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On analysis of a nonlinear fractional system for social media addiction involving Atangana–Baleanu–Caputo derivative

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Abstract

A mathematical model for the dynamic systems of $\mathbb{SMA}$ involving the $\mathbb{ABC}$-fractional derivative is considered in this manuscript. We examine the basic reproduction number and analyze the stability of the equilibrium points. We prove the theoretical results of the existence and Ulam's stability of the solutions for the proposed model using fixed point theory and nonlinear analytic techniques. Using the Adams type predictor–corrector rule for the $\mathbb{ABC}$-fractional integral operator, a numerical scheme is devised for obtaining the approximate solution of the proposed model. Different numerical plots corresponding to various fractional orders are presented. In addition, we demonstrate a numerical simulation for the transmission of social media addiction in two cases with the basic reproduction numbers greater than and less than one.

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1 Introduction

During the last decade, social media ($\mathbb{SM}$) has plentifully influenced the world. $\mathbb{SM}$ is the most popular technology which collects the wide knowledge of all attention and makes up the society or the individual who interacts with communication. People use $\mathbb{SM}$ advantage via internet access in many parts such as business, education, health, science, and amusement [1, 2]. Some of them access information of their curious attention from $\mathbb{SM}$ platforms such as Google. Some find old or new friends, earn money, present work, make advertising products, buy or sell their goods via Facebook, Instagram, and Youtube. Some make a money transaction via bank applications. Some share information via Twitter and play games from various applications [3–5]. Although $\mathbb{SM}$ has become a part of our daily lives, it can negatively cause or affect people's daily lives or relatives in families. One of the significant causes of more serious negative impacts does the social media addiction ($\mathbb{SMA}$). $\mathbb{SMA}$ is a state that is used to refer to people who spend so much time in their daily life on $\mathbb{SM}$ and feel anxious when they cannot make a visit to a $\mathbb{SM}$ platform [6, 7].

In fact, $\mathbb{SMA}$ is a kind of addictive problem, the same as in psychology alcoholism, smoking, game addiction, *etc*. Mathematical models play an important role in the construction to study the dynamic behavior of these problems. For instance, Nyabadza and co-workers [8, 9] formulated methamphetamine transmission in South Africa by building an appropriate mathematical model. In 2018, Ma and co-workers [10] have studied the stability of the synthetic drugs transmission epidemic models with psychological addicts. In 2019, Liu et al. [11] have analyzed a synthetic drug transmission model with treatment and discussed global stability and backward bifurcation of the model. Huo and co-workers [12] introduced a new alcoholism model with treatment and effect of Twitter. The stability of the equilibrium point is determined by using the basic reproductive number and numerical results are conducted. Li and Guo [13] constructed an online game addiction model. They used the basic reproduction number to obtain some properties and analyzed the stability of the equilibria. Pontriagin's maximum principle was employed to solve the optimal control strategy and numerical simulations are presented in their work. In 2020, Samad *et al.* [14] presented and analyzed a mathematical model of the smoking tobacco epidemic in Bangladesh. They derived the basic reproduction number and established the stability theorem for all equilibria. In 2021, Alemneh and Alemu [15] formulated and analyzed a mathematical model for the transmission dynamics of $\mathbb{SMA}$ in the human population as follows:

$$\begin{cases} \frac{d\mathcal{S}}{dt} = \pi + \gamma\eta\mathcal{R} - \beta\sigma\mathcal{A}\mathcal{S} - (\kappa + \mu)\mathcal{S}, \\ \frac{d\mathcal{E}}{dt} = \beta\sigma\mathcal{A}\mathcal{S} - (\delta + \mu)\mathcal{E}, \\ \frac{d\mathcal{A}}{dt} = \alpha\delta\mathcal{E} - (\mu + \epsilon + \rho)\mathcal{A}, \\ \frac{d\mathcal{R}}{dt} = (1-\alpha)\delta\mathcal{E} + \epsilon\mathcal{A} - (\mu + \eta)\mathcal{R}, \\ \frac{d\mathcal{Q}}{dt} = \kappa\mathcal{S} + (1-\gamma)\eta\mathcal{R} - \mu\mathcal{Q}. \end{cases} \tag{1.1}$$

For system (1.1), the human population is divided into five groups representing addiction status. Group 1: the people who are not addicted but susceptible to $\mathbb{SMA}$ are denoted by susceptible populations; $\mathcal{S}(t)$. Group 2: the people who use $\mathbb{SM}$ less frequently but do not grow to the addicted stage are denoted by exposed populations; $\mathcal{E}(t)$. Group 3: the people who are addicted to $\mathbb{SM}$ and spent most of their time on it are denoted by addicted populations; $\mathcal{A}(t)$. Group 4: the people who recovered from $\mathbb{SMA}$ are denoted by recovered populations; $\mathcal{R}(t)$. Group 5: the people who permanently do not use and quit using $\mathbb{SM}$ are denoted by $\mathcal{Q}(t)$. The total number of members of the population is $\mathcal{N} = \mathcal{S} + \mathcal{E} + \mathcal{A} + \mathcal{R} + \mathcal{Q}$. The assumptions of the system are the following: the spread of the problem of $\mathbb{SMA}$ happens within a closed environment, and it does not depend on sex, race, and human social state, members mix homogeneously, and the social media addictive people will transmit to non-addictive people when they are in connecting with the pressure of addictive. Moreover, the differential equations of this system are integrated by using the social media addictive cycle, which starts from entering susceptible individuals into the population with a rate of π. They are motivated by addictive people with the pressure contact rate of β and the probability transmission rate of σ and move to the exposed state. Some susceptible individuals move to a group of people who permanently do not use social media at a rate of κ. The exposed individuals are separated into two groups, one becomes addicted and moves to the addicted group at rate $\alpha\delta$, and another recovered with treatment at a rate $(1-\alpha)\delta$. Some addicted individuals move to the recovered group at a rate of ϵ or died due to the overusing of addiction on social media at a rate of ρ. The recovered individuals become again susceptible individuals at a rate of $\gamma\eta$ or permanently stop using social media at a rate $(1-\gamma)\eta$. Finally, all the people in every compartment have a natural death rate of μ. Alemneh and Alemu also investigated the stability of the equilibrium points and employed Pontryagin's maximum principle for the optimal control system.

More than three centuries have passed, fractional-order derivative models have been applied in several areas of real-world problems such as science, economics, engineering, biology, and epidemiology with various types of fractional calculus such as Liouville–Caputo ($\mathbb{LC}$), Caputo–Katugumpola ($\mathbb{CK}$), Caputo–Fabrizio ($\mathbb{CF}$), and fractal–fractional ($\mathbb{FF}$); see [16–26]. In addition, some of the authors incorporated the fractional-order derivative to addictive problems. In 2017, Singh *et al.* [27] studied and analyzed the existence and uniqueness of the smoking model under the $\mathbb{CF}$ sense. In 2019, Dokuyucu [28] presented a fractional order of an alcoholism model with $\mathbb{CF}$ type and investigated the existence and uniqueness of the model by using a fixed-point theorem. In 2021, Alrabaiah and co-workers [29] have formulated and analyzed a new mathematical model for $\mathbb{LC}$-fractional tobacco smoking with snuffing class. They accomplished a numerical solution of the proposed model via the generalized Adams–Bashforth–Moulton method. The Atangana–Baleanu–Caputo ($\mathbb{ABC}$) fractional derivative operator is one of the most popular fractional derivative operators. A fractional-order derivative was first roused into operation by Atangana and Baleanu [30] under the rule of a generalized Mittag-Leffler function in the part of a non-singular and non-local kernel. In many real-world problems, the $\mathbb{ABC}$-fractional derivative produces better results [31–41].

Based on the best of our knowledge of previous research, no manuscripts have looked into the mathematical model of $\mathbb{SMA}$ with various fractional derivatives. We initiated the $\mathbb{ABC}$-fractional derivative to the $\mathbb{SMA}$ model which is the creativity of this manuscript. Consequently, we are interested in filling this gap by considering the $\mathbb{SMA}$ model studied by [15] under the $\mathbb{ABC}$-fractional derivative with order ϕ. We replace the integer order of model (1.1) with a fractional-order system. Therefore, the classical model (1.1) extend to fractional-order system by replacing the ordinary time derivative d/dt to the $\mathbb{ABC}$-fractional derivative ${}^{\mathbb{ABC}}_{t}\mathfrak{D}^{\phi}_{0}$. It is remarkable that in the classical model (1.1), the dimension of the right-hand side of fractional model has dimensions $(time)^{-1}$, but the dimensions of the left-hand side of $\mathbb{ABC}$-fractional model equal to $(time)^{-\phi}$. In addition, when we convert an integer order system into fractional-order ϕ, we also have to consider all nonnegative parameters in the term of ϕ-exponent for making the equal dimensions of the differential equations. The modified $\mathbb{SMA}$ transmission model with the $\mathbb{ABC}$-fractional derivative suggested a model as follows:

$$\begin{cases} {}^{\mathbb{ABC}}_{t}\mathfrak{D}^{\phi}_{0}\mathcal{S}(t) = \pi^{\phi} + \gamma^{\phi}\eta^{\phi}\mathcal{R} - \beta^{\phi}\sigma^{\phi}\mathcal{A}\mathcal{S} - (\kappa^{\phi} + \mu^{\phi})\mathcal{S}, \\ {}^{\mathbb{ABC}}_{t}\mathfrak{D}^{\phi}_{0}\mathcal{E}(t) = \beta^{\phi}\sigma^{\phi}\mathcal{A}\mathcal{S} - (\delta^{\phi} + \mu^{\phi})\mathcal{E}, \\ {}^{\mathbb{ABC}}_{t}\mathfrak{D}^{\phi}_{0}\mathcal{A}(t) = \alpha^{\phi}\delta^{\phi}\mathcal{E} - (\mu^{\phi} + \epsilon^{\phi} + \rho^{\phi})\mathcal{A}, \\ {}^{\mathbb{ABC}}_{t}\mathfrak{D}^{\phi}_{0}\mathcal{R}(t) = (1-\alpha^{\phi})\delta^{\phi}\mathcal{E} + \epsilon^{\phi}\mathcal{A} - (\mu^{\phi} + \eta^{\phi})\mathcal{R}, \\ {}^{\mathbb{ABC}}_{t}\mathfrak{D}^{\phi}_{0}\mathcal{Q}(t) = \kappa^{\phi}\mathcal{S} + (1-\gamma^{\phi})\eta^{\phi}\mathcal{R} - \mu^{\phi}\mathcal{Q}, \end{cases} \tag{1.2}$$

with the initial conditions $(\mathcal{S},\mathcal{E},\mathcal{A},\mathcal{R},\mathcal{Q}) = (\mathcal{S}_0,\mathcal{E}_0,\mathcal{A}_0,\mathcal{R}_0,\mathcal{Q}_0)$. The descriptions of all parameters are shown in Table 1. The main aim of this manuscript is to analyze the conditions that influence the transmission of this addiction to cease or, opposite, turn into epidemic, based on the number of reproductions. We establish the existence and uniqueness of the solutions for the proposed model via the famous fixed point theorems. The context of various Ulam's stability is provided to discuss the stability analysis. Finally, we use the novel numerical method represented by Alkahtani et al. [42] to find the approximated solutions of the $\mathbb{SMA}$ for different fractional orders.

This paper is organized as follows: in Sect. 2, we present definitions and basic concepts of $\mathbb{ABC}$-fractional differential and integral operators after that we provide fixed point instruments to proof the our existence results. We computed the equilibrium points, the basic reproduction numbers, and established the stability analysis of the proposed model in Sect. 3. In Sect. 4, the uniqueness of the solution for the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ system (1.2) is examined by employing Banach's fixed point theorem and the existence result is proved by Krasnoselskii's fixed point theorem. In Sect. 5, the four types of Ulam's stability concepts of the model (1.2) are investigated. Numerical simulations to support the theoretical results are provided in Sect. 6. Finally, the discussion and conclusion of the proposed model are presented in Sect. 7.

2 Preliminaries

This section presents relevant and necessary essential concepts used in this manuscript.

Definition 2.1 ([30]) Let $f \in \mathcal{C}^1[a,b]$, $a < b$, be a function, and $0 \le \phi \le 1$. Then the $\mathbb{ABC}$-fractional derivative of a function f of order ϕ is defined as follows:

$$ {}^{\mathbb{ABC}}_{\quad t}\mathfrak{D}^{\phi}_{a} f(t) = \frac{\mathbb{AB}(\phi)}{1-\phi}\int_a^t \mathbb{E}_\phi\left[-\frac{\phi}{1-\phi}(t-s)^\phi\right]\frac{d}{dt}f(s)\,ds, \quad t > a > 0, \tag{2.1}$$

where $\mathbb{AB}(\phi) = 1-\phi+\phi/\Gamma(\phi)$ is normalization function, characterized by $\mathbb{AB}(0) = \mathbb{AB}(1) = 1$, and the Mittag-Leffler function $\mathbb{E}_\phi$ is given as

$$\mathbb{E}_\phi(z) = \sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\phi k+1)}, \quad z,\phi \in \mathbb{C}, \mathrm{Re}(\phi) > 0,$$

with $\mathbb{C}$ the set of complex numbers.

