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New generating functions of I-function satisfying Truesdell's F_q-equation

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Abstract

In this paper we first obtain various new forms of the q-analogue of the I-function satisfying Truesdell's ascending and descending F_q-equation. Then we use these forms to obtain new generating functions for the q-analogue of the I-function. Some particular cases of these results in terms of the q-analogue of the I-function, H-function and G-function have also been obtained.

Keywords: F_q-equation; Generating function; Basic analogues of the I-function; H-function

1 Introduction

Many researchers engaged their focus on a different version of calculus which is also called calculus without limits or q-calculus. This form of calculus was initiated in the 1920s of the last century. Basic elements of q-calculus can be found in Kac and Cheung's book [1] entitled "Quantum Calculus". The investigations of q-integrals and q-derivatives of arbitrary order have gained importance due to their various applications in the areas like ordinary and fractional differential equations, solutions of the q-difference (differential) and q-integral equations, and q-transform analysis.

Hypergeometric functions evolved as a natural unification of many known functions [2–5] starting from the seventeenth century to the present day. Functions of this type may also be generalized using the concept of basic number q, resulting in their basic or q-analogues. Over the last 30 years, great interest in q-functions has been witnessed in view of their applications in number theory, statistics and other areas of mathematics and physics. Recent developments in the theory of basic hypergeometric functions have introduced new generalized forms of them. These functions are Mac-Roberts's E-function, Meijer's G-function, Fox's H-function, Saxena's I-function and their q-analogues. The q-analogue of the I-function has been introduced by Dutta et al. [6], in view of the q-gamma function, which is a q-extension of the generalized H-function and I-function earlier defined by Saxena et al. [7] and Farooq et al. [8].

In his effort towards achieving unification of special functions, Truesdell [9] has put forward a theory which yielded a number of results for special functions satisfying the so-called Truesdell's F-equation. Agrawal [10], extended this theory and derived results for a descending F-equation. He obtained various properties like orthogonality, Rodrigue's and Schalafli's formulas for the F-equation. Renu Jain et al. [11] obtained various forms of the I-function which satisfy Truesdell's ascending and descending F-equation. For more of such recent studies, the interested reader is referred to [12–16] and the references therein.

In this paper, on the one hand we derive some identities for the q-analogue of gamma function and on the other we obtain new generating functions which satisfy Truesdell's ascending and descending F_q-equation. To attain the purpose, we have stretched out F-equation to its q-analogue, namely the F_q-equation. We have additionally determined different types of the I-function satisfying Truesdell's ascending and descending F_q-equation. By applying the various structures we have obtained some generating functions of the I-function. Some special cases of these results are also obtained, which validate the outcomes and also yield some new results. We present our main results in Sect. 3 after presenting related mathematical preliminaries in Sect. 2.

2 Mathematical preliminaries

Before going further to this section, it is to be remarked that some literature can be found for a trivial generalization of the I-function, namely the Aleph function. Therefore, Sexena [17] has proved that the so-called "Aleph function" is nothing but another form of the Saxena I-function, which is the last generalization of the hypergeometric functions. Hence, all studies of the Aleph function are, in fact, studies of the earlier function (I-function) unless it indicates some new properties of both I-function and Aleph function. For further details, see [17] and the references therein. Therefore, in our present investigation we confine ourselves to different forms of the I-function.

2.1 Basic analogue of the *I*-function

The basic analogue of the modified I-function was given by Dutta et al. [6] in terms of the Mellin–Barnes type basic contour integral as

$$
\begin{aligned}
&I^{m,n}_{p_i,q_i,\tau_i;r}\left[\left(z;q\left|\begin{matrix}(a_j,A_j)_{1,n} & \dots & [\tau_i(a_{ji},A_{ji})]_{n+1,p_i}\\ (b_j,B_j)_{1,m} & \dots & [\tau_i(b_{ji},B_{ji})]_{m+1,q_i}\end{matrix}\right.\right)\right]\\
&\quad=\frac{1}{2\pi\omega}\\
&\qquad\times\int_L\frac{\prod_{j=1}^{m}G(q^{(b_j-B_js)})\prod_{j=1}^{n}G(q^{(1-a_j-A_js)})}{\sum_{i=1}^{r}\tau_i[\prod_{j=m+1}^{q_i}G(q^{(1-b_{ji}+B_{ji}s)})\prod_{j=n+1}^{p_i}G(q^{(a_{ji}-A_{ji}s)})G(q^s)G(q^{1-s})\sin\pi s]}\pi z^s\,ds,
\end{aligned}
\tag{1}
$$

where $z\neq 0$, $0<q<1$ and $\omega=\sqrt{-1}$. The parameters p_i, q_i are non-negative whole numbers fulfilling the disparity $0\le n\le p_i$, $0\le m\le q_i$ and $\tau_i>0$; $i=1,2,3,\dots,r$ is finite and A_j, B_j, A_{ji}, B_{ji} are +ve real numbers and $a_j,b_j,a_{ji},b_{ji}\in C$. The $C=C_{\omega\gamma\infty}$ is an appropriate shape of the Mellin–Barnes type in the complex s-plane, which keeps running from $\gamma-\omega\infty$ to $\gamma+\omega\infty$ with $\gamma\in C$, in such a way, that all poles of $G(q^{(b_j-B_js)})$; $1\le j\le m$, isolating from those of $G(q^{(1-a_j+A_js)})$; $1\le j\le n$. Every pole of the integrand (1) is simple and void products are translated as unity. The integral converges if $\mathrm{Re}[s\log(z)-\log\sin\pi s]<0$, for substantial estimations of $|s|$ on the contour L, that is, if $|(\arg(z)-w_2w_1^{-1}\log|z|)|<\pi$, where $0<|q|<1$, $\log q=-w=-(w_1+iw_2)$, w, w_1, w_2 are definite quantities, w_1, w_2 are real.

If we take $r = 1$; $\tau_i = 1$ in (1), then it reduces to the q-analogue of H-function defined by Saxena et al. [18]. If we set $A_i = B_j = 1$ for all i and j, then it reduces to the q-analogue of the G-function defined by Saxena et al. [18].

Ahmad et al. [19] defined the basic analogue of the modified I-function in terms of the q-gamma function as follows:

$$\begin{aligned} &I^{m,n}_{p_i,q_i,\tau_i;r}\left[\left(z;q\left|\begin{matrix}(a_j,A_j)_{1,n} & \dots & [\tau_i(a_{ji},A_{ji})]_{n+1,p_i}\\ (b_j,B_j)_{1,m} & \dots & [\tau_i(b_{ji},B_{ji})]_{m+1,q_i}\end{matrix}\right.\right)\right]\\ &\quad=\frac{1}{2\pi\omega}\\ &\qquad\times\int_L\frac{\prod_{j=1}^m\Gamma_q(b_j-B_js)\prod_{j=1}^n\Gamma_q(1-a_j-A_js)\pi z^s}{\sum_{i=1}^r\tau_i[\prod_{j=m+1}^{q_i}\Gamma_q(1-b_{ji}+B_{ji}s)\prod_{j=n+1}^{p_i}\Gamma_q(a_{ji}-A_{ji}s)\Gamma_q(s)\Gamma_q(1-s)\sin\pi s]}\,ds. \end{aligned}\tag{2}$$

By setting $\tau_i = 1$ in (2), we get the results defined in [7, 8].

2.2 Truesdell *F*-equation

If for the function $F(z,\alpha)$,

$$D_z^rF(z,\alpha) = F(z,\alpha + r),\tag{3}$$

then it is said to be satisfying the ascending F_q-condition. However, for $F(z,\alpha)$ satisfying the ascending F-equation, Truesdell [9] has obtained the following generating functions using Taylor's series:

$$F(z + y,\alpha) = \sum_{n=0}^{\infty} y^n\frac{F(z,\alpha + n)}{n!}.\tag{4}$$

The function $G(z,\alpha)$ is said to satisfy the descending F_q-equation if

$$D_z^rG(z,\alpha) = G(z,\alpha - r).\tag{5}$$

For $G(z,\alpha)$ satisfying the descending F-condition, Agrawal [20] has obtained the following generating functions:

$$G(z + y,\alpha) = \sum_{n=0}^{\infty} y^n\frac{G(z,\alpha - n)}{n!}.\tag{6}$$

The q-derivative of Eqs. (3) and (5) can be written, respectively, in the following manner:

$$D_{q,z}^rF(z,\alpha) = F(z,\alpha + r),\tag{7}$$

$$D_{q,z}^rG(z,\alpha) = G(z,\alpha - r).\tag{8}$$

3 Main results

In this section we derive our main results by dividing them into three subsections (Sects. 3.1–3.3). We first obtain the identities involving the gamma function and then we will obtain generating functions satisfying Truesdell's ascending and descending F_q-equations.

3.1 Identities involving gamma function

In this section, we first derive some identities for the q-analogue of the gamma function in order to obtain further results of this paper.

Lemma 3.1 *The following identity for the q-analogue of gamma function holds true*:

$$\prod_{k=0}^{m-1} \Gamma_q\left(\frac{\alpha+r+k}{m}\right) = \frac{(q^\alpha;q)_r}{(1-q)^r} \prod_{k=0}^{m-1} \Gamma_q\left(\frac{\alpha+k}{m}\right). \tag{9}$$

Proof Considering the involved ratio and simplifying in the following manner, we get the required result:

$$\begin{aligned}
\frac{\prod_{k=0}^{m-1} \Gamma_q(\frac{\alpha+r+k}{m})}{\prod_{k=0}^{m-1} \Gamma_q(\frac{\alpha+k}{m}} &= \frac{\Gamma_q(\frac{\alpha+r}{m})}{\Gamma_q(\frac{\alpha}{m})} \frac{\Gamma_q(\frac{\alpha+r+1}{m})}{\Gamma_q(\frac{\alpha+1}{m})} \frac{\Gamma_q(\frac{\alpha+r+2}{m})}{\Gamma_q(\frac{\alpha+2}{m})} \cdots \frac{\Gamma_q(\frac{\alpha+r+m-1}{m})}{\Gamma_q(\frac{\alpha+m-1}{m})} \\
&= \left[\frac{\alpha}{m}\right]_{q,r/m} \left[\frac{\alpha+1}{m}\right]_{q,r/m} \cdots \left[\frac{\alpha+m-1}{m}\right]_{q,r/m} \\
&= \frac{[(q^\alpha;q^m)_{\frac{r}{m}} (q^{\alpha+1};q^m)_{\frac{r}{m}} (q^{\alpha+2};q^m)_{\frac{r}{m}} \cdots (q^{\alpha+m-1};q^m)_{\frac{r}{m}}]}{(1-q)^r} \\
&= \frac{(q^\alpha;q)_r}{(1-q)^r}.
\end{aligned} \tag{10}$$

□

Remark 1 Similarly, by following the same type of calculations, we can also derive the following identities:

$$\prod_{k=0}^{m-1} \Gamma_q\left(1-\frac{\alpha+r+k}{m}\right) = \frac{q^{\frac{2r\alpha+r^2-r}{2m}}(1-q)^r}{(-1)^r(q^\alpha;q)_r} \prod_{k=0}^{m-1} \Gamma_q\left(1-\frac{\alpha+k}{m}\right); \tag{11}$$

$$\prod_{k=0}^{m-1} \Gamma_q\left(\frac{\alpha-r+k}{m}\right) = \frac{q^{\frac{r}{m}(r-2\alpha+1)}(1-q)^r}{(-1)^r(q^{1-\alpha};q)_r} \prod_{k=0}^{m-1} \Gamma_q\left(\frac{\alpha+k}{m}\right); \tag{12}$$

$$\prod_{k=0}^{m-1} \Gamma_q\left(1-\frac{\alpha-r+k}{m}\right) = \frac{(q^{1-\alpha};q)_r}{(1-q)^r} \prod_{k=0}^{m-1} \Gamma_q\left(1-\frac{\alpha+k}{m}\right). \tag{13}$$

Here and in the following, $\triangle(\mu,\alpha)$, denotes the array of μ parameters:

$$\frac{\alpha}{\mu}, \frac{\alpha+1}{\mu}, \dots, \frac{\alpha+\mu-1}{\mu} \quad (\mu = 1,2,3,\dots)$$

and

$$\big(\triangle(\mu,\alpha),\beta\big) \text{ stands for } \left(\frac{\alpha}{\mu},\beta\right), \left(\frac{\alpha+1}{\mu},\beta\right), \dots, \left(\frac{\alpha+\mu-1}{\mu},\beta\right). \tag{14}$$

3.2 Generating functions satisfying Truesdell's ascending F_q-equation

In this section we will derive various forms of the q-analogue of the modified I-function, which satisfy Truesdell's ascending F_q-equation.

Theorem 3.2 *The following form of the I-function satisfies Truesdell's ascending F_q-equation*:

$$\begin{aligned}&\left(q^{\frac{\alpha-1}{2}[\frac{\rho-1}{\rho}]}z\right)^{-\alpha}\\&\quad\times I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left(q^{\alpha h(\lambda-1)}z^{h\lambda};q\left|\begin{matrix}(a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i-\rho},(\triangle(\rho,\alpha),h)\\(\triangle(\lambda,\alpha),h),(b_j,\beta_j)_{\lambda+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i-\rho},(\triangle(\rho,\alpha),h)\end{matrix}\right.\right)\right].\end{aligned}\tag{15}$$

Proof Expecting that the structure (15) is $F(z,\alpha)$, supplanting the basic analogue of the I-function according to its definition (2) and then interchanging the order of integration and differentiation, which is legitimized under the conditions of convergence [19], we see that

$$\begin{aligned}&D^r_{q,z}F(z,\alpha)\\&\quad=\frac{1}{2\pi\omega}\int_L\frac{\prod_{k=0}^{\lambda-1}\Gamma_q(\frac{\alpha+k}{\lambda}-hs)}{\sum_{i=1}^{l}\tau_i[\prod_{j=m+1}^{q_i-\rho}\Gamma_q(1-b_{ji}+B_{ji}s)}\\&\qquad\times\left(\prod_{j=\lambda+1}^{m}\Gamma_q(b_j-\beta_js)\prod_{j=1}^{n}\Gamma_q(1-a_j-\alpha_js)q^{\frac{\alpha(\alpha-1)}{2\rho}-\frac{\alpha(\alpha-1)}{2}}q^{\alpha h(\lambda-1)s}D^r_{q,z}z^{h\lambda s-\alpha}\pi\right)\\&\qquad\bigg/\left(\prod_{k=0}^{\rho-1}\Gamma_q\left(1-\frac{\alpha+k}{\rho}+hs\right)\prod_{j=n+1}^{p_i-\rho}\Gamma_q(a_{ji}-\alpha_{ji}s)\right.\\&\qquad\left.\times\prod_{k=0}^{\rho-1}\Gamma_q\left(\frac{\alpha+k}{\rho}-hs\right)\Gamma_q(s)\Gamma_q(1-s)\sin\pi s\right)ds.\end{aligned}\tag{16}$$

Using the results (9) and (11), we have

$$\prod_{k=0}^{\lambda-1}\Gamma_q\left(\frac{\alpha+k}{\lambda}-hs\right)=\frac{(1-q)^r}{(q^{\alpha-h\lambda s};q)_r}\prod_{k=0}^{\lambda-1}\Gamma_q\left(\frac{\alpha+r+k}{\lambda}-hs\right),\tag{17}$$

$$\prod_{k=0}^{\lambda-1}\Gamma_q\left(1-\frac{\alpha+k}{\lambda}+hs\right)=\frac{(-1)^r(q^{\alpha-h\lambda s};q)_r}{(1-q)^rq^{\frac{2r(\alpha-h\lambda s)+r^2-r}{2\lambda}}}\prod_{k=0}^{\lambda-1}\Gamma_q\left(1-\frac{\alpha+r+k}{\lambda}+hs\right).\tag{18}$$

