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Flat Ride; Problems and Solutions in Vehicle

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Abstract. Flat ride is the condition in which the uncomfortable pitch oscillation of the vehicle body turns into more tolerable bounce oscillation, when a car hits a bump in forward motion. Based on experimental results, Maurice Olley discovered and introduced two conditions for flat ride:
1. The radius of gyration in pitch should be equal to the product of the distance from the mass centers a_1, a_2 of the front and rear wheels of the car ($r^2 = a_1 a_2$).
2. The rear suspension should have around 20% higher rate then the front. The equation $r^2 = a_1 a_2$ makes the car to be considered as two separated uncoupled mass-spring systems of front and rear suspensions.
In this study, we will analytically review the flat ride conditions and provide design charts to satisfy the required conditions. The nonlinear practical model of shock absorbers modifies the conditions which were based on linear models.

Keywords. Flat ride, Maurice Olley, Optimal suspension, Vehicle vibrations, Vehicle dynamics, Suspension design, Suspension optimization.

1 Flat Ride Definition

The excitation inputs from the road to a straight moving car will affect the front wheels first and then, with a time lag, the rear wheels. The general recommendation was that the natural frequency of the front suspension should be lower than that of the rear. So, the rear part oscillates faster to catch up with the front to eliminate pitch and put the car in bounce before the vibrations die out by damping. This is what Olley called the *Flat Ride Tuning* [4, 6–8]. Maurice Olley (1889 – 1983) established guidelines, back in the 1930, for designing vehicles with better ride. These were derived from experiments with a modified car to allow variation of the pitch mass moment. Although the measures of ride were strictly subjective, those guidelines are considered as valid rules of thumb even for modern cars. What is known as Olley's Flat Ride not considering the other prerequisites can be put forward as:

The front suspension should have around 30% *lower rate than the rear.*

An important prerequisite for flat ride was the uncoupling condition, which was introduced by Rowell and Guest for the first time in 1923 [4, 6–9]. Rowell and Guest used the geometry of a bicycle car model to find the condition which sets the bounce and pitch centers of the model located on the springs. Having the condition, the front and rear spring systems of the vehicle can be regarded as two separate one degree of freedom systems.

In this study, using analytical methods, we study the flat ride conditions which has been respected and followed by the car manufacturers' designers since they were introduced for the first time. This article will provide a more reliable scientific and mathematical approach for what are the flat ride design criteria in vehicle dynamic studies.

2 Previous works

Maurice Olley was one of the first pioneers who introduced and studied the concept of flat ride in vehicle dynamics. He was an English engineer born in 1889, who during his life added a lot to the general knowledge of vehicle dynamics and is counted as one of the great automobile engineers of his era. He is one of the founders of modern vehicle dynamics. In his early career in the Rolls Royce design office, he worked under Sir Henry Royce but the majority of his career was spent at Cadillac in the USA and Vauxhall in England.

Olley worked directly for Sir Henry Royce, and was in the United States for some ten years struggling to get off the ground the manufacture of Rolls-Royce cars at Springfield, Massachusetts. The financial crash of 1929 put the skids under the operation. His first task after moving to the Cadillac company in 1930 was suspension and ride. He introduced the Rolls-Royce type of bump rig and began a full program of ride development. He studied the oscillation of wheels and tires and by applying some changes on the rig was soon studying the basic ride motions of the car. In his paper [4] he published the results taken from his experiments using the test rig for the first time.

He developed a bouncing table rig in General Motors proving grounds, on which humans were vibrated vertically at different frequencies and amplitudes. They would have increased the frequency till the person on the table begins to feel uncomfortable. Using this equipment Olley explained the relation between vertical acceleration and comfort over

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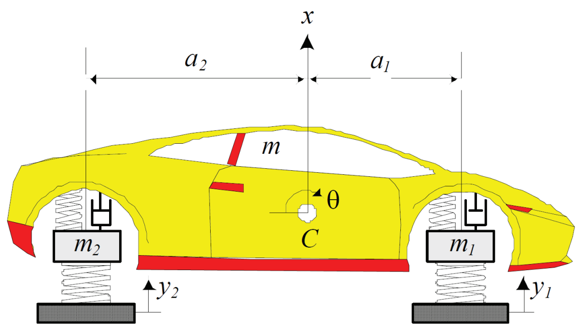

Figure 1. Bicycle car model used for vibration analysis.

a range of frequencies. He generated a curve for passenger comfort, which is very similar to the current ISO2631 standard.

Olley as well as other investigators in well-established car companies realized that the pitch and roll modes of the car body are much more uncomfortable than the bounce mode. The investigators' effort focused on the suspension stiffness and damping rates to be experimentally adjusted to provide acceptable vertical vibrations. However, the strategy about roll and pitch modes were to transform them to bounce. Due to usual geometric symmetry of cars, as well as the symmetric excitation from the road, roll mode are being excited much less than pitch mode. Therefore, lots of investigations have been focused on adjustment of the front and rear suspensions such that pitch mode of vibration transform to the bounce.

In the early 1930s most cars were built with fairly stiff springs at the front and soft at the rear, with a $\frac{r^2}{a_1 a_2}$ ratio in pitch of about 0.8, where r is the pitch radius of gyration of the car and a_1 and a_2 are the distance of the mass center, C, from the front and rear axles, as shown in Figure 1, [1]. However, based on what Olley discovered, such a choice was against the mode transfer desire.

Besides all the important facts that Olley discovered during his experiments, the principle known as the Flat Ride Tuning or Olley's Flat Ride proved to be more industry approved and accepted. After his publications [6–8] in which he advocated this design practice, they became rules of practice.

We can summarize what has been said about ride and comfort in American passenger cars by Olley as the following:

(i) The front spring should be softer than the rear for Flat Ride Tuning. This will promote bouncing of the body rather than pitching motions at least for a greater majority of speeds and bump road situations. The front suspension should have a 30% lower ride rate than the rear suspension, or the spring center should be at least 6.5% of the wheelbase behind the center of gravity. Although this does not explicitly determine the front and rear natural frequencies, since the front-rear weight distribution on passenger cars is close to 50-50, it will generally assure that the rear frequency is greater than the front.

(ii) The ratio $\frac{r^2}{a_1 . a_2}$ normally approaches unity. This reduces vibration interactions between front and rear because the two suspensions can now be considered as two separate systems. As a consequence there will be less resonant build-ups on the road and the pitching frequency will have a magnitude closer to that of bounce.

(iii) The pitch and bounce frequencies should be close together: the bounce frequency should be less than 1.2 times the pitch frequency. For higher ratios interference kicks resulting from the superposition of the two motions. This condition will be met for modern cars because the dynamic index is near unity with the wheels located near the forward and rearward extremes of the chassis.

(iv) Neither frequency should be greater than 1.3 Hz, which means that the effective static deflection of the vehicle should exceed roughly 6 inches.

(v) The roll frequency should be approximately equal to the pitch and bounce frequencies. To minimize roll vibrations the natural frequency in roll needs to be low just as for the bounce and pitch modes.

Rowell and Guest [9] in 1923 identified the value of $\frac{r^2}{a_1 . a_2}$ being associated with vehicles in which the front and rear responses were uncoupled. Olley was able to investigate the issue experimentally and these experiments led him to the belief that pitching motion was extremely important in the subjective assessment of vehicle ride comfort. He built the Cadillac k_2 rig in 1931 which was a 12 cylinder, 7 passenger Cadillac limousine of the period, fitted with front and rear outriggers each of which could carry up to 327kg made up in 27kg weights. To their surprise, under these supposedly ideal conditions, they still got an unsatisfactory ride. This arrangement gave no fixed oscillation centers and the ride had no pattern. However, by fitting all the weights they found that if the front spring static deflections are some 30% greater than the rear then the revolutionary flat ride occurs. Olley's explanation was that because the two ends of the car did not cross a given disturbance at the same instant it was important that the front wheels initiated the slower mode and that the rear wheels initiated the faster mode. This allowed the body movement at the rear to catch up the front and so produce the flat ride.

The condition of Flat Ride is expressed in various detailed forms; however, the main idea states that *the front suspension should have a* 30% *lower ride rate then the rear.* The physical explanation for why this is beneficial in reducing pitch motion is usually argued based on the time history of events following a vehicle hitting a bump. First, the front of the vehicle responds “approximately- in the well-known damped oscillation manner. At some time later, controlled by the wheelbase and the vehicle speed, the rear responds in similar fashion. The net motion of the vehicle is then crudely some summation of these two motions which minimizes the vehicle pitch response, [2].

Confirmation of the effectiveness in pitch reduction of the Olley design was given by Best [1] over a limited range of circumstances. Random road excitation was applied to a half-car computer model, with identical front and rear excitations, considering the time delay generated by the wheelbase and vehicle’s speed. Pitch suppression was associated with the wheelbase filtering effect. Pitch suppression appeared to be necessarily associated with increases in bounce response, leaving in unclear whether or not it is a worthwhile goal, [10].

Sharp and Pilbeam [11] attempted a more fundamental investigation of the phenomenon, primarily by calculating frequency response for the half-car over a wide range of speed and design conditions. At higher speeds, remarkable reductions in pitch response with only small costs in terms of bounce response were shown. At low speeds, the situation is reversed. These behavioral features were shown to be generic insofar as variations in mass center location, pitch inertia and damping level were concerned, and the implifications from the frequency responses were confirmed by simulations with nonlinear asymmetric suspension damping.

Later on Sharp [10] discussed the rear to front stiffness tuning of the suspension system of a car, through reference to a half-car pitch plane mathematical model. He used new results relating to the frequency responses of the bouncing and pitching motions of the car body to show that the pitch minimization mechanism of Olley’s Flat Ride tuning “involves interference between the responses to the front and rear axle inputs. He showed that interference with respect to the rotational motion implies reinforcement with respect to the translational motion, and vice versa. Sharp conclude almost the same facts mentioned by Best and other researchers before him, saying that at higher vehicle speeds, Olley tuning is shown to bring advantage in pitch suppression with a very little disadvantage in terms of body acceleration. At lower speeds, he continues, not only does the pitch tuning bring large vertical acceleration penalties but also suspension stiffness implied are impractical from an attitude control standpoint.

The flat ride problem was revisited by Crolla and King [2]. They generated vehicle vibration response spectra under random road excitations. Some results included the wheelbase filter effect, while others did not. Olley and reverse Olley designs were simulated at speeds of 10, 20, 30, and 40 m/s, with the result that Olley design was good in pitch and bad in bounce in all cases. It was confidently concluded that the rear/front stiffness ratio has virtually no effect on overall levels of ride comfort.

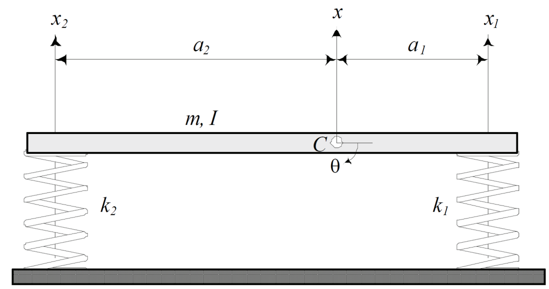

Figure 2. The bicycle model of a car is a beam of mass m and mass moment I, sitting on two springs k_1 and k_2.

In 2004, Odhams and Cebon investigated the tuning of a pitch-plane model of a passenger car with a coupled suspension system and compared it to that of a conventional suspension system, which followed the Rowell and Guest treatment [5]. They believed that there is a significant benefit from coupling front and rear suspensions; coupled suspensions with a "Hydrolastic" or "Hydragas" systems, in which the front and rear suspension struts are connected hydraulically, have proved very effective in some applications. The concluded that the Olley’s flat ride tuning provides a near optimum stiffness choice for conventional suspensions for minimizing dynamic tire forces and is very close to optimal for minimizing horizontal acceleration at the chest (caused by pitching) but not the vertical acceleration.

3 Uncoupling the Car Bicycle Model

Consider the two degree-of-freedom (DOF) system in Figure 2 . A beam with mass m and mass moment I about the mass center C is sitting on two springs k_1 and k_2 to model a car in bounce and pitch motions. The translational coordinate x of C and the rotational coordinate θ are the usual generalized coordinates that we use to measure the kinematics of the beam. The equations of motion and the mode shapes are functions of the chosen coordinates.

The free vibration equations of motion of the system are:

$$\begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & a_2 k_2 - a_1 k_1 \\ a_2 k_2 - a_1 k_1 & a_2^2 k_2 + a_1^2 k_1 \end{bmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix} = 0 \tag{1}$$

To compare the mode shapes of the system practically, we employ the coordinates x_1 and x_2 instead of x and θ. The equations of motion of the system would then be:

$$\begin{bmatrix} \dfrac{m a_2^2 + I}{a_1 + a_2{}^2} & \dfrac{m a_1 a_2 - I}{a_1 + a_2{}^2} \\ \dfrac{m a_1 a_2 - I}{a_1 + a_2{}^2} & \dfrac{m a_1^2 + I}{a_1 + a_2{}^2} \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \tag{2}$$

Let us define the following parameters:

$$I = mr^2 \tag{3}$$

$$\Omega_1^2 = \frac{k_1}{m}\beta \tag{4}$$

$$\Omega_2^2 = \frac{k_2}{m}\beta \tag{5}$$

$$\beta = \frac{l^2}{a_1 a_2} \tag{6}$$

$$\alpha = \frac{r^2}{a_1 a_2} \tag{7}$$

$$\gamma = \frac{a_2}{a_1} \tag{8}$$

$$l = a_1 + a_2 \tag{9}$$

and rewrite the equations as

$$\begin{bmatrix} \alpha+\gamma & 1-\alpha \\ 1-\alpha & \alpha+\frac{1}{\gamma} \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} \Omega_1^2 & 0 \\ 0 & \Omega_2^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \tag{10}$$

Setting

$$\alpha = 1 \tag{11}$$

makes the equations decoupled

$$\begin{bmatrix} \alpha+\gamma & 0 \\ 0 & \alpha+\frac{1}{\gamma} \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} \Omega_1^2 & 0 \\ 0 & \Omega_2^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \tag{12}$$

The natural frequencies ω_i and mode shapes u_i of the system are

$$\omega_1^2 = \frac{1}{\gamma+1}\Omega_1^2 = \frac{l}{a_2}\frac{k_1}{m} \quad u_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{13}$$

$$\omega_2^2 = \frac{\gamma}{\gamma+1}\Omega_2^2 = \frac{l}{a_1}\frac{k_2}{m} \quad u_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{14}$$

They show that the nodes of oscillation in the first and second modes are at the rear and front suspensions respectively.

The decoupling condition $\alpha = 1$ yields

$$r^2 = a_1 a_2 \tag{15}$$

which indicates that the pitch radius of gyration, r, must be equal to the multiplication of the distance of the mass canter C from the front and rear axles. Therefore, by setting $\alpha = 1$, the nodes of the two modes of vibrations appear to be at the front and rear axles. As a result, the front wheel excitation will not alter the body at the rear axle and vice versa. For such a car, the front and rear parts of the car act independently. Therefore, the decoupling condition $\alpha = 1$ allows us to break the initial two DOF system into two independent one DOF systems, where:

$$m_r = m\frac{a_1}{l} = m\varepsilon \tag{16}$$

$$m_f = m\frac{a_2}{l} = m(1-\varepsilon) \tag{17}$$

$$\varepsilon = \frac{a_1}{l} \tag{18}$$

The equations of motion of the independent systems will be:

$$m(1-\varepsilon)\ddot{x}_1 + c_1\dot{x}_1 + k_1 x_1 = k_1 y_1 + c_1\dot{y}_1 \tag{19}$$

$$m\varepsilon\ddot{x}_2 + c_2\dot{x}_2 + k_2 x_2 = k_2 y_2 + c_2\dot{y}_2 \tag{20}$$

The decoupling condition of undamped free system will not necessarily decouple the general damped system. However, if there is no anti-pitch spring or anti-pitch damping between the front and rear suspensions then equations of motion

$$\begin{bmatrix} \alpha+\gamma & 1-\alpha \\ 1-\alpha & \alpha+\frac{1}{\gamma} \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 2\xi_1\Omega_1 & 0 \\ 0 & 2\xi_2\Omega_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} \Omega_1^2 & 0 \\ 0 & \Omega_2^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \\ = \begin{bmatrix} 2\xi_1\Omega_1 & 0 \\ 0 & 2\xi_2\Omega_2 \end{bmatrix} \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} + \begin{bmatrix} \Omega_1^2 & 0 \\ 0 & \Omega_2^2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{21}$$

$$2\xi_1\Omega_1 = \frac{c_1}{m}\beta \tag{22}$$

$$2\xi_2\Omega_2 = \frac{c_2}{m}\beta \tag{23}$$

will be decoupled by $\alpha = 1$.

$$\begin{bmatrix} \alpha+\gamma & 0 \\ 0 & \alpha+\frac{1}{\gamma} \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} \Omega_1^2 & 0 \\ 0 & \Omega_2^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \\ = \begin{bmatrix} 2\xi_1\Omega_1 & 0 \\ 0 & 2\xi_2\Omega_2 \end{bmatrix} \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} + \begin{bmatrix} \Omega_1^2 & 0 \\ 0 & \Omega_2^2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{24}$$

The equations of motion of the independent system may also be written as

$$m(1-\varepsilon)\ddot{x}_1 + c_1\dot{x}_1 + k_1 x_1 = c_1\dot{y}_1 + k_1 y_1 \tag{25}$$

$$m\varepsilon\ddot{x}_2 + c_2\dot{x}_2 + k_2 x_2 = c_2\dot{y}_2 + k_2 y_2 \tag{26}$$

which are consistent with the decoupled equations 36 because of

$$\varepsilon = \frac{1+\gamma}{\gamma\Omega_2^2} \tag{27}$$

4 No Flat Ride Solution for Linear Suspension

The time lag between the front and rear suspension oscillations is a function of the wheelbase, l, and speed of the vehicle, v. Soon after the rear wheels have passed over a step, the vehicle is at the worst condition of pitching. Olley experimentally determined a recommendation for the optimum frequency ratio of the front and rear ends of cars. His suggestion for American cars and roads of 50s was to have the natural frequency of the front approximately 80% of that of the rear suspension.

To examine Olley's experimental recommendation and possibly make an analytical base for flat ride, let us rewrite the equation of motion (25) and (26) as:

$$\ddot{x}_1 + 2\xi_1\dot{x}_1 + \frac{k_1}{m(1-\varepsilon)}x_1 = 2\xi_1\dot{y}_1 + \frac{k_1}{m(1-\varepsilon)}y_1 \tag{28}$$

$$\ddot{x}_2 + 2\xi\xi_1\dot{x}_2 + \frac{kk_1}{m\varepsilon}x_2 = 2\xi\xi_1 y_2 + \frac{kk_1}{m\varepsilon}y_2 \tag{29}$$

where,

$$\xi = \frac{\xi_2}{\xi_1} = \frac{c_1}{c_2}\frac{\varepsilon}{1-\varepsilon} \tag{30}$$

$$k = \frac{k_2}{k_1} = \frac{k_1}{k_2}\frac{\varepsilon}{1-\varepsilon} \tag{31}$$

$$\xi_1 = \frac{c_1}{m(1-\varepsilon)} \tag{32}$$

$$\xi_2 = \frac{c_2}{m\varepsilon} \tag{33}$$

Parameters k and ξ are the ratio of the rear/front spring rates and damping ratios respectively.

The necessity to achieve a flat ride provides that the rear system must oscillate faster to catch up with the front system at a reasonable time. At the time both systems must be at the same amplitude and oscillate together afterwards. Therefore, an ideal flat ride happens if the frequency of the rear system be higher than the front to catch up with the oscillation of the front at a certain time and amplitude. Then, the frequency of the rear must reduce to the value of the front frequency to oscillate in phase with the front. Furthermore, the damping ratio of the rear must also change to keep the same amplitude. Such a dual behavior is not achievable with any linear suspension. Therefore, theoretically, it is impossible to design linear suspensions to provide a flat ride, as the linearity of the front and rear suspensions keep their frequency of oscillation constant.

5 Nonlinear Damper

The force-velocity characteristics of an actual shock absorber can be quite complex. Although we may express the complex behavior using an approximate function, analytic calculation can be quite complicated with little design information. Furthermore, the representations of the exact shock absorber do not greatly affect the behavior of the system. The simplest linear viscous damper model is usually used for linear analytical calculation

$$F_D = cv_D \tag{34}$$

where c is the damping coefficient of the damper.

The bound and rebound forces of the damper are different, in other words the force-velocity characteristics diagram is not symmetric. Practically, a shock absorber compresses much easier than decompression. A reason is that during rebound in which the damper extends back, it uses up the stored energy in the spring. A high compression damping, prevents to have enough spring compression to collect enough potential energy. That is why in order to get a more reliable and close to reality response for analysis on dampers, using bilinear dampers is suggested. It is similar to a linear damper but with different coefficients for the two directions [3].

$$F_D = \begin{cases} c_{DE}v_D & \text{Extension} \\ c_{DC}v_D & \text{Compression} \end{cases} \tag{35}$$

where c_{DE} is the damping coefficient when damper is extended and c_{DC} is the damping coefficient when the damper is compressed.

An ideal dual behavior damper is one which does not provide any damping while being compressed and on the other hand damps the motion while extending.

After using the nonlinear model for the damper, the motion had to be investigated in 3 steps for the front and same for the rear. Ideally, the unit step moves the ground up in no time and therefore the motion of the system begins when $y = 1$ and the suspension is compressed. The first step is right after the wheel hits the step and the damper starts extending. The second step is when the damper starts the compression phase, which the damping coefficient would be equal to zero. The third step is when the damper starts extending again. Each of the equations of motion should be solved for the 3 steps separately in order to find the time and amplitude of the third peak of the motion which have been chosen to be optimal time for the flat ride to happen at.

Figure 3 illustrates the behavior of the car equipped with a nonlinear damper when going over a unit step input.

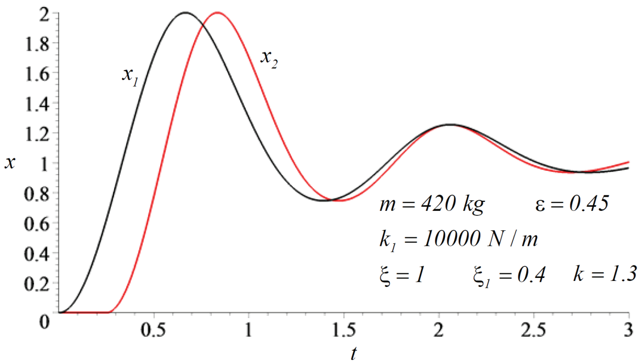

Figure 3. Response of the front and rear suspension of a near flat ride car with ideal nonlinear damper to a unit step input.

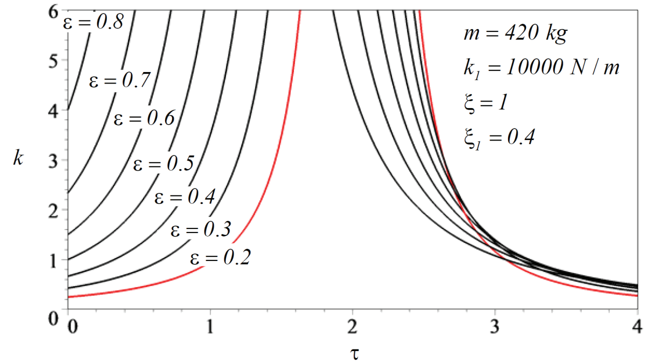

Figure 4. The value of spring ratio $k = k_2/k_1$ versus $\tau = l/v$ to have near flat ride with ideal nonlinear damping, for different $\varepsilon = a_1/l$.

6 Near Flat Ride Solution for Ideal, Nonlinear Damper

The conditions that x_1 and x_2 meet after one and a half oscillations can be shown by equations (36) and (37).

$$x_1 = x_2 \tag{36}$$

$$t_{p1} = t_{p2} \tag{37}$$

The equation resulted from $x_1 = x_2$, (equation 38) has got ξ and ξ_1 as its variables and could be plotted as an explicit function of the variables which interestingly shows that the value for $\xi = \xi_2/\xi_1$ must equal to 1 for any value for damping coefficient of the front suspension ξ_1.

$$\begin{aligned}
EQ1 = {} & \frac{-0.8\times 10^{-18}}{(\zeta_1^2-1)(\zeta_1^2\zeta^2-1)} \\
& \times \Bigg(-0.3172834025\times 10^{18} e^{\frac{-\pi\zeta_1}{\sqrt{1-\zeta_1^2}}} \zeta_1^4\zeta^2 \\
& + 0.3172834025\times 10^{18} e^{\frac{-\pi\zeta_1}{\sqrt{1-\zeta_1^2}}} \zeta_1^2 \\
& + 0.3172834025\times 10^{18} e^{\frac{-\pi\zeta_1}{\sqrt{1-\zeta_1^2}}} \zeta_1^2\zeta^2 \\
& - 0.3172834025\times 10^{18} e^{\frac{-\pi\zeta_1}{\sqrt{1-\zeta_1^2}}} \\
& + 0.130151797\times 10^{9} e^{\frac{-\pi\zeta_1}{\sqrt{1-\zeta_1^2}}} \sqrt{1-\zeta_1^2}\,\zeta_1^3\zeta^2 \\
& - 0.3172834025\times 10^{9} e^{\frac{-\pi\zeta_1}{\sqrt{1-\zeta_1^2}}} \sqrt{1-\zeta_1^2}\,\zeta_1 \\
& + 0.3172834025\times 10^{18} e^{\frac{-\pi\zeta_1\zeta}{\sqrt{1-\zeta_1^2\zeta^2}}} \\
& - 0.3172834025\times 10^{18} e^{\frac{-\pi\zeta_1\zeta}{\sqrt{1-\zeta_1^2\zeta^2}}} \zeta_1^2\zeta^2 \\
& - 0.3172834025\times 10^{18} e^{\frac{-\pi\zeta_1\zeta}{\sqrt{1-\zeta_1^2\zeta^2}}} \zeta_1^2 \\
& + 0.3172834025\times 10^{18} e^{\frac{-\pi\zeta_1\zeta}{\sqrt{1-\zeta_1^2\zeta^2}}} \\
& - 0.130151797\times 10^{9} e^{\frac{-\pi\zeta_1\zeta}{\sqrt{1-\zeta_1^2\zeta^2}}} \sqrt{1-\zeta_1^2\zeta^2}\,\zeta_1^3\zeta \\
& + 0.130151797\times 10^{9} e^{\frac{-\pi\zeta_1\zeta}{\sqrt{1-\zeta_1^2\zeta^2}}} \sqrt{1-\zeta_1^2\zeta^2}\,\zeta_1\zeta \Bigg)
\end{aligned} \tag{38}$$

Therefore, regardless of the value of ξ_1 the rear suspension should have an equal coefficient for the damper. The equation resulted from $t_{P_1} = t_{p_2}$ generates equation 39 to determine $k = k_2/k_1$. Figure 4 illustrates the spring ratio $k = k_2/k_1$ versus $\tau = l/v$, to have near flat ride with ideal nonlinear damping, for different $\varepsilon = a_1/l$.

$$\begin{aligned}
EQ2 = {} & \frac{\pi\sqrt{\frac{-k_1}{m(\varepsilon-1)}}\left(2\sqrt{1-\zeta_1^2}+1-\zeta_1^2\right)m(\varepsilon-1)}{(\zeta_1^2-1)k_1} \\
& -\tau + \frac{\pi\left(2\sqrt{1-\zeta^2\zeta_1^2}+1-\zeta^2\zeta_1^2\right)}{\sqrt{\frac{kk_1}{m\varepsilon}}(\zeta^2\zeta_1^2-1)}
\end{aligned} \tag{39}$$

The average length of a sedan vehicle has been taken equal to 2.6 meters with a normal weight distribution of a front differential vehicle 56/44 heavier at the front. Using the given information some other values can be calculated as: $a_1 = 1144$ mm and $a_2 = 1456$ mm which yields to $\varepsilon = 0.44$. Considering the existing designs for street vehicles, only the small section of $0.1 < \tau < 0.875$ is applied. The mass center of street cars is also limited to $0.4 < \varepsilon < 0.6$.

Figure 5 shows how k varies with τ for $\xi_1 = 0.5$ and different ε to provide a near flat ride with ideal nonlinear damper. For any ε, the required stiffness ratio increases by increasing τ. