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Ultimate Boundedness Control for Uncertain Nonlinear Impulsive Switched Systems: A Fuzzy Approach Based on a Complete Takagi–Sugeno Structure

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Abstract

This paper deals with the ultimate boundedness control of nonlinear impulsive switched systems, which ensures that the state trajectories ultimately converge to a sufficient small region containing the origin. Given a general model, we first propose novel stability criteria that do not need to be satisfied on the entire space and include a condition to guarantee the remaining of trajectories within the ultimate bound, even though the impulse size is not zero at the origin or is not vanishing near the origin. Applying these criteria to a closed-loop system leads to a set of matrix inequalities that may be infeasible when the subsystems are highly nonlinear. Therefore, we redevelop them for an impulsive switched system represented by Takagi–Sugeno (T–S) fuzzy structure with nonlinear consequent parts. Since this structure has fewer rules than the traditional T–S models with linear consequent parts, the number of established stabilization matrix inequalities is sharply reduced. We then derive an optimization problem with linear and bilinear constraints to achieve a fuzzy controller that guarantees convergence to the smallest ultimate bound as well as practical control issues. Finally, a numerical example and a practical-motivated example are given to demonstrate the applicability of the proposed approach.

Keywords Impulsive switched systems · Local sector nonlinearity · Nonlinear control · Parallel distributed compensation · Takagi–Sugeno fuzzy model

1 Introduction

Due to the presence of the switching effect and the impulsive behavior in most of the real-world processes (Haddad et al. 2006; Posa et al. 2015; Wu et al. 2014), the stability and stabilization issues of impulsive switched systems have been extensively studied. These studies generally use the common Lyapunov function (CLF) approach or the multiple Lyapunov functions (MLF) technique, which is less conservative than CLF (Branicky 1998; Gao and Wang 2016; Ghalehnoie et al. 2018; Goldar et al. 2017; Kermani and Sakly 2017; Zhang et al. 2017; Zhao et al. 2017).

Because of the natural complexity of impulsive switched systems along with the existence of inherent nonlinearity and uncertainty in real-world processes, it is difficult to design a stabilization control law for the general nonlinear impulsive switched systems. In the literature, there are two major approaches to overcome this. First, researchers consider some assumptions on nonlinear dynamics and then develop stability and/or stabilization criteria. For example, linear grows condition is assumed by the authors of Li et al. (2019b, c) where they intend to investigate the stability of nonlinear switched systems with particular forms. Li et al. (2019a) also assume a more advanced version of the grows condition to handle the stabilization problem for a class of switched nonlinear systems in the *p*-normal form. However, the most common and most complete assumption is the Lipschitz continuity, which also includes the grows condition. For example, see Feng and Cao (2015), Long (2018), Poznyak et al. (2014), Xu and Teo (2010). Although the nonlinear dynamics of systems usually satisfy these assumptions on their region of attraction, the related coefficients may be very large, and consequently, the proposed stability problem may become infeasible. Second, some other researchers represent the nonlinear system as a Takagi–Sugeno (T–S) fuzzy system where the consequent parts of the rules are linear models (Ai and Chen 2017; Ho and Sun 2007; Sun et al.

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2016; Wang et al. 2012; Wang and Wu 2016; Wu et al. 2014; Xiu and Ren 2005; Zhang et al. 2010, 2011; Zhao et al. 2016; Zheng and Zhang 2017; Zhong and Xu 2011).

During the last decades, the topic of stability and control design for the T–S fuzzy systems has been extensively considered. In the most of them, the consequent part of the rules contains time-invariant linear model for the both continuous- and discrete-time dynamics. For example, we may refer to Wang et al. (2012) where the impulsive control for nonlinear switched systems is considered, to Zhao et al. (2016) that investigates the control of nonlinear switched systems, or to Bouyahya et al. (2017) which consider uncertain T–S fuzzy model for non-switched uncertain nonlinear systems. In Zheng and Zhang (2017), authors extend the conventional T–S fuzzy model by considering vanishing and boundedness nonlinear terms into the jump functions of the consequent parts. Another extension to the traditional impulsive T–S fuzzy systems is given in Ai and Chen (2017) where the continuous-time dynamics of the consequent parts are nonlinear. However, the impulse function is still linear and there is no uncertainty in the local models. Besides, Ai and Chen (2017) do not provide a systematic approach to find the decision variables. Besides, to represent the nonlinear impulsive switched systems with time delay, Zhong and Xu (2011) studied a delay T–S fuzzy system where the consequent parts are linear systems with time delay and then develop exponential stabilization criteria. Moreover, parametric uncertainties for both local linear models and jump functions are given in Sun et al. (2016), Wang and Wu (2016), Zhang et al. (2010, 2011). Note that, in recent years, the idea of using untraditional T–S fuzzy structures has been also considered in different control applications other than switched systems. For example, Wu et al. (2018) use this idea for synchronization of complex networks with partial coupling where each node in the network is modeled by a kind of nonlinear T–S fuzzy structure.

In fact, utilizing the local sector nonlinearity approach (Tanaka and Wang 2001), a nonlinear system can be represented by a T–S fuzzy model with local linear models in the consequent parts. In this approach that all nonlinearities of the nonlinear system are chosen as premise variables, as the number of nonlinearities (or equivalently the number of premise variables) increases, the number of rules increases exponentially. This leads to raised complexity for established stability criteria. On the other hand, by choosing a subset of nonlinearities as premise variables, a fuzzy model with fewer rules is generated in which some of the local models are nonlinear. During the process of obtaining the local models from the original nonlinear system, since the selected nonlinearities are replaced by their minimum or maximum values, the complexity of the obtained models is less than the original system. For example, these local models may have lower Lipschitz coefficients. As a result, it is expected that for a system represented by the nonlinear T–S fuzzy structure, the number of stability conditions is far fewer than when it is modeled by the traditional T–S fuzzy structure. Also, the complexity of these stability conditions may be much lower than those can be derived for the original system.

These features, along with the inherent complexity of real-world impulsive switched systems, motivate us to investigate the stabilization issue for the nonlinear T–S fuzzy structures in the field of impulsive switched systems. Here, if the real-world system, which is modeled in the form of nonlinear T–S fuzzy, contains the parametric uncertainties and distributions, the sub-models obtained in the fuzzy structure also contain similar uncertainties and distributions. So, we introduce a more complete T–S fuzzy model where the continuous- and discrete-time dynamics in the consequent part of the rules include nonlinear dynamics and have different types of nonvanishing uncertainties. According to the author's knowledge, there is no work in this field that uses such complete T–S fuzzy structure.

Given a general model of nonlinear impulsive switched system, we first employ the idea of multiple Lyapunov functions (MLFs) and develop stability criteria that ensure convergence to a sufficiently small ultimate bound instead of convergence to the origin. We consider this kind of convergence because of intrinsic nonvanishing uncertainties in real-world processes. The proposed criteria are different from those found in the literature. Actually, they do not need to be met on the entire state space and also include an additional condition to handle the impulse effect over the ultimate bound as well as to ensure the remaining of state trajectories within the ultimate bound. This additional condition is very useful, especially when the impact size is not zero at or is not vanishing near the origin. Secondly, utilizing the common quadratic Lyapunov functions and the idea of parallel-distribution compensation (PDC) scheme, we redevelop the stabilization criteria as matrix inequalities for an impulsive switched system represented by the introduced T–S fuzzy model. To achieve the gains of the control law along with the smallest ultimate bound, a constrained optimization problem is also proposed where the constraints are often linear. However, there are some bilinear constraints that make the proposed optimization problem difficult to solve. It encourages us to propose a solving approach based on the genetic algorithm (GA) combined with the linear matrix inequality (LMI) techniques. The applicability of our method is also illustrated through several examples.

Briefly, compared with the existing conclusions, the main contributions of this paper are as follows:

1. For a given general model of an impulsive switched systems, the existing methods take into account same stability conditions for the entire state space, whether the trajectories are far from the origin or near the origin. It forces the size of impulses to be decreased continually as trajectories approach the origin and be zero at the origin. In contrast, we propose novel stability criteria that include a condition to handle the impact effect when trajectories reach the ultimate bound. In this way, the size of impulses can be nonzero near/at the origin.

2. We introduce a more complete T–S fuzzy structure that can represent a system with fewer rules than the traditional T–S models. It sharply reduces the number of stability conditions compared with the traditional T–S fuzzy based approaches. Also, this structure includes nonvanishing uncertainties and nonlinear impulses which is suitable to represent a wide range of impulsive switched systems.
3. We present an optimization problem to achieve the control law as well as the smallest ultimate bound. A GA-LMI approach is also proposed to solve it.

The rest of this paper is as follows. Section 2 introduces the problem formulation. In Sect. 3, we describe the main results. Simulation results for several examples are given in Sect. 4. Finally, conclusions are drawn in Sect. 5.

