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Bi-loop Matrix Forgetting Factor-Based Coupled Recursive Least Squares Algorithm for Identification of Multivariable Plants

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Abstract

In this study, a new parameter estimation method based on coupled least squares method for recursive estimation of time-varying parameters in multivariable systems is presented. The bi-loop matrix forgetting factor-based coupled recursive least squares method is employed for estimation of time-varying parameters in which the forgetting factor for each parameter is adjusted according to its variance. This means that the forgetting factors of slow-varying and fast-varying parameters are calculated according to their variances, automatically. The simulation results demonstrate the advantage of the proposed method for estimation of parameters with different variation rates (including slow-varying or fast-varying parameters) in comparison with ordinary coupled least squares method.

Keywords System identification · Recursive least squares algorithm · Coupled least squares algorithm · Multivariable systems · Time-varying parameters · Matrix forgetting factor

1 Introduction

System identification as a useful tool for describing real plants with mathematical models has been studied in the literature (Åström and Eykhoff 1971; Ljung 2010). Parameter estimation is the fundamental part of the parametric identification approaches in which the unknown parameters in the model structure should be estimated. Closed loop system identification necessitates employing recursive parameter estimation methods (Ljung and Gunnarsson 1990). Moreover, recursive system identification is inevitable when some special model structures are employed (Astrom and Wittenmark 1995). Recursive least squares (RLS) algorithm (Astrom and Wittenmark 1995), the projection algorithm (Feng et al. 2013), the stochastic gradient (SG) method (Ding and Chen 2007; Ding 2010) and recursive kernel-based methods (Romeres et al. 2016) are some of the well-known recursive parameter estimation methods. Among them, the RLS algorithm is more common due to its faster convergence rate.

To apply the RLS algorithm for estimation of time-varying parameters, a forgetting factor RLS (FFRLS) algorithm has been employed. The forgetting factor value is usually considered between 0 and 1. When this parameter is close to one, the RLS algorithm shows good stability, but its tracking capability is reduced. In this case, the constant unknown parameters are estimated, accurately. However, to estimate the fast-varying parameters, the forgetting factor should be kept far from one that may affect the stability of the algorithm. To overcome this problem, variable forgetting factor has been utilized. Gauss Newton (Song et al. 2000) and gradient-based (So et al. 2003) variable forgetting factors have been employed in this area. Paleologu et al. (2008) presented a variable forgetting factor RLS algorithm robust against different types of system noise variations. Yazdi et al. (2009) proposed a dynamic forgetting factor for RLS algorithm in which the forgetting factor was changed based on the gradient of the inverse correlation matrix.

The main problem in the estimation of the time-varying parameters in dynamical systems is that different parameters may have different variation rates. This may cause problems in adjusting the forgetting factor in RLS algorithm. To overcome this drawback, several methods have been proposed in the literature. One of the famous methods proposed in

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this area is the matrix forgetting factor RLS (MRLS) algorithm in which different forgetting factors have been utilized for different parameters (Poznyak 1999; Poznyak and Juarez 1999). Dong et al. (2016) employed dual forgetting factors to improve the convergence rate in an output-error model identification. Li et al. (2014) extracted the optimal value for matrix forgetting factor in RLS algorithm. Fraccaroli et al. (2015) employed multiple forgetting factors in RLS algorithm for tracking time-varying parameters with different variation rates. For tracking fast-varying parameters, several improvements in RLS algorithm have been presented. For example, Yu and Shih (2006) proposed a Bi-loop RLS algorithm that could increase the tracking capabilities of the FFRLS algorithm. Zheng and Lin (2003) incorporated the effect of the local and global trend variations of the estimated parameters in the RLS algorithm to decrease the tracking error of fast-varying parameters.

On the other hand, identification of multi-input multi-output (MIMO) plants has been considered in the literature (Rajbman and Sinha 1977; Garcia et al. 2014). Hierarchical identification is one of the common approaches for parameter estimation in multivariable systems. In the hierarchical identification, the system is decomposed into smaller dimension subsystems and the parameters of each of the corresponding subsystems are estimated (Ding and Chen 2005; Ding 2014). The hierarchical identification principle has been employed to replace the noise terms in the information vector with the estimated residuals in a multivariable autoregressive moving average (ARMA) model structure (Bao et al. 2011). Li et al. (2014) employed the maximum-likelihood RLS algorithm to estimate the parameters of the subsystems obtained from the hierarchical identification procedure. Moreover, a hierarchical-based direct closed loop identification method for multivariable plants in the presence of the color noise has been presented (Jin et al. 2014). Applying the RLS algorithm to multivariable plants requires the matrix inversion in each iteration. To avoid this computational effort, the coupling identification principle has been employed to convert the parameter estimation problem in a MIMO system to parameter estimation in some single-input single-output (SISO) subsystems (Ding 2013). The mentioned coupled least squares (CLS) algorithm has been employed for bias compensation-based recursive parameter estimation of a permanent magnet synchronous motor (PMSM) (Shi et al. 2016).

In this paper, the CLS algorithm has been employed to estimate time-varying parameters in multivariable systems. To apply this algorithm, the matrix forgetting factor-based recursive CLS (MCLS) algorithm is established. The forgetting factor for each parameter is determined based on its variance. In other words, an explicit relation for the forgetting factor of each parameter in terms of its variance is proposed. This means that slowly varying parameters have forgetting factors around one, while fast-varying parameters have smaller forgetting factors. Therefore, constant and variable parameters could be estimated, simultaneously with high estimation accuracy. This algorithm then is combined with bi-loop parameter estimation method proposed by Yu and Shih (2006) to build bi-loop MCLS or BMCLS algorithm. The proposed method estimates parameters with different variation rates. For example, the algorithm estimates fast-varying parameters like square signal. The parameters with continuous changes like sinusoidal or exponential signals are estimated as well as constant parameters, too. Simultaneous estimation of these parameters for multivariable plants could be considered as the main contribution of this paper. This makes the CLS algorithm more applicable for estimation of parameters of real multivariable plants with unknown variation rates. Numerical examples show the superiority of the proposed algorithm compared with the ordinary CLS and Bi-loop forgetting factor RLS (BFFCLS) algorithms, as well.

The organization of this paper is as follows. In Sect. 2, the RLS and CLS algorithms are introduced. The matrix forgetting factor RLS algorithm is illustrated in Sect. 3. The proposed BMCLS algorithm with variance-based adjustment of the forgetting factors is given in Sect. 4. Simulation results of the proposed algorithm are presented in Sect. 5. Finally, Sect. 6 concludes the paper.

2 Recursive Least Squares-based Parameter Estimation Methods

In this section, a brief review on the recursive least squares method for SISO systems and its generalized version already presented for MIMO systems by Ding (2013) are presented. The mathematical details are given in the following subsections.

2.1 RLS Algorithm

The linear regression model for a multi-input multi-output system with m outputs and n unknown parameters is defined as (Astrom and Wittenmark 1995)

$$\underline{y}(t) = \Phi(t)\underline{\vartheta} + \underline{v}(t) \tag{1}$$

where $\underline{y}(t) = [y_1(t), y_2(t), ..., y_m(t)]^{\mathrm{T}} \in R^m$ is the system output vector, $\underline{\vartheta} \in R^n$ is the unknown parameters vector and $\Phi(t) = [\phi_1(t)\ldots\phi_m(t)]^{\mathrm{T}} \in R^{m\times n}$ is the regression matrix consists of the past input and output data, $\phi_i(t) \in R^n, i = 1,\ldots,m$ are the regression vectors for each output and $\underline{v}(t) = [v_1(t)\ldots v_m(t)]^{\mathrm{T}} \in R^m$ is a Gaussian noise vector with zero mean. In this section, the unknown parameters $\underline{\vartheta}$ are considered constant, temporarily. According to the recursive least squares

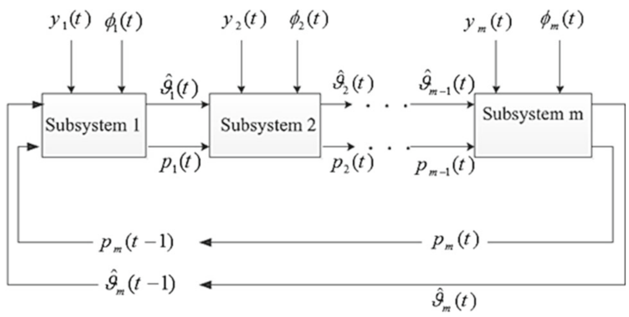

Fig. 1 Block diagram of the CLS algorithm

method, the estimated parameters vector $\hat{\underline{\vartheta}}(t)$ in iteration t could be obtained through the following recursive algorithm (Astrom and Wittenmark 1995)

$$\hat{\underline{\vartheta}}(t) = \hat{\underline{\vartheta}}(t-1) + K(t)[\underline{y}(t) - \Phi(t)\hat{\underline{\vartheta}}(t-1)]. \quad (2)$$

$$K(t) = P(t-1)\Phi^{\mathrm{T}}(t)[I_m + \Phi(t)P(t-1)\Phi^{\mathrm{T}}(t)]^{-1}. \quad (3)$$

$$P(t) = [I_n - K(t)\Phi(t)]P(t-1), \ P(0) = p_0 I_n \quad (4)$$

where $K(t) \in R^{n\times m}$ is the gain matrix, $P \in R^n$ is the covariance matrix, I_n is the $n \times n$ identity matrix, and p_0 is a positive large number.

The expression $I_m + \Phi(t)P(t-1)\Phi^{\mathrm{T}}(t)$ for SISO systems ($m = 1$) is a scalar number which could be easily inversed. However, for MIMO systems it is a $m \times m$ matrix. This means that applying RLS algorithm to MIMO systems requires to inverse a $m \times m$ matrix in each iteration. This leads to some computational complexities. Ding (2013) proposed the CLS algorithm to overcome this drawback.

