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A Novel Moving Average Forecasting Approach Using Fuzzy Time Series Data Set

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Abstract
In this study, we develop a novel moving average forecasting approach based on fuzzy time series data set. The main objective of applying this moving average approach in develop method is to provide better results and enhance the accuracy in forecasted output by reducing the fluctuation in time series data set. The developed method is to define the universe of discourse and partition into equal length of intervals which is based on the average-length method. Further, triangular fuzzy sets are defined and obtain a membership grade of each moving average historical datum rather than actual datum of historical fuzzy time series data. Here, the fuzzification process of moving average historical data to their maximum membership grades obtained into corresponding triangular fuzzy sets. The general suitability of developed model has been examined by implementing in the forecast of student enrollments data at the University of Alabama. Further, the market price of State Bank of India share at Bombay Stock Exchange, India, has also been implemented in the forecast. The developed method of moving average fuzzy time series provides an improved forecasted output with least root mean square error and average forecasting errors which shows that our developed method is more superior than other existing models available in the literature based on fuzzy time series data.

Keywords Fuzzy time series · Triangular fuzzy number · Fuzzy logical relationships · Moving average data · Market prices of BSE

1 Introduction

Conventional fuzzy time series model is one of the most popular time series forecasting models. The fuzzy time series model was initially introduced by Song and Chissom (1993) and applied his model on the historical data to student Enrollments at the University of Alabama. They presented the theory of fuzzy time series to overcome the drawback of the classical time series methods. Several researchers have introduced forecasting models developed to deal with the forecasting problems due to their capability of dealing with the uncertainty and vagueness inherent in the collected data. Song and Chissom (1994) proposed an approach to developing a time-variant fuzzy time series models and after defuzzification they found the forecasted output in more efficient way. A new method for handling the forecasting problem based on fuzzy time series is presented by Hwang et al. (1998). Also, Huarng (2001, 2006) refined his research and studied the length of the interval that affects the forecasted accuracy in fuzzy time series and developed a method with distribution-based length and average-based length to determine the appropriate length of interval. Since then, portioning the universe of discourse has become one of the important issues in the theory of fuzzy time series (FTS). Further, Chen (2002) presented a high-order fuzzy time series model to improve in forecasted outputs. In addition, Lee and Chou (2004) modified the Chen (1996) method by defining the supports of the fuzzy number which denotes the linguistic values of the linguistic variable in more accurate way. Chen and Hsu (2004) proposed first-order and time-variant methods for forecasting the historical time series data sets. Chen and Chung (2006) introduced a method to deal with the forecasting problems based on high-order fuzzy time series based on genetic algorithms, where the length of each interval in the universe of discourse is tuned by using

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genetic algorithms. In the literature, numerous models for the fuzzy time series forecasting have been presented by several researchers which can be utilized to raise the accuracy rates in the forecasted values (Song 2003; Lee et al. 2006; Yolcu et al. 2009). Later, Yu (2005) refined his research and introduced a fuzzy time series model to adjust the lengths of intervals during the formulation of fuzzy logical relationships. Jilani and Burney (2008) presented a new multivariate fuzzy time series forecasting method. Further, Li and Cheng (2007) proposed a deterministic forecasting model to deal with the forecasting problem of fuzzy time series. The model was changed by constructing the fuzzy logical relationship groups and removing the inconsistency of partitioning an interval. Also, Singh (2007a) proposed an improved and versatile method of forecasting based on the concept of fuzzy time series and the model is presented in the form of simple computational algorithm and obtains a forecasted result with least value in terms of average forecasting error (AFE). Also, Singh (2007b) introduced a method of fuzzy time series forecasting based on difference parameters. The introduced method is a simplified computational approach in defuzzification process rather than applying max–min composition operator for the forecasting the fuzzy time series historical data set. Chen et al. (2009) proposed a method to forecast enrollments data based on automatic clustering techniques and fuzzy logical relationships. Wang and Chen (2009) introduced a method to predict the temperature and the Taiwan Futures Exchange (TAIFEX), based on automatic clustering techniques and two-factor high-order fuzzy time series. Also, Aladag et al. (2009) presented a high-order fuzzy time series forecasting model using neural networks. The method gets higher average forecasting accuracy rates than the existing methods.

A novel invariant fuzzy time series forecasting approach is presented by Aladag et al. (2012) in which membership values in the fuzzy relationship matrix are computed by using particle swarm optimization technique. In his study, Gangwar and Kumar (2012) have proposed a computational method of forecasting based on multiple partitioning and higher-order fuzzy time series. The computational method provides a better approach and enhances the accuracy in forecasted values. Eğrioğlu (2012) presented a time-invariant forecasting model based on genetic algorithm. Chen and Kao (2013) proposed a model for forecasting the TAIEX by using the particle swarm optimization techniques, and support vector machine model is based on the slope of one-day and an average of two-day variation of the TAIEX. The integration of cumulative probability distribution method for partitioning the universe of discourse along with a simplified computational approach was presented by Gangwar and Kumar (2014) that uses characteristics of normal distributions to define the length of a linguistic interval. Furthermore, Gangwar and Kumar (2015) presented a modified version of computational algorithm for high-order weighted fuzzy time series with multiple partitioning to enhance the accuracy in forecasted outputs. Fraccaroli et al. (2015) discussed recursive least square method with multiple forgetting scheme. Wang et al. (2016) present intuitionistic fuzzy time series model which is based on intuitionistic fuzzy reasoning. In this model, fuzzy clustering algorithm is used to divide the universe of discourse into unequal length of intervals and a more objective technique for as certaining the membership function and non-membership function of the intuitionistic fuzzy sets. Bisht and Kumar (2016) presented a novel fuzzy time series forecasting method that uses hesitant fuzzy information and aggregation operator on hesitant fuzzy set. Zhao et al. (2016) explained fuzzy adaptive control design model in nonlinear uncertain systems. Further, Chang et al. (2017a, 2017b) presented his model using nonlinear discrete time system. Abhishekh and Kumar (2017) proposed computational method for rice production forecasting based on high-order fuzzy time series. Recently, Abhishekh et al. (2017, 2018a, b) introduced a weighted type 2 fuzzy time series and score function-based intuitionistic fuzzy time series forecasting method to enhance in the forecasted accuracy rates and also presented high-order forecasting model in intuitionistic fuzzy environment. Also, Gautam et al. (2018a, b) gave a solution to the decision making problems in intuitionistic fuzzy environment. Later, Gautam et al. (2018c) introduced a forecasting approach based on high-order fuzzy time series data. Chang and Wang (2018) presented peak-to-peak filtering for networked nonlinear DC motor systems with quantization. Also, Huo et al. (2019) explain his model using stochastic nonlinear systems with input quantization Abhishekh et al. (2019) give a forecasting model based on two factors moving average approach in the fuzzy environment.

