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General decay synchronization of delayed BAM neural networks with reaction–diffusion terms

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Abstract

In this paper, general decay synchronization of delayed bidirectional associative memory neural networks with reaction–diffusion terms is studied. First, a useful lemma is introduced to determine the general decay synchronization of considered systems. Furthermore, a type of nonlinear controller is designed. Then, some sufficient conditions are obtained to insure the general decay synchronization of the drive–response systems via using Lyapunov functional method and Poincaré inequality. Finally, the obtained theoretical results are evaluated by giving one numerical example. The exponential synchronization, polynomial synchronization, and some other types of synchronization can be seen as special cases of the general decay synchronization.

Keywords: BAM neural network; General decay synchronization; Reaction–diffusion terms; Time-delay

1 Introduction

In the last two decades, a great number of scholars from science and engineering community have been paying their attention to the investigation of dynamical behavior and control problems of neural networks [1–9]. Among various neural networks, bidirectional associative memory neural network (BAMNN), introduced by Kosko [10, 11], has been studied wildly since this class of neural network has extensive applications in pattern recognition, complex control, and intelligent processing [12]. In addition, the synchronization of chaotic nonlinear systems has received enormous attention in the past two decades for its significant role in many areas, including biology, climatology, sociology, etc. [3–9]. The authors of [13] studied the global asymptotic stability for continuous BAMNNs by using Lyapunov method, while those of [14] considered the synchronization of memristor-based BAMNNs by using linear matrix inequality technique. In [15], a nonlinear feedback controller is designed for a general decay synchronization of delayed BAMNNs.

As it is known to all, the reaction–diffusion effect cannot be neglected when considering the motion of electrons in an asymmetric electromagnetic field [16]. Therefore it is necessary to introduce a diffusion term in neural networks, where this term is expressed by a partial differential equation [16–19]. In [20], the authors studied the global exponential stability and synchronization of the delayed reaction–diffusion neural networks (RDNNs) under the impulsive control. The authors of [21] discussed the global exponential synchronization of delayed BAMNNs with reaction–diffusion terms. In [22], an adaptive synchronization controller is derived for the global asymptotic synchronization of RDNNs with delays. The authors of [23] investigated the state synchronization of BAM neural networks with reaction–diffusion terms via a feedback control law. In [24], the authors have derived sufficient conditions for the H_∞ synchronization of RDNNs with mixed time-varying delays based on an adaptive controller method. In [25], an adaptive pinning controller is designed to guarantee the tracking synchronization for a class of neural networks with coupled reaction–diffusion terms. In [18], a sufficient condition for the ψ-type stability of RDNNs with bounded distributed delays and time-varying discrete delays was presented. In [26], an appropriate nonlinear controller was utilized, and the decay lag anti-synchronization criteria were designed for multiweighted coupled RDNNs. However, few researches can be found on the general decay synchronization problem of BAMNNs with reaction–diffusion terms. As mentioned in [15], there exist stable systems which are not exponentially stable, but with a general convergence rate.

Inspired by the above discussions, in this paper, we are concerned with the general decay synchronization for a BAMNN model with reaction–diffusion terms and time-varying delays. By using a novel inequality technique and constructing a suitable Lyapunov–Krasovskii-type functional, we obtained some simple sufficient conditions for the general decay synchronization of considered BAMNNs. Finally, we give a numerical example and its simulations to illustrate the effectiveness of the derived results. The polynomial synchronization, asymptotical synchronization, and exponential synchronization can be seen as special cases of the general decay synchronization.

The rest of the paper is organized as follows. In Sect. 2, we will introduce the details of the model and a useful lemma, which plays a critical role in proving the main result of this paper. Then, in Sect. 3, we will design a feedback controller for the general decay synchronization of delayed BAMNNs with reaction–diffusion terms. In Sect. 4, we will give an example to evaluate the effectiveness of theoretical results of this paper. In the final section, we give a brief summary to end up the paper.

2 Preliminaries

In this paper, we consider the following delayed BAMNNs with reaction-0diffusion terms:

$$\begin{cases} \dfrac{\partial u_i}{\partial t} = \displaystyle\sum_{k=1}^{\ell} \frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial u_i}{\partial x_k}\right) - p_i u_i(t,x) \\ \qquad + \displaystyle\sum_{j=1}^{n} b_{ji} f_j\big(v_j(t,x)\big) + \sum_{j=1}^{n} \tilde{b}_{ji} f_j\big(v_j\big(t-\theta_{ji}(t)\big)\big) + I_i, \\ \dfrac{\partial v_j}{\partial t} = \displaystyle\sum_{k=1}^{\ell} \frac{\partial}{\partial x_k}\left(\tilde{D}_{jk}\frac{\partial v_j}{\partial x_k}\right) - q_j v_j(t,x) \\ \qquad + \displaystyle\sum_{i=1}^{m} d_{ij} g_i\big(u_i(t,x)\big) + \sum_{i=1}^{m} \tilde{d}_{ij} g_i\big(u_i\big(t-\tau_{ij}(t)\big)\big) + J_j, \end{cases} \tag{1}$$

with the following Neumann boundary condition:

$$\nabla u_i = 0, \qquad \nabla v_j = 0, \quad x \text{ on } \partial\Omega, t \geq 0, \tag{2}$$

and the following initial condition:

$$u_i(s,x) = \Gamma_i(s,x), \qquad v_j(s,x) = \Lambda_i(s,x), \quad x \text{ in } \Omega, s \leq 0, \tag{3}$$

where $x = (x_1, x_2, \ldots, x_\ell)^T \in \Omega$, Ω is a bounded open subset of $\mathbb{R}^\ell$ with a C^2 boundary $\partial\Omega$, and $|\Omega| > 0$ (where $|\Omega|$ is the volume of Ω); $\nabla v := (\frac{\partial v}{\partial x_1}, \frac{\partial v}{\partial x_1}, \ldots, \frac{\partial v}{\partial x_\ell})^T$ stands for the gradient of function v, u_i and v_j for $i \in \mathcal{I} = \{1,2,\ldots,m\}$, $j \in \mathcal{J} = \{1,2,\ldots,n\}$ denote to the state variables of the ith and jth neuron at time t and in space point x, respectively; $D_{ik} \geq 0$ and $\tilde{D}_{jk} \geq 0$ represent transmission diffusion coefficients along the ith and jth neuron, respectively; p_i and q_j stand for the passive decay rates to the state of ith and jth neuron, respectively; b_{ji}, $\tilde{b}_{ji}$, d_{ij}, and $\tilde{d}_{ij}$ are the connection strengths between the neurons; $f_j(v)$ and $g_i(v)$ correspond to the respective neuron activation functions; $\theta_{ji(t)}$ and τ_{ij} correspond to the continuous time-varying discrete delays, respectively satisfying $0 \leq \theta_{ji}(t) \leq \theta_{ji}$ and $0 \leq \tau_{ij}(t) \leq \tau_{ij}$; I_i and J_j denote to the external inputs on the ith and jth neuron, respectively.