Notations The symmetric positive (or semi-positive) definite matrix A is indicated by $A>0$ (or $A \geq 0$). The ellipsoid $\mathcal{E}(P, r)$ which is associated with the matrix $P>0$ and the scalar $r>0$ is also given by $\{x \in \mathbb{R}^n | x^{\mathrm{T}} P x \leq r\}$. Moreover, the symbol "*" in matrix inequalities denotes matrix's symmetric parts. Note that if dimensions of some matrices are not explicitly stated, it is assumed that they have appropriate dimensions for algebraic operations.

2 Problem Formulation

The following general model can represent a nonlinear impulsive switched system that includes a finite number of subsystems,

$$\begin{cases} \dot{x}(t) = f_i(x(t), u(t)), & t \neq t_k, \quad k \in \mathbb{N}^+ \\ x(t^+) = g_i(x(t), t), & t = t_k, \quad k \in \mathbb{N}^+ \end{cases}, \tag{1}$$

in which continuous- and discrete-time dynamics of the subsystem i are represented by f_i and g_i, respectively. At the instant t, the active subsystem is determined using the switching signal $\sigma(t) : \mathbb{R}^+ \to \{1, 2, \ldots, m\}$, where m is the number of subsystems. This switching signal composes a strictly increasing sequence of switching instants $\{t_1, t_2, \ldots, t_k, \ldots\}$. Here, we assume that the average dwell time τ_{av} is known, and the state $x(t)$ is left continuous at impulse instant t_k,

$$x(t_k) = x(t_k^-) = \lim_{\zeta \to 0^+} x(t_k - \zeta).$$

Using the local sector nonlinearity approach and choosing a subset of nonlinearities (not all of them) as premise variables, the general model (1) can be expressed as follows:

Rule l: IF $\theta_{1i}(t)$ is Z^l_{1i} and … and $\theta_{n_i i}(t)$ is $Z^l_{n_i i}$

$$\text{THEN} \begin{cases} \dot{x}(t) = f^l_i(x, u, t), & t \neq t_k \\ x(t^+) = g^l_i(x, t), & t = t_k \end{cases}, \tag{2}$$

in which

$$f^l_i = (A^l_i + \Delta A^l_i)x + (B^l_i + \Delta B^l_i)u + f^l_{ci}(x, u, t) + \phi^l_{ci},$$
$$g^l_i = (C^l_i + \Delta C^l_i)x + f^l_{di}(x, t) + \phi^l_{di},$$

and $l = 1, 2, \ldots, r_i$, where r_i denotes the number of rules. Moreover, $\theta_{1i}(t), \theta_{2i}(t)$, …, and $\theta_{n_i i}(t)$ are the premise variables and n_i is the number of them. The membership function of the premise variable $\theta_{ji}(t)$ is specified by the fuzzy set Z^l_{ji}. Also, A^l_i, B^l_i and C^l_i are known and constant matrices. $\Delta A^l_i, \Delta B^l_i, \Delta C^l_i, \phi^l_{ci}$ and ϕ^l_{di} are unknown matrices that represent uncertainties in the local model l of the subsystem i. It is assumed that

$$\left[\Delta A^l_i \;\; \Delta B^l_i \;\; \phi^l_{ci} \;\; \Delta C^l_i\right] = D^l_i F^l_i \left[E^l_{ai} \;\; E^l_{bi} \;\; E^l_{\phi i} \;\; E^l_{ci}\right], \tag{3}$$

where $D^l_i, E^l_{ai}, E^l_{bi}, E^l_{\phi i}$ and E^l_{ci} are known and F^l_i is an unknown time-varying matrix such that $(F^l_i)^T F^l_i \leq I$. Also, we suppose the discrete uncertainty $\phi^l_{di}(t)$ is norm-bounded such that

$$(\phi^l_{di})^T \phi^l_{di} = \left\| \phi^l_{di} \right\|^2 \leq (\varepsilon^l_i)^2, \tag{4}$$

where ε^l_i is a real positive scalar. In addition, $f^l_{ci}(x, u, t)$ and $f^l_{di}(x, t)$ represent known nonlinear dynamics that satisfy the following Lipschitz conditions,

$$(f^l_{ci})^T f^l_{ci} \leq x^T (M^l_{ci})^T M^l_{ci} x + u^T (N^l_i)^T N^l_i u, \tag{5}$$
$$(f^l_{di})^T f^l_{di} \leq x^T (M^l_{di})^T M^l_{di} x, \tag{6}$$

where M^l_{ci}, M^l_{di} and N^l_i are known constant matrices.

The input/output mapping of (2) can be obtained using singleton fuzzifier, product inference, and weighted average defuzzification, as follows:

$$\begin{cases} \dot{x}(t) = \sum_{l=1}^{r_i} h^l_i(\Theta_i(t)) f^l_i(x, u, t), & t \neq t_k \\ x(t^+) = \sum_{l=1}^{r_i} h^l_i(\Theta_i(t_k)) g^l_i(x, t), & t = t_k \end{cases}, \tag{7}$$

where

$$\Theta_i(t) = \left[\theta_{1i}(t), \theta_{2i}(t), \ldots, \theta_{n_i i}(t)\right]^T,$$
$$\mu^l_i(\Theta_i(t)) = \prod_{j=1}^{n_i} Z^l_{ji}(\theta_{ji}(t)),$$
$$h^l_i(\Theta_i(t)) = \mu^l_i(\Theta_i(t)) / \sum_{l=1}^{r_i} \mu^l_i(\Theta_i(t)),$$

and we have for all $t \geq 0$,

$$\mu^l_i \geq 0, \quad \sum_{l=1}^{r_i} \mu^l_i > 0, \quad h^l_i \geq 0, \quad \sum_{l=1}^{r_i} h^l_i = 1.$$

In system (7), we define the following mode-dependent control signal,

$$u(t) = u_{\sigma(t)}(t), \tag{8}$$

where $u_i(t), i \in \{1, 2, \dots, m\}$ is the control input which is active when the subsystem i is active. Utilizing the idea of PDC scheme, we introduce the control input $u_i(t)$ as,

$$\begin{aligned} \text{Rule } l: \quad & \text{IF } \theta_{1i}(t) \text{ is } Z^l_{1i} \text{ and } \dots \text{ and } \theta^l_{n_i i}(t) \text{ is } Z^l_{n_i i} \\ & \text{THEN } u(t) = K^l_i x(t), \end{aligned} \tag{9}$$

where $K^l_i(t) \in \mathbb{R}^{m \times n}$ is a known constant matrix. We can also obtain the mapping of control input (9) as follows:

$$u_i(t) = \sum\nolimits_{l=1}^{r_i} h^l_i\big(\Theta_i(t)\big) K^l_i x(t). \tag{10}$$

Substituting (10) in (7) yields the closed-loop of the subsystem i,

$$\begin{cases} \dot{x}(t) = \sum\nolimits_{l,n=1}^{r_i} h^l_i h^n_i \mathcal{F}^{ln}_i x(t) + \sum\nolimits_{l=1}^{r_i} h^l_i H^l_i, & t \neq t_k \\ x(t^+_k) = \sum\nolimits_{l=1}^{r_i} h^l_i \mathcal{G}^l_i, & t = t_k \end{cases} \tag{11}$$

where

$$\mathcal{F}^{ln}_i = \big(A^l_i + \Delta A^l_i\big) + \big(B^l_i + \Delta B^l_i\big) K^n_i,$$
$$H^l_i = f^l_{ci}\left(x, \sum_{n=1}^{r_i} h^n_i K^n_i x\right) + \phi^l_{ci}(t),$$
$$\mathcal{G}^l_i = \big(C^l_i + \Delta C^l_i\big) x\big(t_k\big) + f^l_{di}\big(x, t_k\big) + \phi^l_{di}\big(t_k\big).$$

Before developing the main results, we offer the following lemmas to improve the readability of the paper.

Lemma 1 (Poznyak et al. 2014) *Given real matrices $S_1, S_2 \in \mathbb{R}^{n \times m}$. If $\Lambda > 0$, the following inequality holds,*

$$S^T_1 S_2 + S^T_2 S_1 \le S^T_1 \Lambda S_1 + S^T_2 \Lambda^{-1} S_2.$$

Lemma 2 (de Souza and Li 1999) *Let C, D, E and F be real matrices with appropriate dimensions and F satisfying $\|F\| \le 1$. We have*

(a) *For any scalar >0,*

$$DFE + E^T F^T D^T \le \gamma DD^T + \gamma^{-1} E^T E.$$

(b) *For any real matrix $P > 0$ and real scalar $\gamma > 0$ such that $P - \gamma DD^T > 0$,*

$$(C + DFE)^T P^{-1} (C + DFE) \le C^T \big(P - \gamma DD^T\big)^{-1} C + \gamma^{-1} E^T E.$$

3 Main Results

In this section, we develop an approach to design a stabilizing control input for the fuzzy impulsive switched system (2). Due to nonvanishing uncertainties ϕ^l_{ci} and ϕ^l_{di}, convergence to the origin is not possible (Khalil 2002). Therefore, this paper focuses on the convergence of state trajectories into a sufficient small ultimate bound. In this way, based on the MLFs approach, Theorem 1 provides stability criteria for the general model (1). These criteria are the enhanced version of the criterions presented in Poznyak et al. (2014), Yang et al. (2015), Zheng and Zhang (2017) and are useful to ensure the ultimate boundedness stability when there are different resources of nonvanishing uncertainties within the continuous- and discrete-time dynamics as well as when the jump functions are nonvanishing near the origin. Then, Theorem 2 uses these proposed conditions to develop fuzzy control signal (9) under the fuzzy system (2).

