

Interval-Boundary-Dependent Control for Interval Type-2 T–S Fuzzy Systems

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Abstract

This paper introduces a state-feedback controller for interval type-2 Takagi–Sugeno fuzzy systems that are described as a convex combination of linear subsystems via normalized time-varying fuzzy weighting parameters, which are unknown but belong to intervals. The introduced structure employs lower and upper interval boundaries directly, rather than alternative parameters generated by the interval boundaries in the studies, which does lead to a stabilization condition of parameterized linear matrix inequalities (LMIs), associated only with the parameters and their interval boundaries. The exact relationship among the parameters and their interval boundaries can, therefore, be fully used to convert parameterized LMIs into LMIs, which provides a great feature distinguished from those in the studies. Two illustrative examples demonstrate the effectiveness and robustness of the derived controller.

Keywords Interval type-2 fuzzy system · Stability analysis · Relaxation · Fuzzy control

1 Introduction

In the past decades, the most significant issue for fuzzy systems is how to guarantee the stability and achieve control performance. There have been research efforts on the issue of stability in fuzzy control systems (Mamdani 1994; Wu and Tan 2005; Bezerra et al. 2017). One of the well-known model of fuzzy systems used for control applications is Takagi–Sugeno (T–S) fuzzy model (Takagi and Sugeno 1985; Montagner et al. 2009). The T–S fuzzy models have attracted substantial interest in the control research community, because it offers a general framework for stability analysis and controller synthesis (Liu and Zhang 2003; Teixeira et al. 2003; Feng 2006; Kumar et al. 2006). Therefore, in this paper, we adopt the T–S fuzzy model.

Recently, there has been a growing interest in using a type-2 fuzzy model in control applications due to their ability for model uncertainties (Mendel et al. 2006). The type-2 fuzzy model was introduced by Zadeh (1975) as an extension of the concept of the conventional fuzzy model, that is the type-1 fuzzy model. The type-2 fuzzy models have been shown that it is better to the type-1 fuzzy models on handling with model uncertainties. With the lower and upper membership functions, the uncertainties can be employed and the information of the uncertainties is included in the range of uncertainty (Liang and Mendel 2000; Lam and Seneviratne 2008). Furthermore, a type-2 fuzzy control for a nonlinear system with uncertainties has attracted a lot of interest in recent years (Chaoui et al. 2016; Xie et al. 2017). Even though only interval type-2 fuzzy systems have been applied for control applications, the prospective results have been studied (Hagras 2004; Martinez et al. 2009; Lam and Seneviratne 2008; Biglarbegian et al. 2010; Abbadi et al. 2013). Hagras (2004) and Martinez et al. (2009) designed an interval type-2 fuzzy controller for a mobile robot in dynamic unstructured indoor and outdoor environments. Lam and Seneviratne (2008) investigated the stability analysis of interval type-2 T–S fuzzy control systems. Biglarbegian et al. (2010) introduced an inference mechanism to address the stability of the interval type-2 TSK fuzzy system. Recently, Abbadi et al. (2013) designed an interval type-2 fuzzy controller that has

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the ability to enhance the transient stability and achieve voltage regulation for multimachine power systems.

However, the above approaches require several tuning parameters to be made about the type-2 fuzzy controller using alternative convex parameters generated by the interval boundaries, which makes the conservative results for stability conditions. In addition, the existing studies do not use relationship among lower and upper interval boundaries and the normalized time-varying fuzzy weighting parameters to help guarantee the stability of interval type-2 T–S fuzzy systems. In an attempt to obtain the stability of interval type-2 T–S fuzzy systems, we introduce a new structure of a state-feedback controller. The proposed structure employs lower and upper interval boundaries directly, which does lead to a stabilization condition of parameterized linear matrix inequalities (PLMIs), associated with the parameters and their interval boundaries. Further, the tuning parameters do not addressed in the proposed structure. The exact relationship among the parameters and their interval boundaries can, therefore, be fully used to convert PLMIs into LMIs, which provides a great feature distinguished from those in the studies. Two illustrative examples demonstrate the effectiveness and robustness of the derived controller.

Notation: The notations $X \geq Y$ and $X > Y$ mean that $X - Y$ is positive semidefinite and positive definite, respectively. In symmetric block matrices, $(*)$ is used as an ellipsis for terms that are induced by symmetry. Furthermore, $\text{Sym}(X) = X + X^T$ stands for any matrix X .

2 Preliminaries: System and Controller

The general structure of an interval type-2 T–S fuzzy model is as follows (Mendel et al. 2006):

IF $g_1(x(t))$ is $\tilde{F}_1^i$ and $\dots$ and $g_s(x(t))$ is $\tilde{F}_s^i$,
THEN $\dot{x}(t) = A_i x(t) + B_i u(t)$, (1)

where $i = 1, 2, \dots, p$, $\tilde{F}_j^i$ denotes the interval type-2 fuzzy set of function $g_j(x(t))$ in rule i , $x(t) \in \mathcal{R}^n$ is a state, $u(t) \in \mathcal{R}^m$ is a control input, and p is the number of rules. In the interval type-2 fuzzy model, the lower and upper interval boundaries of the i th rule are given by

$$\theta_i^L(x(t)) = \underline{\mu}_{\tilde{F}_1^i}(g_1(x(t))) \times \dots \times \underline{\mu}_{\tilde{F}_s^i}(g_s(x(t))), \quad (2)$$

$$\theta_i^U(x(t)) = \overline{\mu}_{\tilde{F}_1^i}(g_1(x(t))) \times \dots \times \overline{\mu}_{\tilde{F}_s^i}(g_s(x(t))), \quad (3)$$

where $\underline{\mu}_{\tilde{F}_j^i}$ and $\overline{\mu}_{\tilde{F}_j^i}$ denote the j th ($j = 1, \dots, s$) lower and upper membership functions of rule i , respectively. The overall interval type-2 T–S fuzzy system is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^p \theta_i(x(t)) (A_i x(t) + B_i u(t)), \quad (4)$$

where $\theta_i(x(t))$, a normalized time-varying fuzzy weighting parameter for each rule i and $\eta_i(x(t)) \in [0, 1]$, is a nonlinear function such as

$$\sum_{i=1}^p \theta_i(x(t)) = 1, \quad (5)$$

$$\theta_i(x(t)) = \eta_i(x(t)) \theta_i^L(x(t)) + (1 - \eta_i(x(t))) \theta_i^U(x(t)). \quad (6)$$

In other words, the parameter, $\theta_i(x(t))$, belongs to the interval between the lower interval boundary, $\theta_i^L(x(t))$, and the upper interval boundary, $\theta_i^U(x(t))$, for each rule i .

Let us propose a controller for system (4) as

$$u(t) = \sum_{i=1}^p (K_0 + \theta_i^L(x(t)) \underline{K}_i + \theta_i^U(x(t)) \overline{K}_i) x(t), \quad (7)$$

where K_0 is a constant controller gain and $\underline{K}_i$ and $\overline{K}_i$ are the controller gains associated with the lower and upper membership functions. We shall, henceforth, call it an interval-boundary-dependent (IBD) controller. In view of the above relations, $\theta_i(x(t))$, $\theta_i^L(x(t))$, and $\theta_i^U(x(t))$ accord with the following relationship, for all $i \in [1, p]$:

$$0 \leq \alpha_i \leq \theta_i^L(x(t)) \leq \theta_i(x(t)) \leq \theta_i^U(x(t)) \leq \beta_i \leq 1, \quad (8)$$

where α_i and β_i are nonnegative and positive integers.

For a simple description, θ_i , θ_i^L , θ_i^U , and η_i indicate $\theta_i(x(t))$, $\theta_i^L(x(t))$, $\theta_i^U(x(t))$, and $\eta_i(x(t))$, respectively. Substituting the IBD controller (7) into the overall interval type-2 T–S fuzzy system (4), the closed-loop system is represented as

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^p \theta_i \left\{ A_i x(t) + B_i \sum_{j=1}^p (K_0 + \theta_j^L \underline{K}_j + \theta_j^U \overline{K}_j) x(t) \right\} \\ &= \left\{ \sum_{i=1}^p \theta_i (A_i + B_i K_0) \right. \\ &\quad \left. + \sum_{i=1}^p \sum_{j=1}^p (\theta_i \theta_j^L B_i \underline{K}_j + \theta_i \theta_j^U B_i \overline{K}_j) \right\} x(t). \end{aligned} \quad (9)$$

3 Control Synthesis

Let us choose $V(x(t)) = x^T(t) P x(t)$ as Lyapunov function, where P is a positive definite matrix. Then derivative of $V(x(t))$ is

$$\begin{aligned}\dot{V}(x(t)) &= 2x^T(t)P\dot{x}(t) \\ &= 2x^T(t)P\left\{\sum_{i=1}^p\theta_i(A_i+B_iK_0)\right. \\ &\quad \left.+\sum_{i=1}^p\sum_{j=1}^p(\theta_i\theta_j^L B_i\underline{K}_j+\theta_i\theta_j^U B_i\overline{K}_j)\right\}x(t).\end{aligned}\quad (10)$$

To guarantee the stability of the proposed system, it holds that $\dot{V}(x(t)) < 0$ which leads to the following condition:

$$\begin{aligned}\text{Sym}\left\{\sum_{i=1}^p\theta_i(PA_i+PB_iK_0)\right. \\ \left.+\sum_{i=1}^p\sum_{j=1}^p(\theta_i\theta_j^L PB_i\underline{K}_j+\theta_i\theta_j^U PB_i\overline{K}_j)\right\} < 0.\end{aligned}\quad (11)$$

Pre- and post-multiplying (11) with P^{-1} and its transpose, then we have

$$\begin{aligned}\text{Sym}\left\{\sum_{i=1}^p\theta_i(A_i\bar{P}+B_iL_0)\right. \\ \left.+\sum_{i=1}^p\sum_{j=1}^p(\theta_i\theta_j^L B_i\underline{L}_j+\theta_i\theta_j^U B_i\overline{L}_j)\right\} < 0,\end{aligned}\quad (12)$$

where $\bar{P} \triangleq P^{-1}$, $L_0 \triangleq K_0\bar{P}$, $\underline{L}_j \triangleq \underline{K}_j\bar{P}$, and $\overline{L}_j \triangleq \overline{K}_j\bar{P}$.

To derive the LMI conditions, (12) can be written as follows:

$$\begin{aligned}\mathcal{M} &= \sum_{i=1}^p\theta_i(M_{(0,i)}+M_{(0,i)}^T)+\sum_{i=1}^p\sum_{j=1}^p\theta_i\theta_j^L(\underline{M}_{(i,j)}+\underline{M}_{(i,j)}^T) \\ &\quad +\sum_{i=1}^p\sum_{j=1}^p\theta_i\theta_j^U(\overline{M}_{(i,j)}+\overline{M}_{(i,j)}^T) < 0,\end{aligned}\quad (13)$$

where $M_{(0,i)} \triangleq A_i\bar{P}+B_iL_0$, $\underline{M}_{(i,j)} \triangleq B_i\underline{L}_j$, and $\overline{M}_{(i,j)} \triangleq B_i\overline{L}_j$. Here, θ_i , θ_i^L , and θ_i^U satisfy conditions (5) and (8). Using slack variables for the conditions, we can relax (13) via S-procedure.

For the relaxation technique, we can convert conditions (5) and (8) for all $i \in [1, p]$, respectively, into

$$0 \equiv C_1 \triangleq \begin{bmatrix} I & \theta_1 I & \vdots & \theta_p I & \theta_1^L I & \vdots & \theta_p^L I & \theta_1^U I & \vdots & \theta_p^U I \end{bmatrix}^T \begin{bmatrix} I & -I & \vdots & -I & 0 & \vdots & 0 & 0 & \vdots & 0 \end{bmatrix} \begin{bmatrix} R_0 & R_1^T & \vdots & R_p^T & \underline{R}_1^T & \vdots & \underline{R}_p^T & \overline{R}_1^T & \vdots & \overline{R}_p^T \end{bmatrix}^T \begin{bmatrix} I & \theta_1 I & \vdots & \theta_p I & \theta_1^L I & \vdots & \theta_p^L I & \theta_1^U I & \vdots & \theta_p^U I \end{bmatrix}, \quad (14)$$

$$0 \leq C_{2i} \triangleq -(\theta_i^L - \theta_i)(\theta_i^L - \alpha_i)(X_i + X_i^T), \quad (15)$$

$$0 \leq C_{3i} \triangleq -(\theta_i - \theta_i^L)(\theta_i - \theta_i^U)(Y_i + Y_i^T), \quad (16)$$

$$0 \leq C_{4i} \triangleq -(\theta_i^U - \theta_i)(\theta_i^U - \beta_i)(Z_i + Z_i^T), \quad (17)$$

where $X_i + X_i^T > 0$, $Y_i + Y_i^T > 0$, and $Z_i + Z_i^T > 0$. Here, matrices R_0 , R_i , $\underline{R}_i$, $\overline{R}_i$, X_i , Y_i , and $Z_i \in \mathcal{R}^{n \times n}$ are slack variables to reduce the conservatism of condition (12).

Then, combining conditions (14)–(17) gives

$$\mathcal{N} = C_1 + C_1^T + \sum_{i=1}^p C_{2i} + \sum_{i=1}^p C_{3i} + \sum_{i=1}^p C_{4i} \geq 0, \quad (18)$$

To derive LMI conditions, (18) can be written as follows:

$$\begin{aligned}\mathcal{N} &= N_0 + \sum_{i=1}^p\theta_i(N_{(0,i)}+N_{(0,i)}^T)+\sum_{i=1}^p\theta_i^L(\underline{N}_{(0,i)}+\underline{N}_{(0,i)}^T) \\ &\quad +\sum_{i=1}^p\theta_i^U(\overline{N}_{(0,i)}+\overline{N}_{(0,i)}^T)+\sum_{i=1}^p\theta_i^2N_{(i,i)}+\sum_{i=1}^p(\theta_i^L)^2\underline{N}_{(i,i)} \\ &\quad +\sum_{i=1}^p(\theta_i^U)^2\overline{N}_{(i,i)}+\sum_{i=1}^{p-1}\sum_{j=i+1}^p\theta_i\theta_j(N_{(i,j)}+N_{(i,j)}^T) \\ &\quad +\sum_{i=1}^p\sum_{j=1}^p\theta_i\theta_j^L(\underline{T}_{(i,j)}+\underline{T}_{(i,j)}^T)+\sum_{i=1}^p\sum_{j=1}^p\theta_i\theta_j^U(\overline{T}_{(i,j)}+\overline{T}_{(i,j)}^T) \\ &\quad +\sum_{i=1}^p\theta_i^L\theta_i^U(\tilde{T}_{(i,i)}+\tilde{T}_{(i,i)}^T),\end{aligned}\quad (19)$$

where

$$N_0 \triangleq R_0 + R_0^T, \quad N_{(0,i)} \triangleq R_i - R_0 - \alpha_i X_i - \beta_i Z_i,$$

$$\underline{N}_{(0,i)} \triangleq \underline{R}_i + \alpha_i X_i, \quad \overline{N}_{(0,i)} \triangleq \overline{R}_i + \beta_i Z_i,$$

$$N_{(i,i)} \triangleq -(R_i + R_i^T) - (Y_i + Y_i^T),$$

$$\underline{N}_{(i,i)} \triangleq -(X_i + X_i^T), \quad \overline{N}_{(i,i)} \triangleq -(Z_i + Z_i^T),$$

$$N_{(i,j)} \triangleq -(R_i^T + R_j),$$

$$\underline{T}_{(i,j)} \triangleq \begin{cases} -\underline{R}_i & i \neq j \\ -\underline{R}_i + X_i + Y_i & i = j \end{cases},$$

$$\overline{T}_{(i,j)} \triangleq \begin{cases} -\overline{R}_i & i \neq j \\ -\overline{R}_i + Y_i + Z_i & i = j \end{cases}$$

$$\tilde{T}_{(i,i)} \triangleq -Y_i.$$

By the $\mathcal{S}$ -procedure, if $\mathcal{M} < 0$ whenever $\mathcal{N} \geq 0$ such as θ_i , θ_i^L , and θ_i^U satisfy conditions (5) and (8), then the following sufficient condition is formulated as:

$$\mathcal{L} = \mathcal{M} + \mathcal{N} < 0, \quad (20)$$

where

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_0 + \sum_{i=1}^p \theta_i (\mathcal{L}_{(0,i)} + \mathcal{L}_{(0,i)}^T) \\ & + \sum_{i=1}^p \theta_i^L (\underline{\mathcal{L}}_{(0,i)} + \underline{\mathcal{L}}_{(0,i)}^T) + \sum_{i=1}^p \theta_i^U (\overline{\mathcal{L}}_{(0,i)} + \overline{\mathcal{L}}_{(0,i)}^T) \\ & + \sum_{i=1}^p \theta_i^2 \mathcal{L}_{(i,i)} + \sum_{i=1}^p (\theta_i^L)^2 \underline{\mathcal{L}}_{(i,i)} + \sum_{i=1}^p (\theta_i^U)^2 \overline{\mathcal{L}}_{(i,i)} \\ & + \sum_{i=1}^{p-1} \sum_{j=i+1}^p \theta_i \theta_j (\mathcal{L}_{(i,j)} + \mathcal{L}_{(i,j)}^T) + \sum_{i=1}^p \sum_{j=1}^p \theta_i \theta_j^L (\underline{\mathcal{L}}_{(i,j)} + \underline{\mathcal{L}}_{(i,j)}^T) \\ & + \sum_{i=1}^p \sum_{j=1}^p \theta_i \theta_j^U (\overline{\mathcal{L}}_{(i,j)} + \overline{\mathcal{L}}_{(i,j)}^T) + \sum_{i=1}^p \theta_i^L \theta_i^U (\tilde{\mathcal{L}}_{(i,i)} + \tilde{\mathcal{L}}_{(i,i)}^T), \end{aligned}$$

$$\mathcal{L}_0 = N_0, \quad \mathcal{L}_{(0,i)} = M_{(0,i)} + N_{(0,i)}, \quad \underline{\mathcal{L}}_{(0,i)} = \underline{N}_{(0,i)},$$

$$\overline{\mathcal{L}}_{(0,i)} = \overline{N}_{(0,i)}, \quad \mathcal{L}_{(i,i)} = N_{(i,i)}, \quad \underline{\mathcal{L}}_{(i,i)} = \underline{N}_{(i,i)},$$

$$\overline{\mathcal{L}}_{(i,i)} = \overline{N}_{(i,i)}, \quad \mathcal{L}_{(i,j)} = N_{(i,j)}, \quad \underline{\mathcal{L}}_{(i,j)} = \underline{M}_{(i,j)} + \underline{T}_{(i,j)},$$

$$\overline{\mathcal{L}}_{(i,j)} = \overline{M}_{(i,j)} + \overline{T}_{(i,j)}, \quad \tilde{\mathcal{L}}_{(i,i)} = \tilde{T}_{(i,i)},$$

which can be converted into LMI condition:

$$\mathcal{L} = \Theta^T \Sigma \Theta < 0, \quad (21)$$

where Σ and Θ are shown at the bottom of this page and

$$\mathcal{L}_0 \triangleq R_0 + R_0^T,$$

$$\mathcal{L}_{(0,i)} \triangleq A_i \bar{P} + B_i L_0 + R_i - R_0 - \alpha_i X_i - \beta_i Z_i,$$

$$\underline{\mathcal{L}}_{(0,i)} \triangleq \underline{R}_i + \alpha_i X_i,$$

$$\overline{\mathcal{L}}_{(0,i)} \triangleq \overline{R}_i + \beta_i Z_i,$$

$$\mathcal{L}_{(i,i)} \triangleq -(R_i + R_i^T) - (Y_i + Y_i^T),$$

$$\underline{\mathcal{L}}_{(i,i)} \triangleq -(X_i + X_i^T),$$

$$\overline{\mathcal{L}}_{(i,i)} \triangleq -(Z_i + Z_i^T),$$

$$\mathcal{L}_{(i,j)} \triangleq -(R_i^T + R_j),$$

$$\underline{\mathcal{L}}_{(i,j)} \triangleq \begin{cases} B_i \underline{L}_j - \underline{R}_i & i \neq j \\ B_i \underline{L}_j - \underline{R}_i + X_i + Y_i & i = j \end{cases},$$