Through out the paper, we assume that the neuron activation functions $f_j(v)$, $g_j(v)$ and time-varying delays $\tau_{ij}(t)$, $\sigma_{ij}(t)$ satisfy the following assumptions:

Assumption 1 For any $i \in \mathcal{I}$, $j \in \mathcal{J}$, there exist constants L_j^f and L_i^g such that

$$\begin{aligned}
&\bigl|f_j(v_1) - f_j(v_2)\bigr| \leq L_j^f |v_1 - v_2|, \quad v_1, v_2 \in \mathbb{R}, \\
&\bigl|g_i(v_1) - g_i(v_2)\bigr| \leq L_i^g |v_1 - v_2|, \quad v_1, v_2 \in \mathbb{R}.
\end{aligned}$$

Assumption 2 Time-varying delays $\theta_{ji}(t)$ and $\tau_{ij}(t)$ are differentiable, and there exist constants $\mu_{ji}, \kappa_{ij} \in (0,1)$ such that

$$\begin{aligned}
&0 \leq \dot{\theta}_{ji}(t) \leq \mu_{ji}, \\
&0 \leq \dot{\tau}_{ij}(t) \leq \kappa_{ij}.
\end{aligned}$$

The corresponding system of (1) is given as

$$\begin{aligned}
\frac{\partial \tilde{u}_i}{\partial t} &= \sum_{k=1}^{\ell} \frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial \tilde{u}_i}{\partial x_k}\right) - p_i\tilde{u}_i(t,x) + \sum_{j=1}^{n} b_{ji} f_j\bigl(\tilde{v}_j(t,x)\bigr) \\
&\quad + \sum_{j=1}^{n} \tilde{b}_{ji} f_j\bigl(\tilde{v}_j\bigl(t - \theta_{ji}(t)\bigr)\bigr) + I_i + \phi_i(t,x), \\
\frac{\partial \tilde{v}_j}{\partial t} &= \sum_{k=1}^{\ell} \frac{\partial}{\partial x_k}\left(\tilde{D}_{jk}\frac{\partial \tilde{v}_j}{\partial x_k}\right) - q_j\tilde{v}_j(t,x) + \sum_{i=1}^{m} d_{ij} g_i\bigl(\tilde{u}_i(t,x)\bigr) \\
&\quad + \sum_{i=1}^{m} \tilde{d}_{ij} g_i\bigl(\tilde{u}_i\bigl(t - \tau_{ij}(t)\bigr)\bigr) + J_j + \varphi_j(t,x),
\end{aligned} \tag{4}$$

where $\phi_i(t,x)$ and $\varphi(t,x)$ are the external control inputs to be designed.

Let $e_i(t,x) = \tilde{u}_i(t,x) - u_i(t,x)$ and $z_j(t,x) = \tilde{v}_j(t,x) - v_j(t,x)$, then the error system between (1) and (4) is rewritten as

$$\begin{aligned} \frac{\partial e_i}{\partial t} &= \sum_{k=1}^{\ell} \frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial e_i}{\partial x_k}\right) - p_i e_i(t,x) \\ &\quad + \sum_{j=1}^{n} b_{ji}\tilde{f}_j\big(z_j(t,x)\big) + \sum_{j=1}^{n} \tilde{b}_{ji}\tilde{f}_j\big(z_j\big(t-\theta_{ji}(t)\big)\big) + \phi_i(t,x), \\ \frac{\partial z_j}{\partial t} &= \sum_{k=1}^{\ell} \frac{\partial}{\partial x_k}\left(\tilde{D}_{jk}\frac{\partial z_j}{\partial x_k}\right) - q_j z_j(t,x) \\ &\quad + \sum_{i=1}^{m} d_{ij}\tilde{g}_i\big(e_i(t,x)\big) + \sum_{i=1}^{m} \tilde{d}_{ij}\tilde{g}_i\big(e_i\big(t-\tau_{ij}(t)\big)\big) + \varphi_j(t,x), \end{aligned} \tag{5}$$

where

$$\begin{aligned} &\tilde{f}_j\big(z_j(t,x)\big) = f_j(\tilde{v}_j(t,x) - f_j\big(v_j(t,x)\big), \\ &\tilde{f}_j\big(z_j\big(t-\theta_{ji}(t),x\big)\big) = f_j(\tilde{v}_j\big(t-\theta_{ji}(t),x\big) - f_j\big(v_j\big(t-\theta_{ji}(t),x\big)\big), \\ &\tilde{g}_i\big(e_i(t,x)\big) = g_i(\tilde{u}_i(t,x) - g_i\big(u_i(t,x)\big), \\ &\tilde{g}_i\big(e_i\big(t-\tau_{ji}(t),x\big)\big) = g_i(\tilde{u}_i\big(t-\tau_{ji}(t),x\big) - g_i\big(u_i\big(t-\tau_{ji}(t),x\big)\big). \end{aligned}$$

Definition 1 ([26]) If the function $\psi(t): \mathbb{R}^+ \to (0,+\infty)$ satisfies the following four conditions:

(1) $\psi(t)$ is differentiable and nondecreasing;
(2) $\psi(0) = 1$ and $\psi(+\infty) = +\infty$;
(3) $\tilde{\psi}(t) := \frac{\dot{\psi}(t)}{\psi(t)}$ is nonincreasing;
(4) $\psi(\mu+\nu) \leq \psi(\mu)\psi(\nu)$ for all $\mu,\nu \geq 0$,

then the function $\psi(t)$ is called a ψ-type function.

Definition 2 If there exists a scalar $\varepsilon > 0$ such that

$$\limsup_{t\to\infty} \frac{\ln(\int_\Omega \|e(t,x)\|^2 + \|z(t,x)\|^2\,dx)}{\ln\psi(t)} \leq -\varepsilon,$$

where $e(t,x) = (e_1(t,x), e_2(t,x), \dots, e_m(t,x))^T$, $z(t,x) = (z_1(t,x), z_2(t,x), \dots, z_n(t,x))^T$, and $\psi(t)$ is a ψ-type function defined in Definition 1, then drive–response systems (1) and (4) are said to be general decay synchronized.

Lemma 1 *If there exist a function $\varrho(t) \in C(\mathbb{R}, \mathbb{R}^+)$, a Lyapunov functional $V(t): \mathbb{R}^+ \to \mathbb{R}^+$, and constants r_1, r_2 satisfying the following*:

$$\tilde{\psi}(t) \leq 1, \tag{6}$$

$$\sup_{t\in[0,+\infty)} \int_0^t \psi^\varepsilon(s)\varrho(s)\,ds < +\infty, \tag{7}$$

$$r_1 \int_\Omega \big\|e(t)\big\|^2 + \big\|z(t)\big\|^2\,dx \leq V(t), \tag{8}$$

$$\frac{dV(t)}{dt} + \varepsilon V(t) \le r_2 \varrho(t), \tag{9}$$

where ε is defined in Definition 2, while $\psi(t)$ and $\tilde{\psi}(t)$ are defined in Definition 1, then the drive–response systems (1) and (4) are general decay synchronized.