Theorem 1 *Suppose there exist Lyapunov functions V_i, real positive scalars $\delta \ge 1$, ρ, and $\mu \ge 1$ such that the following conditions hold for all $1 \le i \neq j \le m$,*

$$\dot{V}_i \le -\rho V_i, \quad \forall x \notin \upsilon\big(V_i, 1\big), \tag{12}$$

$$V_j\big(t^+_k\big) \le \mu V_i\big(t_k\big), \quad \forall x \notin \upsilon\big(V_i, 1\big), \tag{13}$$

$$V_j\big(t^+_k\big) \le \delta, \quad \forall x \in \upsilon\big(V_i, 1\big), \tag{14}$$

$$\rho - \ln \mu / \tau_{\text{av}} \ge 0, \tag{15}$$

where τ_{av} is the average dwell time and $\upsilon\big(V_i, 1\big)$ is a closed set as $\upsilon\big(V_i, 1\big) := \big\{x \in \mathbb{R}^n | V_i(x) \le 1\big\}$. Then the impulsive switched system (1) converges to the ultimate bound $\Omega_u := \cup^m_{i=1} \upsilon\big(V_i, \delta\big)$.

Proof Consider a situation where the subsystem i is active and the trajectory is within the $\upsilon\big(V_i, 1\big)$. Since

$$\dot{V}_i \le 0, \quad \forall x \notin \upsilon\big(V_i, 1\big),$$

the trajectory remains in $\upsilon\big(V_i, 1\big)$ until the next switching instant. According to (14), after the impulse instant, the state trajectory remains in Ω_u.

Now suppose another situation, when the subsystem i is alive and the state trajectory $x \in \mathbb{R}^n \backslash \upsilon\big(V_i, 1\big)$. The conditions (11), (12) and (14) ensure that the state trajectory exponentially converges toward the origin. For more details, we may refer to (Yang et al. 2015; Zheng and Zhang 2017). During this convergence, there is a moment that the trajectory reaches $\upsilon\big(V_l, 1\big)$ while the subsystem l is active (l indicates the index of first subsystem for which this condition occurs). As stated at the beginning of the proof, the state trajectory does not leave the ultimate bound Ω_u. This completes the proof. □

As you can see, on a subspace containing the origin, the fulfillment of the common conditions (12) and (13) is not necessary, and a complementary criterion is added to handle the impulse effect on this subspace. Besides, Theorem 1 proves sufficient stability conditions for a general system but does not provide an approach for how to find the Lyapunov functions as well as the appropriate control signal. Theorem 2 eliminates this issue for the closed-loop fuzzy system (11). To this, Theorem 2 uses the common quadratic Lyapunov functions and then reformulates the general conditions proposed in Theorem 1 into the form of matrix inequalities.

Theorem 2 *Consider the fuzzy nonlinear impulsive switched system (11). Suppose there exist real positive definite matrices $L_i > 0$, real matrices W_i^l, real positive scalars $\delta \geq 1$, ρ, $\mu \geq 1$, β_i, $\zeta_i^l, \gamma_{ai}^l, \gamma_{bi}^{ln}, \gamma_{ci}^l, \gamma_{di}^l, \gamma_{\phi i}^l$ where $1 \leq i \neq j \leq m$ and $1 \leq l \leq n \leq r_i$ indicates the index of subsystems and the index of local models, respectively. If the following conditions hold,*

$$\begin{bmatrix} \beta_i I & I \\ * & \beta_i L_i \end{bmatrix} \geq 0, \tag{16}$$

$$\begin{bmatrix} -\Psi_i^{ln} & {}_i\Sigma_{12}^{ln} & {}_i\Sigma_{13}^{ln} & {}_i\Sigma_{14}^{ln} \\ * & {}_i\Sigma_{22}^{ln} & 0 & 0 \\ * & * & {}_i\Sigma_{33}^{ln} & 0 \\ * & * & * & {}_i\Sigma_{44}^{ln} \end{bmatrix} \geq 0, \tag{17}$$

$$\begin{bmatrix} \mu L_i & \sqrt{3} L_i (C_i^l)^T & {}_i\Xi_{13}^l \\ * & L_j - \gamma_{ci}^l D_i^l (D_i^l)^T & 0 \\ * & * & \mathrm{diag}(\gamma_{ci}^l I, I, I) \end{bmatrix} \geq 0, \tag{18}$$

$$\begin{bmatrix} \delta I & \sqrt{3}\beta_j \varepsilon_i^l I & \sqrt{3}\beta_i L_i (C_i^l)^T & {}_i\mathcal{H}_{14}^l \\ * & I & 0 & 0 \\ * & * & L_j - \gamma_{di}^l D_i^l (D_i^l)^T & 0 \\ * & * & * & {}_i\mathcal{H}_{44}^l \end{bmatrix} \geq 0, \tag{19}$$

$$\rho - \ln \mu / \tau_{\mathrm{av}} \geq 0, \tag{20}$$

where

$${}_i\Sigma_{12}^{ln} = \left[L_i (E_{ai}^l)^T \;\; L_i (E_{ai}^n)^T \;\; (E_{bi}^l W_i^n)^T \;\; (E_{bi}^n W_i^l)^T \right],$$

$${}_i\Sigma_{13}^{ln} = \left[\beta_i L_i \sqrt{\left(E_{\phi i}^l\right)^T E_{\phi i}^l} \;\; \beta_i L_i \sqrt{\left(E_{\phi i}^n\right)^T E_{\phi i}^n} \right],$$

$${}_i\Sigma_{14}^{ll} = \left[L_i (M_{ci}^l)^T \;\; L_i (M_{ci}^n)^T \;\; (N_i^l W_i^n)^T \;\; (N_i^n W_i^l)^T \right],$$

$${}_i\Sigma_{22}^{ln} = \mathrm{diag}(\gamma_{ai}^l I, \gamma_{ai}^n I, \gamma_{bi}^{ln} I, \gamma_{bi}^{nl} I),$$

$${}_i\Sigma_{33}^{ln} = \mathrm{diag}\left(\gamma_{\phi i}^l I, \gamma_{\phi i}^n I\right),$$

$${}_i\Sigma_{44}^{ln} = \mathrm{diag}(\zeta_i^l I, \zeta_i^n I, \zeta_i^l I, \zeta_i^n I),$$

$$\begin{aligned} \Psi_i^{ln} &= 2\rho L_i + L_i (A_i^l + A_i^n)^T + (A_i^l + A_i^n) L_i \\ &\quad + (B_i^l W_i^n + B_i^n W_i^l)^T + B_i^l W_i^n + B_i^n W_i^l \\ &\quad + \left(\gamma_{ai}^l + \gamma_{bi}^{ln} + \gamma_{\phi i}^l\right) D_i^l (D_i^l)^T \\ &\quad + \left(\gamma_{ai}^n + \gamma_{bi}^{nl} + \gamma_{\phi i}^n\right) D_i^n (D_i^n)^T + (\zeta_i^l + \zeta_i^n) I, \end{aligned}$$

$${}_i\Xi_{13}^l = \left[\sqrt{3} L_i (E_{ci}^l)^T \;\; \sqrt{3}\beta_j L_i (M_{di}^l)^T \;\; \sqrt{3}\beta_i \beta_j L_i \varepsilon_i^l \right],$$

$${}_i\mathcal{H}_{14}^l = \left[\sqrt{3}\beta_i L_i (E_{ci}^l)^T \;\; \sqrt{3}\beta_i \beta_j L_i (M_{di}^l)^T \right],$$

$${}_i\mathcal{H}_{44}^l = \mathrm{diag}(\gamma_{di}^l I, I),$$

ε_i^l is defined in (3), and τ_{av} is the average dwell time; then, the fuzzy impulsive switched system (11) under the control law (9) converges to $\Omega_u := \cup_{i=1}^m \mathcal{E}(P_i, \delta)$ where $P_i = L_i^{-1}$ and $K_i^l = W_i^l L_i^{-1}$.