In this paper, we present a novel forecasting method based on moving average fuzzy time series data. The proposed method provides a simple computational algorithm and gives better forecasting accuracy rates. In this study, three-period moving average data set is calculated by transforming the actual time series data set. Further, triangular fuzzy sets are defined which are based on moving average historical data set and each moving average datum is fuzzified to their maximum membership grades corresponding to define triangular fuzzy sets. Fuzzy logical relationship groups are established among the fuzzified moving average historical data set. The proposed novel algorithms of moving average have been implemented on forecasting the two time series data sets; the first is University enrollment data, and the second is market price at BSE, India; the performance results have been compared by two statistical parameter MSE and AFE, and then it has been observed that the proposed method gets higher forecasting accuracy rates with different traditional fuzzy time series models available in the literature. The motivation of applying the moving average fuzzy time series forecasting models is to find ways of modeling the prediction of student enrollment, sensex, crop yield and many other real non-deterministic processes. Further, the area-specific crop yield forecasting for a lead year may be applied to help the crop planning and agro-based business planning of the area and can be used in economics and business analysis. Also, the method can be extended to deal with the aspect of meta-heuristic optimization technique such as particle swarm optimization (PSO), neural network (NN) and genetic algorithms (GA) by which the forecasting accuracy of proposed method can be improved.

The rest of the content of this paper is organized as follows: in Sect. 2, the basic definition of fuzzy time series is introduced and formation of moving average approach is provided. In Sect. 3, we develop an algorithm for forecasting based on moving average approach. We illustrate the proposed method by implementing student enrollment of the University of Alabama and the market share price of BSE, India, in Sect. 4, and conclusion is presented in Sect. 5.

2 Basic Definitions

In this section, some basic definitions of fuzzy set, triangular fuzzy number, fuzzy time series, fuzzy logical relationships and moving average approach are briefly presented.

Definition 2.1 (Zadeh 1965) Let $X=\{x_1, x_2, \ldots, x_n\}$ be the universe of discourse. A fuzzy set $\tilde{A}$ can be defined on the universe of discourse X can be represented as follows:

$$\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x)\right) | x \in X \right\} \tag{1}$$

where $\mu_{\tilde{A}}$ denotes the membership function of the fuzzy set $\tilde{A}$, $\mu_{\tilde{A}} : X \to [0, 1]$ and $\mu_{\tilde{A}}\left(x_i\right)$ define the grade of membership of x_i belonging to the fuzzy set $\tilde{A}$ and $\mu_{\tilde{A}}\left(x_i\right) \in [0, 1]$, $1 \leq i \leq n$.

Definition 2.2 (Abhishekh et al. 2018b) A fuzzy set $\tilde{A}$ can be defined on a real line $\mathbb{R}$ with membership function $\mu_{\tilde{A}}(x)$, where $\mu_{\tilde{A}} : \mathbb{R} \to [0, 1]$ is called a fuzzy number, and a fuzzy number is denoted by the triplet $\tilde{A} = (a, b, c)$, which is said to be a triangular fuzzy number (TFN), if its membership function is defined as (Fig. 1)

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & \text{if} \quad a \leq x < b \\ 1, & \text{if} \quad x = b \\ \frac{(c-x)}{(c-b)}, & \text{if} \quad b < x \leq c \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

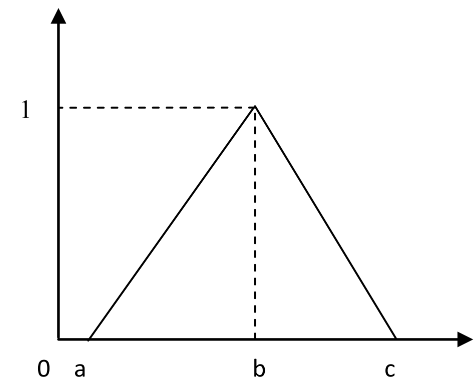

Fig. 1 The membership function of a TFN

Definition 2.3 (Song and Chissom 1994) Let $Y(t)$, $t=0, 1, 2, \ldots$ which is a subset of real number, be a universe of discourse. The fuzzy set $f_i(t)$, $i=1, 2, 3, \ldots$ is defined on the universe of discourse $Y(t)$. If $F(t)$ is the collection of $f_i(t)$, then $F(t)$ is called fuzzy time series defined on $Y(t)$.

Definition 2.4 (Song and Chissom 1994) If there exists a fuzzy logical relationship $R(t-1, t)$ such that $F(t) = F(t-1) \circ R(t-1, t)$ where '∘' refers to max–min composition operator on fuzzy sets and then $F(t)$ is said to be caused by $F(t-1)$. The logical relationship between $F(t)$ and $F(t-1)$ is denoted as $F(t-1) \to F(t)$.

Definition 2.5 (Song and Chissom 1994) Suppose $F(t-1) = \tilde{A}_i$ and $F(t) = \tilde{A}_j$ then a fuzzy logical relationship can be defined as $\tilde{A}_i \to \tilde{A}_j$, where $\tilde{A}_i$ and $\tilde{A}_j$ are called the current state and next state of the fuzzy logical relationships, respectively.

Definition 2.6 (Song and Chissom 1994) If $F(t)$ is caused by more than one fuzzy sets $F(t-n)$, $F(t-n+1)$, ..., $F(t-1)$, then fuzzy logical relationships are represented by $\tilde{A}_{i_1}, \tilde{A}_{i_2}, \ldots, \tilde{A}_{i_n} \to \tilde{A}_j$, where $F(t-n) = \tilde{A}_{i_1}, F(t-n+1) = \tilde{A}_{i_2}, \ldots, F(t-1) = \tilde{A}_{i_n}$. This relationship is called nth-order fuzzy time series model.

2.1 Moving Average

A moving average is a computation to analyze data points by making a series of averages of different subsets of the total data sets. Presenting a series of numbers and a fixed subset size, the first component of the moving average is obtained by getting hold of the norm of the initial fixed subset of the number series. Then, the subset is modified by "shifting forward," that is excluding the first number of the series. This creates a new subset of numbers, which is averaged. This process is repeated over the entire data set. The plot line connecting all the averages (fixed) is the moving average. A moving average of order m can be expressed as

$$T_t \quad \text{or} \quad (M.A.)_t = \frac{1}{m}\sum_{j=-K}^{K} x_{t+j} \tag{3}$$

where $m = 2K + 1$, i.e., the estimate of the trend cycle at time t is obtained by averaging the values of the time series with in K-periods of time t.

3 Developed Method of Forecasting Based on Moving Average Fuzzy Time Series Data

In this section, we develop a novel method of forecasting based on moving average fuzzy time series data. By using the developed method of moving average, we can reduce fluctuations of the time series data and obtain a trend values with a fair degree of accuracy. In the developed method, the period of moving average approach is three and algorithms of average-based length of interval presented by Huarng (2001) is used for partitioning the universe of discourse and defining an appropriate length of intervals in more systematic ways.

Let $x_1, x_2, \ldots, x_n$ be the collection of time series data, and stepwise illustration of the develop method is given as follows:

Step 1 First, we calculate the three-period moving average value from actual time series historical data set
Step 2 Define the universe of discourse $U = [D_{\min} - D_1, D_{\max} + D_2]$ on transformed moving average time series data instead of actual time series data. Here, $D_{\max}$ and $D_{\min}$ are the maximum value and minimum value taken from the three-period moving average data set, respectively, and D_1 and D_2 are two proper positive numbers for easy partitioning the universe of discourse U
Step 3 The point-wise algorithms to define the appropriate length of interval l are as follows:

1. Compute the absolute differences between the three-period moving average time series datum of two consecutive numbers x_{i+1} and x_i ($i = 1, 2, \ldots, n-1$) and after that we calculate the average of first differences.
2. Take the length of interval is one half of the average of first differences.
3. Determine the base of obtaining length of interval by using base mapping Table 1.
4. Consider the length of intervals in round figure, according to the fixed base value.

Table 1 Base mapping table

Range	Base
0.1–1.0	0.1
1.1–10	1
11–100	10
101–1000	100
1001–10,000	1000

Step 4 Partition the universe of discourse U into m equal length of intervals $u_1, u_2, \ldots, u_m$ of length l, where m is the number of intervals and is defined as

$$m = \frac{\left(D_{max} + D_2\right) - \left(D_{min} - D_1\right)}{l} \tag{4}$$

Step 5 Define the triangular fuzzy sets $\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_m$ in accordance with the intervals $u_1, u_2, \ldots, u_m$ and consequently assign the grade of membership for each moving average datum corresponding to their define triangular fuzzy sets
Step 6 Now fuzzify the moving average data by triangular fuzzy set in which moving average datum has maximum membership grades in their corresponding triangular fuzzy sets
Step 7 Establish the fuzzy logical relationships and fuzzy logical relationship groups which is based on the fuzzified moving average historical time series data
Step 8 Calculate the forecasted value of the time series data at time t. Assume $F(t-1) = \tilde{A}_i (i = 1, 2, \ldots, m)$, and then the following case arise

Case 1 If the fuzzy logical relationship groups are obtained $\tilde{A}_i \to \tilde{A}_k$, then forecast value at time t, i.e., $F(t)$ is m_k. Where m_k is the midvalue of the linguistic interval u_k corresponding to triangular fuzzy set $\tilde{A}_k$ in the next state.

Case 2 If the fuzzy logical relationship groups is obtained $\tilde{A}_i \to \tilde{A}_{k_1}, \tilde{A}_{k_2}, \ldots, \tilde{A}_{k_p}$, then the forecast value at time t is $\frac{m_{k_1} + m_{k_2} + \cdots + m_{k_p}}{p}$, where $m_{k_1}, m_{k_2}, \ldots, m_{k_p}$ is midvalue of the linguistic intervals $u_{k_1}, u_{k_2}, \ldots, u_{k_p}$ corresponding to the triangular fuzzy sets $\tilde{A}_{k_1}, \tilde{A}_{k_2}, \ldots, \tilde{A}_{k_p}$, respectively.

Step 9 To evaluate the performance of the forecasting method in fuzzy time series historical data, we calculate the root mean square error (RMSE) and average forecast error (AFE) that are common tools to measuring the forecasting performance. Least value of root mean square error and average forecasting error indicate the better forecasting method. The root mean square error and average forecasting error are defined as follows:

Root mean square error (RMSE)

$$= \sqrt{\frac{\sum_{i=1}^{n}\left((\text{actual value})_i - (\text{forecast value})_i\right)^2}{n}} \tag{5}$$

Forecasting error (%)

$$= \frac{|\text{forecasting value} - \text{actual value}|}{\text{actual value}} \times 100 \tag{6}$$

Average forecasting error (AFE)

$$= \frac{\text{sum of forecasting error}}{\text{number of errors}} \tag{7}$$

4 Numerical Illustration

To examine the performance of the proposed method, the student enrollments at the University of Alabama and market prices of SBI share at BSE India are taken as numerical illustration. Further, the proposed method is compared with other existing methods in the literature in terms of RMSE and AFE and it is observed that the proposed method provides better forecasting accuracy rates. The stepwise interpretation is as follows:

4.1 Forecasting for Student Enrollment at the University of Alabama

The student enrollment data at the University of Alabama is adopted from (Song and Chissom 1993). The stepwise algorithm of the proposed method is illustrated as follows:

Step 1 First calculate the moving average enrollment of period three from actual time series data of enrollment of the University of Alabama (shown in Table 2).

Step 2 *Define the universe of discourse*

From the transformed Table 2 of actual data to moving average enrollment historical data, we find $D_{\max} = 19{,}212$ and $D_{\min} = 13{,}495$ and choose two proper positive numbers as $D_1 = 45$ and $D_2 = 38$. Hence, we define an universe of discourse as $U = [13{,}450, 19{,}250]$ on transformed moving average data set.

Step 3 Now determine the appropriate length (l) of interval as

1. From Table 2, compute the average of the first differences of moving average enrollment data, which is 453.89.
2. Take one half of average of first differences as the length of interval, which is 226.94.
3. From the base mapping Table 1, the calculated length is located in the range [101, 1000], so the base is assigned to be 100.
4. According to base 100, the length of interval is rounded off to 200, which is the appropriate length of interval l.

Step 4 Partition the universe of discourse U in an equal length of intervals by using Eq. (4). The number of interval m is calculated as

$$m = \frac{19{,}250 - 13{,}450}{200} = 29.$$

Hence, there are 29 linguistic intervals of length $l = 200$ which are defined as follows:

Table 2 Actual enrollment data, moving average with their fuzzified enrollment data of University of Alabama

Year	Actual enrollments	Moving average enrollment	Fuzzified enrollment	Year	Actual enrollments	Moving average enrollment	Fuzzified enrollment
1971	13,055	–	–	1982	15,433	15,773	$\tilde{A}_{12}$
1972	13,563	13,495	$\tilde{A}_{1}$	1983	15,497	15,358	$\tilde{A}_{10}$
1973	13,867	14,042	$\tilde{A}_{3}$	1984	15,145	15,268	$\tilde{A}_{9}$
1974	14,696	14,674	$\tilde{A}_{6}$	1985	15,163	15,431	$\tilde{A}_{10}$
1975	15,460	15,156	$\tilde{A}_{9}$	1986	15,984	16,002	$\tilde{A}_{13}$
1976	15,311	15,458	$\tilde{A}_{10}$	1987	16,859	16,998	$\tilde{A}_{18}$
1977	15,603	15,592	$\tilde{A}_{11}$	1988	18,150	17,993	$\tilde{A}_{23}$
1978	15,861	16,090	$\tilde{A}_{13}$	1989	18,970	18,816	$\tilde{A}_{27}$
1979	16,807	16,529	$\tilde{A}_{15}$	1990	19,328	19,212	$\tilde{A}_{29}$
1980	16,919	16,705	$\tilde{A}_{16}$	1991	19,337	19,180	$\tilde{A}_{29}$
1981	16,388	16,247	$\tilde{A}_{14}$	1992	18,876	–	–

u_1 = [13,450, 13,650]	u_2 = [13,650, 13,850]	u_3 = [13,850, 14,050]
u_4 = [14,050, 14,250]	u_5 = [14,250, 14,450]	u_6 = [14,450, 14,650]
u_7 = [14,650, 14,850]	u_8 = [14,850, 15,050]	u_9 = [15,050, 15,250]
u_{10} = [15,250, 15,450]	u_{11} = [15,450, 15,650]	u_{12} = [15,650, 15,850]
u_{13} = [15,850, 16,050]	u_{14} = [16,050, 16,250]	u_{15} = [16,250, 16,450]
u_{16} = [16,450, 16,650]	u_{17} = [16,650, 16,850]	u_{18} = [16,850, 17,050]
u_{19} = [17,050, 17,250]	u_{20} = [17,250, 17,450]	u_{21} = [17,450, 17,650]
u_{22} = [17,650, 17,850]	u_{23} = [17,850, 18,050]	u_{24} = [18,050, 18,250]
u_{25} = [18,250, 18,450]	u_{26} = [18,450, 18,650]	u_{27} = [18,650, 18,850]
u_{28} = [18,850, 19,050]	u_{29} = [19,050, 19,250]	

Step 5 Define a triangular fuzzy sets $\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_{29}$ corresponding to linguistic intervals $u_1, u_2, \ldots, u_{29}$ as follows:

$\tilde{A}_1$ = (13,450, 13,650, 13,850)	$\tilde{A}_2$ = (13,650, 13,850, 14,050)	$\tilde{A}_3$ = (13,850, 14,050, 14,250)
$\tilde{A}_4$ = (14,050, 14,250, 14,450)	$\tilde{A}_5$ = (14,250, 14,450, 14,650)	$\tilde{A}_6$ = (14,450, 14,650, 14,850)
$\tilde{A}_7$ = (14,650, 14,850, 15,050)	$\tilde{A}_8$ = (14,850, 15,050, 15,250)	$\tilde{A}_9$ = (15,050, 15,250, 15,450)
$\tilde{A}_{10}$ = (15,250, 15,450, 15,650)	$\tilde{A}_{11}$ = (15,450, 15,650, 15,850)	$\tilde{A}_{12}$ = (15,650, 15,850, 16,050)
$\tilde{A}_{13}$ = (15,850, 16,050, 16,250)	$\tilde{A}_{14}$ = (16,050, 16,250, 16,450)	$\tilde{A}_{15}$ = (16,250, 16,450, 16,650)
$\tilde{A}_{16}$ = (16,450, 16,650, 16,850)	$\tilde{A}_{17}$ = (16,650, 16,850, 17,050)	$\tilde{A}_{18}$ = (16,850, 17,050, 17,250)
$\tilde{A}_{19}$ = (17,050, 17,250, 17,450)	$\tilde{A}_{20}$ = (17,250, 17,450, 17,650)	$\tilde{A}_{21}$ = (17,450, 17,650, 17,850)
$\tilde{A}_{22}$ = (17,650, 17,850, 18,050)	$\tilde{A}_{23}$ = (17,850, 18,050, 18,250)	$\tilde{A}_{24}$ = (18,050, 18,250, 18,450)
$\tilde{A}_{25}$ = (18,250, 18,450, 18,650)	$\tilde{A}_{26}$ = (18,450, 18,650, 18,850)	$\tilde{A}_{27}$ = (18,650, 18,850, 19,050)
$\tilde{A}_{28}$ = (18,850, 19,050, 19,250)	$\tilde{A}_{29}$ = (19,050, 19,250, 19,250)	

From the definition of triangular fuzzy set, we define the grade of membership for each moving average enrollment datum in accordance with the corresponding triangular fuzzy sets as follows:

Table 3 Fuzzy logical relationships of moving average enrollment data sets

$\tilde{A}_1 \rightarrow \tilde{A}_3$	$\tilde{A}_3 \rightarrow \tilde{A}_6$	$\tilde{A}_6 \rightarrow \tilde{A}_9$	$\tilde{A}_9 \rightarrow \tilde{A}_{10}$
$\tilde{A}_{10} \rightarrow \tilde{A}_{11}$	$\tilde{A}_{11} \rightarrow \tilde{A}_{13}$	$\tilde{A}_{13} \rightarrow \tilde{A}_{15}$	$\tilde{A}_{15} \rightarrow \tilde{A}_{16}$
$\tilde{A}_{16} \rightarrow \tilde{A}_{14}$	$\tilde{A}_{14} \rightarrow \tilde{A}_{12}$	$\tilde{A}_{12} \rightarrow \tilde{A}_{10}$	$\tilde{A}_{10} \rightarrow \tilde{A}_9$
$\tilde{A}_9 \rightarrow \tilde{A}_{10}$	$\tilde{A}_{10} \rightarrow \tilde{A}_{13}$	$\tilde{A}_{13} \rightarrow \tilde{A}_{18}$	$\tilde{A}_{18} \rightarrow \tilde{A}_{23}$
$\tilde{A}_{23} \rightarrow \tilde{A}_{27}$	$\tilde{A}_{27} \rightarrow \tilde{A}_{29}$	$\tilde{A}_{29} \rightarrow \tilde{A}_{29}$	

Table 4 Fuzzy logical relationship groups of moving average enrollment data set

Groups	FLRGs	Groups	FLRGs
Group 1	$\tilde{A}_1 \rightarrow \tilde{A}_3$	Group 9	$\tilde{A}_{14} \rightarrow \tilde{A}_{12}$
Group 2	$\tilde{A}_3 \rightarrow \tilde{A}_6$	Group 10	$\tilde{A}_{15} \rightarrow \tilde{A}_{16}$
Group 3	$\tilde{A}_6 \rightarrow \tilde{A}_9$	Group 11	$\tilde{A}_{16} \rightarrow \tilde{A}_{14}$
Group 4	$\tilde{A}_9 \rightarrow \tilde{A}_{10}, \tilde{A}_{10}$	Group 12	$\tilde{A}_{18} \rightarrow \tilde{A}_{23}$
Group 5	$\tilde{A}_{10} \rightarrow \tilde{A}_{11}, \tilde{A}_9, \tilde{A}_{13}$	Group 13	$\tilde{A}_{23} \rightarrow \tilde{A}_{27}$
Group 6	$\tilde{A}_{11} \rightarrow \tilde{A}_{13}$	Group 14	$\tilde{A}_{27} \rightarrow \tilde{A}_{29}$
Group 7	$\tilde{A}_{12} \rightarrow \tilde{A}_{10}$	Group 15	$\tilde{A}_{29} \rightarrow \tilde{A}_{29}$
Group 8	$\tilde{A}_{13} \rightarrow \tilde{A}_{15}, \tilde{A}_{18}$		

$\tilde{A}_1 = \frac{0.22}{13495}$ $\quad$ $\tilde{A}_2 = \frac{0.04}{14042}$

$\tilde{A}_3 = \frac{0.96}{14042}$ $\quad$ $\tilde{A}_4 = \emptyset$ (null fuzzy set)

$\tilde{A}_5 = \emptyset$ (null fuzzy set) $\quad$ $\tilde{A}_6 = \frac{0.88}{14674}$

$\tilde{A}_7 = \frac{0.12}{14674}$ $\quad$ $\tilde{A}_8 = \frac{0.47}{15156}$

$\tilde{A}_9 = \frac{0.53}{15156} + \frac{0.46}{15358} + \frac{0.91}{15268} + \frac{0.095}{15431}$ $\quad$ $\tilde{A}_{10} = \frac{0.54}{15358} + \frac{0.09}{15268} + \frac{0.905}{15431} + \frac{0.96}{15458} + \frac{0.29}{15592}$

$\tilde{A}_{11} = \frac{0.04}{15458} + \frac{0.71}{15592} + \frac{0.385}{15773}$ $\quad$ $\tilde{A}_{12} = \frac{0.615}{15773} + \frac{0.24}{16002}$

$\tilde{A}_{13} = \frac{0.76}{16002} + \frac{0.8}{16090} + \frac{0.015}{16247}$ $\quad$ $\tilde{A}_{14} = \frac{0.2}{16090} + \frac{0.985}{16247}$

$\tilde{A}_{15} = \frac{0.605}{16529}$ $\quad$ $\tilde{A}_{16} = \frac{0.395}{16529} + \frac{0.725}{16705}$

$\tilde{A}_{17} = \frac{0.275}{16705} + \frac{0.26}{16998}$ $\quad$ $\tilde{A}_{18} = \frac{0.74}{16998}$

$\tilde{A}_{19} = \emptyset$ (null fuzzy set) $\quad$ $\tilde{A}_{20} = \emptyset$ (null fuzzy set)

$\tilde{A}_{21} = \emptyset$ (null fuzzy set) $\quad$ $\tilde{A}_{22} = \frac{0.285}{17993}$

$\tilde{A}_{23} = \frac{0.715}{17993}$ $\quad$ $\tilde{A}_{24} = \emptyset$ (null fuzzy set)

$\tilde{A}_{25} = \emptyset$ (null fuzzy set) $\quad$ $\tilde{A}_{26} = \frac{0.17}{18816}$

$\tilde{A}_{27} = \frac{0.83}{18816}$ $\quad$ $\tilde{A}_{28} = \frac{0.19}{19212} + \frac{0.35}{19180}$

$\tilde{A}_{29} = \frac{0.65}{19180} + \frac{0.81}{19212}$

Step 6 Fuzzify the moving average enrollment data by the triangular fuzzy set $\tilde{A}_i$ ($i = 1, 2, \ldots, 29$) (as shown in Table 2) which is based on the moving average enrollment data having a maximum grades of membership.

Example 1 The moving average value of year 1975 is 15,156. The membership value of datum 15,156 is 0.47 and 0.53 in corresponding triangular fuzzy set $\tilde{A}_8$ and $\tilde{A}_9$, respectively.

We observe that maximum membership grade for enrollment 15,156 is 0.53 which lie in a triangular fuzzy set $\tilde{A}_9$. So moving average value 15,156 of year 1975 is fuzzified by triangular fuzzy set $\tilde{A}_9$ due to their maximum membership grades. Similarly, the fuzzification process can be done for rest year in similar ways which are placed in Table 2.

Step 7 Establish the fuzzy logical relationships and fuzzy logical relationship groups among the fuzzified moving average enrollment data and are placed in Tables 3 and 4.

Step 8 *Calculate the forecasted value*

Using step 8 of the proposed method, the defuzzification process is based on the fuzzy logical relationship groups and the obtained numerical values of the forecasted output are placed in Table 5. For explanatory purpose, we can calculate the forecasted value of year 1977.

Example 2 [Forecast of an year 1977] We have the fuzzified value of year 1976 that is$\tilde{A}_{10}$. From Table 4, fuzzy logical relationship group for $\tilde{A}_{10}$ is $\tilde{A}_{11}, \tilde{A}_9, \tilde{A}_{13}$. Here, in the next state more than one fuzzy set so from case 2 in the defuzzification process we computed the forecasted value of year 1977 as $F(1977) = \frac{m_{11}+m_9+m_{13}}{3}$. Where m_{11}, m_9 and m_{13} are midvalues of linguistic intervals u_{11}, u_9 and u_{13} corresponding to triangular fuzzy sets $\tilde{A}_{11}, \tilde{A}_9$ and $\tilde{A}_{13}$. Hence, the forecast value of year 1977 is 15,550, i.e., $F(1977) = 15{,}550$.

Step 9 To examine the performance comparison of the proposed method with other existing models, we compute with two statistical parameters RMSE and AFE (Table 6).

The trend in the forecasting of enrollment by the above-mentioned methods is illustrated in Fig. 2. The comparative study of RMSE and AFE and the graphical representation of the forecasted values obtained by the proposed method clearly show the superiority of the proposed method over the other existing fuzzy time series methods available in the literature.

4.2 Forecasting of Market Prices of SBI Share

The proposed method is also implemented on the forecasting of market prices of SBI shares at BSE India. The stepwise illustrations of the proposed method are as follows:

Table 5 Forecasted value of enrollment data by the proposed method along with existing models

Year	Actual enrollment	Song and Chissom (1993)	Huarng model (2001)	Gangwar and Kumar (2015)	Wang et al. method (2016)	Qui et al. method (2011)	Wong et al. method (2010)	Bisht and Kumar method (2016)	Proposed method
1971	13,055	–	–	–	–	–	–	–	–
1972	13,563	14,000	–	–	13,500	14,195	–	13,595.67	–
1973	13,867	14,000	–	13,693	14,155	14,424	13,500	13,814.75	13,950
1974	14,696	14,000	14,000	13,693	14,155	14,593	14,500	14,929.79	14,550
1975	15,460	15,500	15,500	14,867	15,539	15,589	15,500	15,541.27	15,150
1976	15,311	16,000	15,500	15,287	15,539	15,645	15,466	15,540.62	15,350
1977	15,603	16,000	16,000	15,376	15,502	15,634	15,392	15,540.62	15,550
1978	15,861	16,000	16,000	15,376	15,502	16,100	15,549	15,540.62	15,950
1979	16,807	16,000	16,000	15,376	16,667	16,188	16,433	16,254.50	16,650
1980	16,919	16,833	17,500	16,523	16,667	17,077	16,656	17,040.41	16,550
1981	16,388	16,833	16,000	16,606	15,669	17,105	16,624	17,040.41	16,150
1982	15,433	16,833	16,000	17,519	15,564	16,369	15,556	16,254.50	15,750
1983	15,497	16,000	16,000	16,606	15,564	15,643	15,524	15,540.62	15,350
1984	15,145	16,000	15,500	15,376	15,564	15,648	15,497	15,540.62	15,550
1985	15,163	16,000	16,000	15,376	15,523	15,622	15,305	15,541.27	15,350
1986	15,984	16,000	16,000	15,287	15,523	15,623	15,308	15,541.27	15,550
1987	16,859	16,000	16,000	15,287	16,799	16,231	16,402	16,254.50	16,650
1988	18,150	16,813	17,500	16,523	18,268	17,090	18,500	17,040.41	17,950
1989	18,970	19,000	19,000	17,519	18,268	18,325	18,534	18,902.30	18,750
1990	19,328	19,000	19,000	19,500	18,780	19,000	19,345	19,357.30	19,150
1991	19,337	19,000	19,500	19,000	19,575	19,000	19,423	19,168.56	19,150
1992	18,876	–	19,000	19,500	18,855	19,000	18,752	19,168.56	19,150