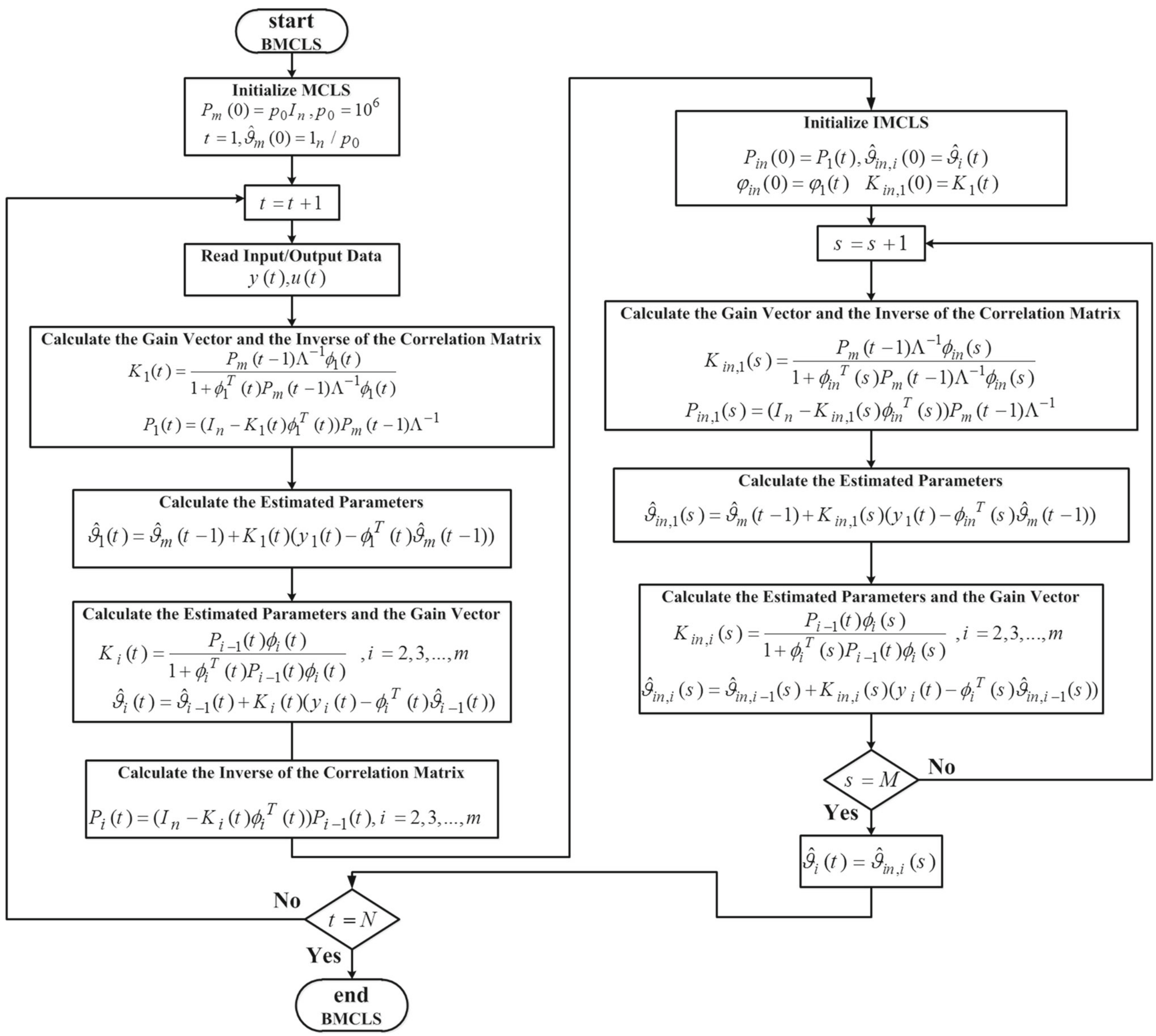

Fig. 2 Flowchart of the BMCLS algorithm

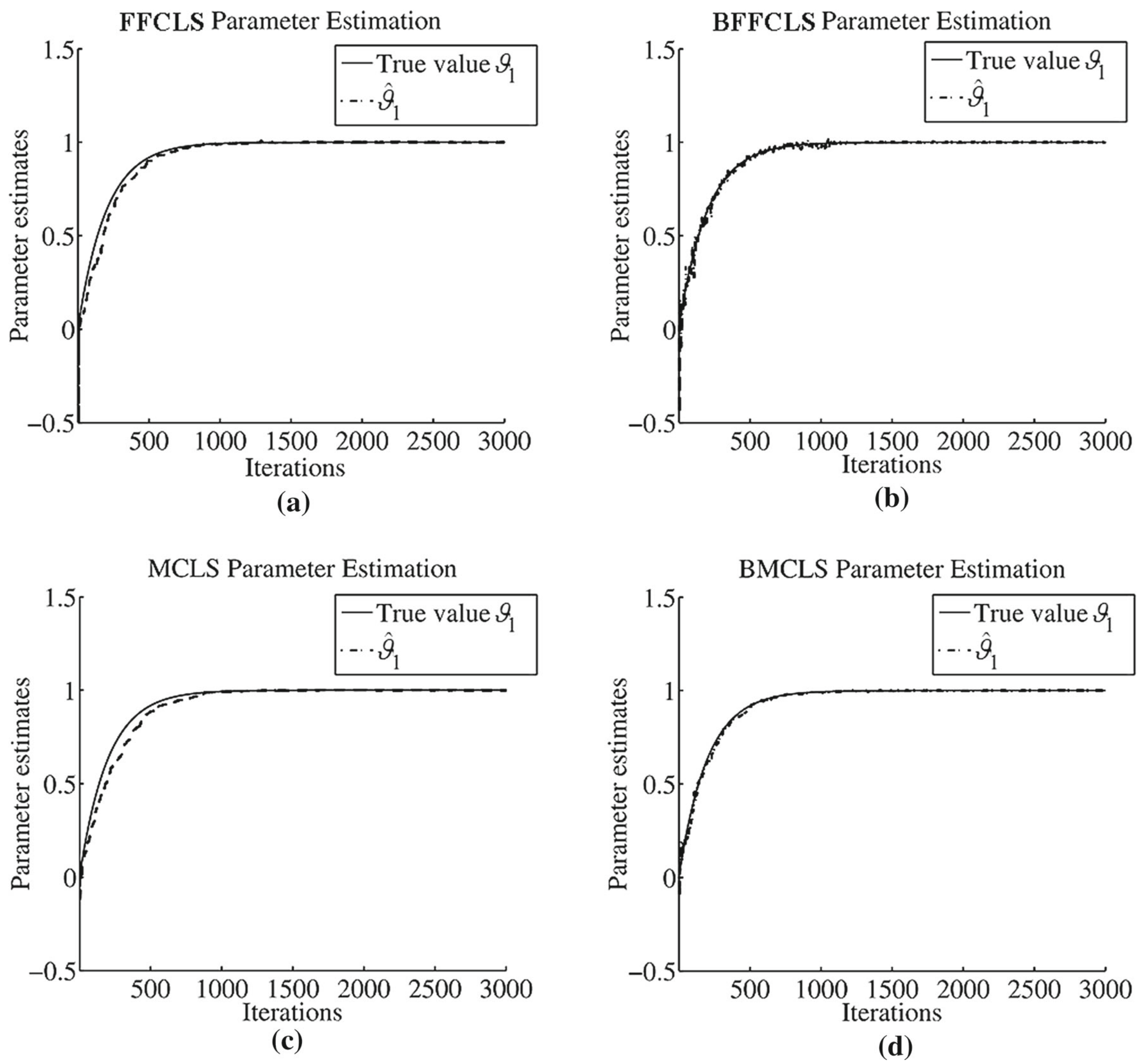

Fig. 3 Exponential parameter estimated with **a** FFCLS, **b** BFFCLS, **c** MCLS and **d** BMCLS algorithms in Example 1

2.2 CLS Algorithm

The relation (1) could be rewritten in the following modified form

$$y_i(t) = \phi_i^{\mathrm{T}}(t)\vartheta + v_i(t), \quad i = 1, 2, ..., m. \tag{5}$$

According to (5), the MIMO system could be decomposed into m SISO subsystems in which the unknown parameters could be estimated through any of them. According to the CLS algorithm, all subsystems are utilized to improve the accuracy of the parameter estimation. In other words, in each iteration, m RLS estimators are employed through a cascade configuration to estimate the unknown parameters. Figure 1 shows the block diagram of the CLS algorithm. As could be seen from Fig. 1, in each iteration, the estimated parameters obtained from each estimator are considered as the initial conditions for the next one. Finally, the estimated vector obtained from the last estimator is considered as the initial condition for the first estimator in the next iteration.

Thus, the CLS parameter estimation algorithm could be described with the following relations (Ding 2013)

$$\hat{\vartheta}_i(t) = \hat{\vartheta}_{i-1}(t) + K_i(t)[y_i(t) - \phi_i^{\mathrm{T}}(t)\hat{\vartheta}_{i-1}(t)]. \tag{6}$$

$$K_i(t) = \frac{P_{i-1}(t)\phi_i(t)}{1 + \phi_i^{\mathrm{T}}(t)P_{i-1}(t)\phi_i(t)}. \tag{7}$$

$$P_i(t) = [I_n - K_i(t)\phi_i^{\mathrm{T}}(t)]P_{i-1}(t), \quad i = 2, 3, ..., m. \tag{8}$$

and

$$\hat{\vartheta}_1(t) = \hat{\vartheta}_m(t-1) + K_1(t)[y_1(t) - \phi_1^{\mathrm{T}}(t)\hat{\vartheta}_m(t-1)]. \tag{9}$$

$$K_1(t) = \frac{P_m(t-1)\phi_1(t)}{1 + \phi_1^{\mathrm{T}}(t)P_m(t-1)\phi_1(t)}. \tag{10}$$

$$P_1(t) = [I_n - K_1(t)\phi_1^{\mathrm{T}}(t)]P_m(t-1),\ P_m(0) = p_0 I_n \tag{11}$$

