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Integrated Fault-Tolerant Stabilization Control for Satellite Attitude Systems with Actuator and Sensor Faults

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Abstract

In this paper, the problem of integrated fault-tolerant stabilization control is studied for the attitude systems of a rigid satellite with both actuator and sensor faults. Firstly, a virtual observer is designed for the faulty attitude systems of rigid satellite in order to estimate the unknown actuator and sensor faults. Because the virtual observer includes unmeasurable information of the attitude systems, the real observer is then presented. On this basis, an integral sliding-mode fault-tolerant stabilization control approach is proposed by using the estimated fault information, and it not only suppresses the disturbance with a disturbance attenuation level γ, but also eliminates the effects of actuator and sensor faults. Finally, the effectiveness of the proposed fault-tolerant stabilization scheme is demonstrated in the attitude systems of a rigid satellite subject to the time-varying actuator and sensor faults.

Keywords Fault-tolerant stabilization · Rigid satellite · Actuator and sensor faults

1 Introduction

The attitude control of satellites is an essential subsystem and has important applications for some space missions such as pointing and formation flying (Zhang et al. 2013; Doroshin 2017; Chunodkar and Akella 2014). Therefore, the corresponding attitude control design has received significant attentions in the past decade, and many effective methods has been investigated for satellites. For example, Ma et al. (2014) propose an adaptive control strategy for the attitude systems of a rigid satellite with external disturbance by using quaternion feedback to compensate actuator failure. In Xiao et al. (2017a), a nonlinear estimator-based control approach is presented for the attitude stabilization of flexible satellites with unknown flexible vibrations. A novel control approach with simple structure is presented in Xiao et al. (2017b) to perform attitude tracking maneuver for rigid satellites with external disturbance torque and uncertain inertia parameters. A fractional-order attitude controller with memory ability is proposed in Guo et al. (2018) to guarantee the attitude stability of a rigid satellite. It is noted that the Euler angle-based attitude model and the quaternion-based attitude model are used to describe the satellite attitude motion in the above-described studies. The Euler angle-based attitude model could intuitively describe the satellite attitude motion, but it has two disadvantages: (i) to calculate a number of complex trigonometric functions and (ii) the attitude angle singularity problem. Taking into account these defects, the Euler angle-based attitude system model is only suitable for small-angle deviation stability control and general maneuver control of in-orbit satellite (Zhang et al. 2013). The quaternion-based attitude model could avoid the attitude angle singular problem and no complicated trigonometry. However, it is difficult to guarantee the normalization conditions of quaternion due to calculation errors, and the quaternion-based attitude system model is suitable for large-angle maneuver control or attitude capture of in-orbit satellite (Xiao et al. 2017a). In past few decades, people have focused on the attitude control of in-orbit satellites that are operating at the small angles. In recent years, with the continuous development of space technology, people are paying more and more attention to the attitude control problem in the process of satellite large-angle maneuvering (Xiao et al. 2017b).

It is well known that the complex satellite attitude control systems inevitably manifest various types of faults in a harsh space environment, which may result in the degraded

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system behavior or even system failure. Therefore, the fault diagnosis (FD) technique for satellite attitude control systems has been an important research topic in recent years (Gao et al. 2013; Cheng et al. 2016; Wang et al. 2015). Generally speaking, the FD strategy divided into three essential tasks: fault detection, fault isolation and fault estimation. It is worth pointing out that all these tasks can be accomplished by fault estimation. Meanwhile, in order to eliminate the impact of faults, fault-tolerant control (FTC) has also become important. Thus, fault estimation and FTC has attracted considerable attentions in recent years. For instance, two observer design method are proposed in Xiao et al. (2015) for the attitude stabilization of a satellite with external disturbances, reaction wheel faults, actuator saturation and unavailable angular velocity. An equivalent idea is introduced to design a sliding-mode observer in Zhang et al. (2017), such that the amplitudes of the faults and disturbances can be detected, identified and estimated. Zhao et al. (2016a, b) focus on the robust control problem of a class of uncertain nonlinear systems, and the considered uncertainty parameters are all dealt with by sliding-mode control technique. An active FTC approach is proposed in Gao et al. (2017) for a class of flexible spacecraft attitude systems with Lipschitz nonlinearity and sensor fault. A dynamic output feedback-based fault-tolerant controller is designed to guarantees the attitude stabilization of flexible spacecraft in sensor faulty case. In Yin et al. (2017b), the fault-tolerant stabilization problem is investigated for a class of nonlinear Markovian jump systems, and a sliding-mode observer design scheme is proposed with the aim of eliminating the effects of unknown actuator fault to the overall closed-loop systems. A descriptor sliding-mode observer approach is proposed in Yin et al. (2017a) for linear continuous-time switched systems, such that the state of the system, disturbances, sensor and actuator faults could be estimated simultaneously. Chen et al. (2016) investigate the fault-tolerant control problem for a class of stochastic Markovian jump systems, and a novel sliding-mode control strategy is proposed by utilizing the state estimation information generated by an adaptive sliding-mode observer, which stabilize the faulty closed-loop systems. In Hamayun et al. (2013), a novel FTC approach scheme is proposed for a linear systems by incorporating integral sliding modes, unknown input observers and a fixed control allocation scheme. In Hamayun et al. (2017), an integrated integral sliding-mode fault-tolerant control scheme is proposed for the linear parameter-varying systems in the output feedback framework. The feedback and observer gains are designed using linear matrix inequalities approach to ensure closed-loop stability nominally and in the situation of actuator faults/failures. In Zhao et al. (2017), a novel fault-tolerant control scheme is developed for a linear systems with time-varying actuator fault, an adaptive unknown input observer is exploited to estimate state and fault simultaneously, and the adaptive output integral-type sliding mode is designed to attenuate unknown bounded uncertainty and tolerate time-varying fault. In Liu et al. (2018), the finite-time sliding-mode controller is designed for a class of singular time-delay system with sensor failures and uncertain nonlinearities, and it guarantees that the closed-loop system is nonsingular sliding-mode finite-time boundedness in both reaching phase and sliding-motion phase. However, it is noted that most of the FTC schemes described above are only solved to the stabilization/tracking problem of all kinds of linear/nonlinear control systems in actuator faulty case or in sensor faulty case, and the fault-tolerant stabilization or tracking control problem of linear/nonlinear in both actuator and sensor faulty cases is always an open problem in the field of automatic control and has not been fully investigated yet, which remains challenging and motivates us to do this study.

Motivated by the preceding discussions, we consider a rigid-body satellite in orbit that runs at a small angle, and an integrate fault-tolerant stabilization control scheme is proposed for the satellite attitude systems described by an Euler angle-based model in both actuator and sensor faulty case. The main contributions of this study could be described by the following: (i) A novel fault estimate observer (FEO) is designed for the considered faulty attitude systems to estimate the sates of system, actuator faults and sensor faults, simultaneously. (ii) An integral sliding-mode fault-tolerant attitude stabilization control scheme is developed by utilizing the information from observer, and it not only attenuates the disturbance with a disturbance attenuation level γ , but also eliminates the influence of both actuator and sensor faults. (iii) Lyapunov stability theory is applied to the closed-loop stability analysis, and the gain matrices for controller and observer could be obtained through the linear matrix inequalities technique. In the future works, we hope that the main results obtained in this paper could be further improved and applied to fault-tolerant attitude control of satellite in large-angle maneuvering stage.

This paper is organized as follows. In Sect. 2, the attitude dynamics of rigid satellite with actuator and sensor faults is describes firstly; then, the control objectives of this paper and some necessary assumptions are formulated. In Sect. 3, a novel observer is designed to estimate states of system, actuator and sensor faults. The main results of integrate sliding-mode FTC strategy are presented, and the closed-loop stability analysis is carried out using Lyapunov stability theory. A numerical example is provided to verity the effectiveness of the proposed fault-tolerant stabilization method in Sect. 4. Finally, the conclusion and some remarks are given in Sect. 5.

Notations The symbol $\star$ stands for the terms induced by symmetry. The symbol $\|\cdot\|$ represents the norm of the matrix.

2 Problem Statement

This paper consider an attitude system model of satellite with reaction wheel as actuator and running in a circular earth orbit, and the attitude dynamic model of rigid satellite is governed by the following differential equation (Gao et al. 2013)

$$J\dot{\omega} + \omega^{\times} J\omega = 3\omega_0^2 \zeta^{\times} J\zeta + u + d \tag{1}$$

where $J = \mathrm{diag}\{J_1, J_2, J_3\}$ is the symmetric inertia matrix of rigid satellite, $\omega = [\omega_x, \omega_y, \omega_z]^{\mathrm{T}}$ is the angular velocity in a body-fixed reference frame and ω_0 is the constant orbital rate. $u = [u_1, u_2, \ldots, u_m]^{\mathrm{T}}$ is the control torque vector generated from the reaction flywheels and $m > 3$ is the number of reaction flywheel. $d = [d_1^{\mathrm{T}}, d_2^{\mathrm{T}}, d_3^{\mathrm{T}}]^{\mathrm{T}}$ is the external disturbance torque, and $\zeta = [-\sin\theta,\ \sin\varphi\cos\theta,\ \cos\varphi\cos\theta]^{\mathrm{T}}$ is the known nonlinear term. φ, θ and ψ are the roll, pitch and yaw attitude angles, respectively. The skew-symmetric matrix $\omega^{\times}$ is given by

$$\omega^{\times} = \begin{bmatrix} 0 & -w_z & w_y \\ w_z & 0 & -w_x \\ -w_y & w_x & 0 \end{bmatrix}$$

For small attitude angles, the attitude dynamics (1) could be rewritten as

$$J_1\dot{\omega}_x - (J_2 - J_3)\omega_y\omega_z + 3\omega_0^{\times}(J_2 - J_3)\varphi = u + d_1 \tag{2}$$

$$J_2\dot{\omega}_y - (J_3 - J_1)\omega_z\omega_x + 3\omega_0^{\times}(J_1 - J_3)\theta = u + d_2 \tag{3}$$

$$J_3\dot{\omega}_z - (J_1 - J_2)\omega_x\omega_y = u + d_3 \tag{4}$$

The kinematic differential equation of an orbiting rigid satellite can be described as Gao et al. (2013)

$$\begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \frac{1}{\cos\theta} \begin{bmatrix} \cos\theta & \sin\varphi\sin\theta & \cos\varphi\sin\theta \\ 0 & \cos\varphi\cos\theta & -\sin\varphi\cos\theta \\ 0 & \sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} + \frac{\omega_0}{\cos\theta} \begin{bmatrix} \sin\psi \\ \cos\theta\cos\psi \\ \sin\theta\sin\psi \end{bmatrix} \tag{5}$$

and this nonlinear system (5) is linearized as

$$\dot{\varphi} = \omega_x + \omega_0\psi,\ \dot{\theta} = \omega_y + \omega_0,\ \dot{\psi} = \omega_z - \omega_0\varphi \tag{6}$$

Choosing the state variable $x = [\varphi, \theta, \psi, \omega_x, \omega_y, \omega_z]^{\mathrm{T}}$, the satellite attitude systems model with parameter uncertainty could be described as

$$\begin{cases} \dot{x} = (A + \Delta A(t))x + g(x,t) + Bu(t) + Dd(t) \\ y = Cx \end{cases} \tag{7}$$

where $\Delta A(t)$ represents the parameter uncertainty of state matrix with appropriate dimension and

$$A = \begin{bmatrix} 0 & 0 & \omega_0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -\omega_0 & 0 & 0 & 0 & 0 & 1 \\ -3\omega_0^2 J_1^{-1}(J_2 - J_3) & 0 & 0 & 0 & 0 & 0 \\ 0 & -3\omega_0^2 J_2^{-1}(J_1 - J_3) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ J_1^{-1} & 0 & 0 \\ 0 & J_2^{-1} & 0 \\ 0 & 0 & J_3^{-1} \end{bmatrix}$$

$$g(x,t) = [0,\ \omega_0,\ 0,\ J_1^{-1}(J_2 - J_3)\omega_y\omega_z,\ J_2^{-1}(J_3 - J_1)\omega_z\omega_x,\ J_3^{-1}(J_1 - J_2)\omega_x\omega_y],$$

$$C = I_6,\quad D = \begin{bmatrix} 0 & 0 & 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 \end{bmatrix}^{\mathrm{T}}$$

There the matrices A and B always satisfy Assumption 2.

To the objective of fault-tolerant attitude stabilization control, the actuator and sensor fault model should be considered in the attitude systems (7). Thus, the faulty satellite attitude systems could be transformed into the following form

$$\begin{cases} \dot{x}(t) = (A + \Delta A(t))x(t) + g(x,t) + Bu(t) + F_{\mathrm{a}} f_{\mathrm{a}}(t) + Dd(t) \\ y_f(t) = Cx(t) + f_{\mathrm{s}}(t) \end{cases} \tag{8}$$

where $f_{\mathrm{a}}(t)$ denotes the unknown actuator fault, which may be loss of effectiveness fault or increased bias torque. $f_{\mathrm{s}}(t)$ denotes the unknown sensor fault, which may be random drift or bias fault. F_{a} is known constant matrix of compatible dimension.

To get the purpose of integrated FTC, the following main objectives should be achieved in this study.

1. For the faulty attitude systems (8), design a novel augmented observer to estimate states of the system, actuator and sensor faults simultaneously.

2. For the faulty attitude systems (8), design an integral sliding-mode controller, based only on output measurements and the obtained fault estimation information, and it could maintain the closed-loop stability in the face of some actuator and sensor faults.

Based on some recent results obtained in Lan and Patton (2018), Li et al. (2018), Gao (2015) and Youssef et al. (2017),

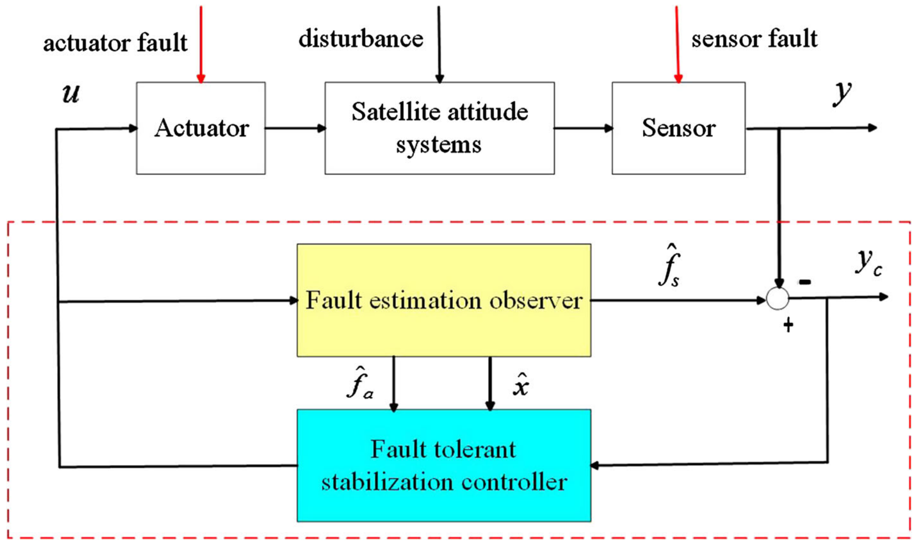

Fig. 1 Integrate fault-tolerant stabilization control scheme for satellite attitude systems

an integrated fault-tolerant stabilization control scheme is developed for the attitude systems of rigid satellite with both actuator and sensor faults in this paper, and the block diagram of FTC is shown in Fig. 1.

Throughout the remainder of this paper, some necessary assumptions and lemmas are given in this position.

Assumption 1 Liu et al. (2018) The satellite is assumed to be running in a small-angle maneuver; therefore, the nonlinearity $g(x,t)$ is assumed to locally Lipschitz in x, i.e., $g(0,t) \neq 0$, and the following inequality holds:

$$\|g(x,t) - g(\tilde{x},t)\| \leq \beta\|x - \tilde{x}\|$$

where β is the known Lipschitz constant, x is the state of the satellite attitude systems and $\tilde{x}$ is the error between the state and its estimate.

Assumption 2 The nonlinearity $g(x,t)$ is bounded.

Assumption 3 The pair (A, C) is observable, and the pair (A, B) is controllable.

Assumption 4 Rahmani (2017)The uncertainty matrix $\Delta A(t)$ is norm-bounded and could be written as

$$\Delta A(t) = M\Gamma(t)H$$

where M and H are known matrices with appropriate dimensions and $\Gamma(t)$ is an unknown matrix satisfying $\Gamma(t)\Gamma^{\mathrm{T}}(t) \leq I$.

Assumption 5 The external disturbance, unknown faults and their time derivative are all bounded.

Lemma 1 *Liu et al. (2018) Let Z, W and diagonal matrix R be the real matrices of appropriate dimensions with $|R| \leq S$, where S is an known real constant matrix, and there exists a scalar $\epsilon > 0$, such that*

$$ZRW + (ZRW)^{\mathrm{T}} \leq \epsilon ZSZ^{\mathrm{T}} + \epsilon^{-1}W^{\mathrm{T}}SW$$

3 Integrate Fault-Tolerant Stabilization Control

3.1 Fault Estimation Observer Design

In this section, a novel observer will be designed to provide the estimation of system state, actuator fault and sensor fault simultaneously, which has the following form

$$\begin{cases} \dot{\hat{\xi}}(t) = \mathcal{A}\hat{\xi}(t) + \mathcal{B}u(t) + \mathcal{L}y_f(t) + F_1 g(\hat{x},t) \\ \hat{z}(t) = \hat{\xi}(t) + \mathcal{C}y(t) \end{cases} \tag{9}$$

where $\hat{\xi}(t)$ is an auxiliary variable, matrices $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{L}$ are observer parameters to be determined later and $\hat{z}(t)$ is the estimation for $x(t)$, $f_{\mathrm{a}}(t)$ and $f_{\mathrm{s}}(t)$. To design observer (9), the faulty attitude systems (8) could be reformulated as

$$\begin{cases} E_1\dot{z}(t) = \bar{A}z(t) + \bar{\Delta}(t)z(t) + g(x,t) + Bu(t) + Dd(t) \\ y(t) = E_2 z(t) \end{cases} \tag{10}$$

where $z(t) = \begin{bmatrix} x^{\mathrm{T}}(t) & f_{\mathrm{s}}^{\mathrm{T}}(t) & f_{\mathrm{a}}^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$, $\bar{A} = \begin{bmatrix} A & 0_{6\times6} & F_{\mathrm{a}} \end{bmatrix}$, $\bar{\Delta}(t) = \begin{bmatrix} \Delta A(t) & 0_{6\times6} & 0_{6\times6} \end{bmatrix}$, $E_1 = \begin{bmatrix} I_{6\times6} & 0_{6\times6} & 0_{6\times6} \end{bmatrix}$, $E_2 = \begin{bmatrix} C & I_{6\times6} & 0_{6\times6} \end{bmatrix}$, $E_3 = \begin{bmatrix} B_1 C & 0_{6\times6} & I_{6\times6} \end{bmatrix}$, and $B_1 \in R^{6\times6}$ is a full-rank matrix; then, it is clear that $\operatorname{rank}\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = 12$, which means it is a full-rank matrix and its inverse exists.

Let $F = \begin{bmatrix} F_1 | & F_2 | & F_3 \end{bmatrix} = \left[\begin{array}{c|c|c} I_{6\times6} & 0_{6\times6} & 0_{6\times6} \\ -C & I_{6\times6} & 0_{6\times6} \\ -B_1C & 0_{6\times6} & I_{6\times6} \end{array}\right]$; then, it can be easily known that

$$\begin{cases} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \begin{bmatrix} F_1 & F_2 & F_3 \end{bmatrix} = I_{18\times18} \\ \begin{bmatrix} F_1 & F_2 & F_3 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = I_{18\times18} \end{cases} \tag{11}$$

which means $\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}^{-1} = \begin{bmatrix} F_1 & F_2 & F_3 \end{bmatrix}$.

Multiplying F_1 to both sides of (10) yields that

$$\begin{aligned} F_1E_1\dot{z}(t) &= F_1\bar{A}z(t) + F_1\bar{\Delta}(t)z(t) + F_1Bu(t) \\ &\quad + F_1g(x,t) + F_1Dd(t) \end{aligned} \tag{12}$$

and by using fact (11), namely $F_1E_1 + F_2E_2 + F_3E_3 = I_{18\times18}$, we have

$$\begin{aligned} \dot{z}(t) &= F_1\bar{A}z(t) + F_1\bar{\Delta}(t)z(t) + F_1Bu(t) + F_1g(x,t) \\ &\quad + F_1Dd(t) + (F_2E_2 + F_3E_3)\dot{z}(t) \end{aligned} \tag{13}$$

Consider the following virtual observer

$$\begin{aligned} \dot{\hat{z}}(t) &= F_1\bar{A}\hat{z}(t) + F_1Bu(t) + F_1g(\hat{x},t) \\ &\quad + (F_2 + F_3B_1)E_2\dot{z}(t) + L(y - E_2\hat{z}(t)) \end{aligned} \tag{14}$$

where $L = [L_1^{\mathrm{T}},\ L_2^{\mathrm{T}},\ L_3^{\mathrm{T}}]^{\mathrm{T}}$ is an unknown gain matrix to be determined later.

Define the following estimation error variable as $e(t) = z(t) - \hat{z}(t)$, and the error dynamics of observer could be obtained as follows

$$\begin{aligned} \dot{e}(t) &= (F_1\bar{A} - LE_2)e(t) + F_1\bar{\Delta}(t)z(t) + F_1\tilde{g} \\ &\quad + \bar{F}\dot{z}(t) + \Omega Dd(t) \\ &= (F_1\bar{A} - LE_2)e(t) + F_1M\Gamma(t)Hx(t) \\ &\quad + F_1\tilde{g} + \bar{F}\dot{z}(t) + \Omega Dd(t) \end{aligned} \tag{15}$$

where $\bar{F} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -B_1 & 0 \\ 0 & 0 & I_{6\times6} \end{bmatrix}$, $\Omega = [I_{6\times6},\ -C^{\mathrm{T}},\ -C^{\mathrm{T}}B_1^{\mathrm{T}}]^{\mathrm{T}}$, $\tilde{g} = g(x,t) - g(\hat{x},t)$.

It is noted that the virtual observer (14) may have poor estimation ability because of the term $(F_2 + F_3B_1)E_2\dot{z}(t)$. By defining $\hat{\xi}(t) = \hat{z}(t) - (F_2 + F_3B_1)E_2z(t)$, such that observer (14) could be further rewritten as

$$\begin{aligned} \dot{\hat{\xi}}(t) &= F_1\bar{A}\hat{z}(t) + F_1Bu(t) + F_1g(\hat{x},t) + L(y(t) - E_2\hat{z}(t)) \\ &= (F_1\bar{A} - LE_2)\hat{z}(t) + F_1Bu(t) + F_1g(\hat{x},t) + Ly(t) \\ &= (F_1\bar{A} - LE_2)\hat{\xi}(t) + F_1Bu(t) + F_1g(\hat{x},t) \\ &\quad + (L + (F_1\bar{A} - LE_2)(F_2 + F_3B_1))y(t) \end{aligned} \tag{16}$$

Then, the fault estimation observer (9) is designed with the following parameters

$$\begin{aligned} &\mathcal{A} = F_1\bar{A} - LE_2,\ \ \mathcal{B} = F_1B,\ \ \mathcal{C} = F_2 + F_3B_1, \\ &\mathcal{L} = L + (F_1\bar{A} - LE_2)(F_2 + F_3B_1) \end{aligned} \tag{17}$$

Observer (16) is the similar to the designed observer (9) with the parameters (17). Finally, we can obtain $\hat{z}(t) = \hat{\xi}(t) + (F_2 + F_3B_1)E_2z(t) = \hat{\xi}(t) + (F_2 + F_3B_1)y(t)$.

Remark 1 The satellite attitude systems considered in this study are suffered from actuator fault, sensor fault, parameter uncertainty and external disturbance simultaneously, so it is difficult to design an observer which estimates the system state and fault with the relatively high accuracy. In this section, a virtual observer is firstly designed for the faulty attitude systems, and the main purpose of the virtual observer is to improve the observation precision of the system state variables and unknown faults. However, it is hard to realize the virtual observer because it includes inaccessible information, namely $(F_2 + F_3B_1)E_2\dot{z}(t)$. As a result, an actual observer based on the virtual observer is derived by eliminating the time derivative term, such that the target of state and fault estimation can be achieved.

Remark 2 It is noted that most existing fault estimation approach, such as Zhang et al. (2013) and Gao et al. (2013, 2017), could only estimate the individual actuator fault or the individual sensor fault. To address this defect, a novel fault estimation observer design approach is proposed in this study, which could estimate the state of the plant, actuator fault and sensor fault simultaneously and could be regarded as the supplement of the existed fault estimation results.

3.2 Integral Sliding-Mode Controller Design

In this section, an integrated sliding-mode fault-tolerant control scheme is proposed using only the measurable output signal, which guarantees the attitude stability of the faulty satellite. Here, an integral sliding-mode surface by using the output feedback information is designed as

$$s(t) = G(y_c(t) - y_c(0)) - \int_0^t u_l(\tau)\mathrm{d}\tau \tag{18}$$

where $G = (CB)^+ - Y(I - CB(CB)^+)$ with an arbitrary matrix $Y \in R^{3\times6}$ and $(CB)^\dagger = ((CB)^\mathrm{T}CB)^{-1}(CB)^\mathrm{T}$. $y_c = y - \hat{f}_s = Cx + e_{f_s}$ is the attitude output signal after sensor fault compensation, and $e_{f_s} = f_s - \hat{f}_s$ is the sensor fault estimation error. u_l is a control input to guarantee disturbance attenuation.

The designed sliding-mode controller has the following form

$$u = u_n + u_l \tag{19}$$

where the linear component is $u_l = -K\hat{x} - \bar{W}\hat{f}_a$ with a design matrix $K \in R^{3\times6}$ and $W = B^\dagger F_a$. $\hat{x}$ and $\hat{f}_a$ are the estimates of the system state and actuator fault, respectively.

The nonlinear component u_n is designed as

$$u_n = \begin{cases} -(\hat{\rho}(t) + \varphi + \kappa)\dfrac{s}{\|s\|}, & s \neq 0 \\ 0, & s = 0 \end{cases}$$

where $\hat{\rho}(t)$ is the estimation of unknown scalar $\rho(t) = \|GCD\|\|d(t)\| + \|GCF_a\|\|f_a\| + \|G\|\|\dot{e}_{f_s}\|$, which has the following adaptive updated algorithm $\dot{\hat{\rho}} = \frac{1}{\kappa}\|s\|$ and φ, κ are the design constants.

Differentiating (18) with respect to time and substituting $\dot{x}(t)$ from (8) obtains

$$\begin{aligned}\dot{s} &= GC(A + \Delta A)x + GCg(x,t) + GCBu + GCF_a f_a \\ &\quad + GCDd + G\dot{e}_{fs} - u_l\end{aligned} \tag{20}$$

In this position, the first main results of this study are given by Theorem 1.

Theorem 1 *Under Assumptions 1–4, suppose the integral sliding-mode surface function is designed described by (18), and the sliding-mode controller u described by (19) guarantees that the sliding-motion is driven on the sliding surface, namely* $s = \dot{s} = 0$.

Proof Select the following Lyapunov function

$$V_s = \frac{1}{2}(s^\mathrm{T}s + \kappa\tilde{\rho}^2) \tag{21}$$

where $\tilde{\rho} = \rho - \hat{\rho}$ is the error variable of parameter estimation. Then, it is derived directly that

$$\begin{aligned}\dot{V}_s &= s^\mathrm{T}\dot{s} - \kappa\tilde{\rho}\dot{\hat{\rho}} \\ &= s^\mathrm{T}(GC(A + \Delta A)x \\ &\quad + GCg(x,t) + GCBu + GCF_a f_a + GCDd \\ &\quad + G\dot{e}_{fs} - u_l) - \kappa\tilde{\rho}\dot{\hat{\rho}} \\ &= s^\mathrm{T}(GC(A + \Delta A)x + GCg(x,t) \\ &\quad + GCBu + GCF_a f_a + GCDd \\ &\quad + G\dot{e}_{fs} + K\hat{x} + \bar{W}\hat{f}_a) - \kappa\tilde{\rho}\dot{\hat{\rho}} \\ &= s^\mathrm{T}(GC(A + \Delta A)x + GCg(x,t) \\ &\quad + GCBu + GCF_a f_a + GCDd \\ &\quad + G\dot{e}_{fs} + K\hat{x} + \bar{W}\hat{f}_a) - \tilde{\rho}\|s\| \\ &\leq \|s\|(\|GCA\|\|x\| + \|GCM\|\|H\|\|x\| + \beta\|GC\|\|x\| \\ &\quad + \|GCD\|\|d(t)\| \\ &\quad + \|GCF_a\|\|f_a\| + \|G\|\|\dot{e}_{fs}\|) - \|s\|(\rho + \varphi + \kappa) \\ &\leq (a\|x\| - \varphi - \kappa)\|s\|\end{aligned} \tag{22}$$

where $a = \|GCA\| + \|GCM\|\|H\| + \beta\|GC\|$. If the parameter is chosen to be $\varphi > a\nu$ with a given scalar $\nu > 0$ and φ should be chosen large enough, then reaching the sliding-mode condition $s^\mathrm{T}\dot{s} \leq -\kappa\|s\|$ is satisfied for $\{x : \|x\| \leq \nu\}$. Then the sliding-mode controller (19) guarantees that $s(t) = \dot{s}(t) = 0$ for all $t \geq 0$. This completes the proof. □

3.3 The Closed-Loop Stability Analysis

In this section, the stability analysis of the closed-loop attitude systems in both actuator and sensor faulty cases is given. Suppose that the system has been successfully controlled to stay in the sliding-mode surface, and the equivalent control law is designed as

$$u_{eq} = -GCAx - GCg(x,t) - GCDd + u_l \tag{23}$$

Substituting (23) into (8) obtains the closed-loop attitude systems as

$$\begin{aligned}\dot{x} &= (A + \Delta A)x + B(-GCAx - GCg(x,t) - GCDd + u_l) \\ &\quad + F_a f_a + g(x,t) + Dd \\ &= ((I - BGC)A - BK)x + Te + \Delta Ax + (I - BGC)Dd \\ &= (\Phi A - BK)x + Te + \Delta Ax + \Phi Dd\end{aligned} \tag{24}$$

where $\Phi = I - BGC$ and $T = \begin{bmatrix} BK & 0 & F_a \end{bmatrix}$.

Meanwhile, consider the error dynamics of observer (15), and the following augmented closed-loop attitude systems are given by

$$\begin{cases} \dot{x} = (\Phi A - BK)x + Te + \Delta Ax + \Phi Dd \\ \dot{e} = (F_1\bar{A} - LE_2)e + F_1 M\Gamma(t)Hx + F_1\tilde{g} + \bar{F}\dot{z} + \Omega Dd \end{cases} \tag{25}$$

In this position, the second main results of this study are given by Theorem 2.

Theorem 2 *Under Assumptions 1–5, for a given positive constant γ, if there exist symmetric positive definite matrices $\bar{Q}$, Q, P_{11}, P_{22}, P_{33}, and matrices $\bar{J}$, $\bar{L}_1$, $\bar{L}_2$, $\bar{L}_3$, and satisfying the following linear matrix inequality*

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & 0 & \Xi_{14} & 0 & \Xi_{16} & QM & 0 & 0 & 0 \\ \star & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} & \Xi_{26} & 0 & \sqrt{\alpha}P_{11}M & 0 & 0 \\ \star & \star & \Xi_{33} & \Xi_{34} & \Xi_{35} & \Xi_{36} & 0 & 0 & \sqrt{\mu}P_{22}CM & 0 \\ \star & \star & \star & \Xi_{44} & \Xi_{45} & \Xi_{46} & 0 & 0 & 0 & \sqrt{\varsigma}P_{33}B_1CM \\ \star & \star & \star & \star & -I & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & -\gamma^2 I & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & -I & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & -I & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & -I & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & -I \end{bmatrix} < 0 \tag{26}$$

where $\Xi_{11} = Q\Phi A + A^{\mathrm{T}}\Phi^{\mathrm{T}}Q - B\bar{J} - \bar{J}^{\mathrm{T}}B^{\mathrm{T}} + H^{\mathrm{T}}H + \alpha^{-1}H^{\mathrm{T}}H - \mu^{-1}H^{\mathrm{T}}H - \varsigma^{-1}H^{\mathrm{T}}H + I$, $\Xi_{12} = B\bar{J}$, $\Xi_{14} = QF_{\mathrm{a}}$, $\Xi_{16} = Q\Phi D_1$, $\Xi_{22} = P_{11}A + A^{\mathrm{T}}P_{11} - \bar{L}_1C - C^{\mathrm{T}}\bar{L}_1^{\mathrm{T}} + \beta^2 + I$, $\Xi_{23} = -\bar{L}_1 - A^{\mathrm{T}}C^{\mathrm{T}}P_{22} - C^{\mathrm{T}}\bar{L}_2^{\mathrm{T}}$, $\Xi_{24} = P_{11}F_{\mathrm{a}} - A^{\mathrm{T}}C^{\mathrm{T}}B_1^{\mathrm{T}}P_{33} - C^{\mathrm{T}}\bar{L}_3^{\mathrm{T}}$, $\Xi_{25} = P_{11}$, $\bar{\Xi}_{26} = P_{11}D_1$, $\Xi_{33} = -\bar{L}_2 - \bar{L}_2^{\mathrm{T}} + I$, $\Xi_{34} = -P_{22}CF_{\mathrm{a}} - \bar{L}_3$, $\Xi_{35} = -P_{22}C$, $\Xi_{36} = -P_{22}D_2 - P_{22}CD_1$, $\Xi_{44} = -P_{33}B_1CF_{\mathrm{a}} - F_{\mathrm{a}}^{\mathrm{T}}C^{\mathrm{T}}B_1^{\mathrm{T}}P_{33} + I$, $\Xi_{45} = -P_{33}B_1C - C^{\mathrm{T}}B_1^{\mathrm{T}}P_{33}$, $\Xi_{46} = P_{33}D_3 - P_{33}B_1CD_1$.

Then the augmented closed-loop attitude systems is asymptotically stable with a disturbance attenuation level γ. Furthermore, the observer gain $L = [L_1^{\mathrm{T}},\ L_2^{\mathrm{T}},\ L_3^{\mathrm{T}}]^{\mathrm{T}}$ and the controller gain K are given by $L_1 = P_{11}^{-1}\bar{L}_1$, $L_2 = P_{22}^{-1}\bar{L}_2$, $L_3 = P_{33}^{-1}\bar{L}_3$, $K = \bar{Q}^{-1}\bar{J}$ where Q is a matrix satisfying

$$QB = B\bar{Q} \tag{27}$$

Proof Consider Lyapunov function candidate

$$V_{ex} = x^{\mathrm{T}}Qx + e^{\mathrm{T}}P_1e \tag{28}$$

where $Q = Q^{\mathrm{T}} > 0$, $P_1 = \mathrm{diag}\{P_{11}, P_{22}, P_{33}\} = P_1^{\mathrm{T}} > 0$.

Calculating the time derivative of V_{ex}, it is obtained as

$$\begin{aligned} \dot{V}_{ex} &= 2x^{\mathrm{T}}Q\dot{x} + 2e^{\mathrm{T}}P_1\dot{e} = 2x^{\mathrm{T}}Q\dot{x} + 2e_x^{\mathrm{T}}P_{11}\dot{e}_x + 2e_{fs}^{\mathrm{T}}P_{22}\dot{e}_{fs} \\ &\quad + 2e_{fa}^{\mathrm{T}}P_{33}\dot{e}_{fa} \\ &= 2x^{\mathrm{T}}Q[(\Phi A - BK)x + BKe_x + F_{\mathrm{a}}e_{fa} \\ &\quad + M\Gamma(t)Hx + \Phi Dd] \\ &\quad + 2e_x^{\mathrm{T}}P_{11}[(A - L_1C)e_x - L_1e_{fs} + F_{\mathrm{a}}e_{fa} \\ &\quad + M\Gamma(t)Hx + \tilde{g} + Dd] \\ &\quad + 2e_{fs}^{\mathrm{T}}P_{22}[(-CA - L_2C)e_x - L_2e_{fs} - CF_{\mathrm{a}}e_{fa} \\ &\quad - CM\Gamma(t)Hx - C\tilde{g} - CDd - B_1\dot{f}_{\mathrm{s}}] \\ &\quad + 2e_{fa}^{\mathrm{T}}P_{33}[(-B_1CA - L_3C)e_x - L_3e_{fs} - B_1CF_{\mathrm{a}}e_{fa} \\ &\quad - B_1CM\Gamma(t)Hx - B_1C\tilde{g} + \dot{f}_{\mathrm{a}} - B_1CDd] \end{aligned} \tag{29}$$

Here, we have a transformation to $2x^{\mathrm{T}}QMF(t)Hx$; therefore, the following inequality could be derived

$$\begin{aligned} &2x^{\mathrm{T}}QMF(t)Hx \\ &\quad = -(M^{\mathrm{T}}Qx - F(t)Hx)^{\mathrm{T}}(M^{\mathrm{T}}Qx - F(t)Hx) \\ &\qquad + x^{\mathrm{T}}QMM^{\mathrm{T}}Qx + x^{\mathrm{T}}H^{\mathrm{T}}F^{\mathrm{T}}(t)F(t)Hx \\ &\quad \le x^{\mathrm{T}}QMM^{\mathrm{T}}Qx + x^{\mathrm{T}}H^{\mathrm{T}}Hx \end{aligned} \tag{30}$$

Under Assumptions 1, 3 and Lemma 1, the following inequalities are easily obtained

$$\tilde{g}^{\mathrm{T}}\tilde{g} \le \beta^2 e_x^{\mathrm{T}}e_x \tag{31}$$

$$\begin{aligned} 2e_x^{\mathrm{T}}P_{11}M\Gamma(t)Hx &\le \alpha e_x^{\mathrm{T}}P_{11}MM^{\mathrm{T}}P_{11}e_x \\ &\quad + \alpha^{-1}x^{\mathrm{T}}H^{\mathrm{T}}Hx \end{aligned} \tag{32}$$

$$\begin{aligned} -2e_{f_s}^{\mathrm{T}}P_{22}CM\Gamma(t)Hx &\le \mu e_{f_s}^{\mathrm{T}}P_{22}CMM^{\mathrm{T}}C^{\mathrm{T}}P_{22}e_{f_s} \\ &\quad - \mu^{-1}x^{\mathrm{T}}H^{\mathrm{T}}Hx \end{aligned} \tag{33}$$

$$\begin{aligned} &-2e_{fa}^{\mathrm{T}}P_{33}B_1CM\Gamma(t)Hx \\ &\quad \le \varsigma e_{fa}^{\mathrm{T}}P_{33}B_1CMM^{\mathrm{T}}C^{\mathrm{T}}B_1^{\mathrm{T}}P_{33}e_{fa} \\ &\qquad - \varsigma^{-1}x^{\mathrm{T}}H^{\mathrm{T}}Hx \end{aligned} \tag{34}$$

Substituting (30)–(34) into (29), we have

$$\begin{aligned}
\dot{V}_{ex} \le\ & 2x^{\mathrm{T}}Q(\Phi A - BK)x + 2x^{\mathrm{T}}QBKe_x + 2x^{\mathrm{T}}QF_{\mathrm{a}}e_{f_{\mathrm{a}}}\\
& + x^{\mathrm{T}}QMM^{\mathrm{T}}Qx + x^{\mathrm{T}}H^{\mathrm{T}}Hx + 2x^{\mathrm{T}}Q\Phi D_1\bar{d}\\
& + 2e_x^{\mathrm{T}}P_{11}(A - L_1C)e_x - 2e_x^{\mathrm{T}}P_{11}L_1e_{f_{\mathrm{s}}}\\
& + 2e_x^{\mathrm{T}}P_{11}F_{\mathrm{a}}e_{f_{\mathrm{a}}} + \alpha e_x^{\mathrm{T}}P_{11}MM^{\mathrm{T}}P_{11}e_x\\
& + \alpha^{-1}x^{\mathrm{T}}H^{\mathrm{T}}Hx + 2e_x^{\mathrm{T}}P_{11}\tilde{g} + 2e_x^{\mathrm{T}}P_{11}D_1\bar{d}\\
& + 2e_{f_{\mathrm{s}}}^{\mathrm{T}}P_{22}(-CA - L_2C)e_x - 2e_{f_{\mathrm{s}}}^{\mathrm{T}}P_{22}L_2e_{f_{\mathrm{s}}}\\
& - 2e_{f_{\mathrm{s}}}^{\mathrm{T}}P_{22}CF_{\mathrm{a}}e_{f_{\mathrm{a}}}\\
& + \mu e_{fs}^{\mathrm{T}}P_{22}CMM^{\mathrm{T}}C^{\mathrm{T}}P_{22}e_{fs} - \mu^{-1}x^{\mathrm{T}}H^{\mathrm{T}}Hx\\
& - 2e_{fs}^{\mathrm{T}}P_{22}C\tilde{g} - 2e_{fs}^{\mathrm{T}}P_{22}CD_1\bar{d}\\
& - 2e_{fs}^{\mathrm{T}}P_{22}D_2\bar{d}\\
& + 2e_{fa}^{\mathrm{T}}P_{33}(-B_1CA - L_3C)e_x - 2e_{fa}^{\mathrm{T}}P_{33}L_3e_{fs}\\
& - 2e_{fa}^{\mathrm{T}}P_{33}B_1CF_{\mathrm{a}}e_{fa}\\
& + \varsigma e_{fa}^{\mathrm{T}}P_{33}B_1CMM^{\mathrm{T}}C^{\mathrm{T}}B_1^{\mathrm{T}}P_{33}e_{fa} - \varsigma^{-1}x^{\mathrm{T}}H^{\mathrm{T}}Hx\\
& - 2e_{fa}^{\mathrm{T}}P_{33}B_1C\tilde{g} + 2e_{fa}^{\mathrm{T}}P_{33}D_3\bar{d}\\
& - 2e_{fa}^{\mathrm{T}}P_{33}B_1CD_1\bar{d} + \beta^2e_x^{\mathrm{T}}e_x - \tilde{g}^{\mathrm{T}}\tilde{g}\\
\le\ & \eta^{\mathrm{T}}\bar{\Xi}\eta - x^{\mathrm{T}}x - e_x^{\mathrm{T}}e_x - \mathrm{e}_{fs}^{\mathrm{T}}e_{fs} - e_{fa}^{\mathrm{T}}e_{fa} + \gamma^2\bar{d}^{\mathrm{T}}\bar{d}\\
\le\ & \eta^{\mathrm{T}}\bar{\Xi}\eta - \xi^{\mathrm{T}}\xi + \gamma^2\bar{d}^{\mathrm{T}}\bar{d}
\end{aligned} \tag{35}$$

where $\eta = [x^{\mathrm{T}},\ e_x^{\mathrm{T}},\ \mathrm{e}_{fs}^{\mathrm{T}},\ e_{fa}^{\mathrm{T}},\ \tilde{g}^{\mathrm{T}},\ \bar{d}^{\mathrm{T}}]^{\mathrm{T}}, \xi = [x^{\mathrm{T}},\ e_x^{\mathrm{T}},\ \mathrm{e}_{fs}^{\mathrm{T}},\ e_{fa}^{\mathrm{T}}]^{\mathrm{T}}$, $D_1 = [D,\ 0_{6\times6},\ 0_{6\times6}]$, $D_2 = [0_{6\times3},\ B_1,\ 0_{6\times6}]$, $\bar{d} = [d^{\mathrm{T}},\ \dot{f}_{\mathrm{s}}^{\mathrm{T}},\ \dot{f}_{\mathrm{a}}^{\mathrm{T}}]$, $D_3 = [0_{6\times3},\ 0_{6\times6},\ I_{6\times6}]$ and

$$\bar{\Xi} = \begin{bmatrix} \bar{\Xi}_{11} & \bar{\Xi}_{12} & 0 & \bar{\Xi}_{14} & 0 & \bar{\Xi}_{16}\\ \star & \bar{\Xi}_{22} & \bar{\Xi}_{23} & \bar{\Xi}_{24} & \bar{\Xi}_{25} & \bar{\Xi}_{26}\\ \star & \star & \bar{\Xi}_{33} & \bar{\Xi}_{34} & \bar{\Xi}_{35} & \bar{\Xi}_{36}\\ \star & \star & \star & \bar{\Xi}_{44} & \bar{\Xi}_{45} & \bar{\Xi}_{46}\\ \star & \star & \star & \star & -I & 0\\ \star & \star & \star & \star & \star & -\gamma^2 I \end{bmatrix}$$

with $\bar{\Xi}_{11} = Q\Phi A + A^{\mathrm{T}}\Phi^{\mathrm{T}}Q - QBK - K^{\mathrm{T}}B^{\mathrm{T}}Q + QMM^{\mathrm{T}}Q + H^{\mathrm{T}}H + \alpha^{-1}H^{\mathrm{T}}H - \mu^{-1}H^{\mathrm{T}}H - \varsigma^{-1}H^{\mathrm{T}}H + I$, $\bar{\Xi}_{12} = QBK$, $\bar{\Xi}_{14} = QF_{\mathrm{a}}$, $\bar{\Xi}_{16} = Q\Phi D_1$, $\bar{\Xi}_{22} = P_{11}A + A^{\mathrm{T}}P_{11} - P_{11}L_1C - C^{\mathrm{T}}L_1^{\mathrm{T}}P_{11} + \alpha P_{11}MM^{\mathrm{T}}P_{11} + \beta^2 + I$, $\bar{\Xi}_{23} = -P_{11}L_1 - A^{\mathrm{T}}C^{\mathrm{T}}P_{22} - C^{\mathrm{T}}L_2^{\mathrm{T}}P_{22}$, $\bar{\Xi}_{24} = P_{11}F_{\mathrm{a}} - A^{\mathrm{T}}C^{\mathrm{T}}B_1^{\mathrm{T}}P_{33} - C^{\mathrm{T}}L_3^{\mathrm{T}}P_{33}$, $\bar{\Xi}_{25} = P_{11}$, $\bar{\Xi}_{26} = P_{11}D_1$, $\bar{\Xi}_{33} = -P_{22}L_2 - L_2^{\mathrm{T}}P_{22} + \mu P_{22}CMM^{\mathrm{T}}C^{\mathrm{T}}P_{22} + I$, $\bar{\Xi}_{34} = -P_{22}CF_{\mathrm{a}} - P_{33}L_3$, $\bar{\Xi}_{35} = -P_{22}C$, $\bar{\Xi}_{36} = -P_{22}D_2 - P_{22}CD_1$, $\bar{\Xi}_{44} = -P_{33}B_1CF_{\mathrm{a}} - F_{\mathrm{a}}^{\mathrm{T}}C^{\mathrm{T}}B_1^{\mathrm{T}}P_{33} + \varsigma P_{33}B_1CMM^{\mathrm{T}}C^{\mathrm{T}}B_1^{\mathrm{T}}P_{33} + I$, $\bar{\Xi}_{45} = -P_{33}B_1C - C^{\mathrm{T}}B_1^{\mathrm{T}}P_{33}$ $\bar{\Xi}_{46} = P_{33}D_3 - P_{33}B_1CD_1$.

By the Schur complement to the inequality (26), $\bar{\Xi} < 0$ is easily obtained. Therefore, integrating both sides of (35) from 0 to ∞ satisfies

$$\int_0^{\infty} \dot{V}_{ex} + \xi^{\mathrm{T}}\xi\, dt < \gamma^2 \int_0^{\infty} \bar{d}^{\mathrm{T}}\bar{d}\ \mathrm{d}t \tag{36}$$

Then under zero initial conditions, the inequality (36) could be rewritten as

$$\int_0^{\infty} \xi^{\mathrm{T}}\xi\, dt < \gamma^2 \int_0^{\infty} \bar{d}^{\mathrm{T}}\bar{d}\ \mathrm{d}t \tag{37}$$

Therefore, the augmented closed-loop systems (25) are asymptotically stable with a given disturbance attenuation level γ. This completes the proof. □

Remark 3 In Li et al. (2018), Gao (2015) and Lee et al. (2018),some observer-based FTC strategies are proposed for linear system with sensor and actuator faults, but the unmatched parameter uncertainty and Lipschitz nonlinearity function are not included in the considered linear system, Therefore, the FTC approach designed in Li et al. (2018), Gao (2015) and Lee et al. (2018) has the poor robustness. On the basis of these results, the unmatched parameter uncertainty and Lipschitz nonlinearity function are introduced in this study and an integrated FTC approach is designed for the attitude systems of rigid satellite with parameter uncertainty, Lipschitz nonlinearity, external disturbance, actuator and sensor faults in this paper. Compared with the FTC result developed in Li et al. (2018), Gao (2015) and Lee et al. (2018), the integrated FTC approach could guarantee the attitude systems asymptotically stable with a disturbance attenuation level γ and has better robustness.

Remark 4 Compared with the sliding-mode controller by using the state feedback information designed in Yin et al. (2017b) and Liu et al. (2018), an integrate sliding-mode controller is designed for the attitude systems of rigid satellite by only utilizing the output feedback information. It is well known that the output information is easier to obtain than the state information in practical engineering application; thus, the integral sliding-mode controller developed in this paper has a wider application value than the sliding-mode control scheme proposed in Yin et al. (2017b) and Liu et al. (2018).