Using the identities (17)–(18) in (16) we get the required Truesdell's ascending F_q-Eq. (7). □

Remark 2 Similarly the following structures (19) to (23) can be shown to fulfill Truesdell's ascending F_q-condition:

$$\begin{aligned}&\left(\frac{q^{\frac{1}{2}[\frac{\alpha}{\lambda}+\alpha-1]}z}{(1-q)}h\right)^{-\alpha}\\&\quad\times I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left(q^{\alpha h(\lambda+1)}z^{h\lambda};q\left|\begin{matrix}(\triangle(\lambda,\alpha+1/2),h),(a_j,\alpha_j)_{\lambda+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i}\\(\triangle(2\lambda,2\alpha),h),(b_j,\beta_j)_{2\lambda+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i}\end{matrix}\right.\right)\right],\end{aligned}\tag{19}$$

$$\left(\frac{q^{\frac{1}{2}[\frac{3\alpha+1}{3\lambda}+\alpha-1]}z}{(1-q)}\right)^{-\alpha} \times I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left(q^{\alpha h(\lambda+1)}z^{h\lambda};q\left|\begin{matrix}(\triangle(\lambda,\alpha+\frac{2}{3}),h),(a_j,\alpha_j)_{\lambda+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i-\lambda},(\triangle(\lambda,\alpha+1/3),h)\\(\triangle(3\lambda,3\alpha),h),(b_j,\beta_j)_{3\lambda+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i}\end{matrix}\right.\right)\right], \tag{20}$$

$$\left(q^{\frac{\alpha-1}{2}}z\right)^{-\alpha}e^{\pi i\alpha}I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left(\left(q^{\alpha}z\right)^{h\lambda};q\left|\begin{matrix}(a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i}\\(\triangle(\lambda,\alpha),h),(b_j,\beta_j)_{\lambda+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i}\end{matrix}\right.\right)\right], \tag{21}$$

$$\left(q^{\frac{\alpha-1}{2}\frac{\lambda-1}{\lambda}}z\right)^{-\alpha}I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left(q^{h\alpha(\lambda-1)}z^{h\lambda};q\left|\begin{matrix}(a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i}\\(\triangle(\lambda,\alpha),h),(b_j,\beta_j)_{\lambda+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i}\end{matrix}\right.\right)\right], \tag{22}$$

$$\left(q^{\frac{\alpha-1}{2}\frac{\lambda-1}{\lambda}+\frac{1}{\rho}}z\right)^{-\alpha}e^{\pi i\alpha} \times I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left(\left(q^{\alpha}z\right)^{h\lambda};q\left|\begin{matrix}(\triangle(\rho,\alpha),h),(a_j,\alpha_j)_{\rho+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i}\\(\triangle(\rho,\alpha),h),(b_j,\beta_j)_{\rho+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i-\lambda}(\triangle(\lambda,\alpha),h)\end{matrix}\right.\right)\right]. \tag{23}$$

Next we use the above forms (15) and (19)–(23) to establish the following new generating functions for the q-analogue of the modified I-functions using Truesdell's ascending F_q-equation technique.

For example, we establish the following generating function (24) by substituting the structures (15) in Truesdell's ascending F_q-condition (7) and supplant z by $\frac{y}{q^{\alpha}}$ and $q^{-\alpha h}y^{h\lambda}$ by x, respectively:

$$\begin{aligned}&\left(1+q^{\alpha}\right)^{-\alpha} \times I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left(\left(1+q^{\alpha}\right)^{h\lambda}x;q\left|\begin{matrix}(a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i-\rho},(\triangle(\rho,\alpha),h)\\(\triangle(\lambda,\alpha),h),(b_j,\beta_j)_{\lambda+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i-\rho},(\triangle(\rho,\alpha),h)\end{matrix}\right.\right)\right]\\&=\sum_{r=0}^{\infty}\frac{(q^{\frac{(-2\alpha-r+1)((\rho-1)/\rho)}{2}+2\alpha})^{r}}{r!}\\&\quad\times I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left((q^{rh(\lambda-1)}x;q\left|\begin{matrix}(a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i-\rho},(\triangle(\rho,\alpha+r),h)\\(\triangle(\lambda,\alpha+r),h),(b_j,\beta_j)_{\lambda+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i-\rho},(\triangle(\rho,\alpha+r),h)\end{matrix}\right.\right)\right].\end{aligned} \tag{24}$$

Similarly, we establish the following generating function (25) by putting the structures (19) in Truesdell's ascending F_q-condition (7) and supplant z by $\frac{y}{q^{\alpha}}$ and $q^{\alpha h}y^{h\lambda}$ by x, respectively:

$$\begin{aligned}&\left(1+q^{\alpha}\right)^{-\alpha}I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left(\left(1+q^{\alpha}\right)^{h\lambda}x;q\left|\begin{matrix}(\triangle(\lambda,\alpha+1/2),h)(a_j,\alpha_j)_{\lambda+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i}\\(\triangle(2\lambda,2\alpha),h),(b_j,\beta_j)_{2\lambda+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i}\end{matrix}\right.\right)\right]\\&=\sum_{r=0}^{\infty}\frac{(q^{\frac{-1}{2}(\frac{2\alpha}{\lambda}+\frac{r}{\lambda}-1)(1-q)})^{r}}{r!}\\&\quad\times I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left((q^{rh(\lambda+1)}x;q\left|\begin{matrix}(\triangle(\lambda,\alpha+r+1/2),h)(a_j,\alpha_j)_{\lambda+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i}\\(\triangle(2\lambda,2(\alpha+r)),h),(b_j,\beta_j)_{2\lambda+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i}\end{matrix}\right.\right)\right].\end{aligned} \tag{25}$$

Following the generating function (26) can be established by substituting the structures (20) in Truesdell's ascending F_q-condition (7) and replacing z by $\frac{y}{q^\alpha}$ and $q^{\alpha h}y^{h\lambda}$ by x, respectively:

$$\begin{aligned}&\left(1+q^\alpha\right)^{-\alpha}\\&\quad\times I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left(\left(1+q^\alpha\right)^{h\lambda}x;q\left|\begin{matrix}(\triangle(\lambda,\alpha+2/3),h)(a_j,\alpha_j)_{\lambda+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i-\lambda}(\triangle(\lambda,\alpha+1/3),h)\\(\triangle(3\lambda,3\alpha),h),(b_j,\beta_j)_{2\lambda+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i}\end{matrix}\right.\right)\right]\\&=\sum_{r=0}^{\infty}\frac{(q^{\frac{-1}{2}(\frac{\alpha}{\lambda}+\frac{r}{2\lambda}+\frac{r}{2}+\frac{1}{6\lambda}-\frac{1}{2})(1-q)})^r}{r!}\\&\quad\times I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left((q^{rh(\lambda+1)}x;q\left|\begin{matrix}(\triangle(\lambda,\alpha+r+2/3),h)(a_j,\alpha_j)_{\lambda+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i-\lambda}(\triangle(\lambda,\alpha+r+1/3),h)\\(\triangle(3\lambda,3(\alpha+r)),h),(b_j,\beta_j)_{2\lambda+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i}\end{matrix}\right.\right)\right].\end{aligned}\tag{26}$$

Similarly, we establish the following generating function (27) by substituting the structures (21) in Truesdell's ascending F_q-condition (7) and supplant z by $\frac{y}{q^\alpha}$ and $y^{h\lambda}$ by x, respectively:

$$\begin{aligned}&\left(1+q^\alpha\right)^{-\alpha}I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left(\left(1+q^\alpha\right)^{h\lambda}x;q\left|\begin{matrix}(a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i}\\(\triangle(\lambda,\alpha),h),(b_j,\beta_j)_{\lambda+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i}\end{matrix}\right.\right)\right]\\&=\sum_{r=0}^{\infty}\frac{(-1)^r(q^{\frac{1-r}{2}})^r}{r!}\\&\quad\times I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left((q^{rh\lambda}x;q\left|\begin{matrix}(a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i}\\(\triangle(\lambda,\alpha+r),h),(b_j,\beta_j)_{\lambda+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i}\end{matrix}\right.\right)\right].\end{aligned}\tag{27}$$

To establish (28) we substitute the structures (22) in Truesdell's ascending F_q-condition (7) and supplant z by $\frac{y}{q^\alpha}$ and $q^{-\alpha h}y^{h\lambda}$ by x, respectively.

$$\begin{aligned}&\left(1+q^\alpha\right)^{-\alpha}I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left(\left(1+q^\alpha\right)^{h\lambda}x;q\left|\begin{matrix}(a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i}\\(b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i-\lambda}(\triangle(\lambda,\alpha),h)\end{matrix}\right.\right)\right]\\&=\sum_{r=0}^{\infty}\frac{(q^{\frac{(\lambda-1)}{\lambda}(-\alpha-\frac{r}{2}+\frac{1}{2})+\alpha})^r}{r!}\\&\quad\times I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left((q^{rh(\lambda-1)}x;q\left|\begin{matrix}(a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i}\\(b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i-\lambda}(\triangle(\lambda,\alpha+r),h)\end{matrix}\right.\right)\right].\end{aligned}\tag{28}$$

Similarly we obtain (29) by putting the structures (23) in Truesdell's ascending F_q-condition (7) and supplant z by $\frac{y}{q^\alpha}$ and $y^{h\lambda}$ by x, respectively:

$$\begin{aligned}&\left(1+q^\alpha\right)^{-\alpha}\\&\quad\times I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left(\left(1+q^\alpha\right)^{h\lambda}x;q\left|\begin{matrix}(\triangle(\rho,\alpha),h),(a_j,\alpha_j)_{\rho+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i}\\(\triangle(\rho,\alpha),h),(b_j,\beta_j)_{\rho+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i-\lambda}(\triangle(\lambda,\alpha),h)\end{matrix}\right.\right)\right]\end{aligned}$$

$$= \sum_{r=0}^{\infty} \frac{(-1)^r (q^{\frac{(}{1}\lambda - \frac{1}{\rho} - 1)(\alpha + \frac{r}{2} - \frac{1}{2}) + \alpha})^r}{r!}$$

$$\times I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left((q^{rh\lambda}x; q \left| \begin{matrix} (\triangle(\rho,\alpha+r),h),(a_j,\alpha_j)_{\rho+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i} \\ (\triangle(\rho,\alpha+r),h),(b_j,\beta_j)_{\rho+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i-\lambda}(\triangle(\lambda,\alpha+r),h) \end{matrix}\right.\right)\right]. \tag{29}$$

Remark 3 The above results yield certain special cases of the generating function for the q-analogue of the I-function, H-function and G-function [7, 8, 18, 21].

3.3 Generating functions satisfying Truesdell's descending F_q-equation

In this section we will derive the different forms of the q-analogue of the modified I-function, which satisfy Truesdell's descending F_q-equation.

Theorem 3.3 *The following form of the q-analogue of the modified I-function satisfies Truesdell's descending F_q-equation*:

$$(q^{\frac{\alpha}{2}[\frac{1-\rho}{\rho}]}z)^{\alpha-1}$$

$$\times I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left(\frac{q^{\alpha h(\lambda+1)}}{z^{h\lambda}}; q \left| \begin{matrix} (\triangle(\lambda,\alpha),h),(a_j,\alpha_j)_{\lambda+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i-\rho},(\triangle(\rho,\alpha),h) \\ (b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i-\rho},(\triangle(\rho,\alpha),h) \end{matrix}\right.\right)\right]. \tag{30}$$

Proof Assuming that the form (30) is $G(z,\alpha)$, replacing a basic analogue of the modified I-function by its statement (2), then exchanging order of differentiation and integration, which is legitimized under the states of combination [19], we observe that

$$\begin{aligned} &D^r_{q,z}G(z,\alpha) \\ &= \frac{1}{2\pi\omega}\int_L \frac{\prod_{j=1}^m \Gamma_q(b_j - \beta_j s)}{\sum_{i=1}^l \tau_i[\prod_{j=m+1}^{q_i-\rho}\Gamma_q(1 - b_{ji} + B_{ji}s)} \\ &\times \left(\prod_{k=0}^{\lambda-1}\Gamma_q(1 - \frac{\alpha+k}{\lambda} - hs)\prod_{j=\lambda+1}^{n}\Gamma_q(1 - a_j + \alpha_j s)q^{\frac{\alpha(\alpha-1)(1-\rho)}{2\rho}}q^{\alpha h(\lambda+1)s}D^r_{q,z}z^{\alpha-1-h\lambda s}\pi\right) \\ &\Big/\left(\prod_{k=0}^{\rho-1}\Gamma_q\left(1 - \frac{\alpha+k}{\rho} + hs\right)\prod_{j=n+1}^{p_i-\rho}\Gamma_q(a_{ji} - \alpha_{ji}s)\right. \\ &\left.\times \prod_{k=0}^{\rho-1}\Gamma_q\left(\frac{\alpha+k}{\rho} - hs\right)\Gamma_q(s)\Gamma_q(1-s)\sin\pi s]\right) ds. \end{aligned} \tag{31}$$

Using results (12) and (13), we have

$$\prod_{k=0}^{\lambda-1}\Gamma_q\left(\frac{\alpha+k}{\lambda} - hs\right) = \frac{(-1)^r(q^{1-(\alpha-h\lambda s)};q)_r}{(1-q)^r q^{\frac{r}{2m}(r-2(\alpha-h\lambda s)+1)}}\prod_{k=0}^{\lambda-1}\Gamma_q\left(\frac{\alpha-r+k}{\lambda} - hs\right), \tag{32}$$

$$\prod_{k=0}^{\lambda-1}\Gamma_q\left(1 - \frac{\alpha+k}{\lambda} + hs\right) = \frac{(1-q)^r}{(q^{1-(\alpha-h\lambda s)};q)_r}\prod_{k=0}^{\lambda-1}\Gamma_q\left(1 - \frac{\alpha-r+k}{\lambda} + hs\right). \tag{33}$$

Using the identities (32) and (33) in (31), we obtain the required Truesdell form of the descending F_q-Eq. (8). □

Remark 4 Similarly, in the same manner, the following forms (34) to (38) can be shown to satisfy the descending F_q-condition:

$$\left(\frac{q^{\frac{-\alpha}{2}[\frac{1+\lambda}{\lambda}]}z}{(1-q)^{\alpha}}\right)^{\alpha-1} \times I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left(\frac{q^{\alpha h(\lambda-1)}}{z^{h\lambda}};q\left|\begin{matrix}(\triangle(2\lambda,2\alpha),h),(a_j,\alpha_j)_{2\lambda+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i}\\(\triangle(\lambda,\alpha+1/2),h),(b_j,\beta_j)_{2\lambda+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i}\end{matrix}\right.\right)\right], \tag{34}$$

$$\frac{(q^{\frac{-\alpha}{2(\alpha-1)}[\frac{3\alpha+1}{3\lambda}+\alpha-1]}z)^{\alpha-1}}{(1-q)^{\alpha}} \times I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left(\frac{q^{\alpha h(\lambda+1)}}{z^{h\lambda}};q\left|\begin{matrix}(\triangle(3\lambda,3\alpha),h),(a_j,\alpha_j)_{3\lambda+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i}\\(\triangle(\lambda,\alpha+2/3),h),(b_j,\beta_j)_{\lambda+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i-\lambda},(\triangle(\lambda,\alpha+1/3),h)\end{matrix}\right.\right)\right], \tag{35}$$

$$(q^{\frac{-\alpha}{2}}z)^{\alpha-1}e^{\pi i\alpha}I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left(\frac{(q^{\alpha h\lambda})}{z^{h\lambda}};q\left|\begin{matrix}(\triangle(\lambda,\alpha),h),(a_j,\alpha_j)_{\lambda+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i}\\(b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i}\end{matrix}\right.\right)\right], \tag{36}$$

$$(q^{\frac{\alpha}{2}[\frac{1-\lambda}{\lambda}]}z)^{\alpha-1}I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left(\frac{q^{h\alpha(\lambda-1)}}{z^{h\lambda}};q\left|\begin{matrix}(a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i-\lambda},(\triangle(\lambda,\alpha),h)\\(b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i}\end{matrix}\right.\right)\right], \tag{37}$$

$$(q^{\frac{\alpha}{2}[\frac{1-\lambda}{\lambda}-\frac{1}{\rho}]}z)^{\alpha-1}e^{\pi i\alpha} \times I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left(\frac{q^{\alpha h\lambda}}{z^{h\lambda}};q\left|\begin{matrix}(\triangle(\rho,\alpha),h),(a_j,\alpha_j)_{\rho+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i-\lambda},(\triangle(\lambda,\alpha),h),\\(\triangle(\rho,\alpha),h),(b_j,\beta_j)_{\rho+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i}\end{matrix}\right.\right)\right]. \tag{38}$$

Next we use the above forms (30) and (34)–(38) and establish the following generating functions for basic analogue of the I-functions by applying the descending F_q-equation technique.

To establish (39) we substitute the structure (30) in Truesdell's descending F_q-Eq. (8) and replace z by $\frac{y}{q^{\alpha}}$ and $\frac{q^{\alpha h+2\alpha h\lambda}}{y^{h\lambda}}$ by x in progression to get the required outcomes:

$$\begin{aligned}&(1+q^{\alpha})^{\alpha-1}\\&\quad\times I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left((1+q^{\alpha})^{-h\lambda}x;q\left|\begin{matrix}(\triangle(\lambda,\alpha),h),(a_j,\alpha_j)_{\lambda+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i-\rho},(\triangle(\rho,\alpha),h)\\(b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i-\rho},(\triangle(\rho,\alpha),h)\end{matrix}\right.\right)\right]\\&=\sum_{r=0}^{\infty}\frac{(q^{(-\alpha+r/2+1/2)((1-\rho)/\rho)}+\alpha)^r}{r!}\\&\quad\times I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left((q^{-rh(\lambda+1)}x;q\left|\begin{matrix}(\triangle(\lambda,\alpha-r),h),(a_j,\alpha_j)_{\lambda+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i-\rho},(\triangle(\rho,\alpha-r),h)\\(b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i-\rho},(\triangle(\rho,\alpha-r),h)\end{matrix}\right.\right)\right].\end{aligned} \tag{39}$$

To establish (40), we substitute the structures (34) in Truesdell's descending F_q-Eq. (8) and replace z by $\frac{y}{q^{\alpha}}$ and $\frac{2\alpha h\lambda-\alpha h}{y^{h\lambda}}$ by x, respectively:

$$\begin{aligned}&(1+q^{\alpha})^{\alpha-1}\\&\quad\times I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left((1+q^{\alpha})^{-h\lambda}x;q\left|\begin{matrix}(\triangle(2\lambda,2\alpha),h)(a_j,\alpha_j)_{2\lambda+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i}\\(\triangle(\lambda,\alpha+1/2),h),(b_j,\beta_j)_{2\lambda+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i}\end{matrix}\right.\right)\right]\end{aligned}$$

$$= \sum_{r=0}^{\infty} \frac{(q^{(\alpha-\frac{1}{2}-\frac{r}{2})(\frac{1+\lambda}{\lambda})+\alpha(1-q)})^r}{r!} \times I_{p_i,q_i;\tau_i;l}^{m,n}\left[\left((q^{rh(1-\lambda)}x;q \,\middle|\, \begin{matrix} (\triangle(2\lambda,2\alpha-2r),h)(a_j,\alpha_j)_{2\lambda+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i} \\ (\triangle(\lambda,\alpha-r+1/2),h),(b_j,\beta_j)_{2\lambda+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i} \end{matrix}\right)\right]. \quad (40)$$