Definition 2.2 ([30]) The $\mathbb{ABC}$-fractional integral of a function $f \in \mathcal{C}^1(a,b)$ is defined as follows:

$$ {}^{\mathbb{AB}}_{\quad t}\mathcal{I}^{\phi}_{a} f(t) = \frac{1-\phi}{\mathbb{AB}(\phi)}f(t) + \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_a^t (t-s)^{\phi-1}f(s)\,ds, \quad t > a > 0.$$

Clearly, if $\phi = 0$ and $\phi = 1$ then we get the initial function and the ordinary integral, respectively. Furthermore, we can calculate the Laplace transform of (2.1) and obtain the following result:

$$\mathcal{L}\left\{{}^{\mathbb{ABC}}_{\quad t}\mathfrak{D}^{\phi}_{a} f(t)\right\}(p) = \frac{\mathbb{AB}(\phi)p^\phi\mathcal{L}\{f(t)\}(p) - p^{\phi-1}f(a)}{(1-\phi)(p^\phi + \frac{\phi}{1-\phi})}. \tag{2.2}$$

Lemma 2.3 ([30]) *The* $\mathbb{AB}$*-fractional derivative and* $\mathbb{AB}$*-fractional integral of a functions* $f \in \mathcal{C}^1(a,b)$ *satisfies the Newton–Leibniz equality*

$$ {}_t^{\mathbb{AB}}\mathcal{I}_a^{\phi}\big({}_t^{\mathbb{ABC}}\mathfrak{D}_a^{\phi} f(t)\big) = f(t) - f(a). $$

Lemma 2.4 ([43]) *For two functions,* $f, g \in \mathcal{H}^1(a,b)$, $a < b$, *the* $\mathbb{AB}$*-fractional derivative of a function* f *and* g *satisfies the following inequality*:

$$ \big\| {}_t^{\mathbb{ABC}}\mathfrak{D}_a^{\phi} f(t) - {}_t^{\mathbb{ABC}}\mathfrak{D}_a^{\phi} g(t) \big\| \le \mathcal{H} \big\| f(t) - g(t) \big\|. $$

Lemma 2.5 (*Generalized mean value theorem* [44]). *Let* $g(t) \in \mathcal{C}[a,b]$, *and let* ${}_t^{\mathbb{ABC}}\mathfrak{D}_a^{\phi} g(t) \in \mathcal{C}[a,b]$ *when* $\phi \in (0,1]$. *Then we have* $g(t) = g(a) + \frac{1}{\Gamma(\phi)} {}_t^{\mathbb{ABC}}\mathfrak{D}_a^{\phi} g(\xi)(t-a)^{\phi}$, *when* $\xi \in [a,t]$, $\forall t \in (a,b]$.

It is easy to see by Lemma 2.5 that, if $g(t) \in [a,b]$, ${}_t^{\mathbb{ABC}}\mathfrak{D}_a^{\phi} g(t) \in [a,b]$, and ${}_t^{\mathbb{ABC}}\mathfrak{D}_a^{\phi} g(t) \ge 0$, $\forall t \in (a,b]$ when $\phi \in (0,1]$, then the function $g(t)$ is nondecreasing, and if ${}_t^{\mathbb{ABC}}\mathfrak{D}_a^{\phi} g(t) \le 0$, $\forall t \in (a,b]$, then the function $g(t)$ is nonincreasing $\forall t \in [a,b]$.

Definition 2.6 (Contraction mapping [45]) Let X be a Banach space. Then the operator $\mathcal{T} : X \to X$ is a contraction if

$$ \|\mathcal{T}x - \mathcal{T}y\| \le L\|x - y\|, \quad \forall x, y, \in X, 0 < L < 1. $$

Lemma 2.7 (Banach's fixed point theorem [45]) *Let* D *be a non-empty closed subset of a Banach space* E. *Then any contraction mapping* $\mathcal{Q}$ *from* D *into itself has a unique fixed point*.

Lemma 2.8 (Krasnoselskii's fixed point theorem [45]) *Let* D *be a non-empty, closed, convex subset of a Banach space* E. *Let* T_1, T_2 *be two operators such that* (i) $T_1x + T_2y \in D$, $\forall x, y \in D$; (ii) T_1 *is compact and continuous*; (iii) T_2 *is a contraction mapping. Then there exists* $z \in D$ *such that* $T_1z + T_2z = z$.

3 Model analysis

3.1 Positivity invariant region

Now, we will discuss the positivity invariant region and steady states of the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2).

The following lemma guarantees the boundedness of the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2).

Lemma 3.1 *The closed set*

$$ \Omega := \left\{ (\mathcal{S}, \mathcal{E}, \mathcal{A}, \mathcal{R}, \mathcal{Q}) \in \mathbb{R}_+^5 : 0 < \mathcal{N}(t) \le \frac{\pi^{\phi}}{\mu^{\phi}} \right\}, \quad \mathcal{N}(t) = \mathcal{S}(t) + \mathcal{E}(t) + \mathcal{A}(t) + \mathcal{R}(t) + \mathcal{Q}(t), $$

is positively invariant with regard to the $\mathbb{ABC}$*-fractional* $\mathbb{SMA}$ *model* (1.2).

Proof Assume that the set $(\mathcal{S},\mathcal{E},\mathcal{A},\mathcal{R},\mathcal{Q})$ with any solution of the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2), and $\mathcal{N}(t) = \mathcal{S}(t) + \mathcal{E}(t) + \mathcal{A}(t) + \mathcal{R}(t) + \mathcal{Q}(t)$ represents the total population. By applying Lemma 2.5, we obtain

$$\begin{cases} {}^{\mathbb{ABC}}_{t}\mathfrak{D}^{\phi}_{0}\mathcal{S}(t) = \pi^{\phi} + \gamma^{\phi}\eta^{\phi}\mathcal{R} \geq 0, \\ {}^{\mathbb{ABC}}_{t}\mathfrak{D}^{\phi}_{0}\mathcal{E}(t) = \beta^{\phi}\sigma^{\phi}\mathcal{A}\mathcal{S} \geq 0, \\ {}^{\mathbb{ABC}}_{t}\mathfrak{D}^{\phi}_{0}\mathcal{A}(t) = \alpha^{\phi}\delta^{\phi}\mathcal{E} \geq 0, \\ {}^{\mathbb{ABC}}_{t}\mathfrak{D}^{\phi}_{0}\mathcal{R}(t) = (1-\alpha^{\phi})\delta^{\phi}\mathcal{E} + \epsilon^{\phi}\mathcal{A} \geq 0, \\ {}^{\mathbb{ABC}}_{t}\mathfrak{D}^{\phi}_{0}\mathcal{Q}(t) = \kappa^{\phi}\mathcal{S} + (1-\gamma^{\phi})\eta^{\phi}\mathcal{R} \geq 0. \end{cases} \tag{3.1}$$

It follows from (3.1) that any of the solutions of (1.2) is nonnegative and remains in $\mathbb{R}^5_+$. Taking into account that all the parameters are positive, by all the equations of the model,

$${}^{\mathbb{ABC}}_{t}\mathfrak{D}^{\phi}_{0}\mathcal{N}(t) = \pi^{\phi} - \mu^{\phi}\mathcal{N} - \rho^{\phi}\mathcal{A} \leq \pi^{\phi} - \mu^{\phi}\mathcal{N}(t). \tag{3.2}$$

Taking the Laplace transform into (3.2), we obtain

$$\begin{aligned} \mathcal{N}(t) \leq {} & \left(\frac{\mathbb{AB}(\phi)}{\mathbb{AB}(\phi) + (1-\phi)\mu^{\phi}}\mathcal{N}(0) + \frac{(1-\phi)\pi^{\phi}}{\mathbb{AB} + (1-\phi)\mu^{\phi}}\right)\mathbb{E}_{\phi,1}\left(-\frac{\phi\mu^{\phi}}{\mathbb{AB}(\phi) + (1-\phi)\mu^{\phi}}t^{\phi}\right) \\ & + \frac{\phi\pi^{\phi}}{\mathbb{AB}(\phi) + (1-\phi)\mu^{\phi}}\mathbb{E}_{\phi,\phi+1}\left(-\frac{\phi\mu^{\phi}}{\mathbb{AB}(\phi) + (1-\phi)\mu^{\phi}}t^{\phi}\right), \end{aligned}$$

where $\mathbb{E}_{\phi_1,\phi_2}$ is the two parameter Mittag-Leffler function, defined by

$$\mathbb{E}_{\phi_1,\phi_2}(z) = \sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\phi_1 k + \phi_2)}.$$

Taking into account the asymptotic behavior of the Mittag-Leffler function, we have

$$\mathbb{E}_{\phi_1,\phi_2}(z) \approx \sum_{K=1}^{\omega}\frac{z^{-K}}{\Gamma(\phi_2 - \phi_1 K)} + O\bigl(|z|^{-1-\omega}\bigr), \quad |z| \to \infty, \frac{\phi_1\pi}{2} < \bigl|\arg(z)\bigr| \leq \pi.$$

It is easily to observe that $\mathcal{N}(t) \to \pi^{\phi}/\mu^{\phi}$ as $t \to \infty$. Then the solution of the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) for initial conditions in Ω stays in Ω for every $t > 0$. Hence, Ω is positively invariant region with regard to the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2). □

All solutions which begin at the boundary of the positivity invariant region Ω converge to this region. We can analyze the flow generated by the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) for consideration because it is biologically and epidemiologically significant.

3.2 Equilibrium points and reproduction numbers

In this subsection, we are going to obtain the equilibrium points of the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2). We are to find equilibrium points and the basic reproduction number of the considered model. There are two species of probable equilibrium points of the model. The primary one is the point where no disease in the group is called the disease-free equilibrium point. For the process of finding the equilibrium point, we will be setting the right-hand side of the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) is equal to zero. Hence, the disease-free equilibrium point of the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) with $\mathcal{E} = \mathcal{A} = 0$ is given by

$$\mathfrak{E}_0 = \left(\mathcal{S}^0, \mathcal{E}^0, \mathcal{A}^0, \mathcal{R}^0, \mathcal{Q}^0\right) = \left(\frac{\pi^\phi}{\kappa^\phi + \mu^\phi}, 0, 0, 0, \frac{\kappa^\phi \pi^\phi}{\mu^\phi(\mu^\phi + \kappa^\phi)}\right).$$

For analyzing the stability of the equilibrium points, the basic reproduction number R_0 of the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) is very important. To find R_0, we only focus on the infectious classes of the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2). The transmission matrix F and transition matrix V for the next-generation matrix method [46, 47] are obtained as

$$F = \begin{pmatrix} 0 & \frac{\beta^\phi \pi^\phi \sigma^\phi}{\kappa^\phi + \mu^\phi} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad V = \begin{pmatrix} \delta^\phi + \mu^\phi & 0 & 0 \\ -\alpha^\phi \delta^\phi & \mu^\phi + \epsilon^\phi + \rho^\phi & 0 \\ -(1-\alpha^\phi)\delta^\phi & -\epsilon^\phi & \eta^\phi + \mu^\phi \end{pmatrix}.$$

Then the next-generation matrix is given by

$$FV^{-1} = \begin{pmatrix} \frac{\beta^\phi \sigma^\phi \pi^\phi \alpha^\phi \delta^\phi}{(\kappa^\phi + \mu^\phi)(\delta^\phi + \mu^\phi)(\mu^\phi + \epsilon^\phi + \rho^\phi)} & \frac{\beta^\phi \sigma^\phi \pi^\phi}{(\kappa^\phi + \mu^\phi)(\mu^\phi + \epsilon^\phi + \rho^\phi)} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{3.3}$$

Therefore, the spectral radius of the next-generation matrix (3.3) provides the number of the basic reproduction number (R_0). Hence,

$$R_0 = \mathfrak{r}\left(FV^{-1}\right) = \frac{\beta^\phi \pi^\phi \alpha^\phi \delta^\phi \sigma^\phi}{(\kappa^\phi + \mu^\phi)(\delta^\phi + \mu^\phi)(\mu^\phi + \epsilon^\phi + \rho^\phi)},$$

where $\mathfrak{r}$ denotes the spectral radius. As we know, R_0 is the information for measuring an infectious disease transmission potential over time. When $R_0 > 1$, then the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) has an endemic equilibrium point $\mathfrak{E}^*$. For finding $\mathfrak{E}^*$, we will be setting this fact that all variables $\mathcal{S}(t)$, $\mathcal{E}(t)$, $\mathcal{A}(t)$, $\mathcal{R}(t)$, and $\mathcal{Q}(t)$ of (1.2) are nonnegative. It can be calculated by equating each equation of (1.2) equal to zero as follows:

$${}^{\mathbb{ABC}}_{t}\mathfrak{D}^{\phi}_{0}\mathcal{S}(t) = {}^{\mathbb{ABC}}_{t}\mathfrak{D}^{\phi}_{0}\mathcal{E}(t) = {}^{\mathbb{ABC}}_{t}\mathfrak{D}^{\phi}_{0}\mathcal{A}(t) = {}^{\mathbb{ABC}}_{t}\mathfrak{D}^{\phi}_{0}\mathcal{R}(t) = {}^{\mathbb{ABC}}_{t}\mathfrak{D}^{\phi}_{0}\mathcal{Q}(t) = 0.$$

Then we obtain $\mathfrak{E}^* = (\mathcal{S}^*, \mathcal{E}^*, \mathcal{A}^*, \mathcal{R}^*, \mathcal{Q}^*)$, where

$$\begin{aligned}
\mathcal{S}^* &= \frac{(\mu^\phi + \delta^\phi)(\mu^\phi + \epsilon^\phi + \rho^\phi)}{\alpha^\phi \beta^\phi \delta^\phi \sigma^\phi}, \\
\mathcal{E}^* &= \frac{\xi_2}{\xi_1}, \\
\mathcal{A}^* &= \frac{\alpha^\phi \delta^\phi \mathcal{E}^*}{\mu^\phi + \epsilon^\phi + \rho^\phi}, \\
\mathcal{R}^* &= \frac{\xi_2 + (\delta^\phi + \mu^\phi)\mathcal{E}^*}{\gamma^\phi \eta^\phi}, \\
\mathcal{Q}^* &= \frac{\kappa^\phi \mathcal{S}^* + (1-\gamma^\phi)\eta^\phi \mathcal{R}^*}{\mu^\phi}.
\end{aligned}$$

Here

$$\xi_1 = \frac{\gamma^\phi \eta^\phi \delta^\phi}{\mu^\phi + \eta^\phi}\left(1 - \alpha^\phi + \frac{\epsilon^\phi \alpha^\phi}{\mu^\phi + \epsilon^\phi + \rho^\phi}\right) - \delta^\phi - \mu^\phi,$$
$$\xi_2 = \frac{(\kappa^\phi + \mu^\phi)(\delta^\phi + \mu^\phi)(\mu^\phi + \epsilon^\phi + \rho^\phi)}{\beta^\phi \sigma^\phi \alpha^\phi \delta^\phi} - \pi^\phi.$$

Next, we will state the theorem and guarantee that $\mathfrak{E}_0$ of the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) is locally asymptotically stable.