$$\overline{\mathcal{L}}_{(i,j)} \triangleq \begin{cases} B_i \overline{L}_j - \overline{R}_i & i \neq j \\ B_i \overline{L}_j - \overline{R}_i + Y_i + Z_i & i = j \end{cases}$$

$$\tilde{\mathcal{L}}_{(i,i)} \triangleq -Y_i.$$

$$\Theta = [I | \theta_1 I \cdots \theta_p I | \theta_1^L I \cdots \theta_p^L I | \theta_1^U I \cdots \theta_p^U I]$$

$$\Sigma = \begin{bmatrix} \mathcal{L}_0 & \mathcal{L}_{(0,1)} & \mathcal{L}_{(0,2)} & \cdots & \mathcal{L}_{(0,p)} & \underline{\mathcal{L}}_{(0,1)} & \underline{\mathcal{L}}_{(0,2)} & \cdots & \underline{\mathcal{L}}_{(0,p)} & \overline{\mathcal{L}}_{(0,1)} & \overline{\mathcal{L}}_{(0,2)} & \cdots & \overline{\mathcal{L}}_{(0,p)} \\ (*) & \mathcal{L}_{(1,1)} & \mathcal{L}_{(1,2)} & \cdots & \mathcal{L}_{(1,p)} & \underline{\mathcal{L}}_{(1,1)} & \cdots & \cdots & \underline{\mathcal{L}}_{(1,p)} & \overline{\mathcal{L}}_{(1,1)} & \cdots & \cdots & \overline{\mathcal{L}}_{(1,p)} \\ \vdots & (*) & \ddots & & \vdots & \vdots & \ddots & & \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & & \ddots & \vdots & \vdots & & \ddots & \vdots \\ (*) & (*) & \cdots & (*) & \mathcal{L}_{(p,p)} & \underline{\mathcal{L}}_{(p,1)} & \cdots & \cdots & \underline{\mathcal{L}}_{(p,p)} & \overline{\mathcal{L}}_{(p,1)} & \cdots & \cdots & \overline{\mathcal{L}}_{(p,p)} \\ (*) & (*) & \cdots & \cdots & (*) & \underline{\mathcal{L}}_{(1,1)} & 0 & \cdots & 0 & \overline{\mathcal{L}}_{(1,1)} & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \cdots & \vdots & (*) & \ddots & \ddots & \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \cdots & \cdots & \vdots & \vdots & \ddots & \ddots & 0 & \vdots & \ddots & \ddots & 0 \\ (*) & (*) & \cdots & \cdots & (*) & (*) & \cdots & (*) & \underline{\mathcal{L}}_{(p,p)} & 0 & \cdots & 0 & \tilde{\mathcal{L}}_{(p,p)} \\ (*) & (*) & \cdots & \cdots & (*) & (*) & \cdots & \cdots & (*) & \mathcal{L}_{(1,1)} & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \cdots & \vdots & \vdots & \cdots & \cdots & \vdots & (*) & \ddots & \ddots & \vdots \\ \vdots & \vdots & \cdots & \cdots & \vdots & \vdots & \cdots & \cdots & \vdots & \vdots & \ddots & \ddots & 0 \\ (*) & (*) & \cdots & \cdots & (*) & (*) & \cdots & \cdots & (*) & (*) & \cdots & (*) & \overline{\mathcal{L}}_{(p,p)} \end{bmatrix},$$

Theorem 1 For all $i \in [1, p]$, suppose that there exist matrices $\bar{P} > 0$, L_0 , $\underline{L}_i$, $\bar{L}_i$, R_0 , R_i , $\underline{R}_i$, $\bar{R}_i$, X_i , Y_i , and Z_i such that

$$X_i + X_i^T > 0, Y_i + Y_i^T > 0, Z_i + Z_i^T > 0, \quad (22)$$

$$\Sigma < 0, \quad (23)$$

where Σ is presented at the bottom of the previous page. Then, the closed-loop system (9) is asymptotically stable for all admissible grades satisfying conditions (5) and (8). Further, the IBD controller is reconstructed as $K_0 = L_0 \bar{P}^{-1}$, $\underline{K}_i = \underline{L}_i \bar{P}^{-1}$, and $\bar{K}_i = \bar{L}_i \bar{P}^{-1}$.

4 Simulation Results

In this section, we demonstrate the performance of the proposed controller using two examples. First example shows stability region as the change of system matrices. Second example shows state trajectory of a nonlinear plant with uncertainties for illustrating effectiveness of the IBD controller.