Table 6 A comparison of RMSE and AFE of the proposed method with existing models

Models	Song and Chissom (1993)	Huarng model (2001)	Gangwar and Kumar (2015)	Wang et al. method (2016)	Qui et al. method (2011)	Wong et al. method (2010)	Eğrioğlu method (2012)	Bisht and Kumar (2016)	Aladag et al. (2009)	Proposed method
RMSE	650.4	476.97	493.56	350.9	511.33	293.08	48,461	428.63	279.41	238.46
AFE	3.22	2.36	2.33	1.72	2.65	1.46	112.30	1.94	1.31	1.29

Step 1 First compute the moving average prices of SBI shares of period three from actual prices historical data set which are placed in Table 7

Step 2 Define the universe of discourse U on transformed moving average prices data set of SBI share rather than on actual data set as $U=[1200, 2500]$

Step 3 Compute the appropriate length of interval l as follows:

1. Calculate the average of absolute differences between the moving average data set of SBI shares prices x_i+1 and x_i ($i=1, 2, \ldots, 23$), which is 96.32.
2. Take length of intervals as one half of the average value, which is 48.16.
3. According to the calculated length, the base for length of interval is 10 by base mapping Table 1.
4. Round off the length 48.16 by the base 10 to 50. Thus, 50 is the appropriate length of an interval.

Step 4 Now partition the universe of discourse into equal length ($l=50$) of intervals. The number of intervals m is computed as

$$m = \frac{2500 - 1200}{50} = 26$$

Hence, 26 linguistic intervals defined of length ($l=50$) are as follows:

$u_1=[1200, 1250]$	$u_2=[1250, 1300]$	$u_3=[1300, 1350]$
$u_4=[1350, 1400]$	$u_5=[1400, 1450]$	$u_6=[1450, 1500]$
$u_7=[1500, 1550]$	$u_8=[1550, 1600]$	$u_9=[1600, 1650]$
$u_{10}=[1650, 1700]$	$u_{11}=[1700, 1750]$	$u_{12}=[1750, 1800]$
$u_{13}=[1800, 1850]$	$u_{14}=[1850, 1900]$	$u_{15}=[1900, 1950]$
$u_{16}=[1950, 2000]$	$u_{17}=[2000, 2050]$	$u_{18}=[2050, 2100]$
$u_{19}=[2100, 2150]$	$u_{20}=[2150, 2200]$	$u_{21}=[2200, 2250]$
$u_{22}=[2250, 2300]$	$u_{23}=[2300, 2350]$	$u_{24}=[2350, 2400]$
$u_{25}=[2400, 2450]$	$u_{26}=[2450, 2500]$	

Step 5 Define the triangular fuzzy sets $\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_{26}$ corresponding to linguistic intervals $u_1, u_2, \ldots, u_{29}$ which are as follows:

$\tilde{A}_1 = (1200, 1250, 1300)$	$\tilde{A}_2 = (1250, 1300, 1350)$	$\tilde{A}_3 = (1300, 1350, 1400)$
$\tilde{A}_4 = (1350, 1400, 1450)$	$\tilde{A}_5 = (140, 1450, 1500)$	$\tilde{A}_6 = (1450, 1500, 1550)$
$\tilde{A}_7 = (1500, 1550, 1600)$	$\tilde{A}_8 = (1550, 1600, 1650)$	$\tilde{A}_9 = (1600, 1650, 1700)$
$\tilde{A}_{10} = (1650, 1700, 1750)$	$\tilde{A}_{11} = (1700, 1750, 1800)$	$\tilde{A}_{12} = (1750, 1800, 1850)$
$\tilde{A}_{13} = (1800, 1850, 1900)$	$\tilde{A}_{14} = (1850, 1900, 1950)$	$\tilde{A}_{15} = (1900, 1950, 2000)$
$\tilde{A}_{16} = (1950, 2000, 2050)$	$\tilde{A}_{17} = (2000, 2050, 2100)$	$\tilde{A}_{18} = (2050, 2100, 2150)$
$\tilde{A}_{19} = (2100, 2150, 2200)$	$\tilde{A}_{20} = (2150, 2200, 2250)$	$\tilde{A}_{21} = (2200, 2250, 2300)$
$\tilde{A}_{22} = (2250, 2300, 2350)$	$\tilde{A}_{23} = (2300, 2350, 2400)$	$\tilde{A}_{24} = (2350, 2400, 2450)$
$\tilde{A}_{25} = (2400, 2450, 2500)$	$\tilde{A}_{26} = (2450, 2500, 2500)$	

Fig. 2 A comparison graph of the actual enrollments versus forecasted enrollments

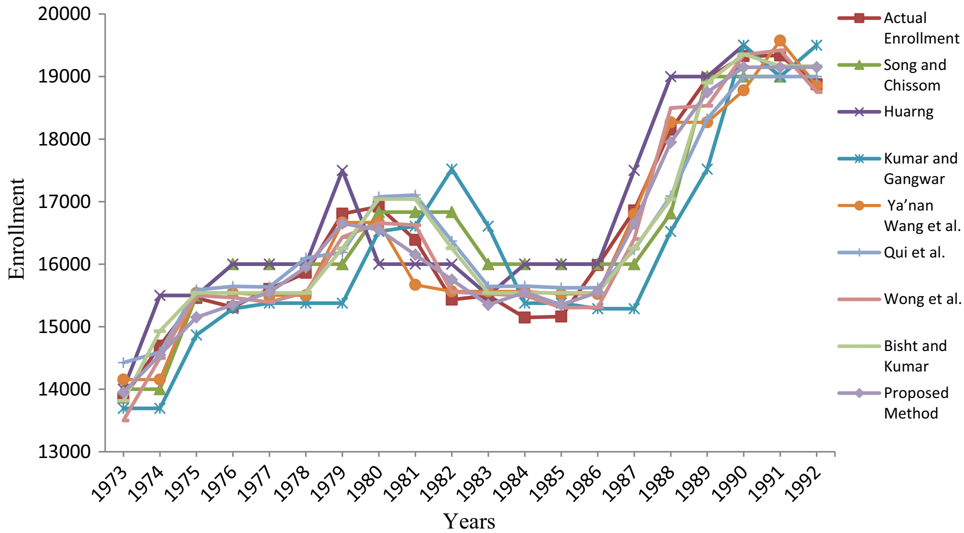

Table 7 Actual prices and transformed moving average prices along with their fuzzified moving average prices of SBI share