To establish (41), we substitute the structures (35) in Truesdell's descending F_q-Eq. (8) and replace z by $\frac{y}{q^{\alpha}}$ and $\frac{q^{\alpha h+2\alpha h\lambda}}{y^{h\lambda}}$ by x, respectively:

$$(1+q^{\alpha})^{\alpha} \times I_{p_i,q_i;\tau_i;l}^{m,n}\left[\left((1+q^{\alpha})^{-h\lambda}x;q \,\middle|\, \begin{matrix} (\triangle(3\lambda,3\alpha),h)(a_j,\alpha_j)_{3\lambda+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i} \\ (\triangle(\lambda,\alpha+2/3),h),(b_j,\beta_j)_{\lambda+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i-\lambda},(\triangle(\lambda,\alpha+1/3),h) \end{matrix}\right)\right]$$
$$= \sum_{r=0}^{\infty} \frac{(q^{2\alpha+\frac{\alpha}{\lambda}-\frac{r}{2\lambda}-\frac{r}{2}+\frac{1}{6\lambda}+\frac{1}{2}(1-q)})^r}{r!} \times I_{p_i,q_i;\tau_i;l}^{m,n}\left[\left((q^{-rh(\lambda+1)}x;q \,\middle|\, \begin{matrix} (\triangle(3\lambda,3\alpha-3r),h)(a_j,\alpha_j)_{3\lambda+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i} \\ (\triangle(\lambda,\alpha-r+2/3),h),(b_j,\beta_j)_{\lambda+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i-\lambda},(\triangle(\lambda,\alpha-r+1/3),h) \end{matrix}\right)\right]. \quad (41)$$

To establish (42) we substitute the structures (36) in Truesdell's descending F_q-Eq. (8) and replace z by $\frac{y}{q^{\alpha}}$ and $\frac{q^{2\alpha h\lambda}}{y^{h\lambda}}$ by x, respectively:

$$(1+q^{\alpha})^{\alpha-1} I_{p_i,q_i;\tau_i;l}^{m,n}\left[\left((1+q^{\alpha})^{-h\lambda}x;q \,\middle|\, \begin{matrix} (\triangle(\lambda,\alpha),h),(a_j,\alpha_j)_{\lambda+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i} \\ (b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i} \end{matrix}\right)\right]$$
$$= \sum_{r=0}^{\infty} \frac{(-1)^r(q^{2\alpha+\frac{r}{2}-1/2})^r}{r!} \times I_{p_i,q_i;\tau_i;l}^{m,n}\left[\left((q^{-rh\lambda}x;q \,\middle|\, \begin{matrix} (\triangle(\lambda,\alpha-r),h),(a_j,\alpha_j)_{\lambda+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i} \\ (b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i} \end{matrix}\right)\right]. \quad (42)$$

To establish (43) we substitute the structures (37) in Truesdell's descending F_q-Eq. (8) and replace z by $\frac{y}{q^{\alpha}}$ and $\frac{q^{2\alpha h\lambda-\alpha h}}{y^{h\lambda}}$ by x, respectively:

$$(1+q^{\alpha})^{\alpha-1} I_{p_i,q_i;\tau_i;l}^{m,n}\left[\left((1+q^{\alpha})^{-h\lambda}x;q \,\middle|\, \begin{matrix} (a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i-\lambda},(\triangle(\lambda,\alpha),h) \\ (b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i} \end{matrix}\right)\right]$$
$$= \sum_{r=0}^{\infty} \frac{(q^{\frac{(1-\lambda)}{\lambda}(-\alpha+\frac{r}{2}+\frac{1}{2})+\alpha})^r}{r!} \times I_{p_i,q_i;\tau_i;l}^{m,n}\left[\left((q^{rh(1-\lambda)}x;q \,\middle|\, \begin{matrix} (a_j,\alpha_j)_{1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i-\lambda},(\triangle(\lambda,\alpha-r),h) \\ (b_j,\beta_j)_{1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i} \end{matrix}\right)\right]. \quad (43)$$

To establish (44) we substitute the structures (38) in Truesdell's descending F_q-Eq. (8) and replace z by $\frac{y}{q^{\alpha}}$ and $\frac{2\alpha h\lambda}{y^{h\lambda}}$ by x, respectively:

$$(1+q^{\alpha})^{\alpha-1} \times I_{p_i,q_i;\tau_i;l}^{m,n}\left[\left((1+q^{\alpha})^{-h\lambda}x;q \,\middle|\, \begin{matrix} (\triangle(\rho,\alpha),h),(a_j,\alpha_j)_{\rho+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i-\lambda},(\triangle(\lambda,\alpha),h) \\ (\triangle(\rho,\alpha),h),(b_j,\beta_j)_{\rho+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i} \end{matrix}\right)\right]$$

$$= \sum_{r=0}^{\infty} \frac{(-1)^r (q^{(\frac{1}{\lambda}-\frac{1}{\rho}-1)(-\alpha+\frac{r}{2}+\frac{1}{2})})^r}{r!}$$

$$\times I^{m,n}_{p_i,q_i;\tau_i;l}\left[\left((q^{-rh\lambda}x;q \left| \begin{matrix} (\triangle(\rho,\alpha-r),h),(a_j,\alpha_j)_{\rho+1,n},[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i-\lambda},(\triangle(\lambda,\alpha-r),h) \\ (\triangle(\rho,\alpha-r),h),(b_j,\beta_j)_{\rho+1,m},[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i} \end{matrix}\right.\right)\right]. \tag{44}$$

Remark 5 These results yield certain special cases of generating functions for the q-analogue of the I-function, H-function and G-function [7, 8, 18, 22].

4 Conclusion

This study led to different types of the I-function satisfying Truesdell's ascending and descending F_q-equation. These forms have been applied to obtain various generating functions for basic analogue of the modified I-function.

The results proved in this paper along with their particular cases are believed to be new. As these functions have well been established as applicable functions these results are likely to contribute significantly in certain applications of the theory of q-calculus.

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