Theorem 3.2 *The disease-free equilibrium point $\mathfrak{E}_0$ of the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model* (1.2) *is locally asymptotically stable if $R_0 < 1$ and unstable otherwise.*

Proof We omit the details of the proof. See Theorem 3.3 in [15]. □

4 Existence results of $\mathbb{SMA}$ transmission mathematical model

In this section, we examine the existence and uniqueness of solutions for the fractional $\mathbb{SMA}$ model with the help of Banach's and Krasnoselskii's fixed point theorems.

For the sake of simplicity, we rewrite the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) as follows:

$$\begin{cases} {}^{\mathbb{ABC}}_{t}\mathfrak{D}^{\phi}_{0}\Theta(t) = \Lambda(t, \Theta(t)), \\ \Theta(0) = \Theta_0 \geq 0, \quad 0 < t < T < \infty, \end{cases} \tag{4.1}$$

where the vector $\Theta(t) = (\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_3, \mathbb{G}_4, \mathbb{G}_5)$ represents the state variables and Λ is a continuous vector function such that

$$\Lambda = \begin{pmatrix} \mathbb{G}_1 \\ \mathbb{G}_2 \\ \mathbb{G}_3 \\ \mathbb{G}_4 \\ \mathbb{G}_5 \end{pmatrix} = \begin{pmatrix} \pi^\phi + \gamma^\phi \eta^\phi \mathcal{R} - \beta^\phi \sigma^\phi \mathcal{A}\mathcal{S} - (\kappa^\phi + \mu^\phi)\mathcal{S} \\ \beta^\phi \sigma^\phi \mathcal{A}\mathcal{S} - (\delta^\phi + \mu^\phi)\mathcal{E} \\ \alpha^\phi \delta^\phi \mathcal{E} - (\mu^\phi + \epsilon^\phi + \rho^\phi)\mathcal{A} \\ (1 - \alpha^\phi)\delta^\phi \mathcal{E} + \epsilon^\phi \mathcal{A} - (\mu^\phi + \eta^\phi) \\ \kappa^\phi \mathcal{S} + (1 - \gamma^\phi)\eta^\phi \mathcal{R} - \mu^\phi \mathcal{Q} \end{pmatrix}, \tag{4.2}$$

with the initial conditions $\Theta_0 = (\mathcal{S}_0, \mathcal{E}_0, \mathcal{A}_0, \mathcal{R}_0, \mathcal{Q}_0)$. Applying the fractional integral of $\mathbb{ABC}$ to both sides of (4.1), we get the integral equation:

$$\Theta(t) = \Theta_0 + \frac{1 - \phi}{\mathbb{AB}(\phi)}\Lambda\bigl(t, \Theta(t)\bigr) + \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t - s)^{\phi - 1}\Lambda\bigl(s, \Theta(s)\bigr)\, ds,$$

where $\mathbb{AB}(\phi)$ is defined as in Definition 2.1. Let us define a Banach space by using $\mathcal{J} = [0, T]$ as $\mathcal{W} = \mathcal{C}(\mathcal{J}, \mathbb{R}^5_+)$ under the norm defined as $\|\Theta\| = \|\mathcal{S}\| + \|\mathcal{E}\| + \|\mathcal{A}\| + \|\mathcal{R}\| + \|\mathcal{Q}\|$ where

$$\sup_{t \in \mathcal{J}}\bigl\{\bigl|\Theta(t)\bigr|\bigr\} = \sup_{t \in \mathcal{J}}\bigl\{\bigl|\mathcal{S}(t)\bigr|\bigr\} + \sup_{t \in \mathcal{J}}\bigl\{\bigl|\mathcal{E}(t)\bigr|\bigr\} + \sup_{t \in \mathcal{J}}\bigl\{\bigl|\mathcal{A}(t)\bigr|\bigr\} + \sup_{t \in \mathcal{J}}\bigl\{\bigl|\mathcal{R}(t)\bigr|\bigr\} + \sup_{t \in \mathcal{J}}\bigl\{\bigl|\mathcal{Q}(t)\bigr|\bigr\}.$$

4.1 Uniqueness result via Banach's fixed point theorem

The existence and uniqueness result of the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ system (1.2) will be investigated by using Banach's fixed point theorem.

Theorem 4.1 *Assume that a quadratic vector function $\Lambda : \mathcal{J} \times \mathbb{R}^5 \to \mathbb{R}$ is continuous such that*:

(H_1) *there exists a positive constant $\mathbb{L}_\Lambda > 0$ such that*

$$\left|\Lambda\big(t, \Theta_1(t)\big) - \Lambda\big(t, \Theta_2(t)\big)\right| \leq \mathbb{L}_\Lambda \left|\Theta_1(t) - \Theta_2(t)\right|,$$

for any $\Theta_1, \Theta_2 \in \mathcal{W}$ and for all $t \in \mathcal{J}$.

If

$$\left(\frac{1-\phi}{\mathbb{AB}(\phi)} + \frac{T^\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\right)\mathbb{L}_\Lambda < 1, \tag{4.3}$$

then the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) has a unique solution on $\mathcal{J}$.

Proof Earlier, we converted the initial value problem (4.1) (which is equivalent to the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2)) into a fixed point problem $\Theta = \mathcal{T}\Theta$. We consider an operator $\mathcal{T} : \mathcal{W} \to \mathcal{W}$ that is defined by

$$(\mathcal{T}\Theta)(t) = \Theta_0 + \frac{1-\phi}{\mathbb{AB}(\phi)}\Lambda\big(t, \Theta(t)\big) + \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\Lambda\big(s, \Theta(s)\big)\,ds. \tag{4.4}$$

Clearly, the initial value problem (4.1) has a solution if and only if the operator $\mathcal{T}$ has fixed points.

Suppose that $\mathbb{K}_1$ is a nonnegative constant such that $\sup_{t\in\mathcal{J}} |\Lambda(t,0)| = \mathbb{K}_1 < +\infty$. Define a bounded, closed, and convex subset B_{r_1} of $\mathcal{W}$, where $B_{r_1} = \{\Theta \in \mathcal{W} : \|\Theta\| \leq r_1\}$, where r_1 is chosen such that

$$r_1 \geq \frac{\|\Theta_0\| + (\frac{1-\phi}{\mathbb{AB}(\phi)} + \frac{T_{\max}^\phi}{\mathbb{AB}(\phi)\Gamma(\phi)})\mathbb{K}_1}{1 - (\frac{1-\phi}{\mathbb{AB}(\phi)} + \frac{T_{\max}^\phi}{\mathbb{AB}(\phi)\Gamma(\phi)})\mathbb{L}_\Lambda}.$$

The proof proceeds in two steps.

Step I. We show that $\mathcal{T}B_{r_1} \subset B_{r_1}$.

For any $\Theta \in B_{r_1}$, we have

$$\begin{aligned}
\left|(\mathcal{T}\Theta)(t)\right| &\leq \|\Theta_0\| + \frac{1-\phi}{\mathbb{AB}(\phi)}\left|\Lambda\big(t,\Theta(t)\big)\right| + \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\left|\Lambda\big(s,\Theta(s)\big)\right|ds \\
&\leq \|\Theta_0\| + \frac{1-\phi}{\mathbb{AB}(\phi)}\left[\left|\Lambda\big(t,\Theta(t)\big) - \Lambda(t,0)\right| + \left|\Lambda(t,0)\right|\right] \\
&\quad + \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\left[\left|\Lambda\big(s,\Theta(s)\big) - \Lambda(s,0)\right| + \left|\Lambda(s,0)\right|\right]ds \\
&\leq \|\Theta_0\| + \frac{1-\phi}{\mathbb{AB}(\phi)}[\mathbb{L}_\Lambda r_1 + \mathbb{K}_1] + \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\,ds[\mathbb{L}_\Lambda r_1 + \mathbb{K}_1] \\
&\leq \|\Theta_0\| + \left(\frac{1-\phi}{\mathbb{AB}(\phi)} + \frac{T_{\max}^\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\right)[\mathbb{L}_\Lambda r_1 + \mathbb{K}_1] \leq r_1,
\end{aligned}$$

which implies that $\mathcal{T}B_{r_1} \subset B_{r_1}$.

Step II. We show that $\mathcal{T}$ is a contraction.

For each $\Theta_1, \Theta_2 \in B_{r_1}$ and for any $t \in \mathcal{J}$, we obtain

$$\begin{aligned}
&\left|(\mathcal{T}\Theta_1)(t) - (\mathcal{T}\Theta_2)(t)\right| \\
&\quad\leq \frac{1-\phi}{\mathbb{AB}(\phi)}\left|\Lambda\big(t,\Theta_1(t)\big) - \Lambda\big(t,\Theta_2(t)\big)\right| \\
&\qquad+ \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\left|\Lambda\big(s,\Theta_1(s)\big) - \Lambda\big(s,\Theta_2(s)\big)\right| ds \\
&\quad\leq \frac{(1-\phi)\mathbb{L}_\Lambda}{\mathbb{AB}(\phi)}\left|\Theta_1(t) - \Theta_2(t)\right| + \frac{\phi\mathbb{L}_\Lambda}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\left|\Theta_1(s) - \Theta_2(s)\right| ds \\
&\quad\leq \left(\frac{1-\phi}{\mathbb{AB}(\phi)} + \frac{T_{\max}^{\phi}}{\mathbb{AB}(\phi)\Gamma(\phi)}\right)\mathbb{L}_\Lambda\|\Theta_1 - \Theta_2\|,
\end{aligned}$$

which implies that

$$\|\mathcal{T}\Theta_1 - \mathcal{T}\Theta_2\| \leq \left(\frac{1-\phi}{\mathbb{AB}(\phi)} + \frac{T_{\max}^{\phi}}{\mathbb{AB}(\phi)\Gamma(\phi)}\right)\mathbb{L}_\Lambda\|\Theta_1 - \Theta_2\|.$$

Since $[(1-\phi)/\mathbb{AB}(\phi) + T_{\max}^{\phi}/(\mathbb{AB}(\phi)\Gamma(\phi))] < 1$, by the conclusion of Banach's fixed point theorem (Lemma 2.7), $\mathcal{T}$ is called a contraction. Hence, $\mathcal{T}$ has a unique fixed point that is a unique solution of the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) on $\mathcal{J}$. □

4.2 Existence result via Krasnoselskii's fixed point theorem

Theorem 4.2 *Assume that* (H_1) *holds and*

(H_2) *there exists positive constant* $\mathbb{M}_\Lambda$, $\mathbb{N}_\Lambda$ *such that*

$$\left|\Lambda\big(t,\Theta(t)\big)\right| \leq \mathbb{M}_\Lambda\left|\Theta(t)\right| + \mathbb{N}_\Lambda,$$

for any $\Theta \in \mathcal{W}$ *and for all* $t \in \mathcal{J}$.

Then there exists at least one solution of the $\mathbb{ABC}$*-fractional* $\mathbb{SMA}$ *model* (1.2), *provided that* $(1-\phi)\mathbb{L}_\Lambda/\mathbb{AB}(\phi) < 1$.

Proof Consider $\mathcal{T}: \mathcal{W} \to \mathcal{W}$ defined by $(\mathcal{T}\Theta)(t) = (\mathcal{T}_1\Theta)(t) + (\mathcal{T}_2\Theta)(t)$, $\Theta \in \mathcal{W}$, $t \in \mathcal{J}$, where

$$(\mathcal{T}_1\Theta)(t) = \Theta_0 + \frac{1-\phi}{\mathbb{AB}(\phi)}\Lambda\big(t,\Theta(t)\big), \tag{4.5}$$

$$(\mathcal{T}_2\Theta)(t) = \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\Lambda\big(s,\Theta(s)\big)\, ds. \tag{4.6}$$

Let $B_{r_2} = \{\Theta \in \mathcal{W} : \|\Theta\| \leq r_2\}$ be a closed convex set with the radius

$$r_2 \geq \frac{\|\Theta_0\| + (\frac{1-\phi}{\mathbb{AB}(\phi)} + \frac{T_{\max}^{\phi}}{\mathbb{AB}(\phi)\Gamma(\phi)})\mathbb{N}_\Lambda}{1 - (\frac{1-\phi}{\mathbb{AB}(\phi)} + \frac{T_{\max}^{\phi}}{\mathbb{AB}(\phi)\Gamma(\phi)})\mathbb{M}_\Lambda}. \tag{4.7}$$

The proof is divided into the following four steps.

Step I. We show that $\mathcal{T}_1\Theta_1 + \mathcal{T}_2\Theta_2 \in B_{r_2}$ for all $\Theta_1, \Theta_2 \in B_{r_2}$.

By the operator (4.5), we get

$$\begin{aligned}
&\left|(\mathcal{T}_1\Theta_1)(t)+(\mathcal{T}_2\Theta_2)(t)\right| \\
&\quad\le \|\Theta_0\| + \frac{1-\phi}{\mathbb{AB}(\phi)}\left|\Lambda\big(t,\Theta_1(t)\big)\right| + \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\left|\Lambda\big(s,\Theta_2(s)\big)\right|ds \\
&\quad\le \|\Theta_0\| + \frac{1-\phi}{\mathbb{AB}(\phi)}[\mathbb{M}_\Lambda r_2+\mathbb{N}_\Lambda] + \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\,ds[\mathbb{M}_\Lambda r_2+\mathbb{N}_\Lambda] \\
&\quad\le \|\Theta_0\| + \left(\frac{1-\phi}{\mathbb{AB}(\phi)}+\frac{T_{\max}^{\phi}}{\mathbb{AB}(\phi)\Gamma(\phi)}\right)\mathbb{N}_\Lambda + \left(\frac{1-\phi}{\mathbb{AB}(\phi)}+\frac{T_{\max}^{\phi}}{\mathbb{AB}(\phi)\Gamma(\phi)}\right)\mathbb{M}_\Lambda r_2 \\
&\quad\le r_2,
\end{aligned}$$

which yields $\|\mathcal{T}_1\Theta_1+\mathcal{T}_2\Theta_2\|\le r_2$. Then $\mathcal{T}_1\Theta_1+\mathcal{T}_2\Theta_2\in B_{r_2}$ for all $\Theta_1,\Theta_2\in B_{r_2}$.