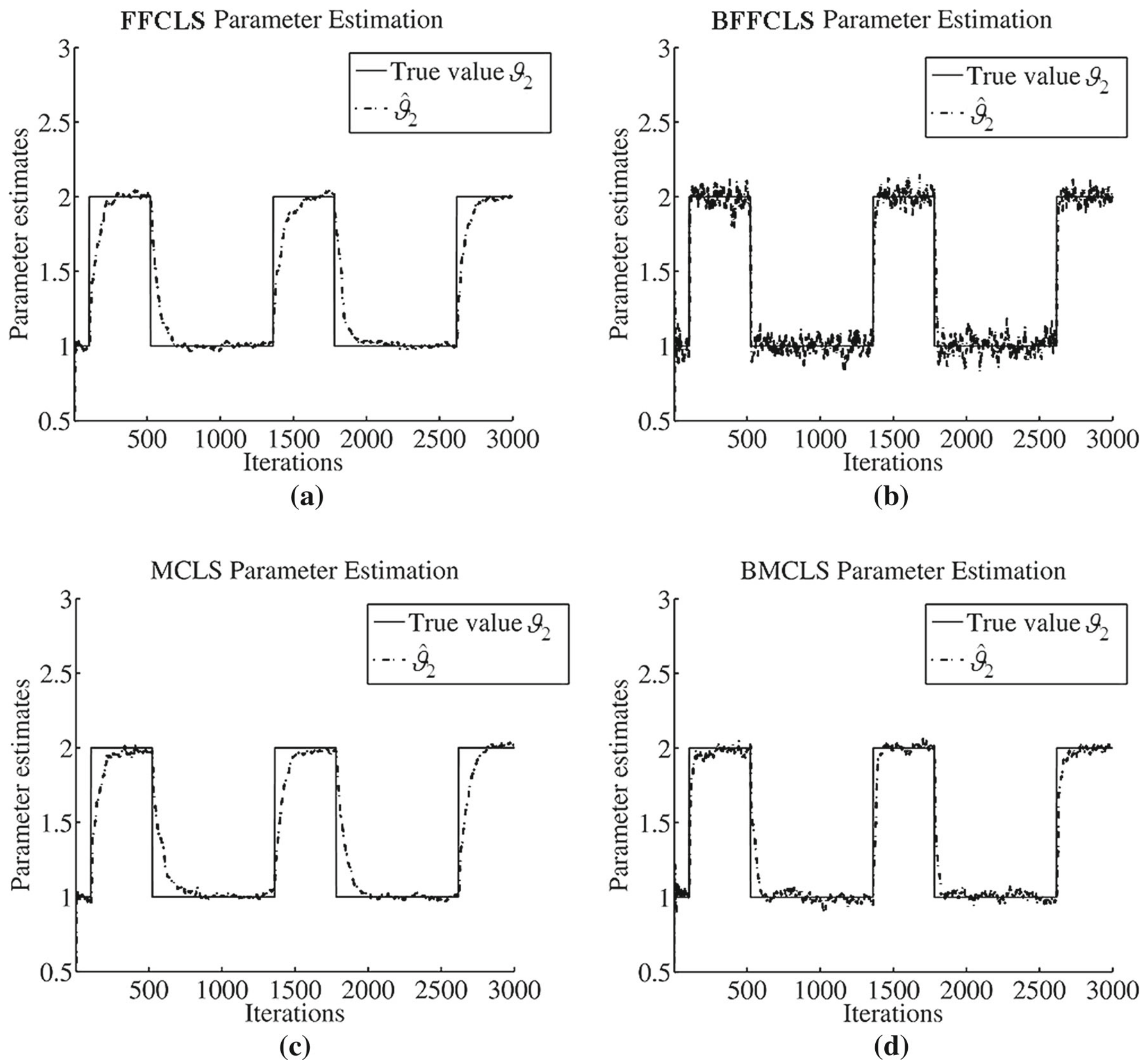

Fig. 4 Square parameter estimated with **a** FFCLS, **b** BFFCLS, **c** MCLS and **d** BMCLS algorithms in Example 1

where $\hat{\vartheta}_i(t) \in R^n$, $K_i(t) \in R^n$ and $P_i(t) \in R^{n\times n}$ are the estimated parameters, the gain vector and the covariance matrix in the $i-th$ subsystem in time t, respectively.

3 RLS Estimation of Time-Varying Parameters

In this section, the forgetting factor RLS and Matrix forgetting factor RLS algorithms for estimation of time-varying parameters in dynamical systems are illustrated.

3.1 FFRLS Algorithm

To apply the RLS algorithm for estimation of time-varying parameters, the effect of the previous data should be considered in the estimation procedure. This could be achieved by incorporating a forgetting factor parameter ($0 < \lambda < 1$) in the estimation procedure. Choosing this parameter to one decreases the tracking capability of the FFRLS algorithm and increases its stability. The parameter adaptation relations in accordance with the FFRLS algorithm are (Astrom and Wittenmark 1995)

$$\underline{\hat{\vartheta}}(t) = \underline{\hat{\vartheta}}(t-1) + K(t)[\underline{y}(t) - \Phi(t)\underline{\hat{\vartheta}}(t-1)]. \tag{12}$$

$$K(t) = P(t-1)\Phi^{\mathrm{T}}(t)[\lambda I_m + \Phi(t)P(t-1)\Phi^{\mathrm{T}}(t)]^{-1}. \tag{13}$$

$$P(t) = \frac{1}{\lambda}[I_n - K(t)\Phi(t)]P(t-1), \;\; P(0) = p_0 I_n. \tag{14}$$

If the forgetting factor λ is considered as 1, the ordinary RLS algorithm relations (2–4) will be obtained.

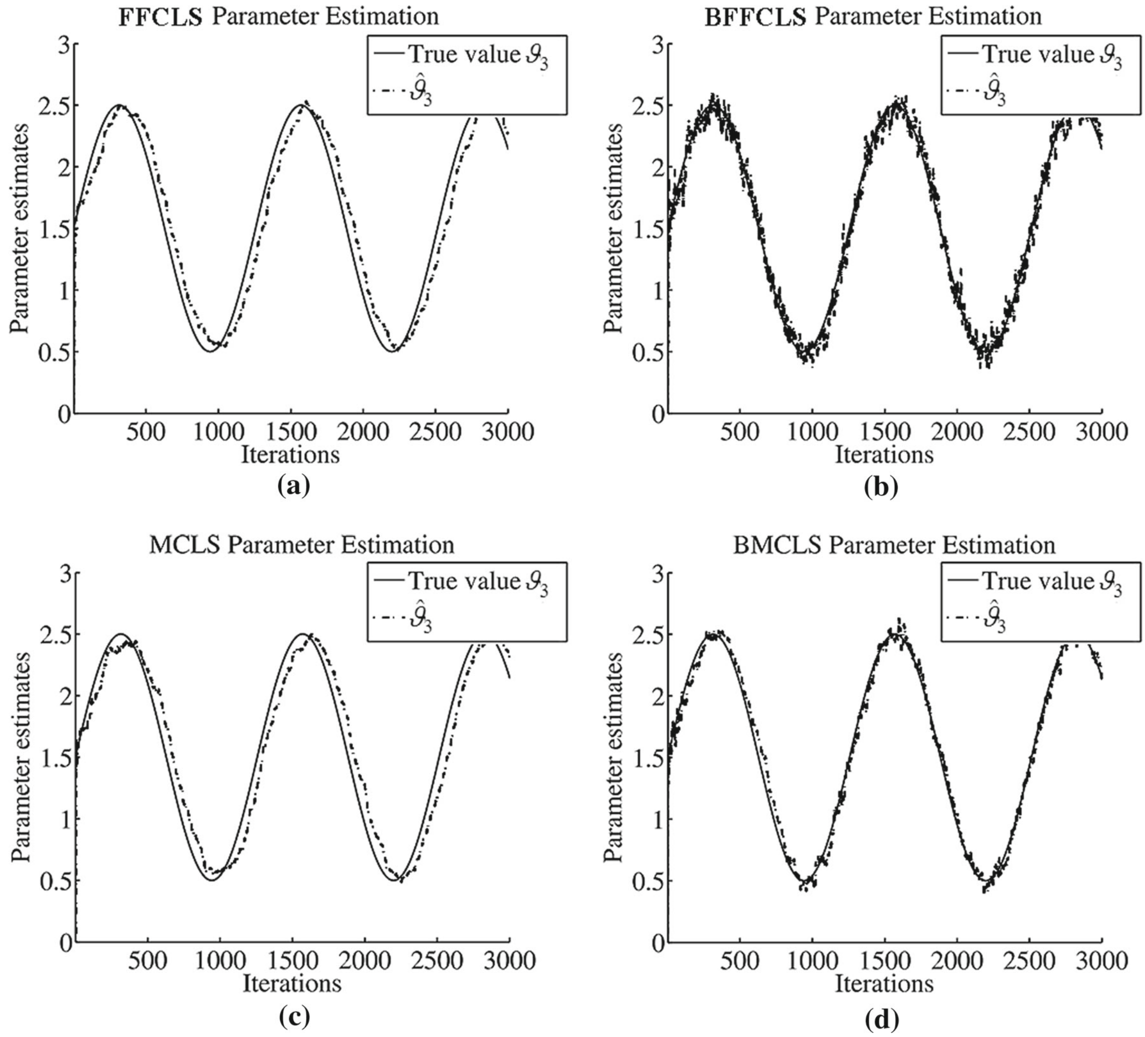

Fig. 5 Sinusoidal parameter estimated with **a** FFCLS, **b** BFFCLS, **c** MCLS and **d** BMCLS algorithms in Example 1

3.2 MRLS Algorithm

In some real plants, the plant parameters could change with time with different rates. For example, the flight conditions of a missile depend on the angle of the attack, the air speed, and the altitude (Apkarian et al. 1995). The variation rates of these three parameters could be different. In another example, in a continuous stirred tank reactor (CSTR), the kinetic rates in the reactor are complex functions of the process states (like the temperature inside the reactor). The heat of reaction and the global coefficient of the heat transfer are other uncertain parameters in a CSTR plant (Bequette 2002). It is obvious that the variation rates of these parameters are not the same. In liquid level control of interacting spherical two-tank system, the area of the spherical tanks changes with respect to change in the flow (Kumar and Meenakshipriya 2012). The variation rate of the area is faster than change in other plant parameters. In these situations, a fixed forgetting factor for all parameters could not be useful. In this case, various forgetting factors for each of the parameters should be utilized. This could be achieved by employing a diagonal matrix forgetting factor Λ instead of a scalar forgetting factor in the FFRLS algorithm. This leads to the following MRLS algorithm (Dong et al. 2016; Li et al. 2014)

$$\hat{\vartheta}(t) = \hat{\vartheta}(t-1) + K(t)[y(t) - \Phi(t)\hat{\vartheta}(t-1)]. \tag{15}$$

$$K(t) = P(t-1)\Lambda^{-1}\Phi^{\mathrm{T}}(t)[I_m + \Phi(t)P(t-1)\Lambda^{-1}\Phi^{\mathrm{T}}(t)]^{-1}. \tag{16}$$

$$P(t) = [I_n - K(t)\Phi(t)]P(t-1)\Lambda^{-1} \tag{17}$$

where $\Lambda = diag(\lambda_1, \ldots, \lambda_n)$ and λ_i is the forgetting factor for $i-th$ parameter.