Proof It can be easily obtained that

$$\frac{d[\psi^{\varepsilon}(t)V(t)]}{dt} = \varepsilon \psi^{\varepsilon-1}\dot{\psi}(t)V(t) + \psi^{\varepsilon}\dot{V}(t),$$

from which we can get following by utilizing (6) and (9):

$$\begin{aligned}\frac{d[\psi^{\varepsilon}(t)V(t)]}{dt} &\le \varepsilon \psi^{\varepsilon}(t)\frac{\dot{\psi}(t)}{\psi(t)}V(t) - \varepsilon \psi^{\varepsilon}(t)V(t) + r_2\psi^{\varepsilon}\varrho(t) \\ &= \varepsilon \psi^{\varepsilon}(t)\big(\tilde{\psi}(t) - 1\big)V(t) + r_2\psi^{\varepsilon}(t)\varrho(t) \\ &\le r_2\psi^{\varepsilon}(t)\varrho(t).\end{aligned}$$

Using the condition $\psi(0) = 1$ in Definition 1 and (7), we get

$$\psi^{\varepsilon}(t)V(t) \le \int_0^t r_2\psi^{\varepsilon}\varrho(t)\,dt - V(0) < +\infty,$$

then, using (8), one has

$$r_1\psi^{\varepsilon}\int_{\Omega}\big\|e(t,x)\big\|^2 + \big\|z(t,x)\big\|^2\,dx < +\infty.$$

Thus there must exist a constant $M > 0$ such that

$$r_1\psi^{\varepsilon}\int_{\Omega}\big\|e(t,x)\big\|^2 + \big\|z(t,x)\big\|^2\,dx \le M,$$

which yields

$$\frac{\ln \int_{\Omega}\|e(t,x)\|^2 + \|z(t,x)\|^2\,dx}{\ln \psi(t)} \le \frac{\ln \frac{M}{r_1}}{\ln \psi(t)} - \varepsilon.$$

Finally,

$$\limsup_{t\to+\infty}\frac{\ln \int_{\Omega}\|e(t,x)\|^2 + \|z(t,x)\|^2\,dx}{\ln \psi(t)} \le \varepsilon.$$

This completes the proof. □

Lemma 2 (Poincaré inequality) *Assume Ω is a bounded open subset of $\mathbb{R}^n$ and suppose $u \in H_0^1(\Omega)$. Then we have*

$$\int_{\Omega} u^2\,dx \le \frac{1}{c}\int_{\Omega}\sum_{k=1}^{\ell}\left(\frac{\partial u}{\partial x_k}\right)^2 dx. \tag{10}$$

3 Main results

In this paper, the external control inputs $\phi_i(t,x)$ and $\varphi_j(t,x)$ are designed as

$$\begin{aligned} \phi_i(t,x) &= -\frac{\eta_i\|e(t,x)\|^2 e_i(t,x)}{(\|e(t,x)\|^2 + \varrho(t)}, \\ \varphi_j(t,x) &= -\frac{\lambda_j\|z(t,x)\|^2 z_j(t,x)}{\|z(t,x)\|^2 + \varrho(t)}, \end{aligned} \tag{11}$$

where η_i for $i \in \mathcal{I}$ and λ_j for $j \in \mathcal{J}$ are positive control gains.

Theorem 1 *Suppose that Assumptions 1 and 2 hold, and there exists a function $\varrho(t) \in C(\mathbb{R}, \mathbb{R}^+)$ satisfying (7) with a ψ-type function satisfying (6), then (4) can be general decay synchronized with system (1) under the feedback controller (11) if the control gains η_i and λ_j satisfy the following inequalities*:

$$\begin{aligned} &c_i D_i + p_i + \eta_i - \sum_{j=1}^{n}\left(\frac{L_i^g}{2}|d_{ij}| + \frac{L_j^f}{2}|b_{ji}| + \frac{L_j^f}{2}|\tilde{b}_{ji}| + \sigma_{ij}\chi_{ij} + \frac{\pi_{ij}}{1-\kappa_{ij}}\right) > 0, \\ &\tilde{c}_j \tilde{D}_j + q_j + \lambda_j - \sum_{i=1}^{m}\left(\frac{L_j^f}{2}|b_{ji}| + \frac{L_i^g}{2}|d_{ij}| + \frac{L_i^g}{2}|\tilde{d}_{ij}| + \tau_{ij}\nu_{ij} + \frac{\omega_{ij}}{1-\mu_{ij}}\right) > 0. \end{aligned} \tag{12}$$

Proof Consider the following Lyapunov functional:

$$V(t) = V_1(t) + V_2(t), \tag{13}$$

where

$$\begin{aligned} V_1(t) &= \int_\Omega \sum_{i=1}^{m}\frac{1}{2}e_i^2(t,x) + \sum_{i=1}^{m}\sum_{j=1}^{n}\int_{t-\sigma_{ij}(t)}^{t}\frac{\pi_{ij}}{(1-\kappa_{ij})}e_i^2(s,x)\,ds \\ &\quad + \sum_{i=1}^{m}\sum_{j=1}^{n}\int_{-\sigma_{ij}}^{0}\int_{t+s}^{t}\chi_{ij}e_i^2(r,x)\,dr\,ds\,dx, \\ V_2(t) &= \int_\Omega \sum_{j=1}^{n}\frac{1}{2}z_i^2(t,x) + \sum_{j=1}^{n}\sum_{i=1}^{m}\int_{t-\tau_{ij}(t)}^{t}\frac{\omega_{ij}}{(1-\mu_{ij})}z_j^2(s,x)\,ds \\ &\quad + \sum_{j=1}^{n}\sum_{i=1}^{m}\int_{-\tau_{ij}}^{0}\int_{t+s}^{t}\nu_{ij}z_i^2(r,x)\,dr\,ds\,dx. \end{aligned} \tag{14}$$