Step II. We show that $\mathcal{T}_1$ is a contraction.

For any $\Theta_1,\Theta_2\in B_{r_2}$, we have

$$\begin{aligned}
\left|(\mathcal{T}_1\Theta_1)(t)-(\mathcal{T}_1\Theta_2)(t)\right| &\le \frac{1-\phi}{\mathbb{AB}(\phi)}\left|\Lambda\big(t,\Theta_1(t)\big)-\Lambda\big(t,\Theta_2(t)\big)\right| \\
&\le \frac{(1-\phi)\mathbb{L}_\Lambda}{\mathbb{AB}(\phi)}\left|\Theta_1(t)-\Theta_2(t)\right|,
\end{aligned}$$

which implies that $\|\mathcal{T}_1\Theta_1-\mathcal{T}_1\Theta_2(t)\|\le[(1-\phi)\mathbb{L}_\Lambda/(\mathbb{AB}(\phi))]\|\Theta_1-\Theta_2\|$. Since $(1-\phi)\mathbb{L}_\Lambda/\mathbb{AB}(\phi)<1$, $\mathcal{T}_1$ is contraction.

Step III. We show that $\mathcal{T}_2$ is continuous and compact.

Let Θ_n be a sequence such that $\Theta_n\to\Theta\in\mathcal{W}$. Then, for any $t\in\mathcal{J}$, we have

$$\begin{aligned}
\left|(\mathcal{T}_2\Theta_n)(t)-(\mathcal{T}_2\Theta)(t)\right| &\le \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\left|\Lambda\big(s,\Theta_n(s)\big)-\Lambda\big(s,\Theta(s)\big)\right|ds \\
&\le \frac{T_{\max}^{\phi}}{\mathbb{AB}(\phi)\Gamma(\phi)}\left\|\Lambda\big(\cdot,\Theta_n(\cdot)\big)-\Lambda\big(\cdot,\Theta(\cdot)\big)\right\|.
\end{aligned}$$

Since Λ is continuous, $\mathcal{T}_2$ is also continuous. Then we get $\|\mathcal{T}_2\Theta_n-\mathcal{T}_2\Theta\|\to0$, as $n\to\infty$. Next, $\mathcal{T}_2$ is uniformly bounded on B_{r_2} ($\mathcal{T}_2$ is relatively compact). For any $\Theta\in B_{r_2}$ and $t\in\mathcal{J}$, one has

$$\left|(\mathcal{T}_2\Theta)(t)\right|\le\frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\left|\Lambda\big(s,\Theta(s)\big)\right|ds\le\frac{T_{\max}^{\phi}}{\mathbb{AB}(\phi)\Gamma(\phi)}[\mathbb{M}_\Lambda r_2+\mathbb{N}_\Lambda].$$

This shows that $\mathcal{T}_2$ is uniformly bounded on B_{r_2}.

Step IV. We show that $\mathcal{T}_2$ is equicontinuous.

Assume that $\tau_1,\tau_2\in\mathcal{J}$ with $0\le\tau_1<\tau_2\le T$ and $\Theta\in B_{r_2}$. Then we have

$$\begin{aligned}
&\left|(\mathcal{T}_2\Theta)(\tau_2)-(\mathcal{T}_2\Theta)(\tau_1)\right| \\
&\quad\le\frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\left|\int_0^{\tau_2}(\tau_2-s)^{\phi-1}\Lambda\big(s,\Theta(s)\big)\,ds-\int_0^{\tau_1}(\tau_1-s)^{\phi-1}\Lambda\big(s,\Theta(s)\big)\,ds\right|
\end{aligned}$$

$$\begin{aligned}&\leq \frac{\phi[\mathbb{M}_\Lambda r_2 + \mathbb{N}_\Lambda]}{\mathbb{AB}(\phi)\Gamma(\phi)}\left|\int_{\tau_1}^{\tau_2}(\tau_2 - s)^{\phi-1}\,ds + \int_0^{\tau_1}\left[(\tau_2 - s)^{\phi-1} - (\tau_1 - s)^{\phi-1}\right]ds\right| \\ &\leq \frac{\mathbb{M}_\Lambda r_2 + \mathbb{N}_\Lambda}{\mathbb{AB}(\phi)\Gamma(\phi)}\big(2|\tau_2 - \tau_1|^\phi\big). \end{aligned} \tag{4.8}$$

Clearly, this being independent of $\Theta \in B_{r_2}$, the right-hand side of (4.8) tends to zero as $\tau_2 \to \tau_1$. Therefore, by the Arzelá–Ascoli theorem, $\mathcal{T}_2 B_{r_2}$ is relatively compact and $\mathcal{T}_2$ is completely continuous. Hence, by Krasnoselskii's fixed point theorem (Lemma 2.8), which implies that the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) has at least one solution on $\mathcal{J}$. □

5 Ulam's stability analysis of $\mathbb{SMA}$ transmission mathematical model

This section is discussing some sufficient conditions for the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) that will correspond to the assumptions of the four types of Ulam's stability as $\mathbb{UH}$ stability, generalized $\mathbb{UH}$ stability, $\mathbb{UHR}$ stability, and generalized $\mathbb{UHR}$ stability.

Firstly, we will state Ulam's stability theorem, which will be used in this section. Let $\varphi > 0$ be a positive real number and $\mathcal{F}_\Lambda : \mathcal{J} \to \mathbb{R}^+$ be a continuous function. We consider

$$\left|{}_t^{\mathbb{ABC}}\mathfrak{D}_0^\phi \xi(t) - \Lambda\big(t, \xi(t)\big)\right| \leq \varphi, \quad \forall t \in \mathcal{J}, \tag{5.1}$$

$$\left|{}_t^{\mathbb{ABC}}\mathfrak{D}_0^\phi \xi(t) - \Lambda\big(t, \xi(t)\big)\right| \leq \varphi\mathcal{F}_\Lambda(t), \quad \forall t \in \mathcal{J}, \tag{5.2}$$

$$\left|{}_t^{\mathbb{ABC}}\mathfrak{D}_0^\phi \xi(t) - \Lambda\big(t, \xi(t)\big)\right| \leq \mathcal{F}_\Lambda(t), \quad \forall t \in \mathcal{J}, \tag{5.3}$$

where $\varphi = \max(\varphi_j)^\mathbb{T}$ for $j = 1, 2, 3, 4, 5$.

Definition 5.1 ($\mathbb{UH}$ Stability) The $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) is called $\mathbb{UHR}$ stable if there exists a real number $C_\Lambda > 0$ such that, for every $\varphi > 0$ and for each solution $\xi \in \mathcal{W}$ of (5.1), there exists a solution $\Theta \in \mathcal{W}$ of the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) with

$$\big|\xi(t) - \Theta(t)\big| \leq C_\Lambda \varphi, \quad t \in \mathcal{J}, \tag{5.4}$$

where $\varphi = \max(\varphi_j)^\mathbb{T}$ and $C_\Lambda = \max(C_{\Lambda_j})^\mathbb{T}$ for $j = 1, 2, 3, 4, 5$.

Definition 5.2 (Generalized $\mathbb{UH}$ Stability) The $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) is called generalized $\mathbb{UH}$ stable if there exists a function $\mathcal{F}_\Lambda \in \mathcal{C}(\mathbb{R}^+, \mathbb{R}^+)$ with $\mathcal{F}_\Lambda(0) = 0$ such that, for each solution $\xi \in \mathcal{W}$ of (5.2), there exists a solution $\Theta \in \mathcal{W}$ of the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) such that

$$\big|\xi(t) - \Theta(t)\big| \leq \mathcal{F}_\Lambda(\varphi), \quad t \in \mathcal{J}, \tag{5.5}$$

where $\varphi = \max(\varphi_j)^\mathbb{T}$ and $\mathcal{F}_\Lambda = \max(\mathcal{F}_{\Lambda_j})^\mathbb{T}$ for $j = 1, 2, 3, 4, 5$.

Definition 5.3 ($\mathbb{UHR}$ Stability) The $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) is called $\mathbb{UHR}$ stable with respect to $\mathcal{F}_\Lambda \in \mathcal{C}(\mathcal{J}, \mathbb{R}^+)$ if there exists a real number $K_{\mathcal{F}_\Lambda} > 0$ such that for each $\varphi > 0$ and for each solution $\xi \in \mathcal{W}$ of (5.2) there exists a solution $\Theta \in \mathcal{W}$ of the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) with

$$\big|\xi(t) - \Theta(t)\big| \leq K_{\mathcal{F}_\Lambda}\varphi\mathcal{F}_\Lambda(t), \quad t \in \mathcal{J}, \tag{5.6}$$

where $\varphi = \max(\varphi_j)^\mathbb{T}$, $K_{\mathcal{F}_\Lambda} = \max(K_{\mathcal{F}_{\Lambda_j}})^\mathbb{T}$, and $\mathcal{F}_\Lambda = \max(\mathcal{F}_{\Lambda_j})^\mathbb{T}$ for $j = 1, 2, 3, 4, 5$.

Definition 5.4 (Generalized $\mathbb{UHR}$ Stability) The $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) is called generalized $\mathbb{UHR}$ stable with respect to $\mathcal{F}_\Lambda \in \mathcal{C}(\mathcal{J}, \mathbb{R}^+)$ if there exists a real number $K_{\mathcal{F}_\Lambda} > 0$ such that, for each solution $\xi \in \mathcal{W}$ of (5.3), there exists a solution $\Theta \in \mathcal{W}$ of the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) with

$$\left|\xi(t) - \Theta(t)\right| \leq K_{\mathcal{F}_\Lambda} \mathcal{F}_\Lambda(t), \quad t \in \mathcal{J}, \tag{5.7}$$

where $K_{\mathcal{F}_\Lambda} = \max(K_{\mathcal{F}_{\Lambda_j}})^{\mathbb{T}}$ and $\mathcal{F}_\Lambda = \max(\mathcal{F}_{\Lambda_j})^{\mathbb{T}}$ for $j = 1, 2, 3, 4, 5$.

Remark 5.5 It is easy to see that (1) Def. 5.1 $\Rightarrow$ Def. 5.2; (2) Def. 5.3 $\Rightarrow$ Def. 5.4; (3) Def. 5.3 for $\mathcal{F}_\Lambda(\cdot) = 1 \Rightarrow$ Def. 5.1.

Remark 5.6 A function $\xi \in \mathcal{W}$ is a solution of (5.1) if and only if there exists a function $w \in \mathcal{W}$ (which depends on ξ) such that the following properties: (i) $|w(t)| \leq \varphi$, $w = \max(w_j)^{\mathbb{T}}$, $\forall t \in \mathcal{J}$. (ii) ${}^{\mathbb{ABC}}_{t}\mathfrak{D}^{\phi}_{0}\xi(t) = \Lambda(t, \xi(t)) + w(t)$, $\forall t \in \mathcal{J}$.

Remark 5.7 A function $\xi \in \mathcal{W}$ is a solution of (5.2) if and only if there exists a function $\nu \in \mathcal{W}$ (which depends on ξ) such that we have the following properties: (i) $|\nu(t)| \leq \varphi \mathcal{F}_\Lambda(t)$, $\nu = \max(\nu_j)^{\mathbb{T}}$, $\forall t \in \mathcal{J}$. (ii) ${}^{\mathbb{ABC}}_{t}\mathfrak{D}^{\phi}_{0}\xi(t) = \Lambda(t, \xi(t)) + \nu(t)$, $\forall t \in \mathcal{J}$.

5.1 The $\mathbb{UH}$ stability and generalized $\mathbb{UH}$ stability results

Lemma 5.8 *Let $\phi \in (0, 1]$. If $\xi \in \mathcal{W}$ is a solution of* (5.1), *then ξ is a solution of the following inequality*:

$$\begin{aligned} &\left|\xi(t) - \mathcal{R}_\xi(t) - \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)} \int_0^t (t-s)^{\phi-1} \Lambda\big(s, \xi(s)\big)\, ds\right| \\ &\quad \leq \left(\frac{1-\phi}{\mathbb{AB}(\phi)} + \frac{T^{\phi}_{\max}}{\mathbb{AB}(\phi)\Gamma(\phi)}\right)\varphi, \end{aligned} \tag{5.8}$$

where $\mathcal{R}_\xi(t) = \xi_0 + \frac{1-\phi}{\mathbb{AB}(\phi)} \Lambda(t, \xi(t))$.