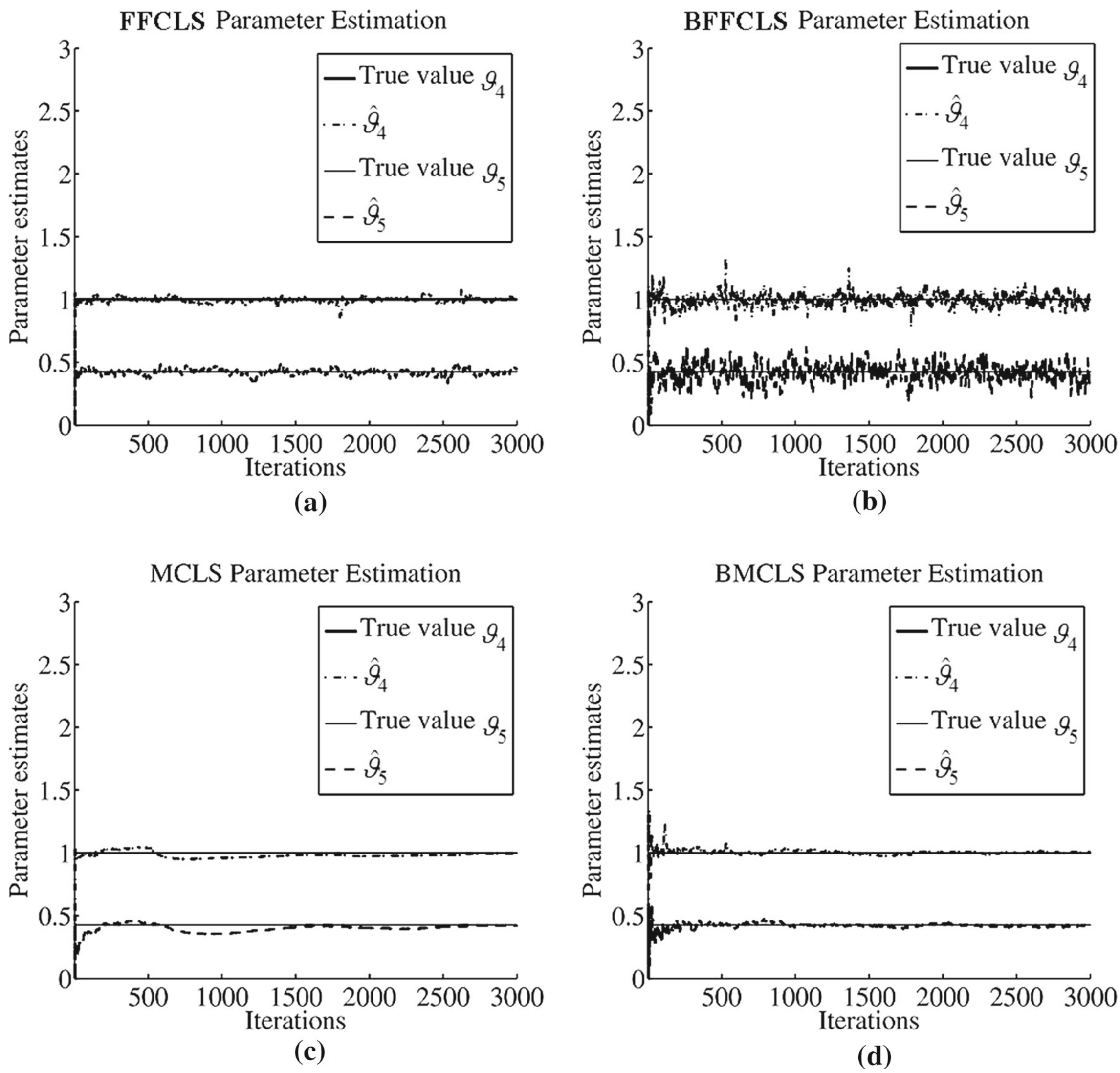

Fig. 6 Constant parameters estimated with **a** FFCLS, **b** BFFCLS, **c** MCLS and **d** BMCLS algorithms in Example 1

4 CLS Algorithm for Estimation of Time-Varying Parameters

In this section, the forgetting factor CLS (FFCLS) algorithm is introduced for parameter estimation of time-varying parameters in multivariable systems. Then, a matrix forgetting factor-based CLS or MCLS algorithm is presented for identification of time-varying parameters with different variation rates. The variance of the parameters is employed to determine their corresponding forgetting factor.

4.1 FFCLS Algorithm

Consider that in the CLS algorithm presented by Ding (2013), the information matrix is updated with a forgetting factor parameter λ. According to Fig. 1, the last obtained information matrix in time $t-1$ is the $P_m^{-1}(t-1)$. Thus, according to the forgetting factor concept, the first stage information matrix in time t ($P_1^{-1}(t)$) should be updated according to the prior information ($P_m^{-1}(t-1)$) and new information ($\phi_1(t)\phi_1^{\mathrm{T}}(t)$) as

$$P_1^{-1}(t) = \lambda P_m^{-1}(t-1) + \phi_1(t)\phi_1^{\mathrm{T}}(t). \tag{18}$$

Recall the matrix inversion lemma (Skogestad and Postlethwaite 2001)

$$(A+BC)^{-1} = A^{-1} - A^{-1}B(I+CA^{-1}B)^{-1}CA^{-1} \tag{19}$$

where A is an invertible matrix and B, C are two arbitrary matrices. Now, according to (18) and (19), we have

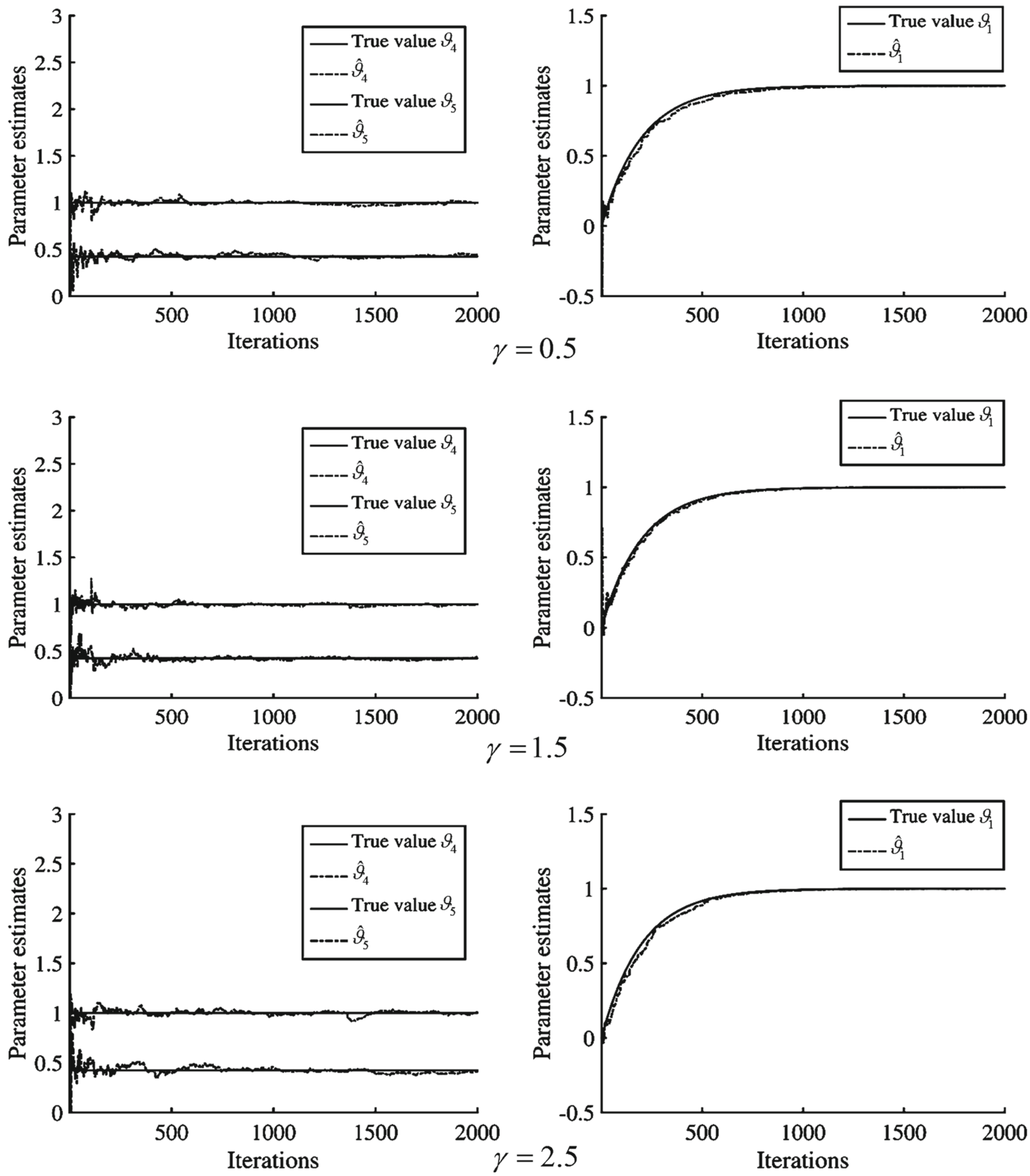

Fig. 7 Constant and exponential parameters estimated with the BMCLS algorithm for $\gamma = 0.5, 1.5, 2.5$ and $\omega_0 = 0.005$ in Example 1

$$P_1(t) = [P_1^{-1}(t)]^{-1} = \frac{1}{\lambda} P_m(t-1) - \frac{P_m(t-1)\phi_1(t)\phi_1^{\mathrm{T}}(t)P_m(t-1)}{\lambda[\lambda + \phi_1^{\mathrm{T}}(t)P_m(t-1)\phi_1(t)]}. \tag{20}$$

Now, consider that the gain vector in (20) is denoted by $K_1(t)$. Or

$$K_1(t) = \frac{P_m(t-1)\phi_1(t)}{\lambda + \phi_1^{\mathrm{T}}(t)P_m(t-1)\phi_1(t)}. \tag{21}$$

Relations (20) and (21) yield

$$P_1(t) = \frac{1}{\lambda}[I_n - K_1(t)\phi_1^{\mathrm{T}}(t)]P_m(t-1). \tag{22}$$

Therefore, in the FFCLS algorithm, relations (6–9) will remain unchanged. However, relations (10) and (11) should be replaced with relations (21) and (22), respectively.