Proof Let us choose a set of quadratic Lyapunov function candidates $V_i = x^T P_i x, i \in \{1, 2, \ldots, m\}$. At first, we show that the upper bound of $\dot{V}_i$ is less than $-\rho V_i$ for all $x \notin \upsilon(V_i, 1) = \mathcal{E}(P_i, 1)$. In this way, we prove that the condition (12) holds. The time derivative of V_i along the trajectories of the fuzzy system (11) is

$$\begin{aligned} \dot{V}_i &= \dot{x}^T P_i x + x^T P_i \dot{x} \\ &= x^T \left(\sum_{l,n=1}^{r_i} h_i^l h_i^n \left(\left(\mathcal{F}_i^{ln}\right)^T P_i + P_i \mathcal{F}_i^{ln} \right) \right) x \\ &\quad + \sum_{l=1}^{r_i} h_i^l \left((H_i^l)^T P_i x + x^T P_i H_i^l \right), \end{aligned}$$

where $\mathcal{F}_i^{ln}$ and H_i^l are defined in (11). The above statement includes the uncertainties ΔA_i^l, ΔB_i^l and $\Delta\phi_{ci}^l$ which are introduced in (3). To calculate the upper bound of $\dot{V}_i$, we consider the following inequalities that can be obtained using Lemma 1,

$$\begin{aligned} (\Delta A_i^l)^T P_i + P_i \Delta A_i^l &= (D_i^l F_i^l E_{ai}^l)^T P_i + P_i D_i^l F_i^l E_{ai}^l \\ &\leq \gamma_{ai}^l P_i D_i^l (D_i^l)^T P_i + (\gamma_{ai}^l)^{-1} (E_{ai}^l)^T E_{ai}^l, \\ (\Delta B_i^l K_i^n)^T P_i + P_i \Delta B_i^l K_i^n &= (D_i^l F_i^l E_{bi}^l K_i^n)^T P_i + P_i D_i^l F_i^l E_{bi}^l K_i^n \\ &\leq \gamma_{bi}^{ln} P_i D_i^l (D_i^l)^T P_i + (\gamma_{bi}^{ln})^{-1} (E_{bi}^l K_i^n)^T E_{bi}^l K_i^n, \\ (\Delta\phi_{ci}^l)^T P_i x + x^T P_i \Delta\phi_{ci}^l &= \left(D_i^l F_i^l E_{\phi i}^l\right)^T P_i x + x^T P_i D_i^l F_i^l E_{\phi i}^l \\ &\leq \gamma_{\phi i}^l x^T P_i D_i^l (D_i^l)^T P_i x + \left(\gamma_{\phi i}^l\right)^{-1} \left(E_{\phi i}^l\right)^T E_{\phi i}^l, \end{aligned}$$

where $\gamma_{ai}^l, \gamma_{bi}^l$ and $\gamma_{\phi i}^l$ are positive real scalars. Besides, using Lemma 2 and then noticing (5) and (10), we have

$$\begin{aligned} (f_{ci}^l)^T P_i x + x^T P_i f_{ci}^l &\leq \zeta_i^l x^T P_i P_i x + (\zeta_i^l)^{-1} (f_{ci}^l)^T f_{ci}^l \\ &\leq x^T \Bigg(\zeta_i^l P_i P_i + (\zeta_i^l)^{-1} (M_{ci}^l)^T M_{ci}^l \\ &\quad + (\zeta_i^l)^{-1} \sum_{n=1}^{r_i} \left\{ (h_i^n)^2 (N_i^l K_i^n)^T N_i^l K_i^n \right\} \\ &\quad + (\zeta_i^l)^{-1} \sum_{1 \leq n < p \leq r_i} \Big\{ h_i^n h_i^p \Big((N_i^l K_i^n)^T N_i^l K_i^p \\ &\quad + (N_i^l K_i^p)^T N_i^l K_i^n \Big) \Big\} \Bigg) x, \end{aligned}$$

where ζ_i^l is a real positive scalar. Since

$$\begin{aligned} &(N_i^l K_i^n)^T N_i^l K_i^p + (N_i^l K_i^p)^T N_i^l K_i^n \\ &\quad \leq (N_i^l K_i^n)^T N_i^l K_i^n + (N_i^l K_i^p)^T N_i^l K_i^p, \end{aligned}$$

one can easily conclude

$$\left(f_{ci}^l\right)^T P_i x + x^T P_i f_{ci}^l \leq x^T\Big(\zeta_i^l P_i P_i + \left(\zeta_i^l\right)^{-1}\left(M_{ci}^l\right)^T M_{ci}^l + \left(\zeta_i^l\right)^{-1}\sum\nolimits_{n=1}^{r_i}\left\{h_i^n\left(N_i^l K_i^n\right)^T N_i^l K_i^n\right\}\Big)x.$$

Therefore, we can achieve the upper bound of $\dot{V}_i$ as follows:

$$\dot{V}_i \leq x^T\Big(\sum\nolimits_{l,n=1}^{r_i} h_i^l h_i^n \Upsilon_i^{ln}\Big)x + \sum\nolimits_{l,n=1}^{r_i}\left\{h_i^l h_i^n\left(\gamma_{\phi i}^l\right)^{-1}\left(E_{\phi i}^l\right)^T E_{\phi i}^l\right\},$$

in which

$$\begin{aligned}\Upsilon_i^{ln} =& \left(A_i^l\right)^T P_i + P_i A_i^l + \left(B_i^l K_i^n\right)^T P_i + P_i B_i^l K_i^n \\ &+ \left(\gamma_{ai}^l + \gamma_{bi}^{ln} + \gamma_{\phi i}^l\right) P_i D_i^l \left(D_i^l\right)^T P_i \\ &+ \left(\gamma_{ai}^l\right)^{-1}\left(E_{ai}^l\right)^T E_{ai}^l + \left(\gamma_{bi}^{ln}\right)^{-1}\left(E_{bi}^l K_i^n\right)^T E_{bi}^l K_i^n \\ &+ \zeta_i^l P_i P_i + \left(\zeta_i^l\right)^{-1}\left(M_{ci}^l\right)^T M_{ci}^l \\ &+ \left(\zeta_i^l\right)^{-1}\left(N_i^l K_i^n\right)^T N_i^l K_i^n.\end{aligned}$$

For all $x \notin \mathcal{E}(P_i, 1)$ where $x^T P_i x > 1$, considering the Rayleigh's inequality, we can rewrite the upper bound as:

$$\begin{aligned}\dot{V}_i &\leq x^T\Big(\sum\nolimits_{l,n=1}^{r_i} h_i^l h_i^n\Big(\Upsilon_i^{ln} + \left(\gamma_{\phi i}^l\right)^{-1} P_i\left(E_{\phi i}^l\right)^T E_{\phi i}^l\Big)\Big)x \\ &\leq x^T\Big(\sum\nolimits_{l,n=1}^{r_i} h_i^l h_i^n\Big(\Upsilon_i^{ln} + \left(\gamma_{\phi i}^l\right)^{-1}\beta_i^2 I\left(E_{\phi i}^l\right)^T E_{\phi i}^l\Big)\Big)x.\end{aligned}$$

where β_i is a real positive scalar such that $\beta_i^2 \geq \lambda_{max}(P_i)$. Note that applying the Schur complement lemma to (16) guarantees $\beta_i^2 \geq \lambda_{max}(P_i)$. Hence, the condition (12), i.e., $\dot{V}_i \leq -\rho V_i$, holds if the following matrix inequality holds,

$$\sum\nolimits_{l,n=1}^{r_i} h_i^l h_i^n \mathcal{M}_i^{ln} \geq 0, \tag{21}$$

where

$$\mathcal{M}_i^{ln} = -\rho P_i - \Upsilon_i^{ln} - \left(\gamma_{\phi i}^l\right)^{-1}\beta_i^2 I\left(E_{\phi i}^l\right)^T E_{\phi i}^l.$$

On the other hand, as seen in Tuan et al. (2001), the inequality (21) is fulfilled if the following condition holds

$$\mathcal{M}_i^{ln} + \mathcal{M}_i^{nl} \geq 0, \quad 1 \leq l \leq n \leq r_i, \tag{22}$$

Noticing to the Schur complement of (17) and then pre- and post-multiplying P_i to it, we can conclude that the condition (22) is fulfilled, and as a result, (12) is held.