Proof Let ξ be a solution of (5.1). In view of Remark 5.6 (2), we have

$$\begin{cases} {}^{\mathbb{ABC}}_{t}\mathfrak{D}^{\phi}_{0}\xi(t) = \Lambda(t, \xi(t)) + w(t), & t \in \mathcal{J}, \\ \xi(0) = \xi_0 \geq 0. \end{cases} \tag{5.9}$$

Then the approximate solution of (5.9) can be written

$$\begin{aligned} \xi(t) = {}& \xi_0 + \frac{1-\phi}{\mathbb{AB}(\phi)} \Lambda\big(t, \xi(t)\big) + \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)} \int_0^t (t-s)^{\phi-1} \Lambda\big(s, \xi(s)\big)\, ds \\ &+ \frac{1-\phi}{\mathbb{AB}(\phi)} w(t) + \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)} \int_0^t (t-s)^{\phi-1} w(s)\, ds. \end{aligned}$$

By using Remark 5.6(i),

$$\begin{aligned}
&\left|\xi(t)-\mathcal{R}_{\xi}(t)-\frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\Lambda\bigl(s,\xi(s)\bigr)\,ds\right| \\
&\quad\le \frac{1-\phi}{\mathbb{AB}(\phi)}\bigl|w(t)\bigr| + \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\bigl|w(s)\bigr|\,ds \\
&\quad\le \left(\frac{1-\phi}{\mathbb{AB}(\phi)}+\frac{T_{\max}^{\phi}}{\mathbb{AB}(\phi)\Gamma(\phi)}\right)\varphi.
\end{aligned}$$

Therefore, the inequality (5.8) is obtained. □

Theorem 5.9 *Assume that $\Lambda:\mathcal{J}\times\mathbb{R}\to\mathbb{R}$ is continuous for every $\Theta\in\mathcal{W}$. If (H_1) and (4.3) are fulfilled, then the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) is $\mathbb{UH}$ stable on $\mathcal{J}$.*

Proof Suppose that $\varphi>0$ and let $\xi\in\mathcal{W}$ be any solution of (5.1). Let $\Theta\in\mathcal{W}$ be the unique solution of the model (4.1),

$$\begin{cases} {}^{\mathbb{ABC}}_{t}\mathfrak{D}_0^{\phi}\Theta(t)=\Lambda(t,\Theta(t)), \quad t\in\mathcal{J}, \\ \Theta(0)=\Theta_0, \end{cases}$$

where

$$\Theta(t)=\Theta_0+\frac{1-\phi}{\mathbb{AB}(\phi)}\Lambda\bigl(t,\Theta(t)\bigr)+\frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\Lambda\bigl(s,\Theta(s)\bigr)\,ds.$$

By using Lemma 5.8 with (H_1), we have

$$\begin{aligned}
\bigl|\xi(t)-\Theta(t)\bigr| &\le \left|\xi(t)-\mathcal{R}_{\Theta}(t)-\frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\Lambda\bigl(s,\Theta(s)\bigr)\,ds\right| \\
&\le \left|\xi(t)-\mathcal{R}_{\xi}(t)-\frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\Lambda\bigl(s,\xi(s)\bigr)\,ds\right| \\
&\quad + \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\bigl|\Lambda\bigl(s,\xi(s)\bigr)\,ds-\Lambda\bigl(s,\Theta(s)\bigr)\bigr|\,ds \\
&\le \left(\frac{1-\phi}{\mathbb{AB}(\phi)}+\frac{T_{\max}^{\phi}}{\mathbb{AB}(\phi)\Gamma(\phi)}\right)\varphi+\frac{\phi\mathbb{L}_{\Lambda}}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\bigl|\xi(s)-\Theta(s)\bigr|\,ds \\
&\le \left(\frac{1-\phi}{\mathbb{AB}(\phi)}+\frac{T_{\max}^{\phi}}{\mathbb{AB}(\phi)\Gamma(\phi)}\right)\varphi+\frac{T_{\max}^{\phi}\mathbb{L}_{\Lambda}}{\mathbb{AB}(\phi)\Gamma(\phi)}\bigl|\xi(t)-\Theta(t)\bigr|.
\end{aligned}$$

This implies that $|\xi(t)-\Theta(t)|\le C_{\Lambda}\varphi$, where

$$C_{\Lambda}=\frac{\frac{1-\phi}{\mathbb{AB}(\phi)}+\frac{T_{\max}^{\phi}}{\mathbb{AB}(\phi)\Gamma(\phi)}}{1-\frac{T_{\max}^{\phi}\mathbb{L}_{\Lambda}}{\mathbb{AB}(\phi)\Gamma(\phi)}}.$$

Hence, the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) is $\mathbb{UH}$ stable. □

Corollary 5.10 *In Theorem 5.9, if we set $\mathcal{F}_{\Lambda}(\varphi)=C_{\Lambda}\varphi$ such that $\mathcal{F}_{\Lambda}(0)=0$, then the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) is generalized $\mathbb{UH}$ stable.*

5.2 The $\mathbb{UHR}$ stability and generalized $\mathbb{UHR}$ stability results

Before proving, we give the following assumption:

(H_3) There exists an increasing function $\mathcal{F}_\Lambda \in \mathcal{W}$ and there exists $\lambda_{\mathcal{F}_\Lambda} > 0$, such that, for any $t \in \mathcal{J}$, we have the following integral inequality:

$$ {}_0^{\mathbb{AB}}\mathcal{I}_t^{\phi} \mathcal{F}_\Lambda(t) \leq \lambda_{\mathcal{F}_\Lambda} \mathcal{F}_\Lambda(t). \tag{5.10}$$

Lemma 5.11 *Let $\phi \in (0,1]$. If $\xi \in \mathcal{W}$ is a solution of (5.2), then ξ is a solution of the following inequality*:

$$ \left| \xi(t) - \mathcal{R}_\xi(t) - \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)} \int_0^t (t-s)^{\phi-1} \Lambda\bigl(s, \xi(s)\bigr)\, ds \right| \leq \varphi \lambda_{\mathcal{F}_\Lambda} \mathcal{F}_\Lambda(t), \tag{5.11}$$

where $\mathcal{R}_\xi(t) = \xi_0 + \frac{1-\phi}{\mathbb{AB}(\phi)} \Lambda(t, \xi(t))$.

Proof Let ξ be a solution of (5.2). In view of Remark 5.7(ii), we have

$$ \begin{cases} {}_t^{\mathbb{ABC}}\mathfrak{D}_0^{\phi} \xi(t) = \Lambda(t, \xi(t)) + \nu(t), \quad t \in \mathcal{J}, \\ \xi(0) = \xi_0 \geq 0. \end{cases} \tag{5.12}$$

Then the solution of (5.12) can be written

$$ \begin{aligned} \xi(t) = \xi_0 &+ \frac{1-\phi}{\mathbb{AB}(\phi)} \Lambda\bigl(t, \xi(t)\bigr) + \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)} \int_0^t (t-s)^{\phi-1} \Lambda\bigl(s, \xi(s)\bigr)\, ds \\ &+ \frac{1-\phi}{\mathbb{AB}(\phi)} \nu(t) + \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)} \int_0^t (t-s)^{\phi-1} \nu(s)\, ds. \end{aligned}$$

By using Remark 5.7(i), we have

$$ \begin{aligned} &\left| \xi(t) - \mathcal{R}_\xi(t) - \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)} \int_0^t (t-s)^{\phi-1} \Lambda\bigl(s, \xi(s)\bigr)\, ds \right| \\ &\quad \leq \frac{1-\phi}{\mathbb{AB}(\phi)} \bigl|\nu(t)\bigr| + \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)} \int_0^t (t-s)^{\phi-1} \bigl|\nu(s)\bigr|\, ds \\ &\quad \leq \varphi \lambda_{\mathcal{F}_\Lambda} \mathcal{F}_\Lambda(t). \end{aligned}$$

Hence, the inequality (5.8) is obtained. □

Theorem 5.12 *Assume that $\Lambda : \mathcal{J} \times \mathbb{R} \to \mathbb{R}$ is continuous for every $\Theta \in \mathcal{W}$. If (H_1), (H_3) and (4.3) are fulfilled, then the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) is $\mathbb{UHR}$ stable on $\mathcal{J}$.*

Proof Let $\varphi > 0$ and $\xi \in \mathcal{W}$ be the solution of (5.3). Let $\Theta \in \mathcal{W}$ be the unique solution of the model (4.1). By using Lemma 5.11, (H_1), and (H_3), we have

$$ \begin{aligned} \bigl|\xi(t) - \Theta(t)\bigr| &\leq \left| \xi(t) - \mathcal{R}_\Theta(t) - \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)} \int_0^t (t-s)^{\phi-1} \Lambda\bigl(s, \Theta(s)\bigr)\, ds \right| \\ &\leq \left| \xi(t) - \mathcal{R}_\xi(t) - \frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)} \int_0^t (t-s)^{\phi-1} \Lambda\bigl(s, \xi(s)\bigr)\, ds \right| \end{aligned}$$

$$
\begin{aligned}
&+\frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\left|\Lambda\big(s,\xi(s)\big)\,ds-\Lambda\big(s,\Theta(s)\big)\right|ds \\
&\le \varphi\lambda_{\mathcal{F}_\Lambda}\mathcal{F}_\Lambda(t)+\frac{\phi\mathbb{L}_\Lambda}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\left|\xi(s)-\Theta(s)\right|ds \\
&\le \varphi\lambda_{\mathcal{F}_\Lambda}\mathcal{F}_\Lambda(t)+\frac{T_{\max}^{\phi}\mathbb{L}_\Lambda}{\mathbb{AB}(\phi)\Gamma(\phi)}\left|\xi(t)-\Theta(t)\right|.
\end{aligned}
$$

This yields the inequality $|\xi(t)-\Theta(t)|\le K_{\mathcal{F}_\Lambda}\varphi\mathcal{F}_\Lambda(t)$, where

$$
K_{\mathcal{F}_\Lambda}=\frac{\lambda_{\mathcal{F}_\Lambda}}{1-\frac{T_{\max}^{\phi}\mathbb{L}_\Lambda}{\mathbb{AB}(\phi)\Gamma(\phi)}}.
$$

Therefore, the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) is $\mathbb{UHR}$ stable. □

Corollary 5.13 *In Theorem 5.12, if we set $\varphi=1$ into $|\xi(t)-\Theta(t)|\le K_{\mathcal{F}_\Lambda}\varphi\mathcal{F}_\Lambda$, then the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) is generalized $\mathbb{UHR}$ stable.*

6 Numberical results

In this section, we introduce a numerical solution scheme for the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) and apply it to obtain a numerical simulation.

6.1 Numerical method

The $\mathbb{SMA}$ model under consideration via $\mathbb{ABC}$-fractional derivative is numerically simulated by using the novel numerical method as proposed in [42]. For this purpose, we look again at the $\mathbb{SMA}$ model in the form of (4.1) and (4.2). Employing the $\mathbb{AB}$-fractional integral operator on both sides of (4.1), we get

$$
\begin{aligned}
\mathcal{S}(t)&=\mathcal{S}_0+\frac{1-\phi}{\mathbb{AB}(\phi)}\mathbb{G}_1(t,\mathcal{S},\mathcal{E},\mathcal{A},\mathcal{R},\mathcal{Q}) \\
&\quad+\frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\mathbb{G}_1(s,\mathcal{S},\mathcal{E},\mathcal{A},\mathcal{R},\mathcal{Q})\,ds, \\
\mathcal{E}(t)&=\mathcal{E}_0+\frac{1-\phi}{\mathbb{AB}(\phi)}\mathbb{G}_2(t,\mathcal{S},\mathcal{E},\mathcal{A},\mathcal{R},\mathcal{Q}) \\
&\quad+\frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\mathbb{G}_2(s,\mathcal{S},\mathcal{E},\mathcal{A},\mathcal{R},\mathcal{Q})\,ds, \\
\mathcal{A}(t)&=\mathcal{A}_0+\frac{1-\phi}{\mathbb{AB}(\phi)}\mathbb{G}_3(t,\mathcal{S},\mathcal{E},\mathcal{A},\mathcal{R},\mathcal{Q}) \\
&\quad+\frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\mathbb{G}_3(s,\mathcal{S},\mathcal{E},\mathcal{A},\mathcal{R},\mathcal{Q})\,ds, \\
\mathcal{R}(t)&=\mathcal{R}_0+\frac{1-\phi}{\mathbb{AB}(\phi)}\mathbb{G}_4(t,\mathcal{S},\mathcal{E},\mathcal{A},\mathcal{R},\mathcal{Q}) \\
&\quad+\frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\mathbb{G}_4(s,\mathcal{S},\mathcal{E},\mathcal{A},\mathcal{R},\mathcal{Q})\,ds, \\
\mathcal{Q}(t)&=\mathcal{Q}_0+\frac{1-\phi}{\mathbb{AB}(\phi)}\mathbb{G}_5(t,\mathcal{S},\mathcal{E},\mathcal{A},\mathcal{R},\mathcal{Q}) \\
&\quad+\frac{\phi}{\mathbb{AB}(\phi)\Gamma(\phi)}\int_0^t (t-s)^{\phi-1}\mathbb{G}_5(s,\mathcal{S},\mathcal{E},\mathcal{A},\mathcal{R},\mathcal{Q})\,ds.
\end{aligned}
$$