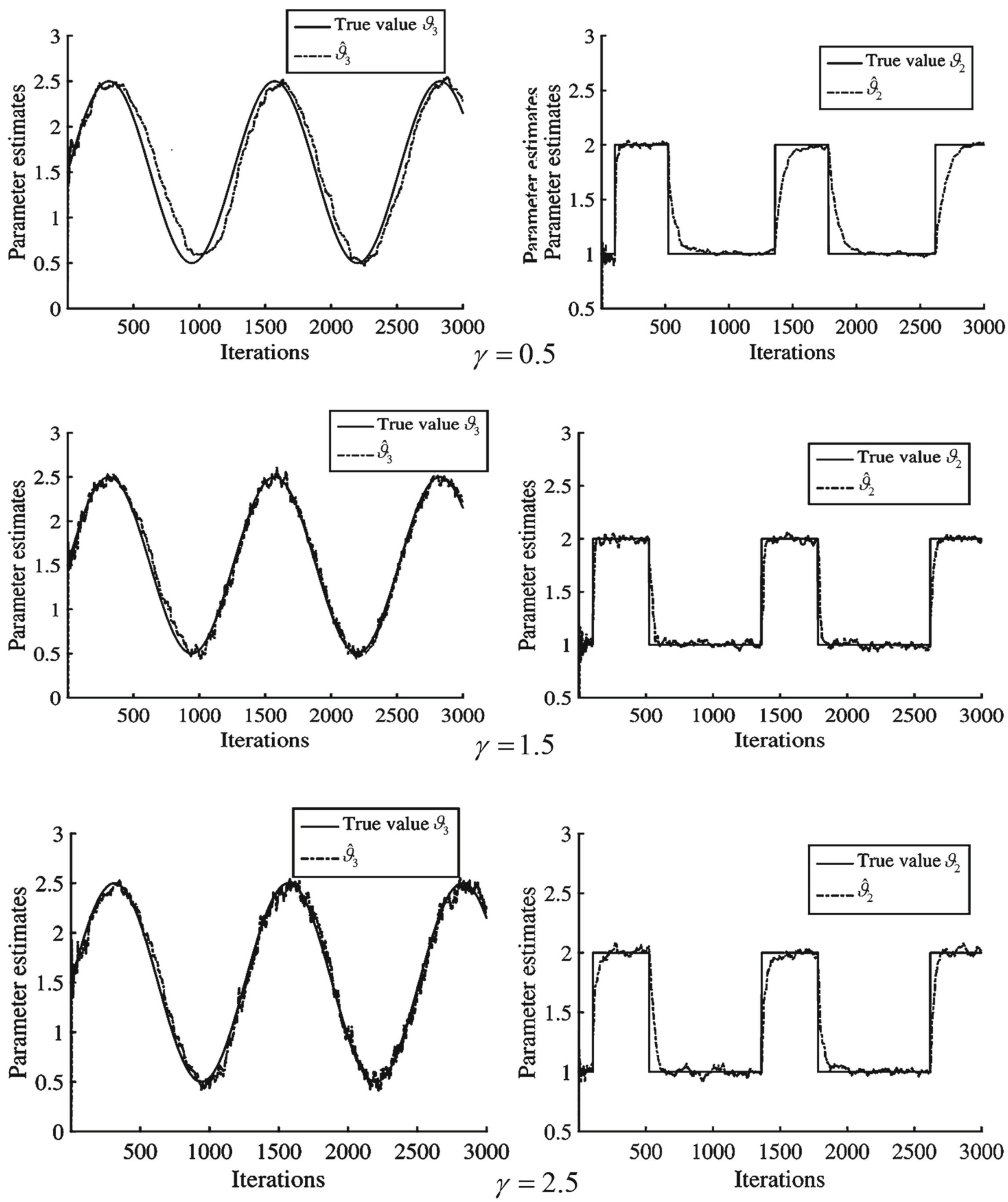

Fig. 8 Sinusoidal and square parameters estimated with the BMCLS algorithm for $\gamma = 0.5, 1.5, 2.5$ and $\omega_0 = 0.005$ in Example 1

4.2 MCLS Algorithm

In the FFCLS algorithm, if the forgetting factor λ is replaced with a forgetting factor matrix Λ, the MCLS algorithm is obtained. To derive the MCLS algorithm relations, relation (18) should be rewritten as

$$P_1^{-1}(t) = \Lambda P_m^{-1}(t-1) + \phi_1(t)\phi_1^{\mathrm{T}}(t). \tag{23}$$

Now, according to (19), relation (23) could be rewritten as

$$P_1(t) = [P_1^{-1}(t)]^{-1} = P_m(t-1)\Lambda^{-1} - \frac{P_m(t-1)\Lambda^{-1}\phi_1(t)\phi_1^{\mathrm{T}}(t)P_m(t-1)\Lambda^{-1}}{1+\phi_1^{\mathrm{T}}(t)P_m(t-1)\Lambda^{-1}\phi_1(t)}. \tag{24}$$

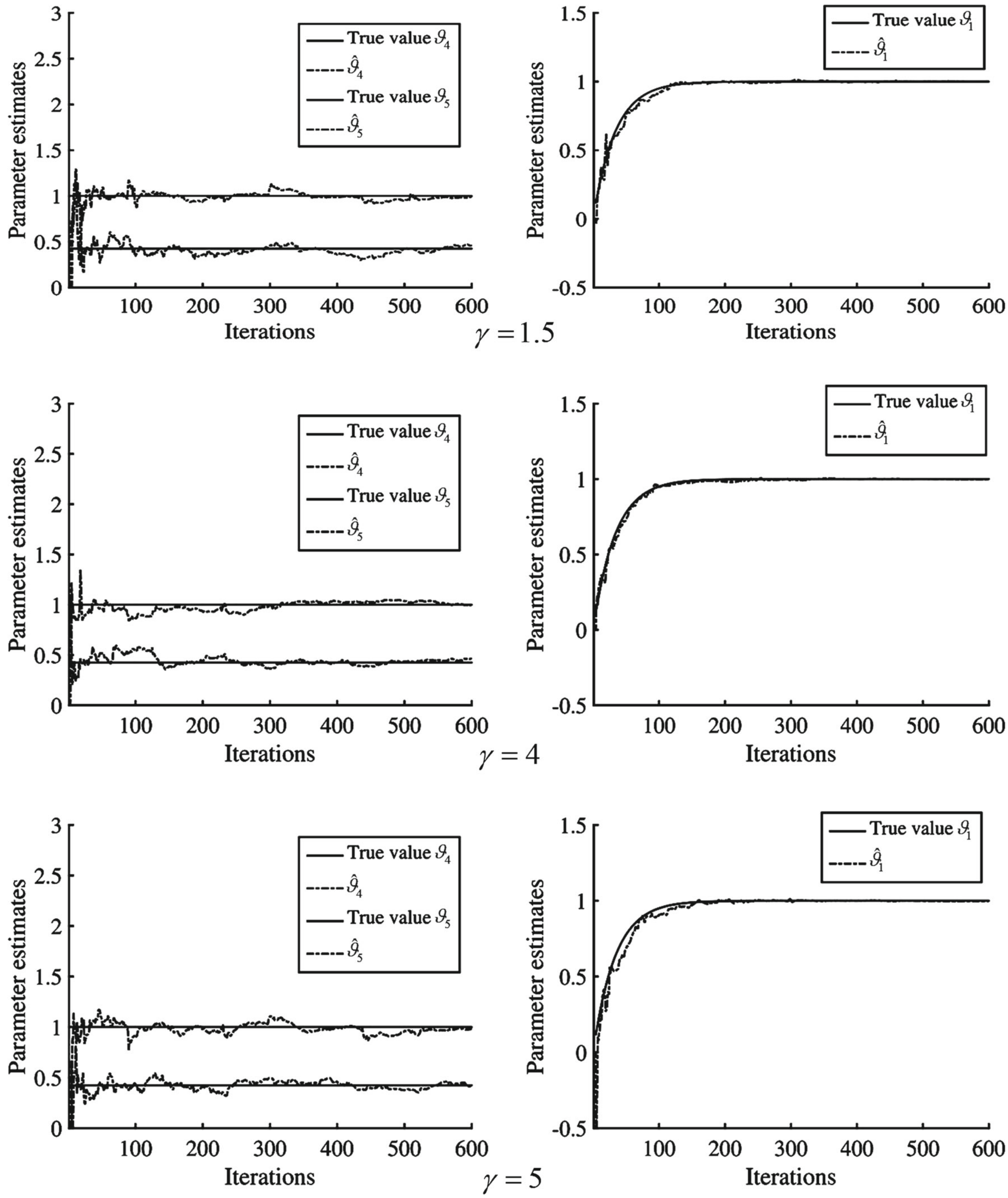

Fig. 9 Constant and exponential parameters estimated with the BMCLS algorithm for $\gamma = 1.5, 4, 5$ and $\omega_0 = 0.03$ in Example 1

Consider that

$$K_1(t) = \frac{P_m(t-1)\Lambda^{-1}\phi_1(t)}{1 + \phi_1^{\mathrm{T}}(t)P_m(t-1)\Lambda^{-1}\phi_1(t)}. \tag{25}$$

Now, relation (24) could be written as

$$P_1(t) = [I_n - K_1(t)\phi_1^{\mathrm{T}}(t)]P_m(t-1)\Lambda^{-1}. \tag{26}$$

This means that for the MCLS algorithm, relations (6–9) could also be utilized while relations (25) and (26) should be employed instead of relations (10) and (11), respectively.

4.3 Variance-based Forgetting Factor Calculation

In Sect. 4.2, the CLS algorithm proposed by Ding (2013) is modified to the MCLS algorithm in which time-varying parameters with different variation rates should be esti-

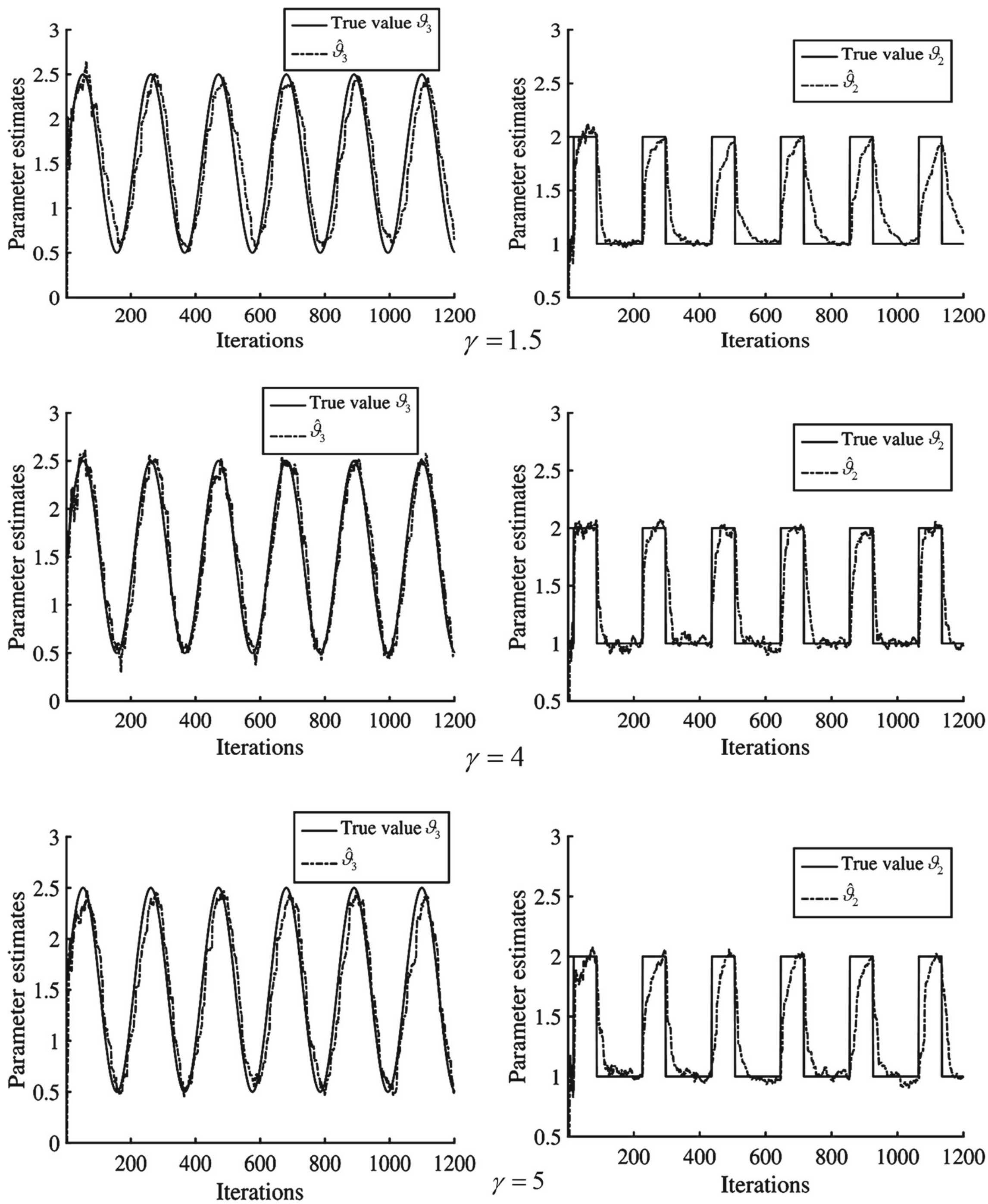

Fig. 10 Sinusoidal and square parameters estimated with the BMCLS algorithm for $\gamma = 1.5, 4, 5$ and $\omega_0 = 0.03$ in Example 1

mated. In this section, a variance-based method for adjusting the forgetting factor in each iteration is proposed. The main idea is that the forgetting factor of each parameter should be adjusted according to its variance. Slow-varying or constant parameters should have forgetting factors 1 and fast-varying parameters should have forgetting factor smaller than 1. This means that for simultaneous tracking of slow-varying and fast-varying parameters, the elements of the forgetting factor matrix should be appropriately adjusted. In this paper, the matrix forgetting factor Λ is considered as a diagonal matrix that its diagonal elements ($\lambda_i, i = 1, \ldots, n$) are calculated according to the variance of each parameter ($\sigma_i^2, i = 1, \ldots, n$). Thus, we have

$$\lambda_i = \alpha + (1 - \alpha) e^{-\gamma \sigma_i^2}, \quad i = 1, \ldots, n \tag{27}$$

Table 1 The relative estimated errors for different values of γ and ω_0

Relative estimated errors	$\omega_0 = 0.005$			$\omega_0 = 0.03$		
	$\gamma = 0.5$	$\gamma = 1.5$	$\gamma = 2.5$	$\gamma = 1.5$	$\gamma = 4$	$\gamma = 5$
Exponential	0.000518	0.000193	0.00036	0.000318	0.00022	0.00024
Square	0.03778	0.02412	0.0286	0.21202	0.08272	0.07595
Sinusoidal	0.07983	0.04741	0.04756	0.1973	0.1219	0.13
Constant ($\hat{\vartheta}_4$)	0.01811	0.00875	0.00963	0.0227	0.0118	0.0252
Constant ($\hat{\vartheta}_5$)	0.04697	0.0186	0.0687	0.0345	0.0289	0.0763

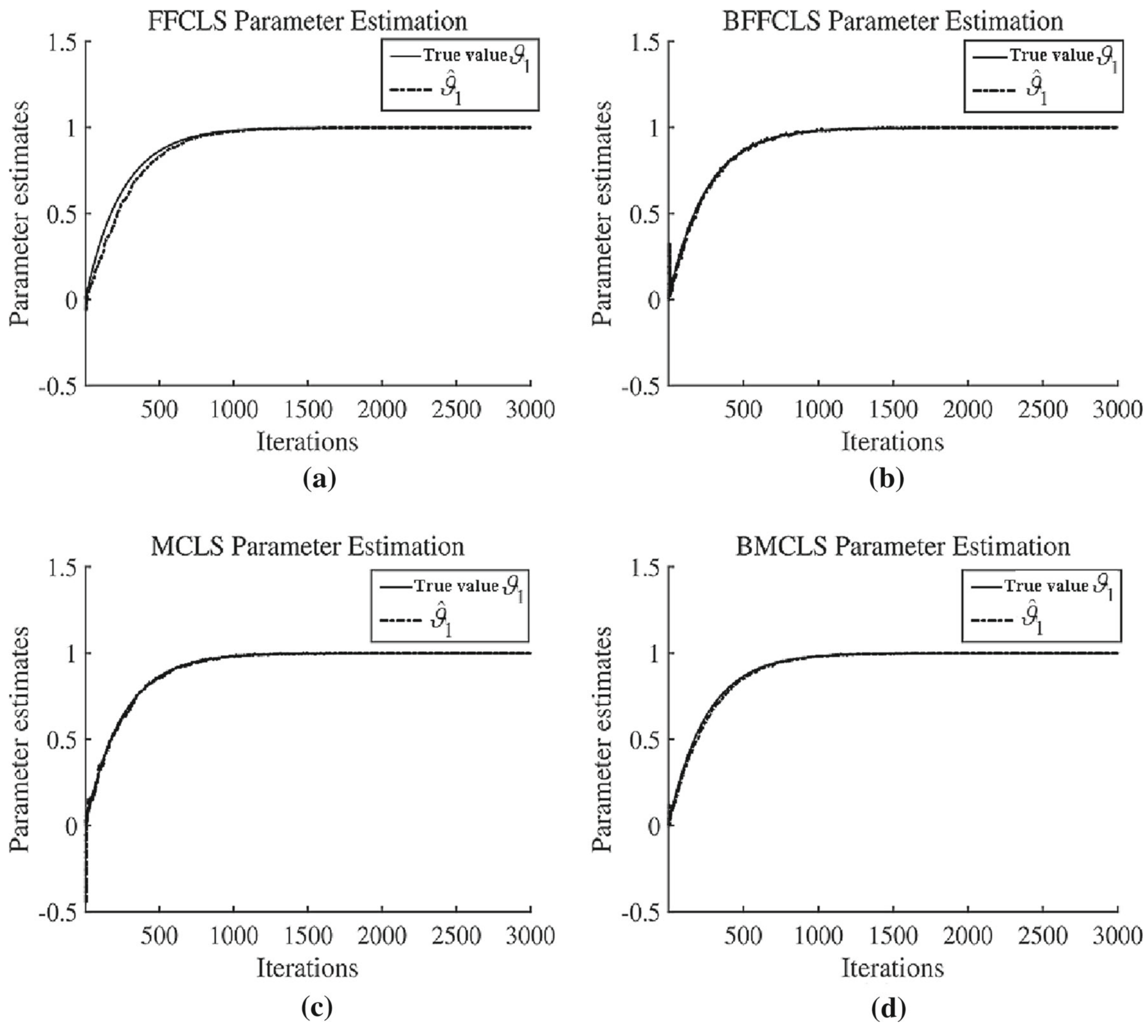

Fig. 11 Exponential parameter estimated with **a** FFCLS, **b** BFFCLS, **c** MCLS and **d** BMCLS algorithms in Example 2

where $0 < \alpha \leq 1$ is an adjustable parameter close to 1 and γ is a positive adjustable parameter for scaling the variance value. According to (27), the forgetting factor of fast-varying parameters with large variance value is close to α while the forgetting factor of slow-varying parameters (or constant parameters) is close to 1.

Remark 1 Relation (27) shows that the forgetting factor of each parameter changes according to its variance during the estimation procedure. When the parameter does not change, its variance is around zero. This means that its corresponding forgetting factor is around 1 (independently of the value of α). According to (27), the forgetting factor is in the range $[\alpha, 1]$. This means that minimum value of λ_i is α (for fast variations of the estimated parameter). Thus, considering α a bit smaller than 1 could be reasonable. This means that for estimating parameters in real plants with fast-varying parameters the value of α could be decreased. On the other hand, the parameter γ determines that how match the forgetting