It is not difficult to prove that there exist positive scalars $\xi > 1$ and $\gamma > 1$ such that

$$\begin{aligned} &\int_\Omega \frac{1}{2}\sum_{i=1}^{m}e_i^2(t,x)\,dx \le V_1(t) \le \int_\Omega \xi\sum_{i=1}^{m}e_i^2(t,x) + \frac{\xi}{\alpha}\sum_{i=1}^{m}\sum_{j=1}^{n}\chi_{ij}\int_{t-\sigma_{ij}}^{t}e_i^2(s,x)\,ds\,dx, \\ &\int_\Omega \frac{1}{2}\sum_{j=1}^{n}z_i^2(t,x)\,dx \le V_2(t) \le \int_\Omega \gamma\sum_{j=1}^{n}z_i^2(t,x) + \frac{\gamma}{\beta}\sum_{j=1}^{n}\sum_{i=1}^{m}\nu_{ij}\int_{t-\tau_{ij}}^{t}z_i^2(s,x)\,ds\,dx, \end{aligned} \tag{15}$$

where $\alpha = \min_{i\in\mathcal{I}}\{\alpha_i\}$, $\beta = \min_{j\in\mathcal{J}}\{\beta_j\}$ with

$$\alpha_i = c_iD_i + p_i + \eta_i - \sum_{j=1}^{n}\left(\frac{L_i^g}{2}|d_{ij}| + \frac{L_j^f}{2}|b_{ji}| + \frac{L_j^f}{2}|\tilde{b}_{ji}| + \sigma_{ij}\chi_{ij} + \frac{\pi_{ij}}{1-\kappa_{ij}}\right),$$

$$\beta_j = \tilde{c}_j\tilde{D}_j + q_j + \lambda_j - \sum_{i=1}^{m}\left(\frac{L_j^f}{2}|b_{ji}| + \frac{L_i^g}{2}|d_{ij}| + \frac{L_i^g}{2}|\tilde{d}_{ij}| + \tau_{ij}\nu_{ij} + \frac{\omega_{ij}}{1-\mu_{ij}}\right).$$

where $D_i = \min_k\{D_{ik}\}$ and $\tilde{D}_j = \min_k\{\tilde{D}_{jk}\}$ for all $k \in \{1,2,\dots,\ell\}$.

Now, calculate the time derivative of $V_1(t)$ and $V_2(t)$:

$$\begin{aligned}
\dot{V}_1 &= \int_\Omega \sum_{i=1}^{m}\Biggl[e_i(t,x)\frac{\partial e_i}{\partial t}(t,x) + \sum_{j=1}^{n}\left(\frac{\pi_{ij}}{(1-\kappa_{ij})}e_i^2(t,x) - \frac{\pi_{ij}(1-\dot{\sigma}_{ij}(t))}{(1-\kappa_{ij})}e_i^2\bigl(t-\sigma_{ij}(t),x\bigr)\right)\Biggr] \\
&\quad + \sum_{j=1}^{n}\chi_{ij}\int_{-\sigma_{ij}}^{0} e_i^2(t,x) - e_i^2(t+s,x)\,ds]\,dx \\
&= \int_\Omega \sum_{i=1}^{m}\Biggl[e_i(t,x)\sum_{k=1}^{\ell}D_{ik}\frac{\partial^2 e_i}{\partial x_k^2} - p_ie_i^2(t,x) + \sum_{j=1}^{n}b_{ji}e_i(t,x)\tilde{f}_j\bigl(z_j(t,x)\bigr) \\
&\quad + \sum_{j=1}^{n}\tilde{b}_{ji}e_i(t,x)\tilde{f}_j\bigl(z_j\bigl(t-\theta_{ji}(t),x\bigr)\bigr) - \frac{\eta_i\|e(t,x)\|^2e_i^2(t,x)}{(\|e(t,x)\|^2 + \varrho(t))} \\
&\quad + \sum_{j=1}^{n}\left(\frac{\pi_{ij}}{(1-\kappa_{ij})}e_i^2(t,x) - \frac{\pi_{ij}(1-\dot{\sigma}_{ij}(t))}{(1-\kappa_{ij})}e_i^2\bigl(t-\sigma_{ij}(t),x\bigr)\right) \\
&\quad + \sum_{j=1}^{n}\sigma_{ij}\chi_{ij}e_i^2(t,x) - \sum_{j=1}^{n}\chi_{ij}\int_{t-\sigma_{ij}}^{t}e_i^2(s,x)\,ds\Biggr]\,dx,
\end{aligned}$$

where

$$\begin{aligned}
\int_\Omega e_i(t,x)\sum_{k=1}^{\ell}D_{ik}\frac{\partial^2 e_i}{\partial x_k^2}\,dz &= \int_{\partial\Omega} e_i(t,z)\sum_{k=1}^{\ell}D_{ik}\frac{\partial e_i}{\partial x_k}\,dx - \int_\Omega\sum_{k=1}^{\ell}D_{ik}\left(\frac{\partial e_i}{\partial x_k}\right)^2dx \\
&= -\int_\Omega\sum_{k=1}^{\ell}D_{ik}\left(\frac{\partial e_i}{\partial x_k}\right)^2dx.
\end{aligned}$$

Since $D_i = \min_k\{D_{ik}\}$ for all $k \in \{1,2,\dots,\ell\}$, one gets the following inequality via Lemma 2:

$$\int_\Omega e_i(t,x)\sum_{k=1}^{\ell}D_{ik}\frac{\partial^2 e_i}{\partial x_k^2}\,dx \le -\int_\Omega c_iD_ie_i^2(t,x)\,dx.$$

Therefore,

$$\begin{aligned}
\dot{V}_1(t) \le \int_\Omega\sum_{i=1}^{m}\Biggl[&-c_iD_ie_i^2(t,x) - p_ie_i^2(t,x) + \sum_{j=1}^{n}b_{ji}e_i(t,x)\tilde{f}_j\bigl(z_j(t,x)\bigr) \\
&+ \sum_{j=1}^{n}\tilde{b}_{ji}e_i(t,x)\tilde{f}_j\bigl(z_j\bigl(t-\theta_{ji}(t),x\bigr)\bigr) - \frac{\eta_i\|e(t,x)\|^2e_i^2(t,x)}{(\|e(t,x)\|^2 + \varrho(t))}
\end{aligned}$$

$$+\sum_{j=1}^{n}\left(\frac{\pi_{ij}}{(1-\kappa_{ij})}e_i^2(t,x)-\frac{\pi_{ij}(1-\dot{\sigma}_{ij}(t))}{(1-\kappa_{ij})}e_i^2\bigl(t-\sigma_{ij}(t),x\bigr)\right)$$
$$\left.+\sum_{j=1}^{n}\sigma_{ij}\chi_{ij}e_i^2(t,x)-\sum_{j=1}^{n}\chi_{ij}\int_{t-\sigma_{ij}}^{t}e_i^2(s,x)\,ds\right]dx.$$

In the light of Assumption 1, we have

$$\begin{aligned}\dot{V}_1(t)\leq{}&\int_{\Omega}-\sum_{i=1}^{m}(c_iD_i+p_i)e_i^2(t,x)+\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{L_j^f}{2}|b_{ji}|\bigl(e_i^2(t,x)+z_j^2(t,x)\bigr)\\&+\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{L_j^f}{2}|\tilde{b}_{ji}|\bigl(e_i^2(t,x)+z_j^2\bigl(t-\theta_{ji}(t),x\bigr)\bigr)-\sum_{i=1}^{m}\eta_ie_i^2(t,x)\\&+\sum_{i=1}^{m}\left(\eta_ie_i^2(t,x)-\frac{\eta_i\|e(t,x)\|^2e_i^2(t,x)}{(\|e(t,x)\|^2+\varrho(t))}\right)+\sum_{i=1}^{m}e_i^2(t,x)\sum_{j=1}^{n}\frac{\pi_{ij}}{1-\kappa_{ij}}\\&+\sum_{i=1}^{m}\sum_{j=1}^{n}\sigma_{ij}\chi_{ij}e_i^2(t,x)-\sum_{i=1}^{m}\sum_{j=1}^{n}\chi_{ij}\int_{t-\sigma_{ij}}^{t}e_i^2(s,x)\,ds\,dx.\end{aligned}$$