Now, we show that (13) is satisfied if (18) is met. Taking into account the jump function of the fuzzy impulsive subsystem i in (11), we have

$$V_j\left(t_k^+\right) = x^T\left(t_k^+\right) P_j x\left(t_k^+\right) = \sum\nolimits_{l=1}^{r_i} h_i^l\left(\mathcal{G}_i^l\right)^T P_j \sum\nolimits_{l=1}^{r_i} h_i^l \mathcal{G}_i^l,$$

where $\mathcal{G}_i^l$ is defined in (11). Using the distributive property,

$$V_j\left(t_k^+\right) = \sum\nolimits_{l,n=1}^{r_i} h_i^l h_i^n \Pi_i^{ln} \tag{23}$$

in which

$$\begin{aligned}\Pi_i^{ln} =& x^T\left(C_i^l + \Delta C_i^l\right)^T P_j\left(C_i^n + \Delta C_i^n\right)x \\ &+ \Big(x^T\left(C_i^l + \Delta C_i^l\right)^T P_j\left(f_{di}^n + \phi_{di}^n\right) \\ &+ \left(f_{di}^n + \phi_{di}^n\right)^T P_j\left(C_i^l + \Delta C_i^l\right)x\Big) \\ &+ \Big(\left(f_{di}^l\right)^T P_j \phi_{di}^n + \left(\phi_{di}^n\right)^T P_j f_{di}^l\Big) \\ &+ \Big(\left(f_{di}^l\right)^T P_j f_{di}^n + \left(\phi_{di}^l\right)^T P_j \phi_{di}^n\Big).\end{aligned}$$

Besides, Lemma 1 yields the following inequalities that are used to develop the upper bound of (23),

$$\begin{aligned}& x^T\left(C_i^l + \Delta C_i^l\right)^T P_j f_{di}^n + \left(f_{di}^n\right)^T P_j\left(C_i^l + \Delta C_i^l\right)x \\ &\quad \leq x^T\left(C_i^l + \Delta C_i^l\right)^T P_j\left(C_i^l + \Delta C_i^l\right)x + \left(f_{di}^n\right)^T P_j f_{di}^n, \\ & x^T\left(C_i^l + \Delta C_i^l\right)^T P_j \phi_{di}^n + \left(\phi_{di}^n\right)^T P_j\left(C_i^l + \Delta C_i^l\right)x \\ &\quad \leq x^T\left(C_i^l + \Delta C_i^l\right)^T P_j\left(C_i^l + \Delta C_i^l\right)x + \left(\phi_{di}^n\right)^T P_j \phi_{di}^n, \\ & \left(f_{di}^l\right)^T P_j \phi_{di}^n + \left(\phi_{di}^n\right)^T P_j f_{di}^l \leq \left(f_{di}^l\right)^T P_j f_{di}^l + \left(\phi_{di}^n\right)^T P_j \phi_{di}^n, \\ & \left(f_{di}^l\right)^T P_j f_{di}^n = \frac{1}{2}\left(f_{di}^l\right)^T P_j f_{di}^n + \left(f_{di}^n\right)^T P_j f_{di}^l \\ &\quad \leq \frac{1}{2}\Big(\left(f_{di}^l\right)^T P_j f_{di}^l + \left(f_{di}^n\right)^T P_j f_{di}^n\Big), \\ & \left(\phi_{di}^l\right)^T P_j \phi_{di}^n = \frac{1}{2}\left(\phi_{di}^l\right)^T P_j \phi_{di}^n + \left(\phi_{di}^n\right)^T P_j \phi_{di}^l \\ &\quad \leq \frac{1}{2}\Big(\left(\phi_{di}^l\right)^T P_j \phi_{di}^l + \left(\phi_{di}^n\right)^T P_j \phi_{di}^n\Big),\end{aligned}$$

In addition, we have

$$\begin{aligned}& \sum\nolimits_{l,n=1}^{r_i} h_i^l h_i^n\left(C_i^l + \Delta C_i^l\right)^T P_j\left(C_i^n + \Delta C_i^n\right) \\ &= \sum\nolimits_{l=1}^{r_i}\left\{\left(h_i^l\right)^2\left(C_i^l + \Delta C_i^l\right)^T P_j\left(C_i^l + \Delta C_i^l\right)\right\} \\ &\quad + \sum\nolimits_{1 \leq l < n \leq r_i}\Big\{h_i^l h_i^n\Big(\left(C_i^l + \Delta C_i^l\right)^T P_j\left(C_i^n + \Delta C_i^n\right) \\ &\qquad + \left(C_i^n + \Delta C_i^n\right)^T P_j\left(C_i^l + \Delta C_i^l\right)\Big)\Big\} \\ &\leq \sum\nolimits_{l=1}^{r_i}\left\{\left(h_i^l\right)^2\left(C_i^l + \Delta C_i^l\right)^T P_j\left(C_i^l + \Delta C_i^l\right)\right\} \\ &\quad + \sum\nolimits_{1 \leq l < n \leq r_i}\Big\{h_i^l h_i^n\Big(\left(C_i^l + \Delta C_i^l\right)^T P_j\left(C_i^l + \Delta C_i^l\right) \\ &\qquad + \left(C_i^n + \Delta C_i^n\right)^T P_j\left(C_i^n + \Delta C_i^n\right)\Big)\Big\} \\ &= \sum\nolimits_{l,n=1}^{r_i} h_i^l h_i^n\left(C_i^l + \Delta C_i^l\right)^T P_j\left(C_i^l + \Delta C_i^l\right),\end{aligned}$$

Using the above inequality, since $\sum_{n=1}^{r_i} h_i^n = 1$, we can conclude

$$\sum_{l,n=1}^{r_i} h_i^l h_i^n x^T \left(C_i^l + \Delta C_i^l\right)^T P_j \left(C_i^n + \Delta C_i^n\right) x \leq \sum_{l=1}^{r_i} h_i^l x^T \left(C_i^l + \Delta C_i^l\right)^T P_j \left(C_i^l + \Delta C_i^l\right) x$$

Therefore, we have the upper bound of $V_j\left(t_k^+\right)$ as:

$$\begin{aligned} V_j\left(t_k^+\right) \leq & \, 3\sum_{l=1}^{r_i} h_i^l x^T \left(C_i^l + \Delta C_i^l\right)^T P_j \left(C_i^l + \Delta C_i^l\right) x \\ & + 3\sum_{l=1}^{r_i} h_i^l \left(f_{di}^l\right)^T P_j f_{di}^l \\ & + 3\sum_{l=1}^{r_i} h_i^l \left(\phi_{di}^l\right)^T P_j \phi_{di}^l \\ \leq & \, 3\sum_{l=1}^{r_i} h_i^l x^T \left(C_i^l + \Delta C_i^l\right)^T P_j \left(C_i^l + \Delta C_i^l\right) x \\ & + 3\sum_{l=1}^{r_i} h_i^l \lambda_{max}\left(P_j\right) \left(f_{di}^l\right)^T f_{di}^l \\ & + 3\sum_{l=1}^{r_i} h_i^l \lambda_{max}\left(P_j\right) \left(\phi_{di}^l\right)^T \phi_{di}^l. \end{aligned}$$

According to (4) and (6), we also have

$$\begin{aligned} V_j\left(t_k^+\right) \leq & \, 3\sum_{l=1}^{r_i} h_i^l x^T \left(C_i^l + \Delta C_i^l\right)^T P_j \left(C_i^l + \Delta C_i^l\right) x \\ & + 3\sum_{l=1}^{r_i} h_i^l \lambda_{max}\left(P_j\right) x^T \left(M_{di}^l\right)^T M_{di}^l x \\ & + 3\sum_{l=1}^{r_i} h_i^l \lambda_{max}\left(P_j\right) \left(\varepsilon_i^l\right)^2. \end{aligned}$$

Noticing to Lemma 2 and considering assumption (3) in which $\Delta C_i^l = D_i^l F_i^l E_{ci}^l$, we can conclude

$$V_j\left(t_k^+\right) \leq \sum_{l=1}^{r_i} h_i^l x^T \Gamma_i^l x + \sum_{l=1}^{r_i} h_i^l \Phi_{di}^l, \tag{24}$$

where $\Phi_{di}^l = 3\beta_j^2 \left(\varepsilon_i^l\right)^2$ and

$$\Gamma_i^l = 3\left(C_i^l\right)^T \left(P_j^{-1} - \gamma_{ci}^l D_i^l \left(D_i^l\right)^T\right)^{-1} C_i^l + 3\left(\gamma_{ci}^l\right)^{-1} \left(E_{ci}^l\right)^T E_{ci}^l + 3\beta_j^2 \left(M_{di}^l\right)^T M_{di}^l.$$

Over the subspace $\mathbb{R}^n \backslash \mathcal{E}\left(P_i, 1\right)$ where $x^T P_i x > 1$, considering the Rayleigh's inequality, we can modify (24) as:

$$\begin{aligned} V_j\left(t_k^+\right) & \leq \sum_{l=1}^{r_i} h_i^l x^T \Gamma_i^l x + \sum_{l=1}^{r_i} h_i^l \Phi_{di}^l x^T P_i x \\ & \leq \sum_{l=1}^{r_i} h_i^l x^T \left(\Gamma_i^l + \beta_i^2 \Phi_{di}^l I\right) x, \end{aligned}$$

Hence, (13) is fulfilled if the following condition holds:

$$\sum_{l=1}^{r_i} h_i^l x^T \left(\Gamma_i^l + \beta_i^2 \Phi_{di}^l I\right) x \leq \mu x^T P_i x,$$

or equivalently,

$$\mu P_i - \Gamma_i^l - \beta_i^2 \Phi_{di}^l I \geq 0, \quad 1 \leq l \leq r_i$$

which is satisfied when the condition (18) holds. To check, use the Schur complement of (18) and then pre- and post-multiply it by P_i^{-1}.

Finally, we show that (14) is fulfilled if (19) holds. According to the Schur complement of (19), we have

$$\begin{aligned} & \left(\delta - 3\beta_j^2 \left(\varepsilon_i^l\right)^2\right) I \\ & \geq 3\beta_i^2 P_i^{-1} \left(C_i^l\right)^T \left(P_j^{-1} - \gamma_{ci}^l D_i^l \left(D_i^l\right)^T\right)^{-1} C_i^l P_i^{-1} \\ & \quad + 3\left(\gamma_{ci}^l\right)^{-1} \beta_i^2 P_i^{-1} \left(E_{ci}^l\right)^T E_{ci}^l P_i^{-1} \\ & \quad + 3\beta_j^2 \beta_i^2 P_i^{-1} \left(M_{di}^l\right)^T M_{di}^l P_i^{-1}. \end{aligned}$$

Then, pre- and post-multiplying it by $\beta_i^{-1} P_i$ yields:

$$\begin{aligned} & 3x^T \left(C_i^l\right)^T \left(P_j^{-1} - \gamma_{ci}^l D_i^l \left(D_i^l\right)^T\right)^{-1} C_i^l x \\ & \quad + 3x^T \left(\gamma_{ci}^l\right)^{-1} \left(E_{ci}^l\right)^T E_{ci}^l x + 3x^T \beta_j^2 \left(M_{di}^l\right)^T M_{di}^l x \\ & \leq \left(\delta - 3\beta_j^2 \left(\varepsilon_i^l\right)^2\right) \beta_i^{-2} x^T P_i P_i x^T. \end{aligned}$$

On the other hand, for all $x \in \mathcal{E}\left(P_i, 1\right)$ where $x^T P_i x < 1$,

$$x^T P_i P_i x^T \leq \lambda_{max}\left(P_i\right) x^T P_i x \leq \beta_i^2.$$

Since $\delta - 3\beta_j^2 \left(\varepsilon_i^l\right)^2 \geq 0$, we have for all $1 \leq l \leq r_i$,

$$\begin{aligned} & 3h_i^l x^T \left(C_i^l\right)^T \left(P_j^{-1} - \gamma_{ci}^l D_i^l \left(D_i^l\right)^T\right)^{-1} C_i^l x \\ & \quad + 3h_i^l x^T \left(\gamma_{ci}^l\right)^{-1} \left(E_{ci}^l\right)^T E_{ci}^l x + 3h_i^l x^T \beta_j^2 \left(M_{di}^l\right)^T M_{di}^l x \\ & \leq \left(h_i^l \delta - 3h_i^l \beta_j^2 \left(\varepsilon_i^l\right)^2\right), \end{aligned}$$

and as a result, we can conclude for all $x \in \mathcal{E}\left(P_i, 1\right)$,

$$\sum_{l=1}^{r_i} h_i^l x^T \Gamma_i^l x + \sum_{l=1}^{r_i} h_i^l \Phi_{di}^l \leq \delta,$$

and thus, according to (24), it is verified that $V_j\left(t_k^+\right) \leq \delta$ for all $x \in \upsilon\left(V_i, 1\right)$. This completes the proof. □

Remark This paper assumes that a nonlinear switched system is presented using the introduced T–S fuzzy structure. However, choosing the right subset of nonlinearities as the premise variables plays a crucial role in the feasibility of the conditions stated in the above theorem and is application dependent. However, intelligent selection of this subset may be an open area in this research.

For practical issues, it is considered to achieve a control signal that ensures convergence to a desired ultimate bound. Suppose the desired bound $\mathcal{E}(Q, 1)$ where Q is a given symmetric positive definite matrix. It is enough to have

$$\mathcal{E}\left(P_i, 1\right) \subseteq \mathcal{E}(Q, 1), \quad 1 \leq i \leq m,$$

which is satisfied if the following condition holds (Poznyak et al. 2014),

$$P_i^{-1} = L_i \leq Q^{-1}, \quad 1 \leq i \leq m. \tag{25}$$

To achieve a practical control signal that guarantees convergence to the smallest possible ultimate bound within the desired ellipsoid $\mathcal{E}(Q, 1)$, we should minimize the size of ellipsoids $\mathcal{E}\left(L_i^{-1}, \delta\right)$ for all $i \in \{1, 2, \ldots, m\}$ which is equivalent to minimizing the objective function $\sum_{i=1}^{m} \text{trace}\left\{L_i\right\} + \delta$ subject to Theorem 2 and (25).

Remark To reduce the size of the ellipsoid $\mathcal{E}\left(L_i^{-1}, \delta\right)$, it is convenient to minimize the sum of the squares of the ellipsoid's semiaxes. Length of the ellipsoid's semiaxes also depends on the eigenvalues of L_i^{-1} as well as to the value of δ. So, simultaneous minimization of trace$\left\{L_i\right\}$ and δ is recommended that results in a convex problem. For more details, we may refer to Poznyak et al. (2014).

Remark This optimization problem contains linear and bilinear constraints that is solvable using an augmented Lagrangian solver such as PENBMI (Kocvara and Stingl 2012). However, the efficiency of PENBMI is critical to the selection of initial point.

Note that if the parameters ρ, μ and β_i be known, the constraints of the proposed optimization problem are reduced to linear ones and can be solved by employing the LMI methods. However, selecting proper values for these parameters requires high computational cost by much iteration. For more details, we may refer to Poznyak et al. (2014), Wang and Wu (2016). So, we develop a GA-based approach that searches the validity space of the parameters ρ, μ and β_i and finds a solution for this optimization problem. To find the validity spaces, according to (16), one can obtain $P_i \leq \beta_i^2 I$. So,

$$\mathcal{E}\left(\beta_i^2 I, 1\right) \subseteq \mathcal{E}\left(P_i, 1\right) \subseteq \mathcal{E}(Q, 1).$$

Hence, the radius of the ball $\mathcal{E}\left(\beta_i^2 I, 1\right)$ must be less than the smallest radius of the desired ellipsoid $\mathcal{E}(Q, 1)$,

$$\beta_i^2 \geq 1/r_{min}^{Q}, \quad 1 \leq i \leq m,$$

where r_{min}^{Q} is the length of the smallest axis of $\mathcal{E}(Q, 1)$. Furthermore, the decay rate between two sequential switching instants can be controlled by the parameter ρ. So,

$$\rho \geq 2\alpha_{\text{des.}},$$

where $\alpha_{\text{des.}}$ is the desired decay rate. This can be easily verified in Poznyak et al. (2014), Tanaka and Wang (2001), Yang et al. (2015), Zheng and Zhang (2017). Also, according to (20),

$$\mu \leq \exp\left(\rho \tau_{\text{av}}\right).$$

Considering the above validity spaces, the parameters ρ, μ and β_i are chosen as genes in chromosomes and a random initial population is created. For any chromosome in a population, the value of the selected parameters is known and we just solve the below linear optimization problem utilizing the LMI methods,

$$\begin{aligned} &\text{min:} \quad \textstyle\sum_{i=1}^{m} \text{trace}\left\{L_i\right\} + \delta \\ &\text{s.t.:} \quad (16), (17), (18), (19), (25), \quad L_i \geq 0 \end{aligned} \tag{26}$$

If (26) be feasible for the given chromosome, the value of the objective function and the other decision variables in Theorem 2 are given. It should be mentioned that the feasibility of (26) should be considered as nonlinear constraint during the progress of genetic algorithm. In the current population, after all the chromosomes are evaluated, the next population will be generated using mutation and crossover operations. The evaluation of the current generation and creation of the next one continue until one of the stopping criteria met.

Remark If the proposed constraints in (26) lead to a limited search space, the GA may fail to find the feasible initial population. In these cases, the author suggests to tune GA parameters more accurately, or to select another subset of nonlinearities as premise variables and develop a new T–S fuzzy model with less complex sub-models than the current T–S model.

4 Illustrative Examples

In this section, a numerical example for impulsive switched systems and a practical-motivated impulsive system are given to illustrate the validity of the proposed approach.

Example 1 Consider the following nonlinear impulsive switched system that contains two subsystems,
Subsystem 1:

$$\begin{cases} \begin{cases} 100\dot{x}_1 = f_1^1(x(t), u(t)) + \phi_1(t) \\ 100\dot{x}_2 = f_2^1(x(t), u(t)) + \phi_2(t) \end{cases}, & t \neq t_k \\ \begin{cases} 100x_1^+ = g_1^1(x(t)) + 10\phi_1(t) \\ 100x_2^+ = g_2^1(x(t)) + 10\phi_2(t) \end{cases}, & t = t_k \end{cases},$$

Subsystem 2:

$$\begin{cases} \begin{cases} 100\dot{x}_1 = f_1^2(x(t), u(t)) + \phi_1(t) \\ 100\dot{x}_2 = f_2^2(x(t), u(t)) + \phi_2(t) \end{cases}, & t \neq t_k \\ \begin{cases} 100x_1^+ = g_1^2(x(t)) + 10\phi_1(t) \\ 100x_2^+ = g_2^2(x(t)) + 10\phi_2(t) \end{cases}, & t = t_k \end{cases},$$

with,

$$
\begin{aligned}
f_1^1 &= \left(a_{11}^1 + 3z_1\right)x_1 + \left(a_{12}^1 + 4z_1\right)x_2 + \left(b_{11}^1 + 9z_1\right)u_1 \\
&\quad + \left(b_{12}^1 - 2z_1\right)u_2 + z_1x_1x_2, \\
f_2^1 &= \left(a_{22}^1 - 6z_1\right)x_2 + \left(b_{22}^1 + 3z_1\right)u_2 + z_1x_2^2, \\
g_1^1 &= \left(c_{11}^1 + z_1\right)x_1 + \sin\left(x_1\right), \\
g_2^1 &= \left(c_{22}^1 + z_1\right)x_2 + \sin\left(x_2\right), \\
f_1^2 &= \left(a_{11}^2 + z_2\right)x_1 - \left(a_{12}^2 - z_2\right)x_2 + \left(b_{11}^2 - 10z_2\right)u_1 \\
&\quad - u_2 + z_2x_1x_2, \\
f_2^2 &= 2x_1 + \left(5z_2 + a_{22}^2\right)x_2 - \left(b_{22}^2 + 10z_2\right)u_2 + z_2x_2^2, \\
g_1^2 &= \left(c_{11}^2 + z_2\right)x_1 + \sin\left(x_1\right), \\
g_2^2 &= \left(c_{22}^2 + z_2\right)x_2 + \sin\left(x_2\right),
\end{aligned}
$$

where

$$
\begin{aligned}
&a_{11}^1 = -20 \pm 0.1, \quad a_{12}^1 = 1 \pm 0.1, \quad a_{22}^1 = 16 \pm 0.1, \\
&b_{11}^1 = -21 \pm 0.1, \quad b_{12}^1 = 1 \pm 0.1, \quad b_{22}^1 = 22 \pm 0.1, \\
&c_{11}^1 = c_{22}^1 = c_{11}^2 = c_{22}^2 = 130 \pm 0.1, \\
&a_{11}^2 = +4 \pm 0.1, \quad a_{12}^2 = 2 \pm 0.1, \quad a_{22}^2 = -35 \pm 0.1, \\
&b_{11}^2 = 30 \pm 0.1, \quad b_{22}^2 = 30 \pm 0.1, \\
&\phi_1 = 0.1\sin(t), \quad \phi_2 = 0.1\cos(t), \\
&z_1 = \frac{\sin^2\left(x_1 + 0.5\right)\exp\left(\sin^2\left(x_1 + 0.5\right)\right)}{\exp(1)}, \quad z_2 = \cos^2\left(x_2 + 0.5\right).
\end{aligned}
$$

Noticing the local sector nonlinearity approach, we can represent the above system as (2) where

$$
\begin{aligned}
&Z_1^1 = 1 - z_1, \quad Z_1^2 = z_1, \quad Z_2^1 = 1 - z_2, \quad Z_2^2 = z_2, \\
&A_1^1 = \begin{bmatrix} -0.2 & 0.01 \\ 0 & 0.16 \end{bmatrix}, \quad B_1^1 = \begin{bmatrix} -0.21 & 0.01 \\ 0 & 0.22 \end{bmatrix}, \\
&A_1^2 = \begin{bmatrix} -0.17 & 0.05 \\ 0 & 0.1 \end{bmatrix}, \quad B_1^2 = \begin{bmatrix} -0.12 & -0.01 \\ 0 & 0.25 \end{bmatrix}, \\
&A_2^1 = \begin{bmatrix} 0.04 & -0.02 \\ 0.02 & -0.35 \end{bmatrix}, \quad B_2^1 = \begin{bmatrix} 0.3 & -0.01 \\ 0 & -0.3 \end{bmatrix}, \\
&A_2^2 = \begin{bmatrix} 0.05 & -0.01 \\ 0.02 & -0.3 \end{bmatrix}, \quad B_2^2 = \begin{bmatrix} 0.2 & -0.01 \\ 0 & -0.4 \end{bmatrix}, \\
&f_{c1}^1 = f_{c2}^1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad f_{c1}^2 = f_{c2}^2 = 0.01\begin{bmatrix} x_1x_2 \\ x_2^2 \end{bmatrix}, \\
&\phi_{c1}^1 = \phi_{c1}^2 = \phi_{c2}^1 = \phi_{c2}^2 = 0.001\begin{bmatrix} \sin(t) \\ \cos(t) \end{bmatrix}, \\
&C_1^1 = C_1^2 = C_2^1 = C_2^2 = \begin{bmatrix} 1.3 & 0 \\ 0 & 1.3 \end{bmatrix}, \\
&f_{d1}^1 = f_{d1}^2 = f_{d2}^1 = f_{d2}^2 = 0.01\begin{bmatrix} \sin\left(x_1\right) \\ \sin\left(x_2\right) \end{bmatrix}, \\
&\phi_{d1}^1 = \phi_{d1}^2 = \phi_{d2}^1 = \phi_{d2}^2 = 0.01\begin{bmatrix} \sin(t) \\ \cos(t) \end{bmatrix}.
\end{aligned}
$$

Taking into account the region of validity $\left|x_2\right| \le 10$, we have

$$
\begin{aligned}
&M_{c1}^1 = M_{c2}^1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad M_{c1}^2 = M_{c2}^2 = 0.1\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\
&N_1^1 = N_1^2 = N_2^1 = N_2^2 = 0,
\end{aligned}
$$

and also,

$$
\begin{aligned}
&M_{d1}^1 = M_{d1}^2 = M_{d2}^1 = M_{d2}^2 = 0.01\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\
&\varepsilon_1^1 = \varepsilon_1^2 = \varepsilon_2^1 = \varepsilon_2^2 = 0.001.
\end{aligned}
$$

Besides, we suppose

$$
\begin{aligned}
&D_1^1 = D_1^2 = D_2^1 = D_2^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\
&F_1^1 = F_1^2 = F_2^1 = F_2^2 = \begin{bmatrix} \sin(t) & 0 \\ 0 & \cos(t) \end{bmatrix}, \\
&E_{a1}^1 = E_{a1}^2 = E_{b1}^1 = E_{b1}^2 = 0.001\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \\
&E_{a2}^1 = E_{a2}^2 = 0.001\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \\
&E_{b2}^1 = E_{b2}^2 = 0.001\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \\
&E_{c1}^1 = E_{c1}^2 = E_{c2}^1 = E_{c2}^2 = 0.001\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},
\end{aligned}
$$

Considering $\tau_{av} = 1$ s and $Q = 10^4\mathrm{diag}(1, 1)$, then applying (27), we have

$$
\begin{aligned}
&L_1 = 10^{-5}\begin{bmatrix} 0.2610 & -0.0000 \\ -0.0000 & 0.2610 \end{bmatrix}, \\
&L_2 = 10^{-5}\begin{bmatrix} 0.2632 & -0.0000 \\ -0.0000 & 0.2632 \end{bmatrix}, \\
&\rho = 2.5601, \quad \mu = 12.9369, \quad \delta = 12.1642,
\end{aligned}
$$

and the control signal in the form of (9) where

$$
\begin{aligned}
&K_1^1 = \begin{bmatrix} 19.6731 & -0.0264 \\ 0.2456 & -19.7375 \end{bmatrix}, \\
&K_1^2 = \begin{bmatrix} 20.6003 & 0.4287 \\ 0.0889 & -17.1902 \end{bmatrix}, \\
&K_2^1 = \begin{bmatrix} -19.6903 & 0.4350 \\ 0.2081 & 18.8977 \end{bmatrix}, \\
&K_2^2 = \begin{bmatrix} -20.0361 & 0.4842 \\ 0.3037 & 16.1091 \end{bmatrix}.
\end{aligned}
$$

The state response of this simulation is shown in Fig. 1 where the states converge toward the origin. This convergence as well as the guaranteed ultimate bound is also illustrated in Fig. 2.

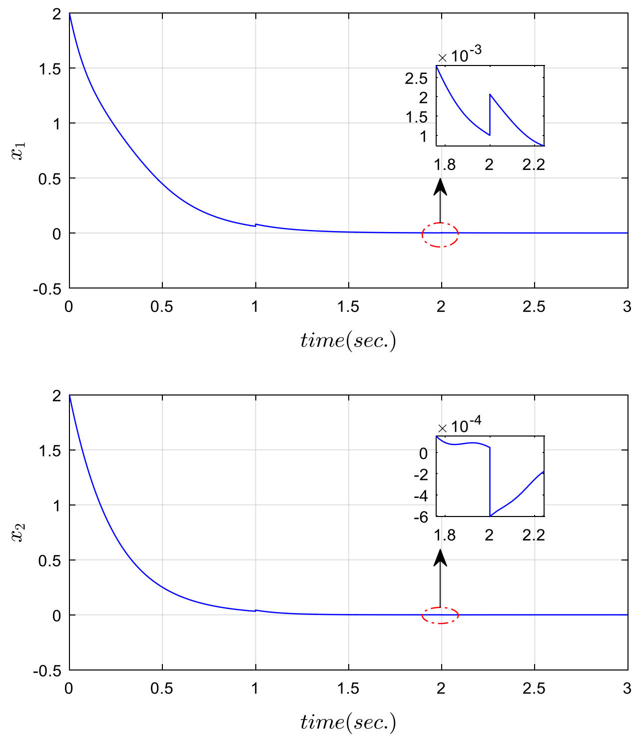

Fig. 1 State response of the presented numerical example

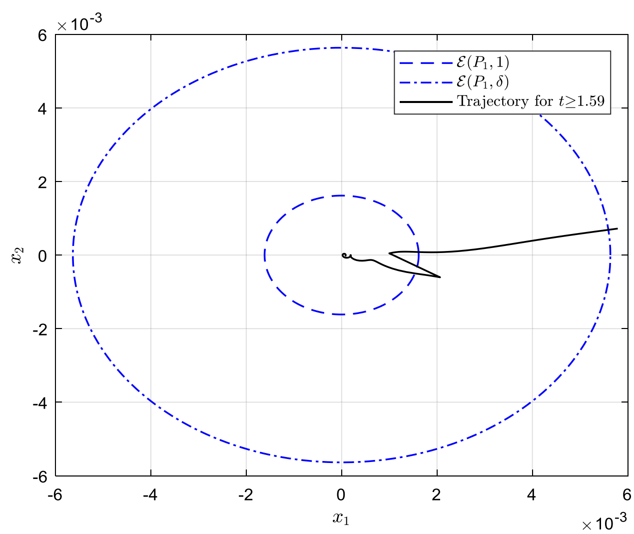

Fig. 2 Trajectory converges to the guaranteed ultimate bound

Example 2 Consider the ball-and-beam system in Fig. 3. Let $x = \left(r, \dot{r}, \theta, \dot{\theta}\right)^T$ be the state vector of the system. Then, the system can be represented by the following state-space model,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ B\left(x_1 x_4^2 - g \sin x_3\right) \\ x_4 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u, \tag{27}$$

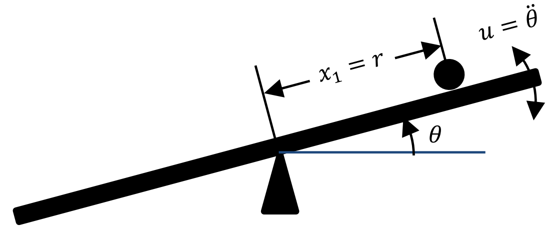

Fig. 3 Ball-and-beam system

where the control signal u is the acceleration of θ and $B = 0.7143$, $g = g_0 + \Delta g = 9.8 \pm 0.01\ \mathrm{m/s^2}$ are parameters. We also consider $x_1 \in [-1, 1], x_3 \in [-1, 1], x_4 \in [-1, 1]$ and x_2 has no limit.

By choosing $\theta\left(x_3\right) = \sin\left(x_3\right)/x_3$ as the premise variable, system (27) is exactly modeled using the T–S fuzzy system (2) where

$$Z^1 = \frac{\theta\left(x_3\right) - \check{\theta}}{\hat{\theta} - \check{\theta}}, \quad Z^2 = \frac{\hat{\theta} - \theta\left(x_3\right)}{\hat{\theta} - \check{\theta}},$$

$$\hat{\theta} = \max_{x_3 \in [-1,1]} \theta\left(x_3\right), \quad \check{\theta} = \min_{x_3 \in [-1,1]} \theta\left(x_3\right),$$

$$A^1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -Bg_0\hat{\theta} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},$$

$$A^2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -Bg_0\check{\theta} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},$$

$$f_c^1 = f_c^2 = B\left[0 \ \ x_1 x_4^2 \ \ 0 \ \ 0\right]^T, \quad \phi_c^1 = \phi_c^2 = 0,$$

in which f_c^1 and f_c^2 satisfy (5) with,

$$M^1 = M^2 = \mathrm{diag}(B, 0, 0, 0),$$

and $N^1 = N^2 = 0$. To represent uncertainties, we consider

$$D^1 = D^2 = I,$$

$$F^1 = F^2 = \mathrm{diag}(\sin(t), \cos(t), \sin(t), \cos(t)),$$

$$E_a^1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -B\Delta g\hat{\theta} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$E_a^2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -B\Delta g\check{\theta} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$E_b^1 = E_b^2 = E_\phi^1 = E_\phi^2 = 0.$$

As seen, (27) has no impulse effect. Hence, we also assume that the velocity of the ball and the velocity of θ are mutated due to collision with an obstacle. So, we also consider,

$$C^1 = C^2 = \mathrm{diag}(1, 1.2, 1, 1.2),$$
$$E_c^1 = E_c^2 = 0.05 \times \mathrm{diag}(0, 1, 0, 1),$$
$$f_d^1 = f_d^2 = 0.01\begin{bmatrix} 0 \\ \sin(x_2) \\ 0 \\ \sin(x_4) \end{bmatrix}, \quad \phi_d^1 = \phi_d^2 = 0,$$

where $M_d^1 = M_d^2 = 0.01\mathrm{diag}(0, 1, 0, 1), \quad \varepsilon^1 = \varepsilon^2 = 0.$

Considering the impulse sequence $\{1, 2, 3, \dots\}$ and $Q = 10^3 \times I$, then solving the optimization problem (26), the gains of the fuzzy controller (9) and other obtained data are as follows:

$$K^1 = \begin{bmatrix} 165.109 & 99.078 & -163.040 & -18.480 \end{bmatrix},$$
$$K^2 = \begin{bmatrix} 161.991 & 97.901 & -156.747 & -18.801 \end{bmatrix},$$
$$L = 10^{-3}\begin{bmatrix} 0.0142 & -0.0319 & -0.0025 & -0.0241 \\ * & 0.1188 & 0.0368 & 0.0636 \\ * & * & 0.0309 & -0.0559 \\ * & * & * & 0.9016 \end{bmatrix},$$
$$\rho = 2.6100, \quad \mu = 13.5960, \quad \delta = 2.9727.$$

Figure 4 shows the state response of the impulsive ball-and-beam system (27) under the fuzzy control signal (9) for a period of 4 s with impulse instances.

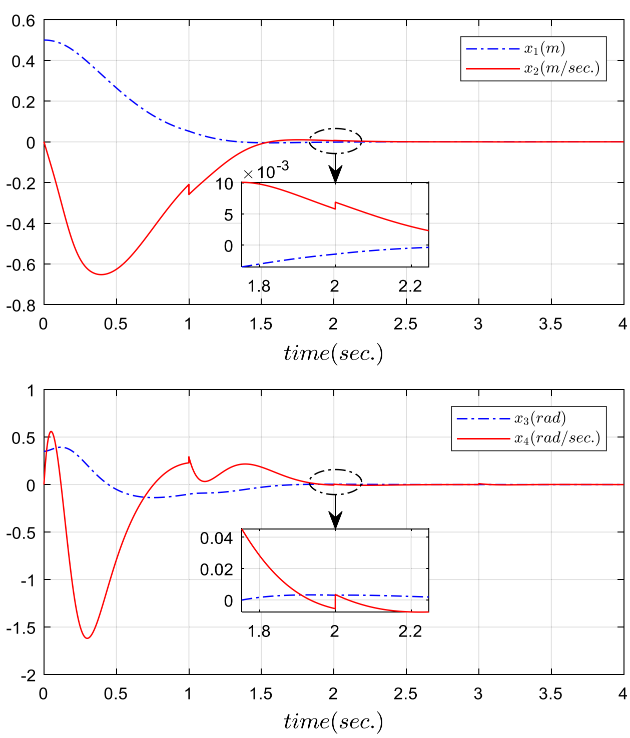

Fig. 4 State response of the impulsive ball-and-beam system under the proposed fuzzy control law

5 Conclusion

This paper investigates the ultimate boundedness stabilization of nonlinear impulsive switched systems. At first, we propose the sufficient stability conditions for the general model of the nonlinear impulsive switched systems with uncertainties. Due to different resources of uncertainties in continuous- and discrete-time dynamics for real-world processes, convergence to the origin is difficult. Therefore, contrary to the most of the existing literature, we modify the stability criteria, such that they ensure convergence to an ultimate bound rather than to the origin. Secondly, to design a fuzzy control law for a general nonlinear impulsive switched system which is described using a novel T–S fuzzy model, the proposed novel criteria are then reformulated as matrix inequalities. Besides, we introduce an optimization problem to find gains of the controller along with the smallest ultimate bound. Since some of these matrix inequalities are not linear, we also introduce a GA-LMI approach to solve the optimization problem.

In addition, introducing a T–S fuzzy system that includes uncertain nonlinear local models is another contribution of this paper. Unlike the traditional T–S fuzzy systems where all local models are linear, the introduced model needs a fewer number of rules to describe a nonlinear system, and as a result, it severely reduces the number and the complexity of stabilization criteria. Furthermore, this model can also be applied to nonlinear impulsive system without any switching effect or to switched systems without impulsive behaviors. As the next step in this research, we hope to study delay in the states and the asynchronous control signal.

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