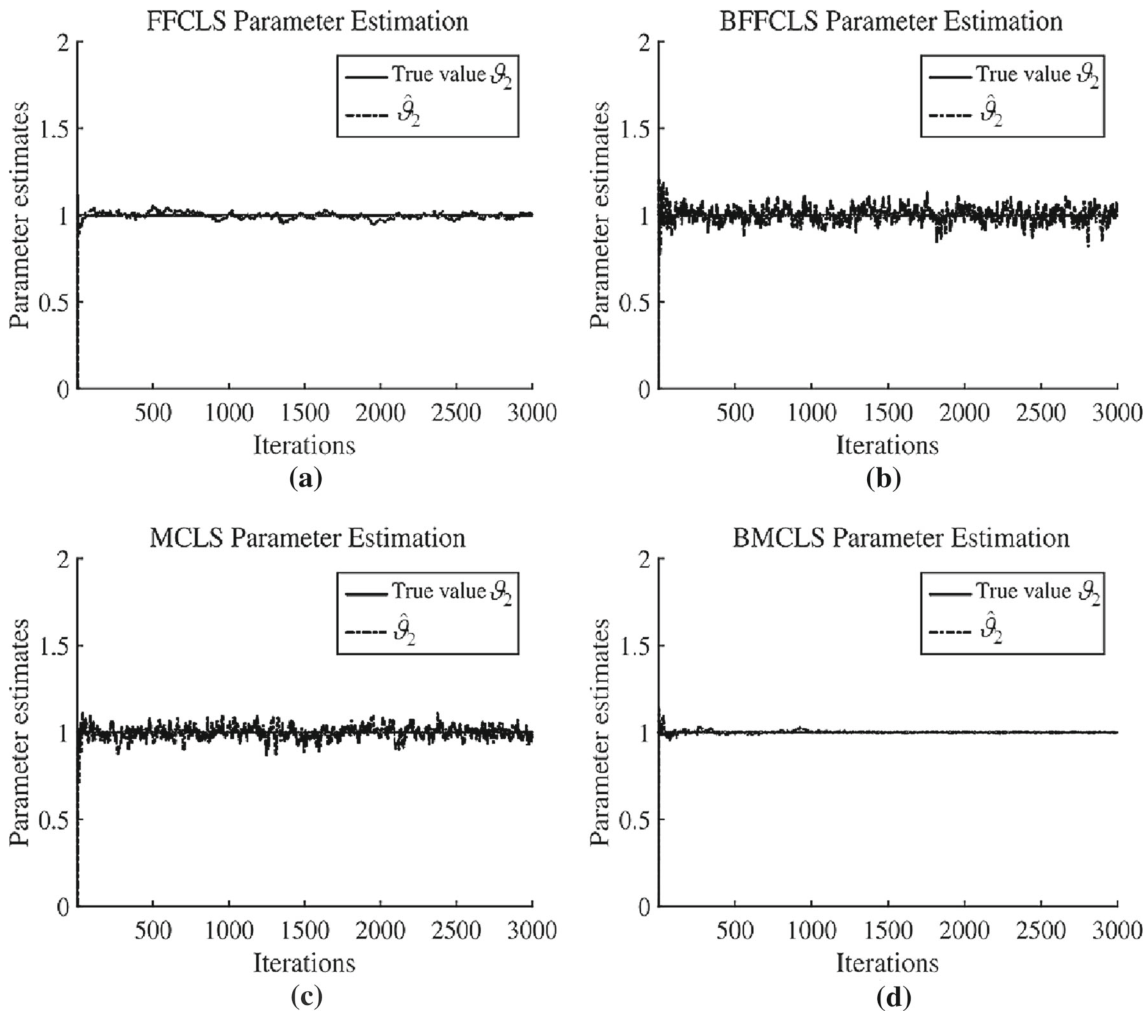

Fig. 12 Constant parameter estimated with **a** FFCLS, **b** BFFCLS, **c** MCLS and **d** BMCLS algorithms in Example 2

factor should be decreased when the variance of the estimated parameter is increased. In Sect. 5, the effect of the parameterγ on the estimation error will be investigated.

4.4 BMCLS Algorithm

To improve the performance of the FFRLS algorithm, a Bi-loop FFRLS or BLFFRLS algorithm was proposed (Yu and Shih 2006). In this algorithm, a nested loop was employed in which the outputs of the FFRLS algorithm employed in the outer loop were utilized as the initial parameters values of the inner loop. Resetting initial conditions during the in-between time interval gives improvement in tracking time-varying parameters. To track the fast-varying parameters, the in-between recursive procedure should be executed M times. In other words, the inner loop is employed for tracking fast-varying parameters. The estimated parameters in the inner loop are employed by the outer loop (FRLS loop). In this paper, the MCLS algorithm is combined with the Bi-loop algorithm to build the BMCLS algorithm. The flowchart of the BMCLS algorithm is shown in Fig. 2. The final estimated parameter obtained from the inner loop ($\hat{\vartheta}_{\mathrm{in},1}(M)$) could be employed to improve the tracking abilities of the MCLS algorithm (in the outer loop). This is the combination of the BLFFRLS algorithm given by Yu and Shih (2006) and the MCLS algorithm.

5 Simulation Results

In this section, the performance of the proposed parameter estimation methods is investigated through a numerical example.

Example 1 In this example, a two-input two-output system with the following dynamics is considered

$$d(z)\underline{y}(t) = Q(z)\underline{u}(t) + \underline{v}(t) \tag{28}$$

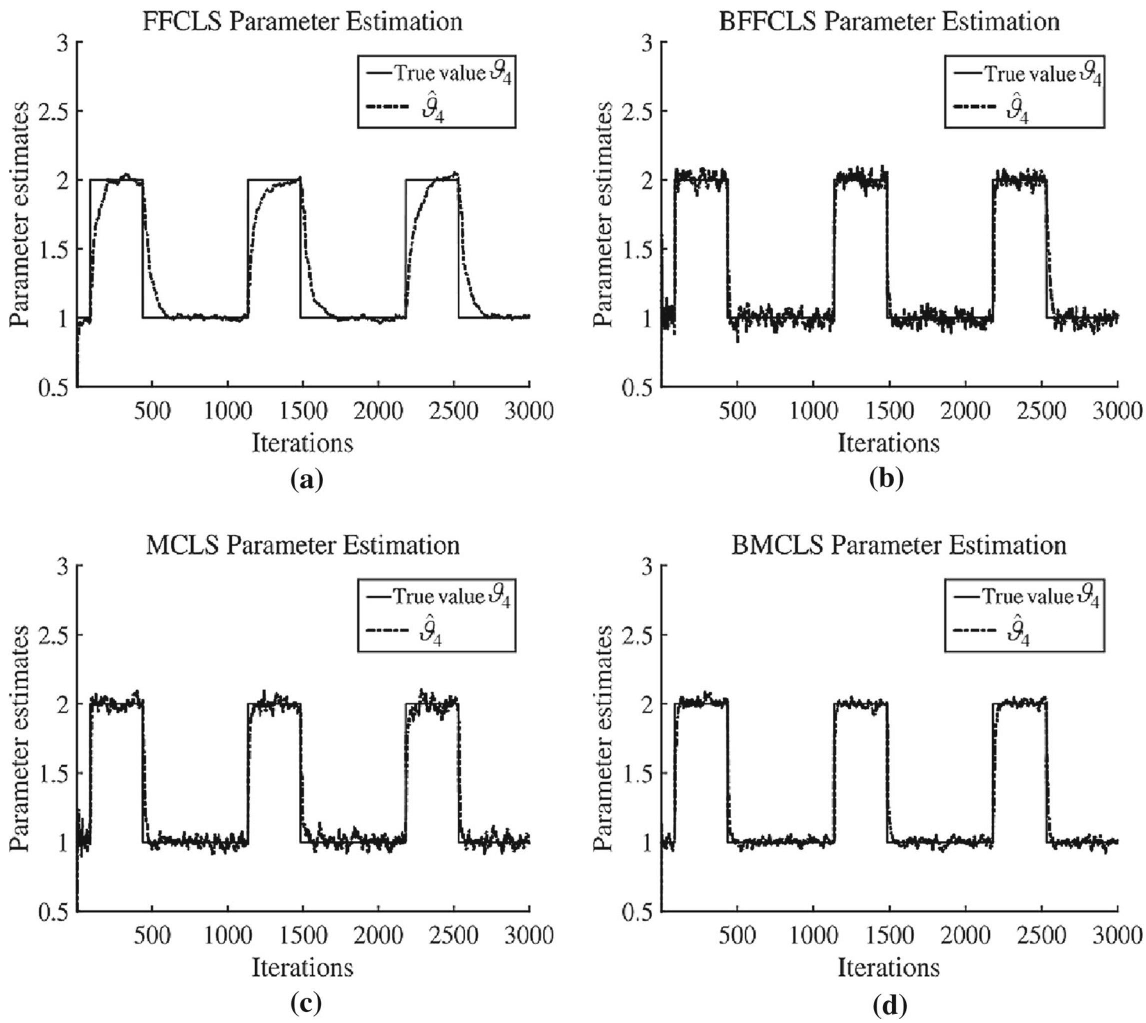

Fig. 13 Square parameter estimated with **a** FFCLS, **b** BFFCLS, **c** MCLS and **d** BMCLS algorithms in Example 2

where $\underline{u}(t) = [u_1(t) \quad u_2(t)]^{\mathrm{T}}$, $\underline{y}(t) = [y_1(t) \quad y_2(t)]^{\mathrm{T}}$ and $\underline{v}(t) = [v_1(t) \quad v_2(t)]^{\mathrm{T}}$ are the system input, output and the Gaussian noise, respectively. Moreover, $d(z)$ and $Q(z)$ are defined as

$$d(z) = 1 + \vartheta_1^\circ(t) z^{-1},\ Q(z) = \begin{bmatrix} \vartheta_2^\circ(t) & \vartheta_4^\circ \\ \vartheta_3^\circ(t) & \vartheta_5^\circ \end{bmatrix} z^{-1} \tag{29}$$

where

$$\vartheta_1^\circ(t) = 1 - e^{-\omega_0 t}, t \geq 0,\ \vartheta_2^\circ(t) = \begin{cases} 1 & \sin(\omega_0 t) < 0.5 \\ 2 & \text{Otherwise} \end{cases},$$
$$\vartheta_3^\circ(t) = 1.5 + \sin(\omega_0 t),\ \vartheta_4^\circ = 1,\ \vartheta_5^\circ = 0.425 \tag{30}$$

$\omega_0 = 0.005$ is considered. System (28) could be described with linear regression model (1) where

$$\Phi(t) = \begin{bmatrix} -y_1(t-1) & u_1(t-1) & 0 & u_2(t-1) & 0 \\ -y_2(t-1) & 0 & u_1(t-1) & 0 & u_2(t-1) \end{bmatrix},$$
$$\underline{\vartheta} = \left[\, \vartheta_1 \ \vartheta_2 \ \vartheta_3 \ \vartheta_4 \ \vartheta_5 \,\right]^{\mathrm{T}} \tag{31}$$

For estimation of the unknown parameters, $v_1(t)$ and $v_2(t)$ are considered as Gaussian noises with variance 0.16 and 0.25, respectively. For estimation of parameters with the FFCLS algorithm, the fixed forgetting factor $\lambda = 0.98$ is considered. In the MCLS algorithm, $\alpha = 0.98$ and $\gamma = 1.5$ are considered. In the BMCLS algorithm, $M = 10$ is considered. In all simulations, $p_0 = 10^6 I_5$, $\hat{\vartheta}(0) = 0$ are considered (I_5 is the 5×5 identity matrix).