Here we used the following inequality:

$$\begin{aligned}\sum_{i=1}^{m}\left(\eta_ie_i^2(t,x)-\frac{\eta_i\|e(t,x)\|^2e_i^2(t,x)}{(\|e(t,x)\|^2+\varrho(t))}\right)&=\sum_{i=1}^{m}\frac{\eta_i\varrho(t)e_i^2(t,x)}{\|e(t,x)|^2+\varrho(t)}\\&\leq\max_{i\in\mathcal{I}}\{\eta_i\}\frac{\varrho(t)}{\|e(t,x)\|^2+\varrho(t)}\sum_{i=1}^{m}e_i^2(t,x)\\&=\eta\frac{\varrho(t)\|e(t,x)\|^2}{\|e(t,x)\|^2+\varrho(t)}\leq\eta\varrho(t).\end{aligned}$$

Due to $0\leq ab/(a+b)\leq a$ for any $a>0$ and $b>0$, where $\eta=\max_{i\in\mathcal{I}}\{\eta_i\}>0$,

$$\begin{aligned}\dot{V}_1(t)\leq{}&\int_{\Omega}\sum_{i=1}^{m}\left[-c_iD_i-p_i-\eta_i+\sum_{j=1}^{n}\left(\frac{L_j^f}{2}|b_{ji}|+\frac{L_j^f}{2}|\tilde{b}_{ji}|+\frac{\pi_{ij}}{1-\kappa_{ij}}+\sigma_{ij}\chi_{ij}\right)\right]e_i^2(t,x)\\&+\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{L_j^f}{2}|b_{ji}|z_j^2(t,x)+\eta\varrho(t)-\sum_{i=1}^{m}\sum_{j=1}^{n}\chi_{ij}\int_{t-\sigma_{ij}}^{t}e_i^2(s,x)\,ds\,dx.\end{aligned}$$

Similarly,

$$\begin{aligned}\dot{V}_2(t)\leq{}&\int_{\Omega}\sum_{j=1}^{n}\left[-\tilde{c}_j\tilde{D}_j-q_j-\lambda_j+\sum_{i=1}^{m}\left(\frac{L_i^g}{2}|d_{ij}|+\frac{L_i^g}{2}|\tilde{d}_{ij}|+\frac{\omega_{ij}}{1-\mu_{ij}}+\tau_{ij}\nu_{ij}\right)\right]z_j^2(t,x)\\&+\sum_{j=1}^{n}\sum_{i=1}^{m}\frac{L_i^g}{2}|d_{ij}|e_i^2(t,x)+\lambda\varrho(t)-\sum_{j=1}^{n}\sum_{i=1}^{m}\nu_{ij}\int_{t-\tau_{ij}}^{t}z_j^2(s,x)\,ds\,dx,\end{aligned}$$

where $\lambda = \max_{j\in\mathcal{J}}\{\lambda_j\} > 0$. Then we have

$$\begin{aligned}
\dot{V}(t) \le{}& \int_\Omega \sum_{i=1}^{m}\Biggl[-c_iD_i - p_i - \eta_i + \sum_{j=1}^{n}\biggl(\frac{L_i^g}{2}|d_{ij}| + \frac{L_j^f}{2}|b_{ji}| + \frac{L_j^f}{2}|\tilde{b}_{ji}| + \sigma_{ij}\chi_{ij} \\
&+ \frac{\pi_{ij}}{1-\kappa_{ij}}\biggr)\Biggr]e_i^2(t,x) + \eta\varrho(t) - \sum_{i=1}^{m}\sum_{j=1}^{n}\chi_{ij}\int_{t-\sigma_{ij}}^{t} e_i^2(s,x)\,ds \\
&+ \sum_{j=1}^{n}\Biggl[-\tilde{c}_j\tilde{D}_j - q_j - \lambda_j + \sum_{i=1}^{m}\biggl(\frac{L_j^f}{2}|b_{ji}| + \frac{L_i^g}{2}|d_{ij}| + \frac{L_i^g}{2}|\tilde{d}_{ij}| + \tau_{ij}\nu_{ij} \\
&+ \frac{\omega_{ij}}{1-\mu_{ij}}\biggr)\Biggr]z_j^2(t,x) + \lambda\varrho(t) - \sum_{j=1}^{n}\sum_{i=1}^{m}\nu_{ij}\int_{t-\tau_{ij}}^{t} z_j^2(s,x)\,ds\,dx \\
\le{}& \int_\Omega -\alpha\sum_{i=1}^{m} e_i^2(t,x) + \eta\varrho(t) - \sum_{i=1}^{m}\sum_{j=1}^{n}\chi_{ij}\int_{t-\sigma_{ij}}^{t} e_i^2(s,x)\,ds \\
&- \beta\sum_{j=1}^{n} z_j^2(t,x) + \lambda\varrho(t) - \sum_{j=1}^{n}\sum_{i=1}^{m}\nu_{ij}\int_{t-\tau_{ij}}^{t} z_j^2(s,x)\,ds\,dx.
\end{aligned}$$

Then, there exists a small enough δ satisfying $\delta\xi \le \alpha$ and $\delta\gamma \le \beta$ such that

$$\begin{aligned}
\dot{V}(t) + \delta V(t) \le{}& \int_\Omega -\alpha\sum_{i=1}^{m} e_i^2(t,x) + \eta\varrho(t) - \sum_{i=1}^{m}\sum_{j=1}^{n}\chi_{ij}\int_{t-\sigma_{ij}}^{t} e_i^2(s,x)\,ds \\
&+ \delta\Biggl(\xi\sum_{i=1}^{m} e_i^2(t,x) + \frac{\xi}{\alpha}\sum_{i=1}^{m}\sum_{j=1}^{n}\chi_{ij}\int_{t-\sigma_{ij}}^{t} e_i^2(s,x)\,ds\Biggr) \\
&- \beta\sum_{j=1}^{n} z_j^2(t,x) + \lambda\varrho(t) - \sum_{j=1}^{n}\sum_{i=1}^{m}\nu_{ij}\int_{t-\tau_{ij}}^{t} z_j^2(s,x)\,ds \\
&+ \delta\Biggl(\gamma\sum_{j=1}^{n} z_i^2(t,x) + \frac{\gamma}{\beta}\sum_{j=1}^{n}\sum_{i=1}^{m}\nu_{ij}\int_{t-\tau_{ij}}^{t} z_i^2(s,x)\,ds\Biggr)dx \\
={}& (\delta\xi - \alpha)\sum_{i=1}^{m} e_i^2(t,x) + (\eta+\lambda)\varrho(t) - \biggl(\frac{\xi\delta}{\alpha} - 1\biggr)\sum_{i=1}^{m}\sum_{j=1}^{n}\chi_{ij}\int_{t-\sigma_{ij}}^{t} e_i^2(s,x)\,ds \\
&+ (\delta\gamma - \beta)\sum_{j=1}^{n} z_j^2(t,x) - \biggl(\frac{\gamma\delta}{\beta} - 1\biggr)\sum_{j=1}^{n}\sum_{i=1}^{m}\nu_{ij}\int_{t-\tau_{ij}}^{t} z_j^2(s,x)\,ds \\
\le{}& \int_\Omega (\eta+\lambda)\varrho(t)\,dx \\
={}& (\eta+\lambda)|\Omega|\varrho(t),
\end{aligned}$$