Adapting the Adams type predictor–corrector tool represented by [42] to obtain the numerical approximation of the right-hand side of the system. The first step of the algorithm, under the assumption that the solution is in the closed interval $[0, T]$, this interval addressed by setting $h = T/N$, $t_k = hk$ $(k = 0, 1, 2, \ldots, N)$. Consequently, the corrector schemes of variable order integral form of $\mathbb{ABC}$-fractional derivative are given as follows:

$$\begin{aligned}
\mathcal{S}_{k+1} &= \mathcal{S}_0 + \frac{(1-\phi)h^{\phi}}{\mathbb{AB}(\phi)\Gamma(\phi+2)}\mathbb{G}_1\big(t_{k+1}, \mathcal{S}^p_{k+1}, \mathcal{E}^p_{k+1}, \mathcal{A}^p_{k+1}, \mathcal{R}^p_{k+1}, \mathcal{Q}^p_{k+1}\big) \\
&\quad + \frac{\phi h^{\phi}}{\mathbb{AB}(\phi)\Gamma(\phi+2)}\sum_{j=0}^{k}\Xi_{j,k+1}\mathbb{G}_1(t_j, \mathcal{S}_j, \mathcal{E}_j, \mathcal{A}_j, \mathcal{R}_j, \mathcal{Q}_j), \\
\mathcal{E}_{k+1} &= \mathcal{E}_0 + \frac{(1-\phi)h^{\phi}}{\mathbb{AB}(\phi)\Gamma(\phi+2)}\mathbb{G}_2\big(t_{k+1}, \mathcal{S}^p_{k+1}, \mathcal{E}^p_{k+1}, \mathcal{A}^p_{k+1}, \mathcal{R}^p_{k+1}, \mathcal{Q}^p_{k+1}\big) \\
&\quad + \frac{\phi h^{\phi}}{\mathbb{AB}(\phi)\Gamma(\phi+2)}\sum_{j=0}^{k}\Xi_{j,k+1}\mathbb{G}_2(t_j, \mathcal{S}_j, \mathcal{E}_j, \mathcal{A}_j, \mathcal{R}_j, \mathcal{Q}_j), \\
\mathcal{A}_{k+1} &= \mathcal{A}_0 + \frac{(1-\phi)h^{\phi}}{\mathbb{AB}(\phi)\Gamma(\phi+2)}\mathbb{G}_3\big(t_{k+1}, \mathcal{S}^p_{k+1}, \mathcal{E}^p_{k+1}, \mathcal{A}^p_{k+1}, \mathcal{R}^p_{k+1}, \mathcal{Q}^p_{k+1}\big) \\
&\quad + \frac{\phi h^{\phi}}{\mathbb{AB}(\phi)\Gamma(\phi+2)}\sum_{j=0}^{k}\Xi_{j,k+1}\mathbb{G}_3(t_j, \mathcal{S}_j, \mathcal{E}_j, \mathcal{A}_j, \mathcal{R}_j, \mathcal{Q}_j), \\
\mathcal{R}_{k+1} &= \mathcal{R}_0 + \frac{(1-\phi)h^{\phi}}{\mathbb{AB}(\phi)\Gamma(\phi+2)}\mathbb{G}_4\big(t_{k+1}, \mathcal{S}^p_{k+1}, \mathcal{E}^p_{k+1}, \mathcal{A}^p_{k+1}, \mathcal{R}^p_{k+1}, \mathcal{Q}^p_{k+1}\big) \\
&\quad + \frac{\phi h^{\phi}}{\mathbb{AB}(\phi)\Gamma(\phi+2)}\sum_{j=0}^{k}\Xi_{j,k+1}\mathbb{G}_4(t_j, \mathcal{S}_j, \mathcal{E}_j, \mathcal{A}_j, \mathcal{R}_j, \mathcal{Q}_j), \\
\mathcal{Q}_{k+1} &= \mathcal{Q}_0 + \frac{(1-\phi)h^{\phi}}{\mathbb{AB}(\phi)\Gamma(\phi+2)}\mathbb{G}_5\big(t_{k+1}, \mathcal{S}^p_{k+1}, \mathcal{E}^p_{k+1}, \mathcal{A}^p_{k+1}, \mathcal{R}^p_{k+1}, \mathcal{Q}^p_{k+1}\big) \\
&\quad + \frac{\phi h^{\phi}}{\mathbb{AB}(\phi)\Gamma(\phi+2)}\sum_{j=0}^{k}\Xi_{j,k+1}\mathbb{G}_5(t_j, \mathcal{S}_j, \mathcal{E}_j, \mathcal{A}_j, \mathcal{R}_j, \mathcal{Q}_j),
\end{aligned}$$

where

$$\Xi_{j,k+1} = \begin{cases} k^{\phi+1} - (k-\phi)(k+1)^{\phi}, & \text{if } j = 0, \\ (k-j+2)^{\phi+1} + (k-j)^{\phi+1} - 2(k-j+1)^{\phi+1}, & \text{if } 1 \le j \le k. \end{cases}$$

Further, the predictor terms $\mathcal{S}^p_{k+1}$, $\mathcal{E}^p_{k+1}$, $\mathcal{A}^p_{k+1}$, $\mathcal{R}^p_{k+1}$, $\mathcal{Q}^p_{k+1}$ are described as

$$\begin{aligned}
\mathcal{S}^p_{k+1} &= \mathcal{S}_0 + \frac{1-\phi}{\mathbb{AB}(\phi)}\mathbb{G}_1(t_k, \mathcal{S}_k, \mathcal{E}_k, \mathcal{A}_k, \mathcal{R}_k, \mathcal{Q}_k) \\
&\quad + \frac{\phi}{\mathbb{AB}(\phi)\Gamma^2(\phi)}\sum_{j=0}^{k}\omega_{j,k+1}\mathbb{G}_1(t_j, \mathcal{S}_j, \mathcal{E}_j, \mathcal{A}_j, \mathcal{R}_j, \mathcal{Q}_j), \\
\mathcal{E}^p_{k+1} &= \mathcal{E}_0 + \frac{1-\phi}{\mathbb{AB}(\phi)}\mathbb{G}_2(t_k, \mathcal{S}_k, \mathcal{E}_k, \mathcal{A}_k, \mathcal{R}_k, \mathcal{Q}_k) \\
&\quad + \frac{\phi}{\mathbb{AB}(\phi)\Gamma^2(\phi)}\sum_{j=0}^{k}\omega_{j,k+1}\mathbb{G}_2(t_j, \mathcal{S}_j, \mathcal{E}_j, \mathcal{A}_j, \mathcal{R}_j, \mathcal{Q}_j),
\end{aligned}$$

$$\mathcal{A}^p_{k+1} = \mathcal{A}_0 + \frac{1-\phi}{\mathbb{AB}(\phi)}\mathbb{G}_3(t_k,\mathcal{S}_k,\mathcal{E}_k,\mathcal{A}_k,\mathcal{R}_k,\mathcal{Q}_k) + \frac{\phi}{\mathbb{AB}(\phi)\Gamma^2(\phi)}\sum_{j=0}^{k}\omega_{j,k+1}\mathbb{G}_3(t_j,\mathcal{S}_j,\mathcal{E}_j,\mathcal{A}_j,\mathcal{R}_j,\mathcal{Q}_j),$$

$$\mathcal{R}^p_{k+1} = \mathcal{R}_0 + \frac{1-\phi}{\mathbb{AB}(\phi)}\mathbb{G}_4(t_k,\mathcal{S}_k,\mathcal{E}_k,\mathcal{A}_k,\mathcal{R}_k,\mathcal{Q}_k) + \frac{\phi}{\mathbb{AB}(\phi)\Gamma^2(\phi)}\sum_{j=0}^{k}\omega_{j,k+1}\mathbb{G}_4(t_j,\mathcal{S}_j,\mathcal{E}_j,\mathcal{A}_j,\mathcal{R}_j,\mathcal{Q}_j),$$

$$\mathcal{Q}^p_{k+1} = \mathcal{Q}_0 + \frac{1-\phi}{\mathbb{AB}(\phi)}\mathbb{G}_5(t_k,\mathcal{S}_k,\mathcal{E}_k,\mathcal{A}_k,\mathcal{R}_k,\mathcal{Q}_k) + \frac{\phi}{\mathbb{AB}(\phi)\Gamma^2(\phi)}\sum_{j=0}^{k}\omega_{j,k+1}\mathbb{G}_5(t_j,\mathcal{S}_j,\mathcal{E}_j,\mathcal{A}_j,\mathcal{R}_j,\mathcal{Q}_j),$$

where

$$\omega_{j,k+1} = \frac{h^\phi}{\phi}\big((k+1-j)^\phi - (k-j)^\phi\big), \quad 0 \le j \le k.$$

6.2 Numerical simulations

In this subsection, we demonstrate numerical simulations for the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) by using the Adam type predictor–corrector rule for the $\mathbb{ABC}$-fractional operator [42] as said in the earlier subsection. We use nonnegative parameters to obtain these numerical results as shown in Table 1.

If we set $\beta = 0.30$ and $\phi = 0.998$, then $R_0 = 0.3836 < 1$ is obtained and the transmission of the addiction to cease stops, which is the result of the numerical simulation of the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) as shown in Fig. 1 with $N = 2000$, and $(\mathcal{S}_0,\mathcal{E}_0,\mathcal{A}_0,\mathcal{R}_0,\mathcal{Q}_0) = (100,1,5,0,10)$. The disease-free equilibrium point $\mathfrak{E}_0 = (8.2906,0,0,0,1.6635)$ in this case. We notice that the number of exposed and addicted populations rapidly increases and decreases to zero over time, since when exposed and addicted populations recover, the number of recovering populations increases, and when the addict's transmission is stopped, the number of recovered populations decreases to zero. Moreover, the number of people who permanently do not use and quit using social media population rapidly increases and decreases to zero over time, because when susceptible and recovered pop-

Table 1 The description of parameters of the $\mathbb{SMA}$ model (1.2)

Parameter	Description of the parameter	Value	Source
π	Recruitment rate of susceptible individuals	0.5	Assumed
μ	Natural death rate	0.05	[48]
β	Transmission rate of addiction to the susceptible individuals	0.1–0.8	[48]
σ	Contact rate of susceptible individuals with addicted individuals	0.2	[48]
α	Proportion of exposed individuals that join addicted class	0.7	[48]
ρ	Induce death rate	0.01	Assumed
δ	Individuals that leave exposed class	0.25	[48]
ϵ	Addicted individuals that join recovered class due to the treatment	0.7	[49]
κ	Susceptible individuals that don't use and/or quit from using $\mathbb{SM}$	0.01	Assumed
γ	Proportion of recovered individuals susceptible to $\mathbb{SMA}$	0.35	[50]
η	Individuals that leave recoverd class	0.4	[49]

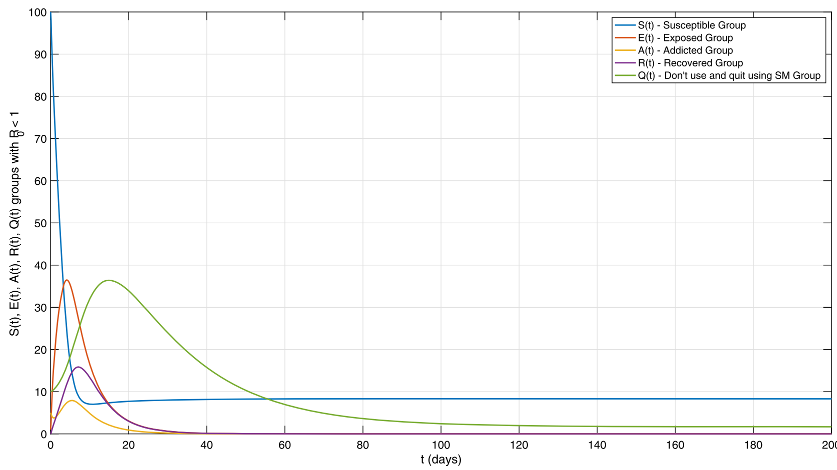

Figure 1 Plots of the result of the model (1.2) for $\phi = 0.998$ in the case $R_0 < 1$

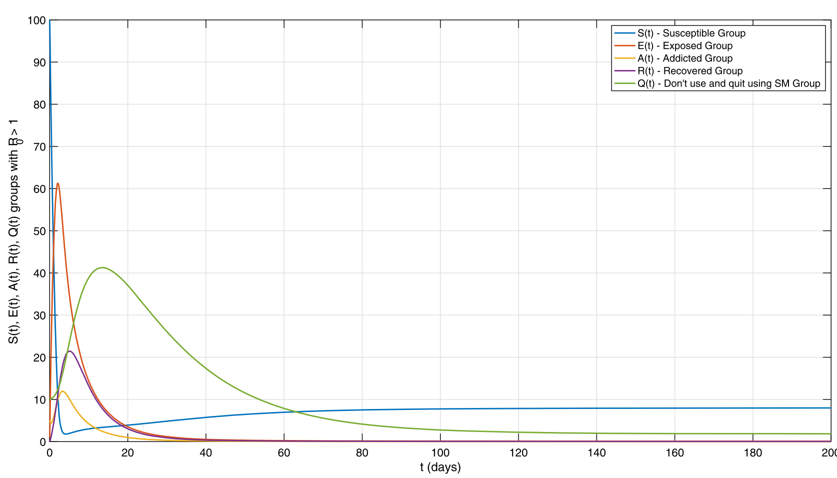

Figure 2 Plots of the result of the model (1.2) for $\phi = 0.998$ in the case $R_0 > 1$

ulations with the decrease in the number of addicts again increased the number of people who permanently do not use and quit using social media population is balanced stable at 1.6635. On the other hand, the susceptible population with the addiction decreased, which with the decrease in the number of people who are exposed and permanently do not use and quit using social media populations again increased the number of the good-quality population and is balanced stable at 8.2906.

Furthermore, if we set $\beta = 0.80$ and $\phi = 0.998$, then $R_0 = 1.0209 > 1$. The endemic equilibrium point is $\mathfrak{E}^* = (8.1212, 0.0450, 0.0104, 0.0236, 1.7517)$. The numerical results of the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2) in this case are shown in Fig. 2. This figure shows that when $R_0 > 1$ the number of the exposed and addicted populations primarily increase radically after passing the highest point of the addiction with the transmission of the addiction continues to stabilize at 0.0450 and 0.0104, respectively. Moreover, as the number of exposed and addicted populations decreases, the number of the recovered population increases, then decreases and stabilizes at 0.0236. With the recovery of exposed and addicted populations, the number of people who permanently do not use and quit using social media population also increases and eventually decreases tend to a stable point of 1.7517. On the other hand, as the number of the people who are exposed and permanently do not use and quit using social media populations increases, the number of the susceptible population decreases and then stabilizes with a little increase at 8.1212.

6.3 The effect of fractional derivative orders

In this subsection, we consider the effect of fractional derivative orders on the results of the $\mathbb{ABC}$-fractional $\mathbb{SMA}$ model (1.2). For this simulation, we apply the numerical scheme stated in Sect. 6.1 Numerical Method and the parameters as given in Table 1 with $\beta = 0.80$. The numerical simulations of the system (1.2) are shown in Fig. 3–Fig. 7 for different fractional orders $\phi = \{0.94, 0.96, 0.98, 0.998, 1.00\}$.