Months	Actual SBI share prices	Moving average SBI share prices	Fuzzified SBI share prices	Months	Actual SBI share prices	Moving average SBI share prices	Fuzzified SBI share prices
Apr.-08	1819.95	–	–	Apr.-09	1355.00	1459.42	$\tilde{A}_5$
May-08	1840.00	1718.88	$\tilde{A}_{10}$	May-09	1891.00	1727.00	$\tilde{A}_{11}$
June-08	1496.70	1634.73	$\tilde{A}_9$	June-09	1935.00	1888.67	$\tilde{A}_{14}$
July-08	1567.50	1567.70	$\tilde{A}_7$	July-09	1840.00	1887.30	$\tilde{A}_{14}$
Aug.-08	1638.90	1608.13	$\tilde{A}_8$	Aug.-09	1886.90	1987.30	$\tilde{A}_{16}$
Sept.-08	1618.00	1608.93	$\tilde{A}_8$	Sept.-09	2235.00	2207.30	$\tilde{A}_{20}$
Oct.-08	1569.90	1520.97	$\tilde{A}_6$	Oct.-09	2500.00	2376.33	$\tilde{A}_{24}$
Nov.-08	1375.00	1423.30	$\tilde{A}_4$	Nov.-09	2394.00	2422.92	$\tilde{A}_{24}$
Dec.-08	1325.00	1358.80	$\tilde{A}_3$	Dec.-09	2374.75	2361.33	$\tilde{A}_{23}$
Jan.-09	1376.40	1302.43	$\tilde{A}_2$	Jan.-10	2315.25	2249.98	$\tilde{A}_{21}$
Feb.-09	1205.90	1238.18	$\tilde{A}_1$	Feb.-10	2059.95	2165.08	$\tilde{A}_{19}$
Mar.-09	1132.25	1231.13	$\tilde{A}_1$	Mar.-10	2120.05		

Table 8 Fuzzy logical relationships of fuzzified moving average prices of SBI share

$\tilde{A}_{10} \to \tilde{A}_9$	$\tilde{A}_9 \to \tilde{A}_7$	$\tilde{A}_7 \to \tilde{A}_8$	$\tilde{A}_8 \to \tilde{A}_8$
$\tilde{A}_8 \to \tilde{A}_6$	$\tilde{A}_6 \to \tilde{A}_4$	$\tilde{A}_4 \to \tilde{A}_3$	$\tilde{A}_3 \to \tilde{A}_2$
$\tilde{A}_2 \to \tilde{A}_1$	$\tilde{A}_1 \to \tilde{A}_1$	$\tilde{A}_1 \to \tilde{A}_5$	$\tilde{A}_5 \to \tilde{A}_{11}$
$\tilde{A}_{11} \to \tilde{A}_{14}$	$\tilde{A}_{14} \to \tilde{A}_{14}$	$\tilde{A}_{14} \to \tilde{A}_{16}$	$\tilde{A}_{16} \to \tilde{A}_{20}$
$\tilde{A}_{20} \to \tilde{A}_{24}$	$\tilde{A}_{24} \to \tilde{A}_{24}$	$\tilde{A}_{24} \to \tilde{A}_{23}$	$\tilde{A}_{23} \to \tilde{A}_{21}$
$\tilde{A}_{21} \to \tilde{A}_{19}$			

Compute the grade of membership for each transformed moving average datum of the SBI share corresponding to its defined triangular fuzzy sets as follows:

$$\tilde{A}_1 = \frac{0.62}{1231.13} + \frac{0.76}{1238.18} \qquad \tilde{A}_2 = \frac{0.95}{1302.43}$$

$$\tilde{A}_3 = \frac{0.05}{1302.43} + \frac{0.82}{1358.80} \qquad \tilde{A}_4 = \frac{0.18}{1358.80} + \frac{0.53}{1423.30}$$

$$\tilde{A}_5 = \frac{0.47}{1423.30} + \frac{0.81}{1459.42} \qquad \tilde{A}_6 = \frac{0.19}{1459.42} + \frac{0.58}{1520.97}$$

$$\tilde{A}_7 = \frac{0.42}{1520.97} + \frac{0.65}{1567.7} \qquad \tilde{A}_8 = \frac{0.35}{1567.70} + \frac{0.8374}{1608.13} + \frac{0.8214}{1608.93} + \frac{0.30}{1634.73}$$

$$\tilde{A}_9 = \frac{0.1626}{1608.13} + \frac{0.1786}{1608.93} + \frac{0.69}{1634.73} \qquad \tilde{A}_{10} = \frac{0.62}{1718.88} + \frac{0.46}{1727}$$

$$\tilde{A}_{11} = \frac{0.38}{1718.88} + \frac{0.54}{1727} \qquad \tilde{A}_{12} = \emptyset \text{ (null fuzzy set)}$$

$$\tilde{A}_{13} = \frac{0.25}{1887.30} + \frac{0.23}{1888.67} \qquad \tilde{A}_{14} = \frac{0.75}{1887.3} + \frac{0.77}{1888.67}$$

$\tilde{A}_{15} = \frac{0.25}{1987.30}$	$\tilde{A}_{16} = \frac{0.75}{1987.30}$
$\tilde{A}_{17} = \emptyset$ (null fuzzy set)	$\tilde{A}_{18} = \emptyset$ (null fuzzy set)
$\tilde{A}_{19} = \frac{0.698}{2165.08}$	$\tilde{A}_{20} = \frac{0.301}{2165.08} + \frac{0.85}{2207.30} + \frac{0.0004}{2249.98}$
$\tilde{A}_{21} = \frac{0.15}{2207.3} + \frac{0.9996}{2249.98}$	$\tilde{A}_{22} = \emptyset$ (null fuzzy set)
$\tilde{A}_{23} = \frac{0.77}{2361.33} + \frac{0.47}{2376.33}$	$\tilde{A}_{24} = \frac{0.23}{2361.33} + \frac{0.53}{2376.33} + \frac{0.54}{2422.92}$
$\tilde{A}_{25} = \frac{0.46}{2422.92}$	$\tilde{A}_{26} = \emptyset$ (null fuzzy set)

Step 6 Fuzzify the transformed moving average prices data of SBI shares by triangular fuzzy set in which moving average price of the SBI share has a maximum grade of membership, so the fuzzified moving average data are placed in Table 7.

Table 9 Fuzzy logical relationship groups of fuzzified moving average prices of SBI share

Groups	FLRGs	Groups	FLRGs
Group 1	$\tilde{A}_1 \to \tilde{A}_1, \tilde{A}_5$	Group 10	$\tilde{A}_{10} \to \tilde{A}_9$
Group 2	$\tilde{A}_2 \to \tilde{A}_1$	Group 11	$\tilde{A}_{11} \to \tilde{A}_{14}$
Group 3	$\tilde{A}_3 \to \tilde{A}_2$	Group 12	$\tilde{A}_{14} \to \tilde{A}_{14}, \tilde{A}_{16}$
Group 4	$\tilde{A}_4 \to \tilde{A}_3$	Group 13	$\tilde{A}_{16} \to \tilde{A}_{20}$
Group 5	$\tilde{A}_5 \to \tilde{A}_{11}$	Group 14	$\tilde{A}_{20} \to \tilde{A}_{24}$
Group 6	$\tilde{A}_6 \to \tilde{A}_4$	Group 15	$\tilde{A}_{21} \to \tilde{A}_{19}$
Group 7	$\tilde{A}_7 \to \tilde{A}_8$	Group 16	$\tilde{A}_{23} \to \tilde{A}_{21}$
Group 8	$\tilde{A}_8 \to \tilde{A}_8, \tilde{A}_6$	Group 17	$\tilde{A}_{24} \to \tilde{A}_{24}, \tilde{A}_{23}$
Group 9	$\tilde{A}_9 \to \tilde{A}_7$		