Remark 5 Note that the equality constraint (27) is difficult to solve using MATLAB LMI toolbox. However, by using the method presented in Youssef et al. (2017), for a positive scalar $\bar{\gamma}$, it can be converted into the following optimization problem to solve using the LMI toolbox

$$\text{Minimize } \bar{\gamma} \text{ subject to (26) and } \begin{bmatrix} \bar{\gamma}I & QB - B\bar{Q}\\ B^{\mathrm{T}}Q^{\mathrm{T}} - \bar{Q}^{\mathrm{T}}B^{\mathrm{T}} & \bar{\gamma}I \end{bmatrix} > 0 \tag{38}$$

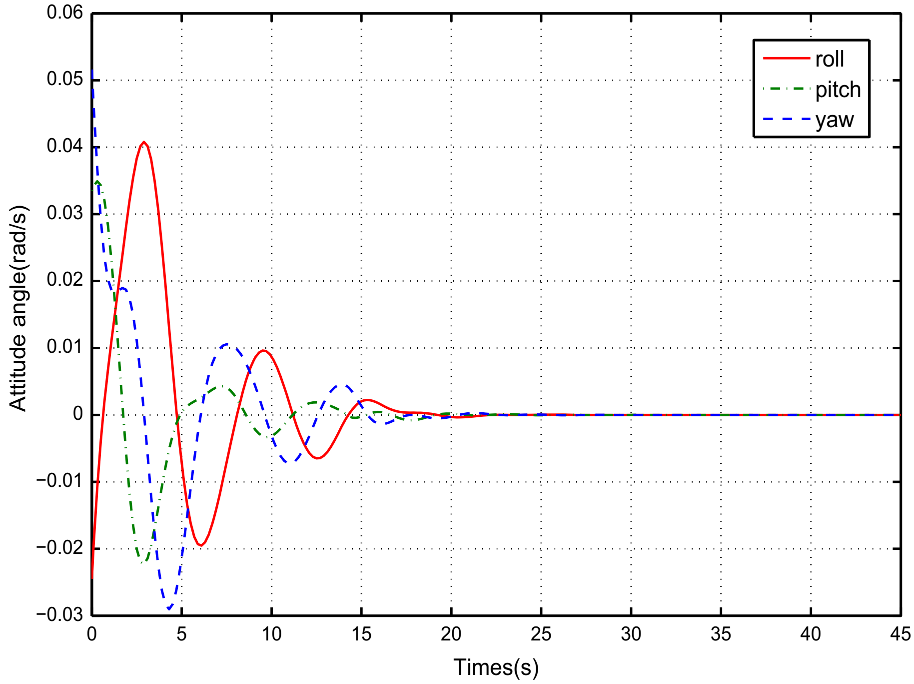

Fig. 2 Attitude angle output curve in fault-free case using attitude stabilization approach in Xiao et al. (2017a)

4 Simulation Example

To demonstrate the effectiveness of the proposed integrated fault-tolerant stabilization control scheme, simulation results of a satellite attitude control systems are presented in this section.

The inertia matrix is assumed as $J = \text{diag}\{18.55, 20.49, 23.56\}$ and the orbital angular velocity as $\omega_0 = 0.0014\,\text{rad/s}$. The initial attitude angles are selected as $\varphi(0) = -0.0245$, $\theta(0) = 0.0335$ and $\psi(0) = 0.0516$, and the initial angular velocities are selected as $\omega_x = 0.0316$, $\omega_y = 0.0424$ and $\omega_z = 0.056$.

The satellite has three reaction flywheels as actuators at each principle axis, gyroscopes and star sensors as sensors are used to measure angular velocity and attitude angle respectively. In the severe space environment, actuators and sensors are unavoidably susceptible to possible faults. In the simulation, we consider additive time-varying bias actuator fault and sensor fault occurred in the satellite attitude systems. For actuator fault, it is assumed that the second actuator is prone to fault, and other actuators are fault-free; the actuator vector is defined as $f_{\text{a}} = [0,\ f_{\text{a2}},\ 0,\ 0,\ 0,\ 0]^{\text{T}}$ with

$$f_{\text{a2}}(t) = \begin{cases} 0\ \ \text{rad/s}, & 0 \le t < 12\ \text{s} \\ 0.04\ \ \text{rad/s}, & 12\ s \le t < 20\ \text{s} \\ 0.05\sin(t)\ \ \text{rad/s}, & t \ge 30\ \text{s} \end{cases}$$

For sensor fault, it is assumed that only the first star sensor is prone to time-varying fault, while other sensors are fault-free. Define the sensor fault vector as $f_{\text{s}} = [f_{\text{s1}},\ 0,\ 0,\ 0,\ 0,\ 0]^{\text{T}}$ with

$$f_{\text{s1}}(t) = \begin{cases} 0\ \ \text{rad}, & 0 \le t < 12\ \text{s} \\ 0.03\sin(t)\ \ \text{rad}, & t \ge 12\ \text{s} \end{cases}$$

In the design of integral sliding-mode controller (19), the adaptive gain parameter κ is selected as 1.2. By means of MATLAB linear matrix inequality toolbox, and choosing $\alpha = 0.5$, $\mu = 0.2$, $\beta = 0.5$, $\gamma = 0.7$, $\bar{\gamma} = 0.9$, the unknown observer gain matrix and the integral sliding-mode controller gain matrix could be solved from the inequality (26) and (37), which are given by the next page.

To illustrate the superiority of the integrated FTC approach proposed this study, some necessary simulation comparisons are given in the following. A normal attitude stabilization control approach of satellite is proposed in Xiao et al. (2017a), which can be used to stabilize the rigid satellite attitude systems considered in this study, and the attitude angle output curve and the control input torque curve are depicted in Figs. 2 and 3, respectively. It is easily seen that the attitude stabilization control approach developed in Xiao et al. (2017a) guarantees that the closed-loop attitude systems in actuator and sensor healthy case have good dynamic performance. When the unknown actuator fault and sensor fault considered in this study occur in satellite attitude systems, the attitude stabilization control approach designed in Xiao et al. (2017a) is still used; the corresponding attitude angle output curve and the control input torque curve are depicted in Figs. 4 and 5; it

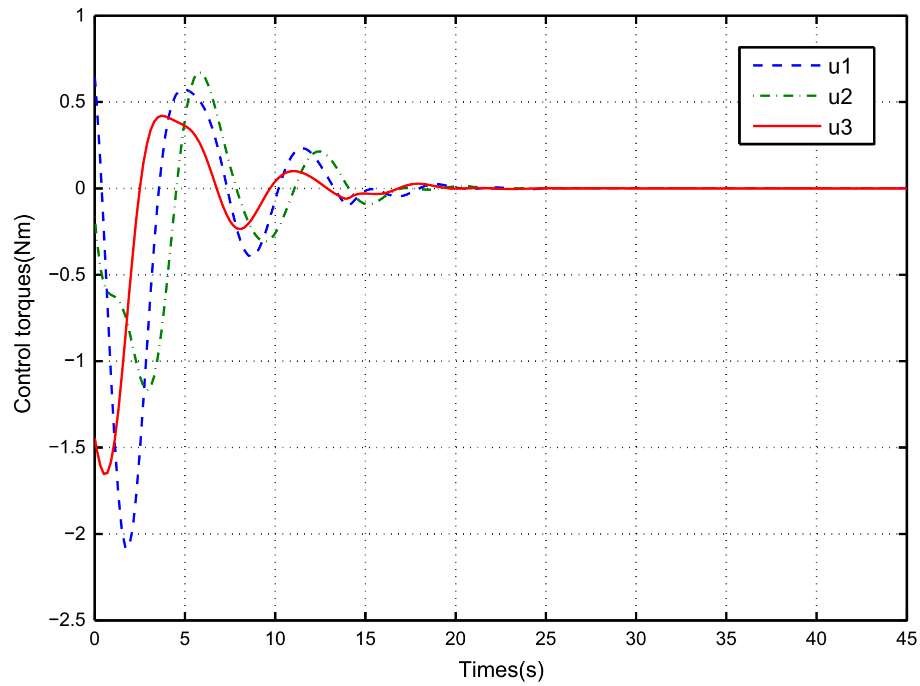

Fig. 3 Control input torque curve in fault-free case using attitude stabilization approach in Xiao et al. (2017a)

is not difficult to find that the attitude stabilization approach could not guarantee the closed-loop attitude systems in actuator and sensor faulty case which has the acceptable dynamic performance; the attitude angle curve and the control input torque curve shows unstable phenomenon, which fully illustrate the necessity of designing fault-tolerant attitude control scheme for satellite attitude systems.

$$L1 = 10^2\begin{bmatrix} -0.1198 & 0.0192 & 0.1121 & -1.0239 & -0.0047 & 0.0262 \\ 0.0061 & 0.3343 & -0.0409 & -0.0348 & -1.1295 & -0.0179 \\ 0.1254 & -0.0404 & -0.0075 & -0.0025 & -0.0237 & -0.9108 \\ -1.0037 & -0.0360 & -0.0006 & 0.0457 & 0.0189 & 0.1362 \\ -0.0028 & -1.1378 & -0.0155 & 0.0309 & 0.1203 & -0.0563 \\ 0.0239 & -0.0067 & -0.9224 & 0.1192 & -0.0566 & 0.0241 \end{bmatrix}$$

$$L2 = 10^3\begin{bmatrix} -1.2172 & 0.9783 & -2.0050 & -2.2483 & 1.6308 & -1.3966 \\ 0.9119 & -0.3703 & 1.8213 & 0.9272 & -2.4504 & 0.7030 \\ -1.8676 & 1.8144 & -1.6881 & -1.3205 & 1.3129 & -3.9580 \\ -2.1541 & 0.9227 & -1.3212 & -0.6776 & 1.2412 & -1.4894 \\ 1.5715 & -2.4817 & 1.3826 & 1.2886 & -0.9692 & 2.4932 \\ -1.3620 & 0.7532 & -4.0658 & -1.5630 & 2.5216 & -1.2421 \end{bmatrix}$$

$$L3 = 10^2\begin{bmatrix} -0.0046 & 0.0035 & -0.0042 & -0.0004 & 0.0002 & -0.0008 \\ -0.0022 & 0.0012 & -0.0014 & -0.0000 & -0.0005 & 0.0003 \\ -0.0281 & 0.0122 & -0.0112 & -0.0007 & -0.0009 & -0.0030 \\ 1.0803 & -0.4771 & 0.4359 & 0.0235 & 0.0490 & 0.0836 \\ -0.0223 & 0.0113 & -0.0102 & -0.0002 & -0.0017 & -0.0006 \\ -0.0137 & 0.0050 & -0.0034 & -0.0007 & -0.0005 & -0.0012 \end{bmatrix}$$

$$K = \begin{bmatrix} -49.5645 & 5.9053 & -8.6881 & -270.9268 & 157.3410 & -126.4720 \\ 3.6891 & 2.0305 & -1.7437 & 136.3118 & -34.6346 & -116.6039 \\ -30.0092 & -2.7538 & -2.2670 & -116.5872 & -128.1343 & 36.9429 \end{bmatrix}$$

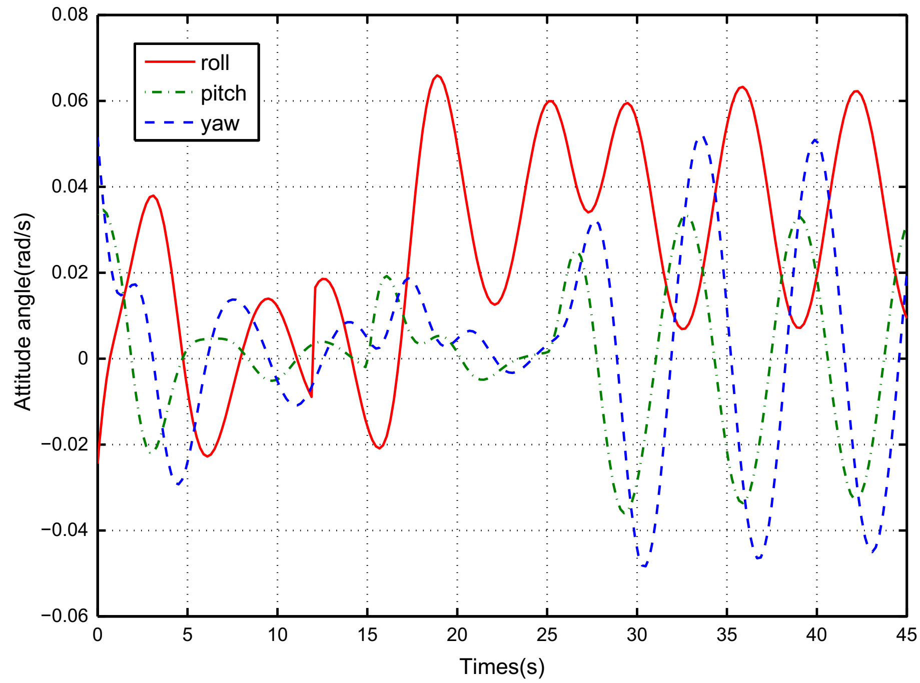

Fig. 4 Attitude angle output curve in faulty case using attitude stabilization approach in Xiao et al. (2017a)

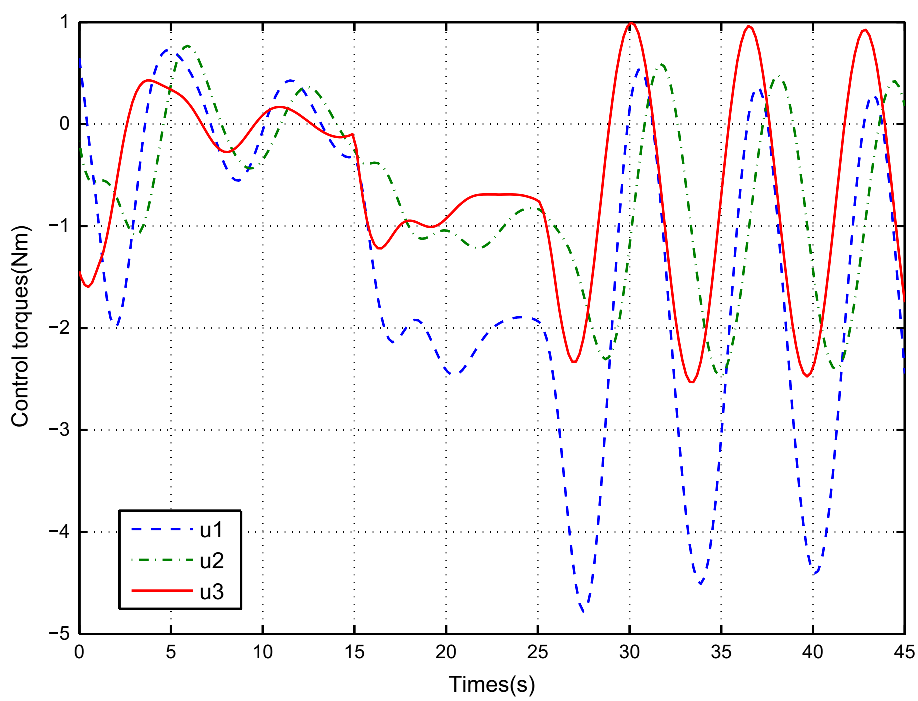

Fig. 5 Control input torque curve in faulty case using attitude stabilization approach in Xiao et al. (2017a)

In this position, the integrated fault-tolerant attitude control scheme designed in this paper is used to stabilize the rigid satellite in both actuator and sensor faulty cases, the unknown actuator and sensor faults could be estimated accurately by utilizing the designed observer, and the corresponding simulation results are given in Figs. 6 and 7. It is easily known that the designed observer has good fault estimation capability, and it laid a good foundation for the implementation of the subsequent fault-tolerant control scheme. By utilizing the designed sliding-mode fault-tolerant stabilization control scheme (19), the corresponding attitude angle output curve and control input torque curve are depicted in Figs. 8 and 9, respectively. It is not difficult to find that the effects of actuator and sensor faults considered in this paper to the closed-loop attitude systems could be compensated, such that the closed-loop attitude systems in faulty case have also the satisfactory dynamic performance.

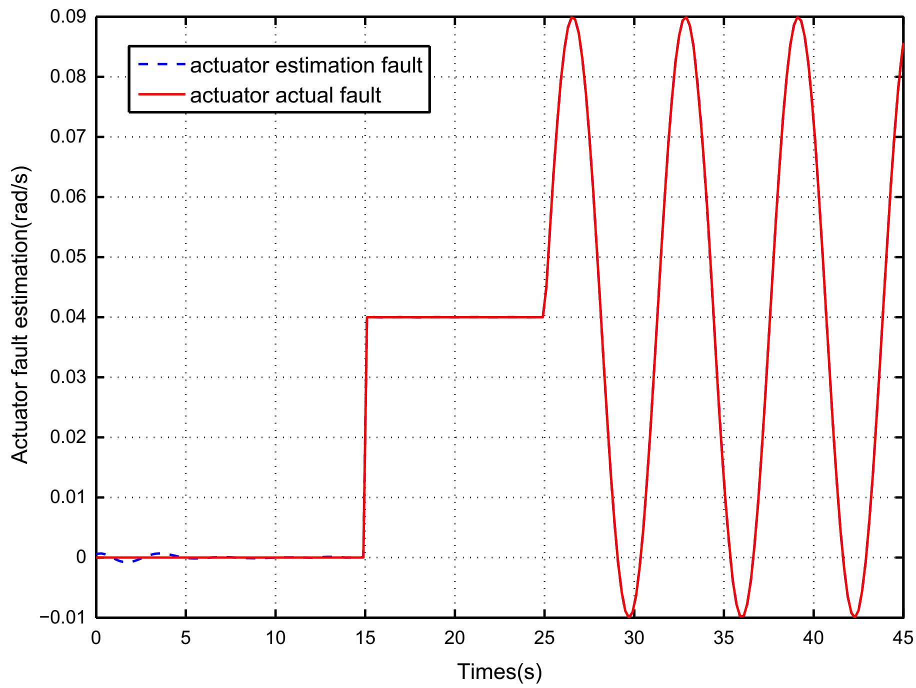

Fig. 6 Actual actuator fault and its estimation curve

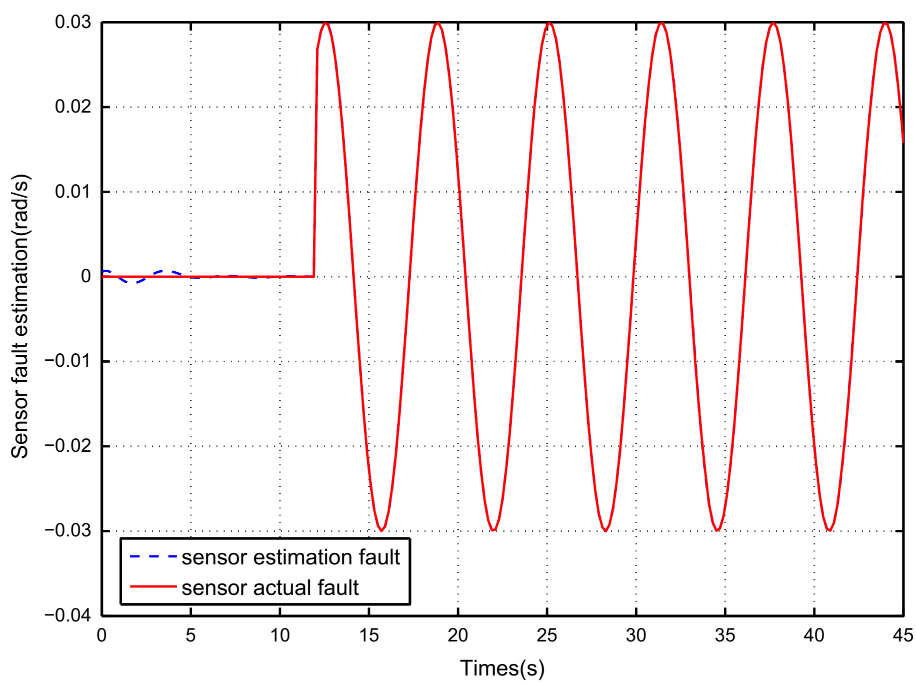

Fig. 7 Actual sensor fault and its estimation curve

In order to make a comparison, the fault-tolerant stabilization control strategy proposed in Xiao et al. (2015) is also simulated, and the attitude angles curve and the control input torque curve are depicted in Figs. 10 and 11. It is not difficult to find from Figs. 10 and 11 that the attitude angle output curves and control input–control torque curves have the greater oscillation under the fault-tolerant schemes developed in Xiao et al. (2015). Meanwhile, it is clearly seen from Figs. 8 to 9 that simulation results using integrated fault-tolerant stabilization scheme developed in this paper have better dynamic performance than those obtained in Figs. 10 and 11. To further emphasize the superiority of the proposed method, the control performance comparisons using two dif-