As we can see from Fig. 3 the susceptible population decreases with various fractional orders ϕ increasing and approaching 1 and then it becomes stable for all fractional orders at $\mathcal{S}^* = 8.1212$. Fig. 4–Fig. 5 show that the exposed and addicted populations rapidly increase and decrease to $\mathcal{E}^* = 0.0450$ and $\mathcal{A}^* = 0.0104$ with various fractional orders ϕ decreasing and approaching 1. Fig. 6–Fig. 7 show that the number of people who are recovered and permanently do not use and quit using $\mathbb{SM}$ population rapidly increases and decreases to $\mathcal{R}^* = 0.0236$ and $\mathcal{Q}^* = 1.7517$ with various fractional orders ϕ increases approaching 1. The main point of this manuscript is that tiny changes in the fractional derivative order do not affect the overall behavior of the resultant functions; only the numerical simulations are affected. In addition, the absolute errors of the numerical results of the population in five groups for all fractional orders comparing with $\phi = 1$ in the case of $\beta = 0.30$ are shown in Table 2–Table 6 and in the case of $\beta = 0.80$ are shown in Table 7–Table 11.

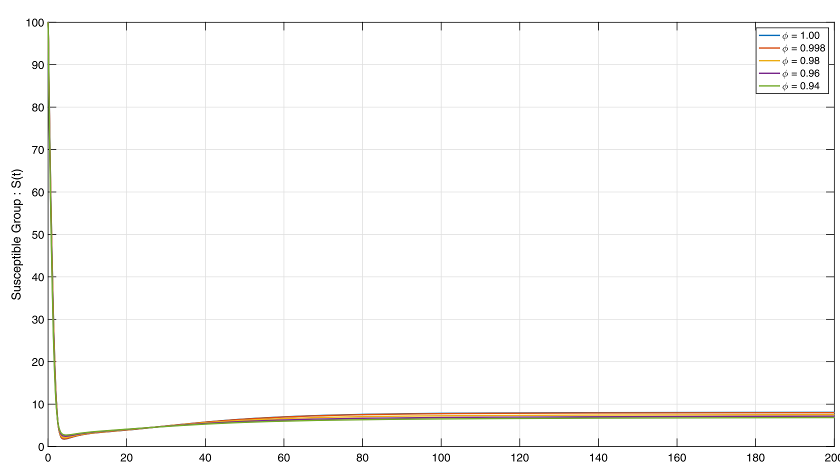

Figure 3 The quantity of $\mathcal{S}(t)$ via $\phi = 0.94, 0.96, 0.98, 0.998, 1.00$

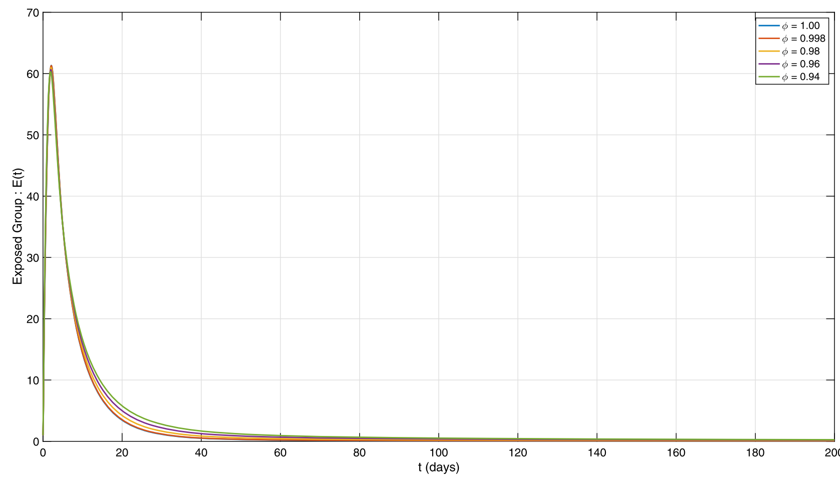

Figure 4 The quantity of $\mathcal{E}(t)$ via $\phi = 0.94, 0.96, 0.98, 0.998, 1.00$

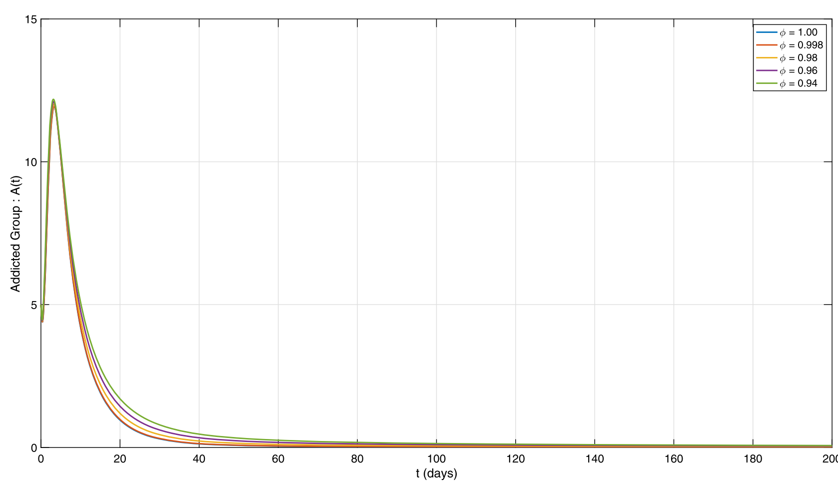

Figure 5 The quantity of $\mathcal{A}(t)$ via $\phi = 0.94, 0.96, 0.98, 0.998, 1.00$

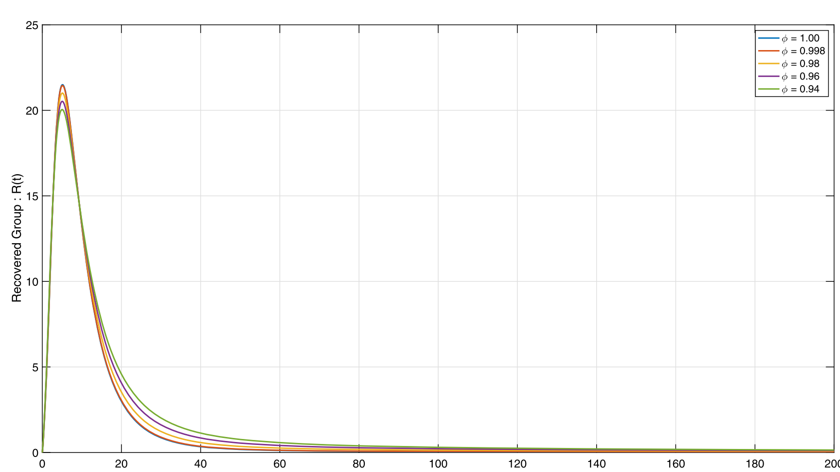

Figure 6 The quantity of $\mathcal{R}(t)$ via $\phi = 0.94, 0.96, 0.98, 0.998, 1.00$

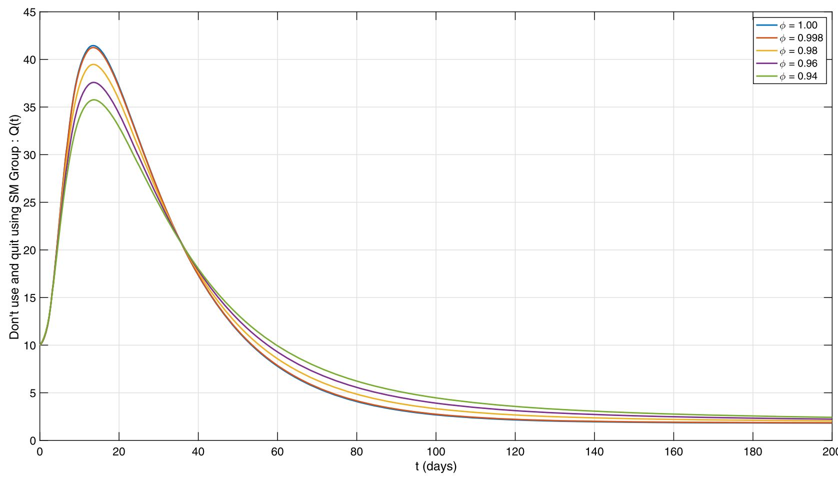

Figure 7 The quantity of $\mathcal{Q}(t)$ with $\phi = 0.94, 0.96, 0.98, 0.998, 1.00$

Table 2 The values of $|\mathcal{S}_1 - \mathcal{S}_\phi|$ for $\phi = \{0.94, 0.96, 0.98, 0.998\}$

$n \times 100$	t	$\lvert\mathcal{S}_1 - \mathcal{S}_{0.94}\rvert$	$\lvert\mathcal{S}_1 - \mathcal{S}_{0.96}\rvert$	$\lvert\mathcal{S}_1 - \mathcal{S}_{0.98}\rvert$	$\lvert\mathcal{S}_1 - \mathcal{S}_{0.998}\rvert$
0	0	0.0000	0.0000	0.0000	0.0000
1	10	0.7861	0.5441	0.2809	0.0287
2	20	0.7332	0.5485	0.3071	0.0339
3	30	0.8142	0.6232	0.3560	0.0399
4	40	0.8049	0.6112	0.3452	0.0383
5	50	0.7067	0.5238	0.2879	0.0311
6	60	0.5638	0.4032	0.2132	0.0222
7	70	0.4087	0.2778	0.1387	0.0136
8	80	0.2593	0.1614	0.0719	0.0061
9	90	0.1236	0.0591	0.0150	0.0000
10	100	0.0041	0.0287	0.0326	0.0051
11	110	0.0999	0.1033	0.0722	0.0092
12	120	0.1898	0.1666	0.1051	0.0126
13	130	0.2675	0.2205	0.1327	0.0154
14	140	0.3350	0.2666	0.1560	0.0177
15	150	0.3939	0.3063	0.1759	0.0197
16	160	0.4454	0.3408	0.1930	0.0214
17	170	0.4909	0.3709	0.2079	0.0228
18	180	0.5313	0.3975	0.2209	0.0241
19	190	0.5673	0.4211	0.2323	0.0252
20	200	0.5996	0.4421	0.2425	0.0262

7 Conclusion

In this manuscript, we considered a fractional-order $\mathbb{SMA}$ model in the $\mathbb{ABC}$-derivative sense. The equilibrium points and the system's basic reproduction number (1.2) have been determined, and the necessary circumstances for the system's stability at the equilibrium points have been examined. The existence results of the solutions for the proposed model (1.2) were investigated by applying Banach's and Krasnoselskii's fixed point theorems. The stability of the solutions was established by employing the various versions of Ulam's stability, such as $\mathbb{UH}$ stability, generalized $\mathbb{UH}$ stability, $\mathbb{UHR}$ stability, and generalized $\mathbb{UHR}$ stability. The novel numerical method, especially the Adams-type predictor–corrector technique, illustrates the approximate solutions for the different fractional order ϕ. A numerical simulation for transmission of addiction in the cases $R_0 < 1$ and $R_0 > 1$ is demon-

Table 3 The values of $|\mathcal{E}_1 - \mathcal{E}_\phi|$ for $\phi = \{0.94, 0.96, 0.98, 0.998\}$

$n \times 100$	t	$\lvert\mathcal{E}_1 - \mathcal{E}_{0.94}\rvert$	$\lvert\mathcal{E}_1 - \mathcal{E}_{0.96}\rvert$	$\lvert\mathcal{E}_1 - \mathcal{E}_{0.98}\rvert$	$\lvert\mathcal{E}_1 - \mathcal{E}_{0.998}\rvert$
0	0	0.0000	0.0000	0.0000	0.0000
1	10	0.7769	0.4692	0.2087	0.0184
2	20	1.8557	1.2096	0.5902	0.0576
3	30	1.1562	0.7214	0.3354	0.0312
4	40	0.6322	0.3730	0.1632	0.0143
5	50	0.3540	0.1990	0.0828	0.0070
6	60	0.2137	0.1166	0.0473	0.0039
7	70	0.1406	0.0759	0.0307	0.0025
8	80	0.0999	0.0541	0.0221	0.0019
9	90	0.0755	0.0411	0.0170	0.0014
10	100	0.0597	0.0328	0.0137	0.0012
11	110	0.0489	0.0271	0.0114	0.0010
12	120	0.0411	0.0230	0.0097	0.0008
13	130	0.0353	0.0198	0.0085	0.0007
14	140	0.0309	0.0174	0.0075	0.0007
15	150	0.0273	0.0155	0.0067	0.0006
16	160	0.0245	0.0139	0.0060	0.0005
17	170	0.0221	0.0126	0.0055	0.0005
18	180	0.0201	0.0115	0.0050	0.0004
19	190	0.0185	0.0106	0.0046	0.0004
20	200	0.0171	0.0098	0.0043	0.0004

Table 4 The values of $|\mathcal{A}_1 - \mathcal{A}_\phi|$ for $\phi = \{0.94, 0.96, 0.98, 0.998\}$

$n \times 100$	t	$\lvert\mathcal{A}_1 - \mathcal{A}_{0.94}\rvert$	$\lvert\mathcal{A}_1 - \mathcal{A}_{0.96}\rvert$	$\lvert\mathcal{A}_1 - \mathcal{A}_{0.98}\rvert$	$\lvert\mathcal{A}_1 - \mathcal{A}_{0.998}\rvert$
0	0	0.0000	0.0000	0.0000	0.0000
1	10	0.4406	0.2808	0.1338	0.0128
2	20	0.6111	0.3939	0.1900	0.0184
3	30	0.3656	0.2257	0.1039	0.0096
4	40	0.1999	0.1171	0.0509	0.0044
5	50	0.1139	0.0638	0.0265	0.0022
6	60	0.0706	0.0386	0.0157	0.0013
7	70	0.0479	0.0260	0.0106	0.0009
8	80	0.0351	0.0191	0.0079	0.0007
9	90	0.0273	0.0150	0.0062	0.0005
10	100	0.0222	0.0123	0.0051	0.0004
11	110	0.0186	0.0104	0.0044	0.0004
12	120	0.0160	0.0090	0.0038	0.0003
13	130	0.0140	0.0079	0.0034	0.0003
14	140	0.0124	0.0070	0.0030	0.0003
15	150	0.0112	0.0063	0.0027	0.0002
16	160	0.0101	0.0058	0.0025	0.0002
17	170	0.0093	0.0053	0.0023	0.0002
18	180	0.0086	0.0049	0.0021	0.0002
19	190	0.0079	0.0045	0.0020	0.0002
20	200	0.0074	0.0042	0.0018	0.0002

strated and the results display that in two cases the system is stable at its equilibrium points. We analyzed the dynamic behavior of the $\mathbb{SMA}$ system with ϕ approaching 1. Finally, the system responses were predicted for various fractional derivative orders, demonstrating that a few changes in the fractional derivative order did not affect the overall behavior of the function, just the numerical simulations that occur.