and we have

$$\dot{V}(t) + \delta V(t) \le (\eta+\lambda)|\Omega|\varrho(t). \tag{16}$$

Therefore, by Lemma 1, the drive–response systems (1) and (4) achieve general decay synchronization under the nonlinear feedback controller (11). The convergence rate of $e(t)$ and $z(t)$ approaching zero is δ. The proof is completed. □

Remark 1 In this paper, we firstly studied the general decay synchronization of BAMNN with reaction–diffusions terms and time-varying delay by introducing a novel nonlinear feedback controller and using some inequality techniques. It it not difficult to see that the results obtained in [13, 16, 21, 23] can be seen as special cases of our results when the general decay function is chosen as $\psi(t) = e^{\alpha t}$ or $\psi(t) = (1+t)^{\alpha}$ for any $\alpha > 0$. From this viewpoint, our results are more general and have better applicability.

4 Numerical examples

In this section, a numerical example is provided to validate the effectiveness of the established theoretical results in this paper.

Example For $\ell = 1$, $n = m = 2$, consider the following BAMNN with reaction–diffusion terms,

$$\begin{aligned} \frac{\partial u_i}{\partial t} &= D_{i1}\frac{\partial}{\partial x_k}\left(\frac{\partial u_i}{\partial x_k}\right) - p_i u_i(t,x) + \sum_{j=1}^{2} b_{ji} f_j\big(v_j(t,x)\big) + \sum_{j=1}^{2} \tilde{b}_{ji} f_j\big(v_j\big(t-\theta_{ji}(t)\big)\big) + I_i, \\ \frac{\partial v_j}{\partial t} &= \tilde{D}_{j1}\frac{\partial}{\partial x_k}\left(\frac{\partial v_j}{\partial x_k}\right) - q_j v_j(t,x) + \sum_{i=1}^{2} d_{ij} g_i\big(u_i(t,x)\big) + \sum_{i=1}^{2} \tilde{d}_{ij} g_i\big(u_i\big(t-\tau_{ij}(t)\big)\big) + J_j, \end{aligned} \tag{17}$$

for $x \times t \in [-5,5] \times [0,1]$, with the following Neumann boundary condition:

$$\left.\frac{\partial u_i}{\partial x}\right|_{x=\pm 5} = 0, \qquad \left.\frac{\partial v_j}{\partial x}\right|_{x=\pm 5} = 0, \quad \text{for } t \in [0,1], \tag{18}$$

where the parameters of system (17) are taken as $D_{i1} = \tilde{D}_{j1} = 0.1$, $p_i = 1$, $q_j = 1.05$, $b_{11} = 1.8$, $b_{12} = -0.15$, $b_{21} = -5.2$, $b_{22} = 3.5$, $\tilde{b}_{11} = -1.7$, $\tilde{b}_{12} = -0.12$, $\tilde{b}_{21} = -0.26$, $\tilde{b}_{22} = -2.5$, $d_{11} = 1.71$, $d_{12} = -0.1425$, $d_{21} = -4.94$, $d_{22} = 3.325$, $\tilde{d}_{11} = -1.87$, $\tilde{d}_{12} = -0.132$, $\tilde{d}_{21} = -0.286$, $\tilde{d}_{22} = -2.75$, $I_i = J_j = 0$, neuron activation functions are chosen as $f_i(v) = g_j(v) = \tanh(v)$, and time-varying delays are chosen as $\theta_{ji}(t) = \tau_{ij}(t) = \frac{e^t}{1+e^t}$ for $i,j \in \{1,2\}$. The initial conditions of system (17) are chosen as

$$\begin{aligned} &u_1(s,x) = 0.2\cos(\pi x) + 0.2, \qquad u_2(s,x) = 0.5\cos(\pi x) + 0.5, \\ &v_1(s,x) = 0.4\cos(\pi x) + 0.3, \qquad v_2(s,x) = 0.7\cos(\pi x) + 0.2 \\ &\quad \text{for } s \times x \in [-1,0] \times [-5,5]. \end{aligned}$$

Matlab simulations of the drive system (17) under the above initial conditions are presented in Figs. 1–8, where we can see that the drive system (17) has a chaotic attractor.

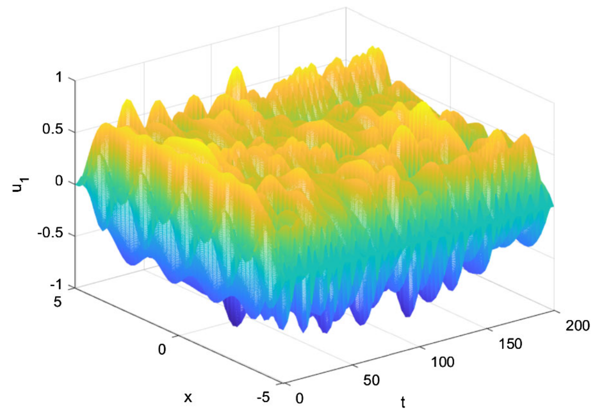

Figure 1 Chaotic attractor of u_1 in (17)

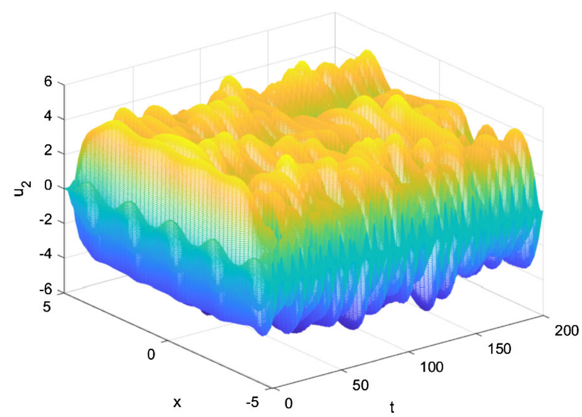

Figure 2 Chaotic attractor of u_2 in (17)

