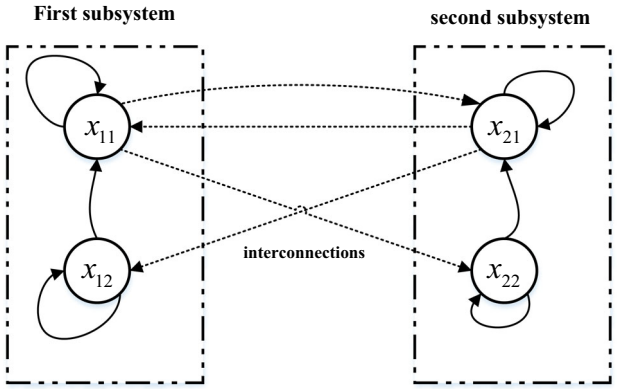


Fig. 2 A connection graph of (46) and (47)

From Assumption 1 can be seen that matrix A_{ij} can be rewritten as $A_{ij} = B_i H_{ij} C_j$. Using Eqs. (45) and (46), concluded that:

$$\|H_{12}\| \leq 1 \quad \|H_{21}\| \leq 1, \tag{50}$$

Parameters $\rho_i = 0.7$ (as in Assumption 3), and $\mu_1 = \mu_2 = 1.2(\mu_i > 1)$ and $\beta_{12} = \beta_{21} = 1$ are also selected in the sliding term $R_i(t)$ (39).
Fourth step; equivalent of the interconnections matrix: Adaptive laws (44) are used to estimate an equivalent of the matrix interconnections $\hat{\dot{A}}_{ij}$ by:

$$\hat{\dot{A}}_{ij}^{\mathrm{T}} = \left(\hat{x}_j(t)\, s_i^{\mathrm{T}}(t)\, C_i\right) \tag{51}$$

Fifth step; the controller design: The sliding surface of the controllers is considered as:

$$\begin{aligned} \delta_1(t) &\triangleq y_1(t) \\ \delta_2(t) &\triangleq y_2(t) \end{aligned} \tag{52}$$

Parameters of the sliding term $G_i(t)$ in (40), $\rho_i = 1.5$ (as in Assumption 3) and $\xi_i = 1.2$ ($\xi_i > 1$) are also tuned.
Sixth step; select appropriate amount for k_{ij}, k_j: In Eq. (50), $k_{ij} = k_j = 2$ according to $\|k_{ij}\| = \|k_j\| \geq \beta_{ij}$ in (41).

Figure 3 shows the behavior of each subsystem in response to initial conditions. Figure 4 shows the time response of the convergence error between the actual and that of the estimated state for each subsystem.

Control input of each subsystem is also seen in Fig. 5.

By selecting parameter $\rho_i = 1.5$ and comparing with the simulation results in Liu (2016), it is observed that in the proposed method (assuming unknown interconnections), the state convergence error is similar to (Liu 2016). However, in Liu (2016) the interconnections between subsystems are assumed known. From the comparison, it can be concluded that the proposed method is efficient for the interconnected systems with unknown interconnections.

Case 2: A connected system with two subsystems of (53) and (54) and disturbance in (55) is considered. Let the first subsystem equation to be as follows:

$$\begin{cases} \dot{x}_1(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & 0 & -3 \end{bmatrix} x_1(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_1(t) \\ \quad + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} x_2(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} f_1(x_i, t) \\ y_1(t) = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} x_1(t) \end{cases} \tag{53}$$

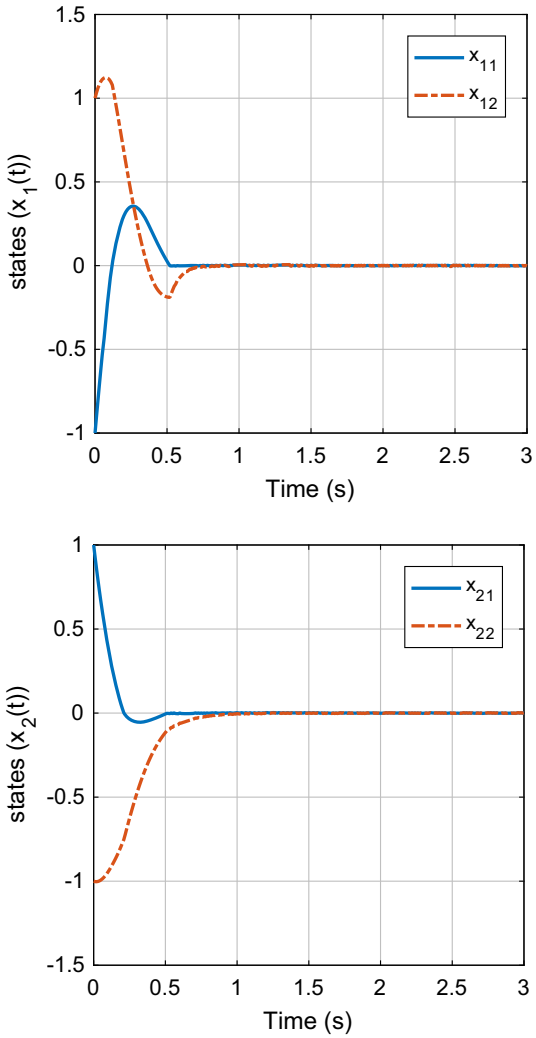


Fig. 3 Time response of each subsystem

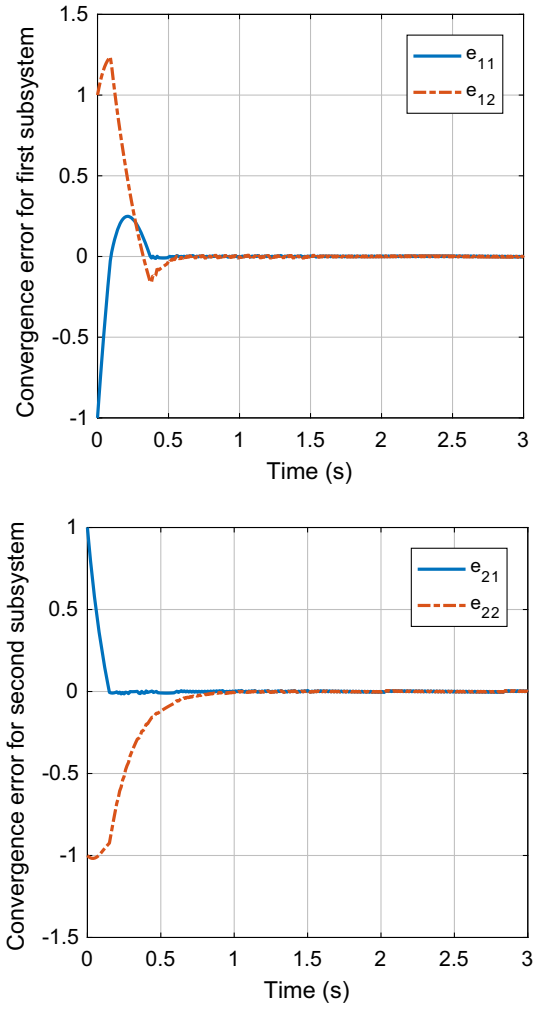


Fig. 4 Time response of the convergence error

While the state space equation of the second subsystem is given by:

$$\begin{cases} \dot{x}_2(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & 4 & -1 \end{bmatrix} x_2(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_2(t) \\ \quad + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} x_1(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} f_2(x_i, t) \\ y_2(t) = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} x_2(t) \end{cases} \tag{54}$$

The state vectors are seen as $x_1(t) = \left[x_{11}(t)\ x_{12}(t)\ x_{13}(t)\right]^{\mathrm{T}}$ and $x_2(t) = \left[x_{21}(t)\ x_{22}(t)\ x_{23}(t)\right]^{\mathrm{T}}$, where $f_i(x_i, t)$ is the

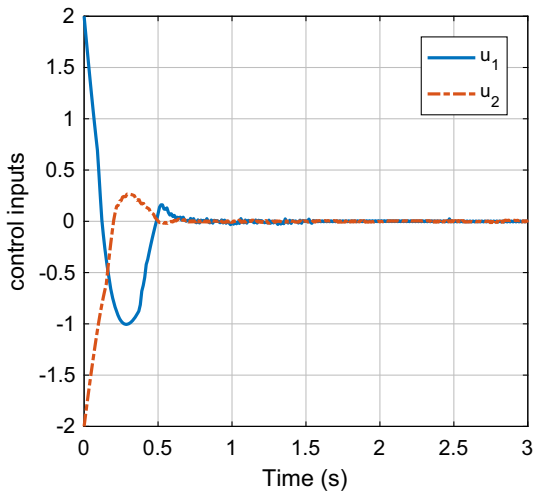


Fig. 5 The control input, applied to each subsystem

distributed disturbance term of the *i*th subsystem which is as follows:

$$\begin{aligned} f_1(x_i, t) &= 0.15\cos(x_{12}) \\ &\quad + 0.4\text{sign}(\sin(2t)) \\ f_2(x_i, t) &= 0.2\sin(x_{21}) + 0.1\cos(x_{22}) \\ &\quad + 0.4\text{sign}(\sin(2t)) \end{aligned} \tag{55}$$

Initial conditions are selected as $x_1(0) = \begin{bmatrix} 1 & -1 & -0.5 \end{bmatrix}^{\mathrm{T}}$ and $x_2(0) = \begin{bmatrix} -1 & 1 & 0.5 \end{bmatrix}^{\mathrm{T}}$ for each subsystem. Modes of subsystems are $\lambda_1 = \{1, -2, -2\}$ and $\lambda_2 = \{-2, -1, 2\}$, respectively. Due to existence of one unstable mode on each subsystem, they are unstable, regardless of the existence of interconnections. However, decentralized controller design steps are in continuing:

First step; the assumptions validity: Each subsystem is seen completely observable and controllable (with the study of pairs of (A_1, C_1) and (A_2, C_2)) and also (A_1, B_1) and (A_2, B_2) matrices, respectively), as well as the assumption of $C_i B_i \neq 0$.

Second step; the answer possibility: During the observer (9) and controller (14) designs, the LMI inequality problem in (37) leads to an answer, (according to Lemma 2), using the observer gains $L_1 = \begin{bmatrix} 2.5126 & 1.6344 & 0.1552 \\ 3.9734 & 2.7678 & 1.7884 \end{bmatrix}^{\mathrm{T}}$ and $L_2 = \begin{bmatrix} 0.0685 & 1.3611 & 3.0795 \\ 4.0672 & 4.4327 & 0.1011 \end{bmatrix}^{\mathrm{T}}$, and the controller gains $k_1 = [\,22.0495\ 13.2535\ 2.1207\,]$ and $k_2 = [\,25.0218\ 22.3701\ 4.5884]$ (according to Remark 1). Furthermore, the LMI answer concludes $\dot{V}_1(Z(t)) < 0$.

Third step; the observer design: To design the observer, the sliding surface is chosen as in (7) together with using $\|H_{12}\| \leq 1$ and $\|H_{21}\| \leq 1$, parameters of the sliding term $R(t)$ in (39) (β_{ij}, μ_i and ρ_i) are also chosen as follows:

$$\begin{aligned} &\rho_i = 0.7, \quad \mu_1 = \mu_2 = 1.2, \\ &\beta_{12} = \beta_{21} = 1.2 \end{aligned} \tag{56}$$

Fourth step; an equivalent of the interconnections matrix: Adaptive law in (44) is used to estimate an equivalent of the interconnection matrix $\hat{\hat{A}}_{ij}$.

Fifth step; the controller design: The sliding surface of controller in (10) and the sliding term $G(t)$ (40) is selected as:

$$\rho_i = 0.7 \quad \xi_1 = \xi_2 = 1.2(\xi_i > 1) \tag{57}$$

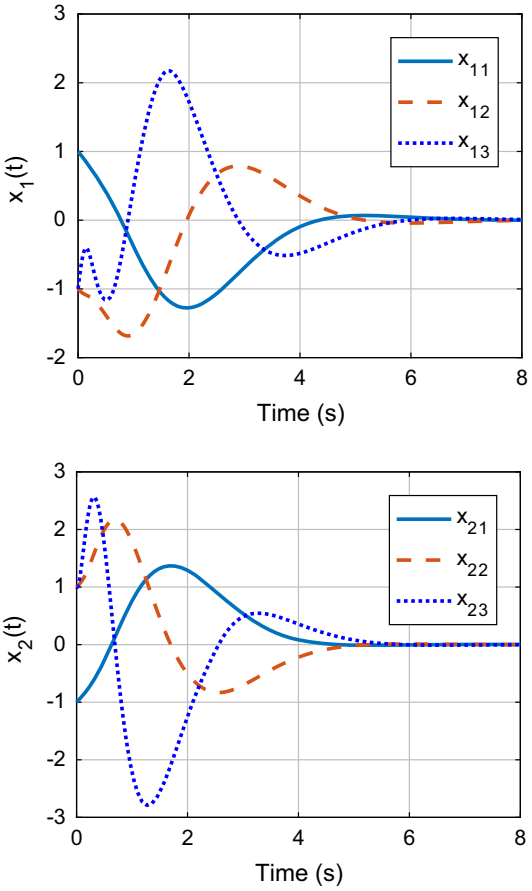


Fig. 6 Time response of states of each subsystem

Sixth step; select a gains of k_{ij}, k_j: both k_j and k_{ij} in (50) are selected as:

$$k_{ij} = k_j = [\,1.5\ 1.5] \tag{58}$$

The convergence error in response to nonzero initial conditions (zero input), using the proposed observer-based adaptive controller for the first and second subsystems, is depicted in Figs. 6 and 7, respectively. From the simulation, outcome can be seen that the proposed observer-based adaptive decentralized controller stabilizes each (unstable) subsystem and eliminates the disturbance effects. Furthermore, zero convergence error can be achieved even in the presence of unknown interconnection on unstable subsystems.

The triggering control is shown in Fig. 8.

7 Conclusion

In this manuscript, a decentralized adaptive observer-based sliding mode controller is presented for a class of connected systems. By using the output data and combination of the

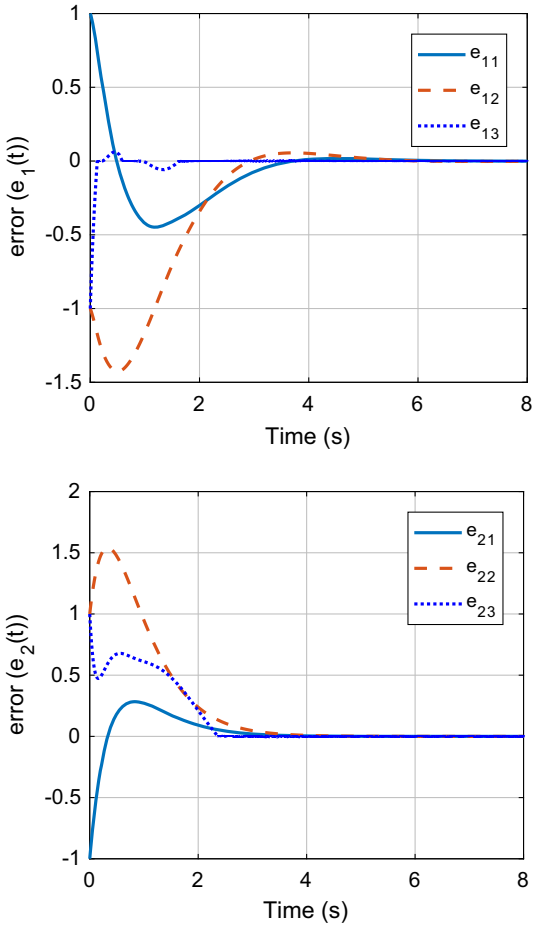


Fig. 7 Time response of convergence error

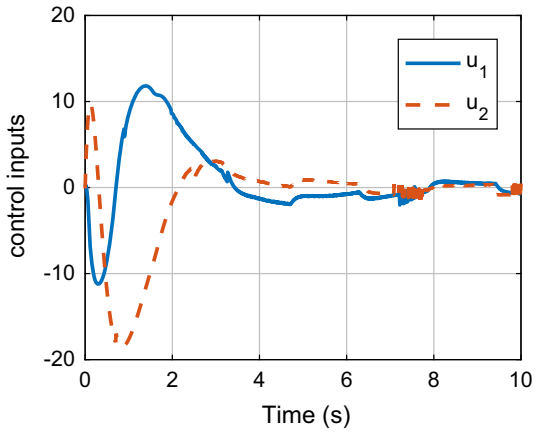


Fig. 8 The control input, applied to each subsystem

Luenberger and sliding mode observer, estimation of non-available states is made possible together with observing the disturbance effects. Furthermore, an equivalent of the interconnection matrices is adaptively built. In order to stabilize the overall system, through a Lyapunov approach, a decentralized sliding mode controller is proposed, based on the observed states. The LMI technique provides a stability margin to find an adaptive solution where effects of bounded disturbance are reduced.

Simulation results of two (stable and unstable) connected systems and comparison of states convergence error of the proposed method and Liu (2016) verify that the proposed decentralized adaptive observer-based sliding mode controller is effective to provide stability of the closed-loop system, in the presence of no knowledge about of the interconnections and or the bounded disturbance.

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