This study would be a new way to explore the mathematical model of $\mathbb{SMA}$ with $\mathbb{ABC}$-fractional derivative. For extension of this work, the researcher may develop and apply this $\mathbb{SMA}$ model with the other types of fractional-order derivative operators.

Table 5 The values of $|\mathcal{R}_1 - \mathcal{R}_\phi|$ for $\phi = \{0.94, 0.96, 0.98, 0.998\}$

$n \times 100$	t	$\lvert\mathcal{R}_1 - \mathcal{R}_{0.94}\rvert$	$\lvert\mathcal{R}_1 - \mathcal{R}_{0.96}\rvert$	$\lvert\mathcal{R}_1 - \mathcal{R}_{0.98}\rvert$	$\lvert\mathcal{R}_1 - \mathcal{R}_{0.998}\rvert$
0	0	0.0000	0.0000	0.0000	0.0000
1	10	0.3700	0.2530	0.1293	0.0131
2	20	1.2369	0.8260	0.4129	0.0412
3	30	0.9083	0.5816	0.2778	0.0265
4	40	0.5125	0.3109	0.1399	0.0126
5	50	0.2899	0.1675	0.0717	0.0062
6	60	0.1759	0.0986	0.0411	0.0035
7	70	0.1164	0.0644	0.0267	0.0023
8	80	0.0832	0.0460	0.0192	0.0016
9	90	0.0632	0.0352	0.0148	0.0013
10	100	0.0504	0.0282	0.0120	0.0010
11	110	0.0415	0.0235	0.0100	0.0009
12	120	0.0352	0.0200	0.0086	0.0008
13	130	0.0304	0.0174	0.0075	0.0007
14	140	0.0267	0.0153	0.0066	0.0006
15	150	0.0238	0.0137	0.0059	0.0005
16	160	0.0214	0.0123	0.0054	0.0005
17	170	0.0194	0.0112	0.0049	0.0004
18	180	0.0178	0.0103	0.0045	0.0004
19	190	0.0164	0.0095	0.0042	0.0004
20	200	0.0152	0.0088	0.0039	0.0003

Table 6 The values of $|\mathcal{Q}_1 - \mathcal{Q}_\phi|$ for $\phi = \{0.94, 0.96, 0.98, 0.998\}$

$n \times 100$	t	$\lvert\mathcal{Q}_1 - \mathcal{Q}_{0.94}\rvert$	$\lvert\mathcal{Q}_1 - \mathcal{Q}_{0.96}\rvert$	$\lvert\mathcal{Q}_1 - \mathcal{Q}_{0.98}\rvert$	$\lvert\mathcal{Q}_1 - \mathcal{Q}_{0.998}\rvert$
0	0	0.0000	0.0000	0.0000	0.0000
1	10	2.9460	1.9437	0.9588	0.0944
2	20	3.8530	2.6189	1.3357	0.1360
3	30	1.3208	0.8887	0.4509	0.0459
4	40	0.4132	0.2819	0.1407	0.0137
5	50	1.2436	0.8329	0.4154	0.0412
6	60	1.5414	1.0256	0.5100	0.0506
7	70	1.5670	1.0373	0.5142	0.0510
8	80	1.4662	0.9655	0.4768	0.0472
9	90	1.3159	0.8615	0.4231	0.0417
10	100	1.1551	0.7511	0.3665	0.0359
11	110	1.0025	0.6471	0.3134	0.0305
12	120	0.8661	0.5547	0.2663	0.0257
13	130	0.7480	0.4751	0.2261	0.0216
14	140	0.6476	0.4079	0.1924	0.0182
15	150	0.5630	0.3517	0.1644	0.0154
16	160	0.4921	0.3049	0.1412	0.0131
17	170	0.4326	0.2659	0.1221	0.0113
18	180	0.3825	0.2334	0.1063	0.0097
19	190	0.3402	0.2060	0.0931	0.0085
20	200	0.3042	0.1830	0.0820	0.0074

Table 7 The values of $|\mathcal{S}_1 - \mathcal{S}_\phi|$ for $\phi = \{0.94, 0.96, 0.98, 0.998\}$

$n \times 100$	t	$\lvert\mathcal{S}_1 - \mathcal{S}_{0.94}\rvert$	$\lvert\mathcal{S}_1 - \mathcal{S}_{0.96}\rvert$	$\lvert\mathcal{S}_1 - \mathcal{S}_{0.98}\rvert$	$\lvert\mathcal{S}_1 - \mathcal{S}_{0.998}\rvert$
0	0	0.0000	0.0000	0.0000	0.0000
1	10	0.3407	0.2299	0.1164	0.0118
2	20	0.1893	0.1422	0.0813	0.0093
3	30	0.1010	0.0570	0.0198	0.0008
4	40	0.4687	0.3241	0.1652	0.0164
5	50	0.7818	0.5588	0.2982	0.0313
6	60	1.0055	0.7317	0.4008	0.0435
7	70	1.1503	0.8474	0.4738	0.0529
8	80	1.2366	0.9190	0.5227	0.0601
9	90	1.2832	0.9595	0.5535	0.0654
10	100	1.3043	0.9790	0.5712	0.0693
11	110	1.3099	0.9851	0.5795	0.0721
12	120	1.3064	0.9828	0.5814	0.0740
13	130	1.2981	0.9759	0.5792	0.0753
14	140	1.2874	0.9664	0.5743	0.0759
15	150	1.2758	0.9558	0.5679	0.0761
16	160	1.2643	0.9449	0.5607	0.0760
17	170	1.2532	0.9342	0.5532	0.0757
18	180	1.2427	0.9241	0.5457	0.0752
19	190	1.2330	0.9145	0.5384	0.0745
20	200	1.2240	0.9056	0.5313	0.0738

Table 8 The values of $|\mathcal{E}_1 - \mathcal{E}_\phi|$ for $\phi = \{0.94, 0.96, 0.98, 0.998\}$

$n \times 100$	t	$\lvert\mathcal{E}_1 - \mathcal{E}_{0.94}\rvert$	$\lvert\mathcal{E}_1 - \mathcal{E}_{0.96}\rvert$	$\lvert\mathcal{E}_1 - \mathcal{E}_{0.98}\rvert$	$\lvert\mathcal{E}_1 - \mathcal{E}_{0.998}\rvert$
0	0	0.0000	0.0000	0.0000	0.0000
1	10	2.3330	1.5683	0.7911	0.0798
2	20	2.3304	1.5394	0.7615	0.0753
3	30	1.6432	1.0617	0.5124	0.0494
4	40	1.1764	0.7514	0.3586	0.0342
5	50	0.9012	0.5763	0.2766	0.0266
6	60	0.7311	0.4719	0.2306	0.0228
7	70	0.6169	0.4034	0.2019	0.0207
8	80	0.5343	0.3540	0.1818	0.0194
9	90	0.4711	0.3159	0.1663	0.0186
10	100	0.4206	0.2850	0.1533	0.0179
11	110	0.3792	0.2591	0.1421	0.0174
12	120	0.3446	0.2370	0.1320	0.0168
13	130	0.3153	0.2180	0.1230	0.0163
14	140	0.2900	0.2014	0.1149	0.0157
15	150	0.2681	0.1869	0.1076	0.0152
16	160	0.2490	0.1740	0.1009	0.0147
17	170	0.2321	0.1626	0.0949	0.0142
18	180	0.2170	0.1524	0.0894	0.0136
19	190	0.2036	0.1433	0.0844	0.0132
20	200	0.1915	0.1350	0.0798	0.0127

Table 9 The values of $|\mathcal{A}_1 - \mathcal{A}_\phi|$ for $\phi = \{0.94, 0.96, 0.98, 0.998\}$

$n \times 100$	t	$\lvert\mathcal{A}_1 - \mathcal{A}_{0.94}\rvert$	$\lvert\mathcal{A}_1 - \mathcal{A}_{0.96}\rvert$	$\lvert\mathcal{A}_1 - \mathcal{A}_{0.98}\rvert$	$\lvert\mathcal{A}_1 - \mathcal{A}_{0.998}\rvert$
0	0	0.0000	0.0000	0.0000	0.0000
1	10	0.8805	0.5864	0.2929	0.0293
2	20	0.7437	0.4835	0.2354	0.0229
3	30	0.4924	0.3122	0.1479	0.0140
4	40	0.3401	0.2125	0.0991	0.0093
5	50	0.2546	0.1587	0.0742	0.0070
6	60	0.2034	0.1277	0.0606	0.0058
7	70	0.1700	0.1080	0.0523	0.0052
8	80	0.1463	0.0941	0.0467	0.0048
9	90	0.1285	0.0836	0.0425	0.0046
10	100	0.1144	0.0751	0.0390	0.0044
11	110	0.1030	0.0682	0.0360	0.0042
12	120	0.0935	0.0623	0.0334	0.0041
13	130	0.0855	0.0572	0.0311	0.0039
14	140	0.0786	0.0529	0.0290	0.0038
15	150	0.0727	0.0490	0.0272	0.0037
16	160	0.0675	0.0457	0.0255	0.0035
17	170	0.0629	0.0427	0.0240	0.0034
18	180	0.0589	0.0400	0.0226	0.0033
19	190	0.0553	0.0376	0.0213	0.0032
20	200	0.0521	0.0355	0.0202	0.0031

Table 10 The values of $|\mathcal{R}_1 - \mathcal{R}_\phi|$ for $\phi = \{0.94, 0.96, 0.98, 0.998\}$

$n \times 100$	t	$\lvert\mathcal{R}_1 - \mathcal{R}_{0.94}\rvert$	$\lvert\mathcal{R}_1 - \mathcal{R}_{0.96}\rvert$	$\lvert\mathcal{R}_1 - \mathcal{R}_{0.98}\rvert$	$\lvert\mathcal{R}_1 - \mathcal{R}_{0.998}\rvert$
0	0	0.0000	0.0000	0.0000	0.0000
1	10	0.2888	0.2104	0.1142	0.0122
2	20	1.5940	1.0751	0.5428	0.0547
3	30	1.1569	0.7549	0.3678	0.0357
4	40	0.7942	0.5076	0.2421	0.0230
5	50	0.5826	0.3703	0.1761	0.0168
6	60	0.4574	0.2922	0.1407	0.0137
7	70	0.3772	0.2435	0.1197	0.0120
8	80	0.3215	0.2102	0.1059	0.0110
9	90	0.2802	0.1854	0.0957	0.0104
10	100	0.2481	0.1659	0.0875	0.0100
11	110	0.2223	0.1499	0.0806	0.0096
12	120	0.2010	0.1365	0.0746	0.0092
13	130	0.1831	0.1250	0.0693	0.0089
14	140	0.1679	0.1152	0.0646	0.0086
15	150	0.1548	0.1066	0.0603	0.0083
16	160	0.1434	0.0990	0.0565	0.0080
17	170	0.1334	0.0924	0.0530	0.0077
18	180	0.1245	0.0864	0.0499	0.0074
19	190	0.1166	0.0811	0.0470	0.0072
20	200	0.1096	0.0764	0.0444	0.0069

Table 11 The values of $|\mathcal{Q}_1 - \mathcal{Q}_\phi|$ for $\phi = \{0.94, 0.96, 0.98, 0.998\}$

$n \times 100$	t	$\lvert\mathcal{Q}_1 - \mathcal{Q}_{0.94}\rvert$	$\lvert\mathcal{Q}_1 - \mathcal{Q}_{0.96}\rvert$	$\lvert\mathcal{Q}_1 - \mathcal{Q}_{0.98}\rvert$	$\lvert\mathcal{Q}_1 - \mathcal{Q}_{0.998}\rvert$
0	0	0.0000	0.0000	0.0000	0.0000
1	10	5.1605	3.5009	1.7808	0.1808
2	20	4.2916	2.8933	1.4625	0.1477
3	30	1.2904	0.8236	0.3936	0.0378
4	40	0.6949	0.5294	0.2967	0.0323
5	50	1.7232	1.2216	0.6462	0.0676
6	60	2.1497	1.5028	0.7865	0.0817
7	70	2.2375	1.5545	0.8111	0.0844
8	80	2.1499	1.4882	0.7768	0.0813
9	90	1.9822	1.3688	0.7163	0.0758
10	100	1.7875	1.2322	0.6476	0.0696
11	110	1.5938	1.0972	0.5798	0.0636
12	120	1.4144	0.9727	0.5172	0.0581
13	130	1.2545	0.8621	0.4615	0.0532
14	140	1.1150	0.7658	0.4127	0.0490
15	150	0.9946	0.6828	0.3706	0.0453
16	160	0.8912	0.6118	0.3342	0.0421
17	170	0.8025	0.5509	0.3029	0.0393
18	180	0.7262	0.4986	0.2759	0.0368
19	190	0.6603	0.4535	0.2524	0.0346
20	200	0.6030	0.4144	0.2319	0.0326

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