Example 1 Consider a simple fuzzy system (Lam and Seneviratne 2008) in the following form:

IF $x_1(t)$ is $\tilde{F}_1^i$,

$$\text{THEN } \dot{x}(t) = A_i x(t) + B_i u(t), \quad i = 1, 2, 3 \quad (24)$$

where

$$A_1 = \begin{bmatrix} 2.78 & -5.63 \\ 0.01 & 0.33 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.2 & -3.22 \\ 0.35 & 0.12 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -a & -6.63 \\ 0.45 & 0.15 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 8 \\ 0 \end{bmatrix}, \quad B_3 = \begin{bmatrix} -b+6 \\ -1 \end{bmatrix},$$

in which $20 \leq a \leq 30$ and $20 \leq b \leq 35$. Assume that the above system is operating in the domain of $x_1(t) \in [-10 \ 10]^T$. In this example, θ_i^L and θ_i^U are regarded as the lower and upper membership functions, because the number of fuzzy sets is one. Therefore, the lower and upper interval boundaries are chosen as

$$\theta_1^L(x_1(t)) = 0.95 - \left(0.925 / (1 + e^{-(x_1(t)+3.5)/8}) \right),$$

$$\theta_3^L(x_1(t)) = 0.025 + \left(0.925 / (1 + e^{-(x_1(t)-3.5)/8}) \right),$$

$$\theta_2^L(x_1(t)) = 1 - \theta_1^L(x_1(t)) - \theta_3^L(x_1(t)),$$

$$\theta_1^U(x_1(t)) = 0.95 - \left(0.925 / (1 + e^{-(x_1(t)+4.5)/8}) \right),$$

$$\theta_3^U(x_1(t)) = 0.025 + \left(0.925 / (1 + e^{-(x_1(t)-4.5)/8}) \right),$$

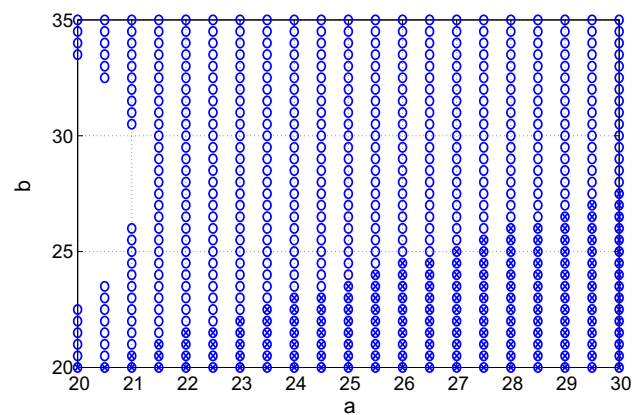


Fig. 1 Stability region for the proposed (○) and published methods (×)

$$\theta_2^U(x_1(t)) = 1 - \theta_1^U(x_1(t)) - \theta_3^U(x_1(t)).$$

For Theorem 1, $\alpha_i = 0$ and $\beta_i = 1$ for $i \in [1, 3]$. Figure 1 illustrates the stability regions for the proposed method and previous method in Lam and Seneviratne (2008). From Fig. 1, it is obviously that the proposed method achieves a larger stability region than the stability region by Lam and Seneviratne (2008). It is important results that the proposed IBD controller with a well-organized relaxation technique for the lower and upper interval boundaries gives a much better performance than that of the previous study (Lam and Seneviratne 2008).

Example 2 Consider an inverted pendulum model subject to uncertainties, which is referred from Ma and Sun (2001):

$$\dot{x}_2(t) = \frac{g \sin(x_1(t)) - \frac{am_p L x_2^2(t) \sin(2x_1(t))}{2} - a \cos(x_1(t))u(t)}{4L/3 - am_p L \cos^2(x_1(t))},$$

where $x_1(t)$ is the angular displacement of the pendulum; $g = 9.8 \text{ m/s}^2$ is the acceleration of gravity, m_p and M_c are the mass of the pendulum and cart in $m_p^{\min} = 2 \leq m_p \leq 5 = m_p^{\max}$ and $M_c^{\min} = 8 \leq M_c \leq 18 = M_c^{\max}$, respectively; $a = 1/(m_p + M_c)$; $2L = 1 \text{ m}$; and $u(t)$ is the force applied to the cart. A plant rule for the inverted pendulum subject to uncertainties is obtained in the following form (Lam and Seneviratne 2008):

IF $x_1(t)$ is $\tilde{F}_1^i$, **IF** $x_1(t)$ is $\tilde{F}_2^i$,

$$\text{THEN } \dot{x}(t) = A_i x(t) + B_i u(t), \quad i = 1, 2, 3, 4 \quad (25)$$

where

$$A_1 = A_2 = \begin{bmatrix} 0 & 1 \\ f_{1\min} & 0 \end{bmatrix}, \quad A_3 = A_4 = \begin{bmatrix} 0 & 1 \\ f_{1\max} & 0 \end{bmatrix},$$