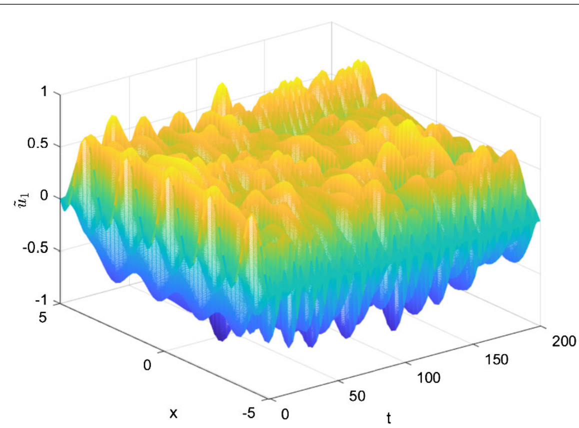

Figure 3 Chaotic attractor of $\tilde{u}_1$ in (17)

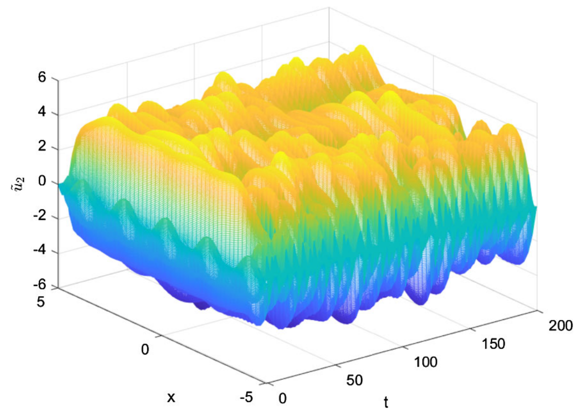

Figure 4 Chaotic attractor of $\tilde{u}_2$ in (17)

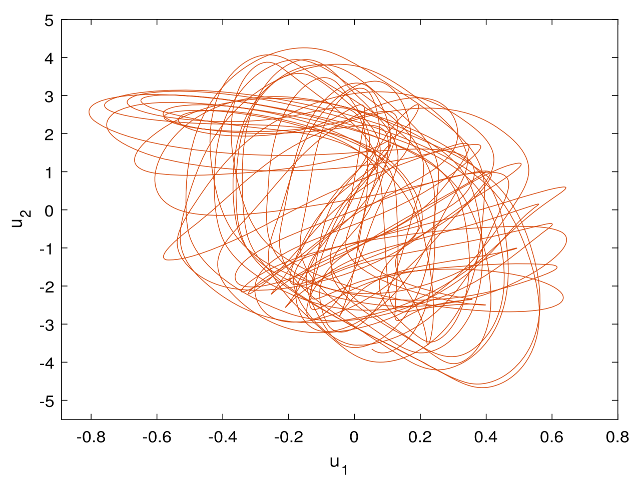

Figure 5 Chaotic attractor of u_1 and u_2 in (17) with $x = -4$

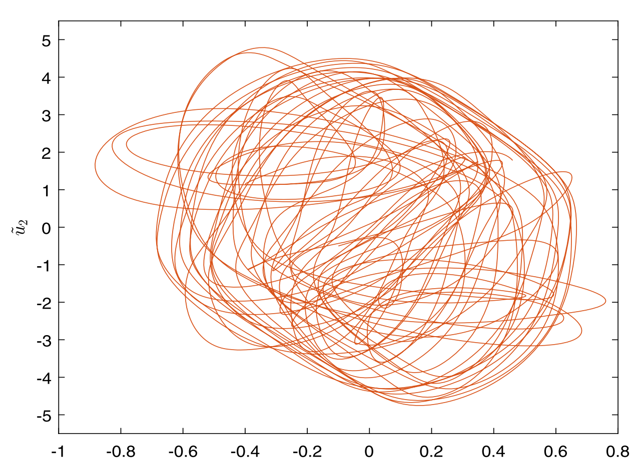

Figure 6 Chaotic attractor of $\tilde{u}_1$ and $\tilde{u}_2$ in (17) with $x = -1$

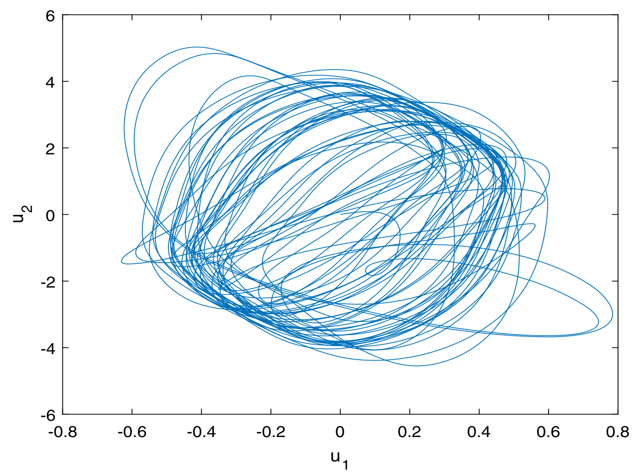

Figure 7 Chaotic attractor of u_1 and u_2 in (17) with $x = 1$

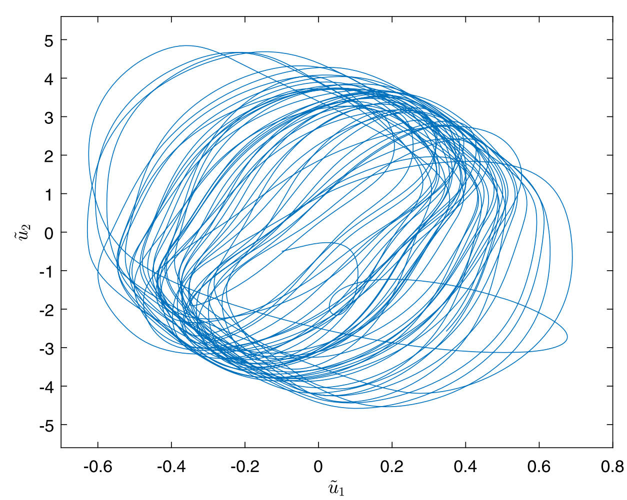

Figure 8 Chaotic attractor of $\tilde{u}_1$ and $\tilde{u}_2$ in (17) with $x = 3$