The estimated parameters for the FFCLS, BFFCLS, MCLS and BMCLS algorithms are shown in Figs. 3, 4, 5 and 6. As could be seen from Fig. 3, the best tracking of the exponential parameter is achieved with the BMCLS algorithm. Figure 4 shows that the fastest tracking capability for the square parameter is attained with the BMCLS algorithm. Figure 5 shows that by using the BMCLS algo-

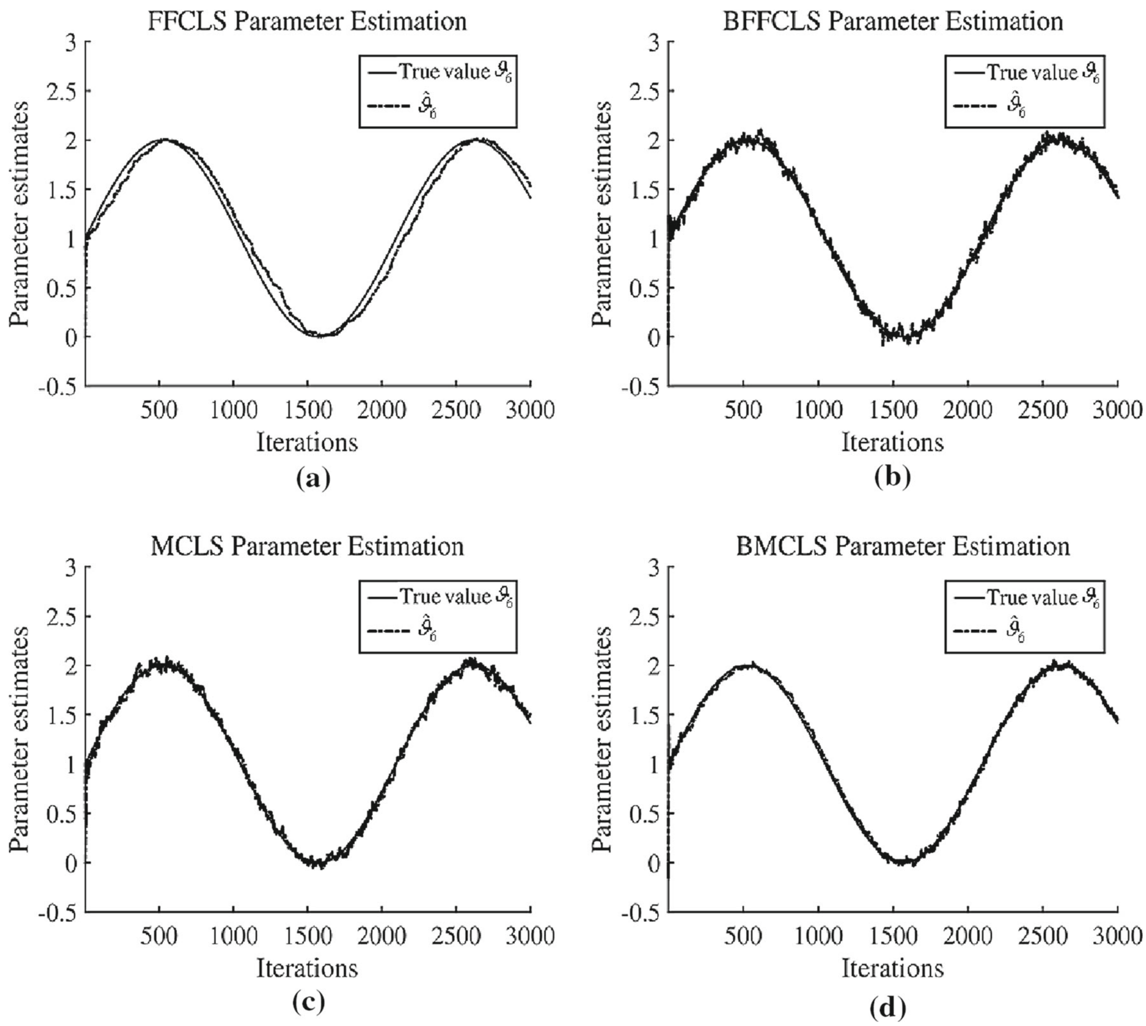

Fig. 14 Sinusoidal parameter estimated with **a** FFCLS, **b** BFFCLS, **c** MCLS and **d** BMCLS algorithms in Example 2

rithm, the sinusoidal parameter is identified with smallest error and distortion (comparing with other algorithms). Moreover, as could be seen from Fig. 6, the constant parameters are better estimated comparing with the MCLS and BMCLS algorithms. Finally, constant and variable parameters (including slow-varying or fast-varying ones) could be estimated through the BMCLS algorithm, simultaneously.

Now, to show the performance of the proposed method for systems with large number of outputs, the following example is presented.

To verify the effect of the parameter γ, all the parameters are estimated for three different values of $\gamma(0.5, 1.5, 2.5)$. The simulation results obtained from the BMCLS algorithm are shown in Fig. 7 (for constant and exponential parameters) and Fig. 8 (for sinusoidal and square parameters). It is obvious that $\gamma = 1.5$ gives superior results.

Remark 2 According to (27), the appropriate value for γ for slow-varying parameters should be considered smaller than the corresponding one for fast-varying parameters. This means that for greater values of ω_0 in (30), a greater value for γ should be selected. The estimated constant and exponential parameters with different values of γ and $\omega_0 = 0.03$ are shown in Fig. 9 while the estimated square and sinusoidal parameters are shown in Fig. 10. As could be seen form Figs. 9 and 10, $\gamma = 4$ leads to better results comparing with other values of γ. Table 1 shows the relative estimated errors (ε) for estimated parameters with different values of γ and ω_0. Consider that $\varepsilon = \frac{\left\|\hat{\underline{\vartheta}}(t)-\underline{\vartheta}(t)\right\|_2}{\left\|\underline{\vartheta}(t)\right\|_2}$ where $\hat{\underline{\vartheta}}(t)$ and $\underline{\vartheta}(t)$ are the estimated and the true values, respectively. Table 1 shows that $\gamma = 4$ is a reasonable choice for $\omega_0 = 0.03$ (that is greater than the corresponding value for $\omega_0 = 0.005$).

Example 2 Consider that in system (28) the input, output and the Gaussian noise are defined as $\underline{u}(t) = \left[u_1(t) \ldots u_5(t)\right]^{\mathrm{T}}$, $y(t) = \left[y_1(t) \ldots y_5(t)\right]^{\mathrm{T}}$ and $\underline{v}(t) = \left[v_1(t) \ldots v_5(t)\right]^{\mathrm{T}}$, respectively. Moreover, $d(z)$ and $Q(z)$ are changed as

$$d(z) = 1+\vartheta_1^\circ(t)z^{-1},\ Q(z) = \begin{bmatrix} \vartheta_2^\circ & \vartheta_3^\circ & 0 & 0 & 0 \\ 0 & 0 & \vartheta_4^\circ(t) & \vartheta_5^\circ & 0 \\ 0 & 0 & 0 & \vartheta_6^\circ(t) & \vartheta_7^\circ \\ \vartheta_8^\circ & 0 & 0 & \vartheta_9^\circ & 0 \\ 0 & \vartheta_{10}^\circ & 0 & 0 & \vartheta_{11}^\circ \end{bmatrix} z^{-1} \tag{32}$$

where

$$\begin{aligned} &\vartheta_1^\circ(t) = 1 - e^{-0.004t},\ t \geq 0, \\ &\vartheta_2^\circ = \vartheta_3^\circ = \vartheta_5^\circ = \vartheta_7^\circ = \vartheta_8^\circ = \vartheta_9^\circ = \vartheta_{10}^\circ = \vartheta_{11}^\circ = 1, \\ &\vartheta_4^\circ(t) = \begin{cases} 1 & \sin(0.006t) < 0.5 \\ 2 & \text{Otherwise} \end{cases},\ \vartheta_6^\circ(t) = 1 + \sin(0.003t) \end{aligned} \tag{33}$$

Now, the linear regression model (1) for this example could be written as

$$\Phi(t) = \begin{bmatrix} -y_1(t-1) & -y_2(t-1) & -y_3(t-1) & -y_4(t-1) & -y_5(t-1) \\ u_1(t-1) & 0 & 0 & 0 & 0 \\ u_2(t-1) & 0 & 0 & 0 & 0 \\ 0 & u_3(t-1) & 0 & 0 & 0 \\ 0 & u_4(t-1) & 0 & 0 & 0 \\ 0 & 0 & u_4(t-1) & 0 & 0 \\ 0 & 0 & u_5(t-1) & 0 & 0 \\ 0 & 0 & 0 & u_1(t-1) & 0 \\ 0 & 0 & 0 & u_4(t-1) & 0 \\ 0 & 0 & 0 & 0 & u_2(t-1) \\ 0 & 0 & 0 & 0 & u_5(t-1) \end{bmatrix}^{\mathrm{T}},$$
$$\vartheta = \left[\, \vartheta_1 \ \ldots \ \vartheta_{11} \,\right]^{\mathrm{T}} \tag{34}$$

Gaussian noises with variance 0.16 and $p_0 = 10^6 I_{11}$, $\hat{\vartheta}(0) = 0$ (I_{11} is the 11×11 identity matrix) are considered for the simulations. In the FFCLS algorithm, the fixed forgetting factor $\lambda = 0.98$ is selected. In the MCLS algorithm, $\alpha = 0.98$, $\gamma = 1.5$ are considered. In the BMCLS algorithm, $M = 10$ is selected.

The estimated parameters for the FFCLS, BFFCLS, MCLS and BMCLS algorithms are given in Figs. 11, 12, 13 and 14. Figure 11 shows that the BFFCLS, MCLS and BMCLS algorithms provide better performance for tracking exponential parameter comparing with the FFCLS algorithm. However, their tracking ability is approximately the same. As could be seen from Fig. 12, the best tracking of the constant parameter is obtained with the BMCLS algorithm (consider that only one of the constant parameters is shown). Figures 13 and 14 show that the fastest tracking capability for the square and the sinusoidal parameter is achieved with the BMCLS algorithm. Thus, for simultaneous estimation of the constant and time-varying parameters (including slow and fast-varying ones), the BMCLS algorithm could be considered as a reasonable choice.

6 Conclusion

In this paper, some algorithms are introduced for estimation of time-varying parameters in multivariable systems. These algorithms employ the CLS algorithm (which has better performance and higher calculation speed compared to the RLS). Incorporating the matrix forgetting factor in the CLS algorithm gives the MCLS algorithm in which the forgetting factor of each parameter is obtained in terms of its variance. Combining this algorithm with the BLFFRLS algorithm leads to the BMCLS algorithm that contains the advantages of both of them. Simulation results with different kinds of parameters (including constant, exponential, square and sinusoidal parameters) demonstrate that the BMCLS algorithm estimates both slow-varying and fast-varying parameters for multivariable systems.

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