$$\begin{aligned}
B_1 &= B_3 = \begin{bmatrix} 0 \\ f_{2\min} \end{bmatrix}, B_2 = B_4 = \begin{bmatrix} 0 \\ f_{2\max} \end{bmatrix}, \\
x(t) &= [x_1(t) \ x_2(t)]^T, \\
x_1(t) &\in [x_{1\min} \ x_{1\max}] = \left[-\frac{5\pi}{12} \ \frac{5\pi}{12}\right], \\
x_2(t) &\in [x_{2\min} \ x_{2\max}] = [-5 \ 5], \\
f_{1\min} &= 8.81 \leq f_1(x(t)) \leq f_{1\max} = 20.7, \\
f_{2\min} &= -0.18 \leq f_2(x_1(t)) \leq f_{2\max} = -0.017, \\
f_1(x(t)) &= \frac{g - am_p L x_2^2(t) \cos(x_1(t))}{4L/3 - am_p L \cos^2(x_1(t))} \left(\frac{\sin(x_1(t))}{x_1(t)} \right), \\
f_2(x_1(t)) &= \frac{-a \cos(x_1(t))}{4L/3 - am_p L \cos^2(x_1(t))},
\end{aligned}$$

From the above plant rule, the type-2 T-S fuzzy model is that

$$\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^4 \theta_i (A_i x(t) + B_i u(t)) \\
&= \sum_{i=1}^4 \left\{ \eta_i \theta_i^L + (1 - \eta_i) \theta_i^U \right\} (A_i x(t) + B_i u(t)), \quad (26)
\end{aligned}$$

where, for all i ,

$$\theta_i^L = \underline{\mu}_{\tilde{F}_1^i}(x(t)) \times \underline{\mu}_{\tilde{F}_2^i}(x(t)), \theta_i^U = \overline{\mu}_{\tilde{F}_1^i}(x(t)) \times \overline{\mu}_{\tilde{F}_2^i}(x(t)),$$

and the lower and upper membership functions for each rule are chosen as:

$$\begin{aligned}
&- x_2(t) = x_{2\max}, m_p = m_p^{\max}, \text{ and } M_c = M_c^{\min} \\
&\quad \underline{\mu}_{\tilde{F}_1^1}(x(t)) = \underline{\mu}_{\tilde{F}_1^2}(x(t)) = \frac{f_{1\max} - f_1(x(t))}{f_{1\max} - f_{1\min}} \\
&\quad \underline{\mu}_{\tilde{F}_1^3}(x(t)) = \underline{\mu}_{\tilde{F}_1^4}(x(t)) = \frac{f_1(x(t)) - f_{1\min}}{f_{1\max} - f_{1\min}} \\
&- x_2(t) = 0, m_p = m_p^{\max}, \text{ and } M_c = M_c^{\min} \\
&\quad \overline{\mu}_{\tilde{F}_1^1}(x(t)) = \overline{\mu}_{\tilde{F}_1^2}(x(t)) = \frac{f_{1\max} - f_1(x(t))}{f_{1\max} - f_{1\min}} \\
&\quad \overline{\mu}_{\tilde{F}_1^3}(x(t)) = \overline{\mu}_{\tilde{F}_1^4}(x(t)) = \frac{f_1(x(t)) - f_{1\min}}{f_{1\max} - f_{1\min}} \\
&- m_p = m_p^{\min} \text{ and } M_c = M_c^{\min} \\
&\quad \underline{\mu}_{\tilde{F}_2^1}(x_1(t)) = \underline{\mu}_{\tilde{F}_2^2}(x_1(t)) = \frac{f_{2\max} - f_2(x_1(t))}{f_{2\max} - f_{2\min}} \\
&\quad \underline{\mu}_{\tilde{F}_2^3}(x_1(t)) = \underline{\mu}_{\tilde{F}_2^4}(x_1(t)) = \frac{f_2(x_1(t)) - f_{2\min}}{f_{2\max} - f_{2\min}} \\
&- m_p = m_p^{\max} \text{ and } M_c = M_c^{\max} \\
&\quad \overline{\mu}_{\tilde{F}_2^1}(x_1(t)) = \overline{\mu}_{\tilde{F}_2^2}(x_1(t)) = \frac{f_{2\max} - f_2(x_1(t))}{f_{2\max} - f_{2\min}} \\
&\quad \overline{\mu}_{\tilde{F}_2^3}(x_1(t)) = \overline{\mu}_{\tilde{F}_2^4}(x_1(t)) = \frac{f_2(x_1(t)) - f_{2\min}}{f_{2\max} - f_{2\min}}
\end{aligned}$$

For Theorem 1, $\alpha_i = 0$ and $\beta_i = 1$ for $i \in [1, 4]$, and the solutions are as follows:

$$\begin{aligned}
K_0 &= [5.0735 \ 1.3488] \times 10^3, \\
\underline{K}_1 &= [-622.9989 \ -162.9032],
\end{aligned}$$