Table 10 Forecasted output of SBI share prices of the proposed method along with existing methods

Months	Actual SBI prices	Chen method (1996)	Huarng method (2011)	Gangwar and Kumar (2015)	Pathak and Singh (2011)	Bisht and Kumar (2016)	Chen and Tanuwijaya (2011)	Proposed method
Apr.-08	1819.95	–	–	–	–	–	–	–
May-08	1840.00	1900	1855	1725.98	1770.00	1877.657	1954.81	–
June-08	1496.70	1900	1855	1725.98	1832.50	1877.657	1954.81	1625
July-08	1567.50	1500	1575	1512.39	1470.00	1466.360	1753.33	1525
Aug.-08	1638.90	1500	1505	1512.39	1570.00	1466.360	1553.95	1575
Sept.-08	1618.00	1600	1610	1574.35	1670.00	1533.504	1553.95	1525
Oct.-08	1569.90	1600	1610	1574.35	1603.33	1533.504	1553.95	1525
Nov.-08	1375.00	1500	1505	1512.39	1670.00	1466.360	1553.95	1375
Dec.-08	1325.00	1433	1482	1305.52	1382.50	1520.652	1541.06	1325
Jan.-09	1376.40	1433	1365	1665.90	1332.50	1520.652	1541.06	1275
Feb.-09	1205.90	1433	1482	1305.52	1332.50	1520.652	1541.06	1225
Mar.-09	1132.25	1433	1155	1294.27	1195.00	1144.718	1132.25	1325
Apr.-09	1355.00	1300	1365	1294.27	1145.00	1322.446	134.58	1325
May-09	1891.00	1433	1482	1665.90	1357.50	1520.652	1541.06	1725
June-09	1935.00	1900	1890	2006.51	1882.50	1877.657	1954.81	1875
July-09	1840.00	1900	1890	2006.51	1970.00	1895.491	2043.13	1925
Aug.-09	1886.90	1900	1855	1725.98	1470.00	1877.657	1954.81	1925
Sept.-09	2235.00	1900	1855	2006.51	1970.00	1877.657	1954.81	2175
Oct.-09	2500.00	2300	2485	2520.00	2245.00	2311.382	2500.00	2375
Nov.-09	2394.00	2300	2415	2420.00	2470.00	2374.204	2396.00	2350
Dec.-09	2374.75	2300	2345	2365.99	2395.00	2352.723	2354.23	2350
Jan.-10	2315.25	2300	2205	2365.99	2395.00	2352.723	2354.23	2225
Feb.-10	2059.95	2300	2205	2020.00	2295.00	2311.382	2043.74	2125
Mar.-10	2120.05	2100	2135	2120.00	2070.00	2166.247	2133.76	–

Table 11 A comparison of RMSE and AFE of the proposed method with other methods

Models	Chen method (1996)	Huarng method (2011)	Gangwar and Kumar (2015)	Pathak and Singh (2011)	Bisht and Kumar (2016)	Chen and Tanuwijaya (2011)	Abhishekh et al. (2017)	Abhishekh et al. (2018a)	Proposed method
RMSE	187.26	164.04	131.28	205.96	179.03	179.0	171.81	112.25	86.10
AFE	8.26	6.29	6.30	8.95	7.86	7.77	6.41	5.85	4.19

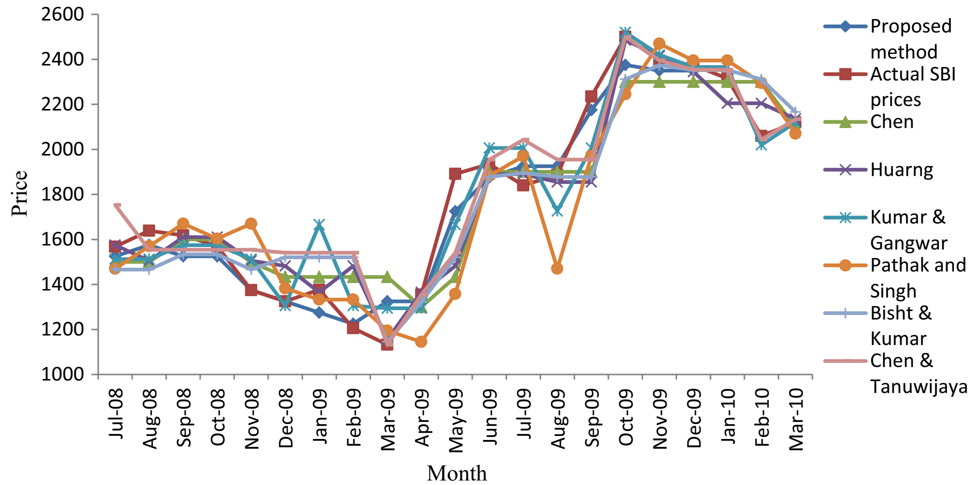

Fig. 3 A comparison graph of the actual market price versus forecasted price of SBI share data

Step 7 Establish the fuzzy logical relationships (FLRs) from the fuzzified moving average prices data of SBI share and then create the fuzzy logical relationship groups (FLRGs) to forecasting. Computed FLRs and FLRGs are placed in Tables 8 and 9, respectively.

Step 8 By using step 8 of the proposed method, we calculate the forecasted output of SBI share prices along with different methods and are placed in Table 10.

Step 9 Compute the RMSE and AFE of the proposed method and compared with existing methods of fuzzy time series, which shows that our proposed method can achieve better forecasting rate in terms of two statistical parameters RMSE and AFE which are placed in Table 11.

The trend in the forecasting of the market price of SBI shares by the other mentioned methods and the proposed method is illustrated in Fig. 3. The comparative study of the proposed method with existing methods available in the literature in terms of RMSE and AFE shows that proposed method can achieve higher forecasting accuracy rates.

5 Conclusion

Moving average approach in fuzzy time series models can generally give better result in forecasted evaluation compared to first-order fuzzy time series models. In this paper, we propose a novel fuzzy time series forecasting method that uses moving average fuzzy information in data analysis. Using the moving average approach, here we define the universe of discourse and partition the transformed moving average data rather than actual data set by using the average-based length method. The fuzzification process of transformed moving average historical data is based on its maximum membership grades in corresponding triangular fuzzy sets. Further, we establish a fuzzy logical relationship groups among the fuzzified moving average data set, and from comparison Tables 6 and 11 it is observed that the proposed method has higher forecasting rates with respect to other fuzzy time series models available in the literature. Also, from Figs. 2 and 3 it is clearly shown that the forecasted output obtained by the proposed method is significantly in close accordance with the actual time series historical data set.

In future work, the proposed method can be extended to deal with the aspect of meta-heuristic optimization technique such as particle swarm optimization (PSO), neural network (NN) and genetic algorithms (GA) by which the forecasting accuracy of proposed method can be improved. It could also apply to other problems by using two or more factors such as sensex forecasting and temperature prediction in further research.

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