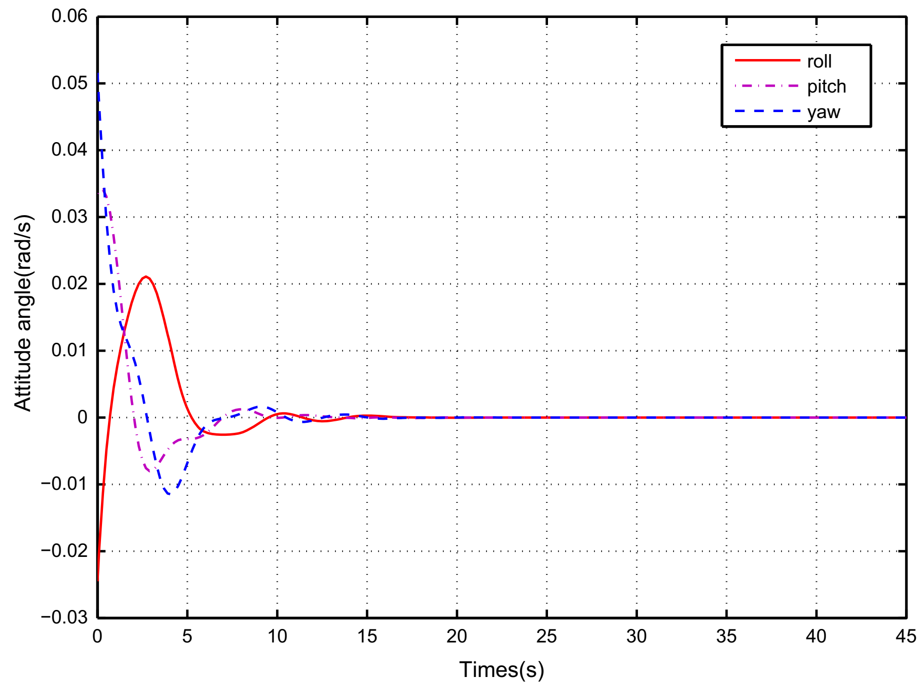

Fig. 8 Attitude angle output curve in faulty case using integrated FTC designed in this paper

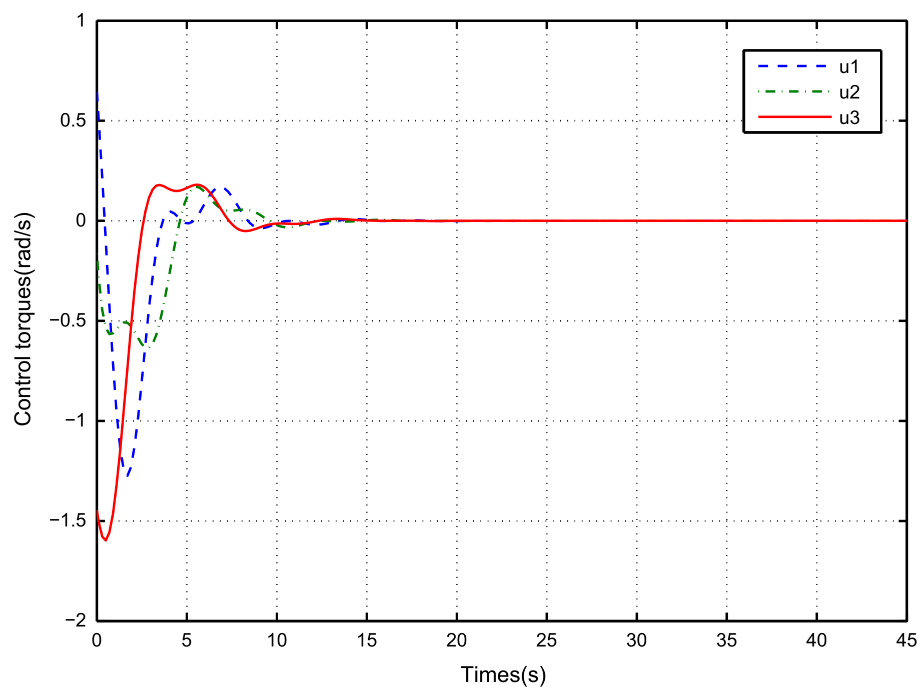

Fig. 9 Control input torque curve in faulty case using integrated FTC designed in this paper

ferent FTC schemes are also given in Table 1, which includes the convergence time and steady precision.

By the comparison of above cases, it can be found that the proposed integrated fault-tolerant stabilization control approach could accomplish the quick attitude stabilization for satellite attitude systems in the presence of actuator and sensor faults, parameter uncertainty, Lipschitz nonlinearity and external disturbance. Simulation results demonstrate the superior fault estimation and accommodation performance compared with the existing approaches reported in recent years.

5 Conclusions

In this paper, an integrated fault-tolerant attitude stabilization control strategy is proposed for a rigid satellite with unmatched uncertainty, Lipschitz nonlinearity, external dis-

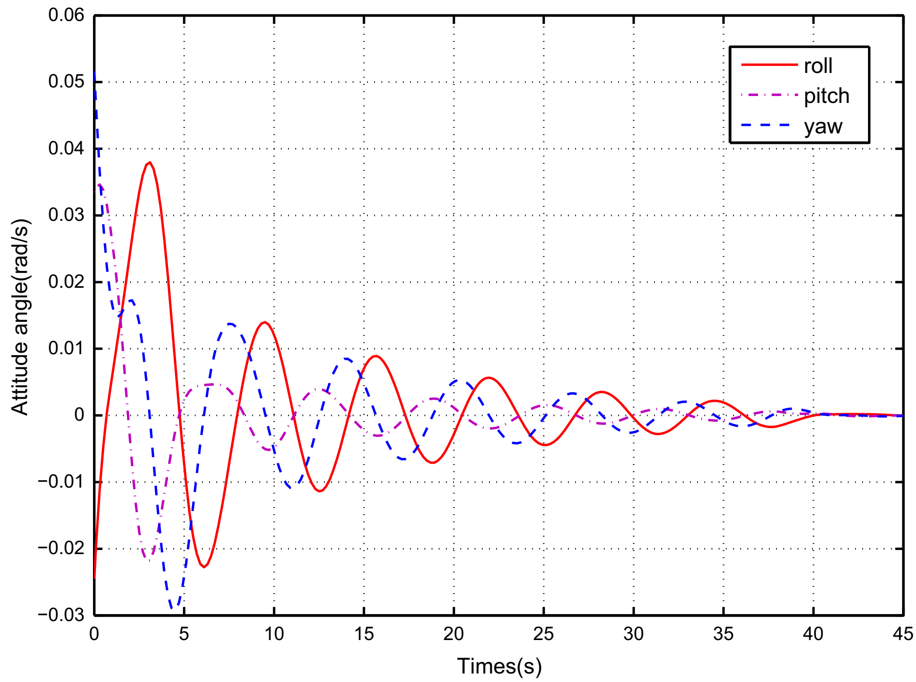

Fig. 10 Attitude angle output curve in faulty case using FTC designed in Xiao et al. (2015)

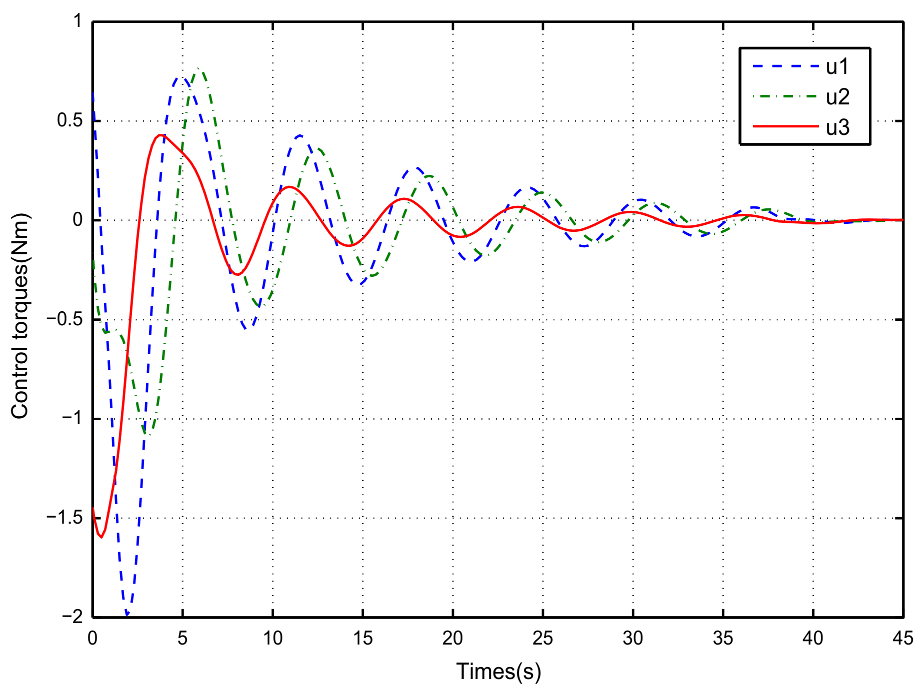

Fig. 11 Control input torque curve in faulty case using FTC designed in Xiao et al. (2015)

turbance, along with unknown actuator and sensor faults. A novel fault estimation approach is firstly given, which provides the estimation information of unknown actuator and sensor faults. Then an integral slide mode-based fault-tolerant stabilization controller is designed using only the measurable attitude angle output, which guarantees the faulty closed-loop attitude systems are asymptotically stable with a given disturbance attenuation level γ. Finally, simulation results show that the integrated design strategy leads to a good fault-tolerant performance. The limitation of the FTC approach developed in this paper is that it is only suitable for the rigid satellite and works in small-angle maneuvering stage; how to design a fault-tolerant attitude controller,

Table 1 Comparison of control performance using two different FTC scheme

Control scheme	Steady precision
FTC designed in this paper	5.5×10^{-4}
FTC designed in Xiao et al. (2015)	7.1×10^{-3}

which is suitable for the satellite in orbit and works in the large-angle maneuvering stage, will be our future work.

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