The corresponding response system for the drive system (17) is given as

$$\begin{aligned}
\frac{\partial \tilde{u}_i}{\partial t} &= D_{i1}\frac{\partial}{\partial x}\left(\frac{\partial \tilde{u}_i}{\partial x}\right) - p_i\tilde{u}_i(t,x) + \sum_{j=1}^{2} b_{ji}f_j\big(\tilde{v}_j(t,x)\big) \\
&\quad + \sum_{j=1}^{2} \tilde{b}_{ji}f_j\big(\tilde{v}_1\big(t-\theta_{11}(t)\big)\big) + I_i + \phi_i, \\
\frac{\partial \tilde{v}_j}{\partial t} &= \tilde{D}_{j1}\frac{\partial}{\partial x}\left(\frac{\partial v_j}{\partial x}\right) - q_jv_j(t,x) + \sum_{i=1}^{2} d_{ij}g_i\big(\tilde{u}_i(t,x)\big) \\
&\quad + \sum_{i=1}^{2} \tilde{d}_{ij}g_i\big(\tilde{u}_i\big(t-\tau_{11}(t)\big)\big) + J_j + \varphi_j,
\end{aligned} \tag{19}$$

for $x \times t \in [-5,5] \times [0,1]$, with the following Neumann boundary condition:

$$\left.\frac{\partial \tilde{u}_1}{\partial x}\right|_{x=\pm 1} = 0, \qquad \left.\frac{\partial \tilde{v}_1}{\partial x}\right|_{x=\pm 1} = 0, \quad \text{for } t \in [0,1], \tag{20}$$

where $\phi_i(t,x)$ and $\varphi_j(t,x)$ are given as follows:

$$\begin{aligned} \phi_i(t,x) &= -\frac{\eta_i \|e(t,x)\|^2 e_i(t,x)}{(\|e(t,x)\|^2 + \varrho(t)}, \\ \varphi_j(t,x) &= -\frac{\lambda_j \|z(t,x)\|^2 z_j(t,x)}{\|z(t,x)\|^2 + \varrho(t)}, \end{aligned} \tag{21}$$

where $\eta_1 = 5.6$, $\eta_2 = 6.8$, $\lambda_1 = 6.2$, $\lambda_2 = 7.1$, $\varrho(t) = e^{-0.1t}$. The other parameters in (19) are the same as of system (17). The initial conditions of (19) are taken as

$$\begin{aligned} &\tilde{u}_1(s,x) = 0.8\cos(\pi x) + 0.5, \qquad \tilde{u}_2(s,x) = -0.5\cos(\pi x) + 0.1, \\ &\tilde{v}_1(s,x) = 1.2\cos(\pi x) + 0.5, \qquad \tilde{v}_2(s,x) = -0.5\cos(\pi x) + 0.7 \\ &\quad \text{for } s \times x \in [-1,0] \times [-5,5]. \end{aligned}$$

It is not difficult to estimate that $L^f{}_i = L_j^g = 1$ and $\mu_{ji} = \kappa_{ij} = 1$. Thus, Assumptions 1 and 2 are satisfied. Letting $\psi(t) = e^t$, inequalities (6), (7), and (12) are satisfied. Therefore, according to Theorem 1, the drive–response systems (17) and (19) can achieve general decay synchronization under the controller (19). The time evolution of synchronization errors between master–slave systems (17) and (19) are show in Figs. 9–12, from where we can see that $e_i(t,x)$ and $z_j(t,x)$ are very close to 0 and maintain this situation as time t increases.

Remark 2 Unlike the synchronization studies for neural networks such as [13, 16, 20, 21, 23], the convergence rate of the synchronization error can be controlled by appropriately choosing the function $\varrho(t)$ in the paper, this is because general decay synchronization en-

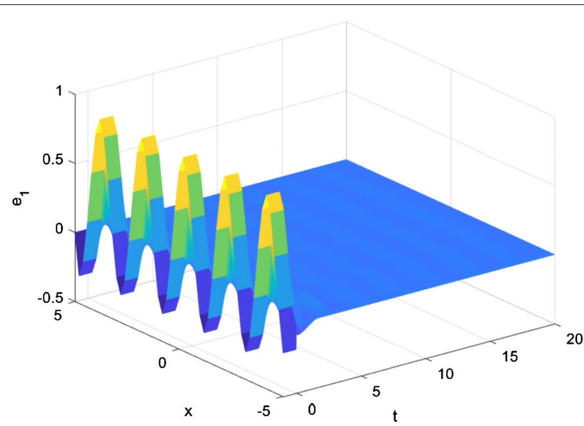

Figure 9 Evaluation of synchronization error e_1

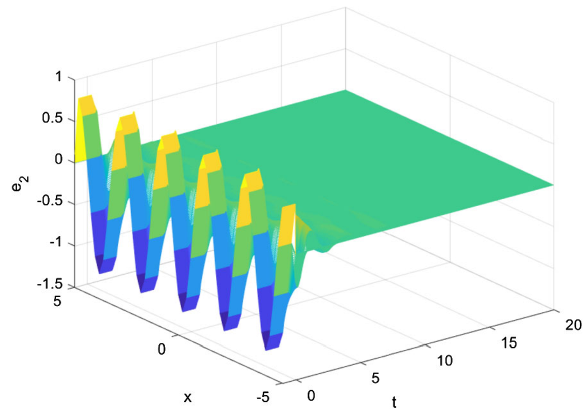

Figure 10 Evaluation of synchronization error e_2

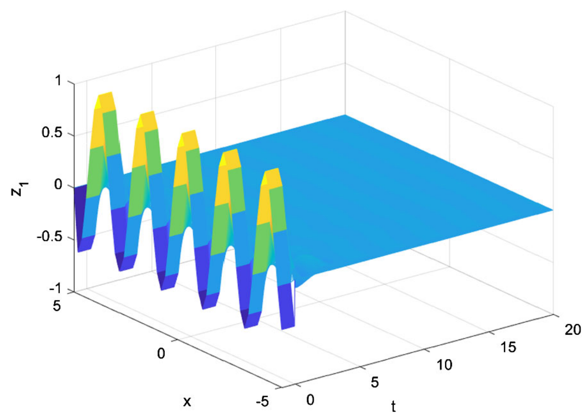

Figure 11 Evaluation of synchronization error z_1

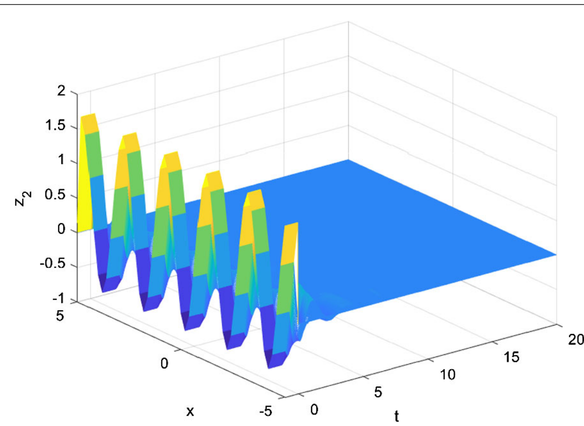

Figure 12 Evaluation of synchronization error z_2

ables us to estimate to convergence rate of synchronization error via defining a more general convergence rate. From this viewpoint, our results are more general and have better applicability.

5 Conclusions

In this paper, first, we introduce a useful lemma to determine the general decay synchronization of the considered drive–response system, and then the nonlinear feedback controllers are designed for synchronization of delayed reaction–diffusion bidirectional associative memory neural networks with Neumann boundary condition via applying Lyapunov functional method and Poincaré inequality, and useful new conditions are obtained. This result generalizes the previous results to some extent [15, 21, 26]. In addition, one numerical example is given to show the effectiveness of the proposed model.

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The authors declare that they have no competing interests.

Authors' contributions
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