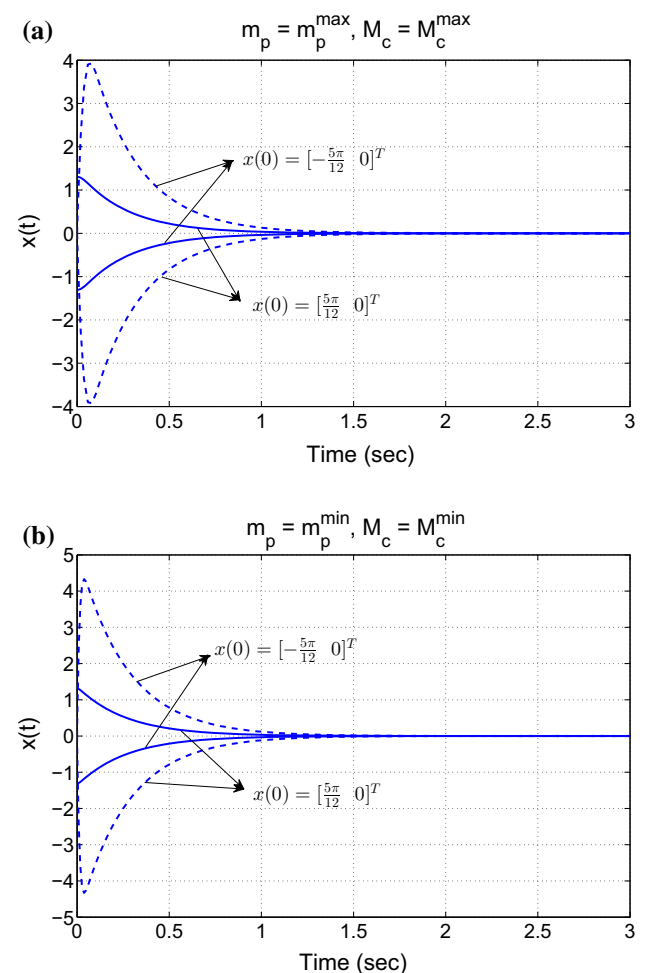


Fig. 2 State trajectories of inverted pendulum model for **a** maximum masses and **b** minimum masses (solid line: $x_1(t)$, dashed line: $x_2(t)$)

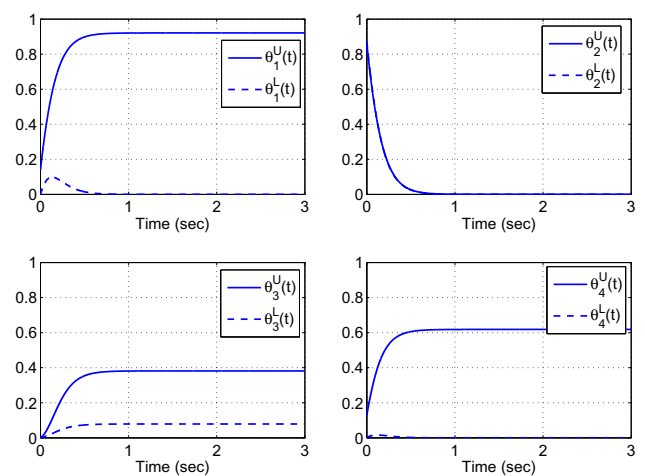


Fig. 3 Lower and upper interval boundaries (θ_i^L, θ_i^U)

$$\begin{aligned}\underline{K}_2 &= [-73.5112 \quad -19.3494], \\ \underline{K}_3 &= [-177.5482 \quad -46.6854], \\ \underline{K}_4 &= [-64.0299 \quad -16.9178], \\ \overline{K}_1 &= [2.8494 \quad 0.7654] \times 10^3, \\ \overline{K}_2 &= [2.7184 \quad 0.7271] \times 10^3, \\ \overline{K}_3 &= [2.6561 \quad 0.7108] \times 10^3, \\ \overline{K}_4 &= [2.7209 \quad 0.7277] \times 10^3.\end{aligned}$$

The state trajectories of the inverted pendulum model that has uncertain mass are depicted in Fig. 2. To illustrate the robustness of the proposed controller, we choose boundary values ($m_p^{\min}$, $m_p^{\max}$, $M_c^{\min}$, $M_c^{\max}$) of the mass of the pendulum and cart, m_p and M_c . In addition, Fig. 2 shows convergence of the state trajectories under various initial conditions. Figure 3 shows the plots of the lower and upper interval boundaries, θ_i^L and θ_i^U for all i . This figure implies that θ_i^L and θ_i^U satisfy the condition of (8). From the figures, it can be seen that the proposed IBD controller designed by Theorem 1 achieves the stability of the inverted pendulum with different system parameter variations in the plant model. Additionally, the proposed controller results in less conservative results because we use the larger mass ranges than of Lam and Seneviratne (2008).

5 Conclusion

In this paper, we proposed the new structure of a state-feedback controller for interval type-2 Takagi–Sugeno (T–S) fuzzy systems. The introduced structure employed lower and upper interval boundaries directly, which did lead to a stabilization condition of parameterized LMIs, associated only with the normalized time-varying fuzzy weighting parameters and their interval boundaries. The exact relationship among the parameters and their interval boundaries could, therefore, be fully used to convert parameterized LMIs into LMIs, which provides a great feature distinguished from those in the studies. Simulation results illustrated the effectiveness and robustness of the IBD controller. The future work is to research stabilizing robust controllers of the robot manipulator that is modeled by the interval type-2 T–S fuzzy system, which will guarantee the desired performance.

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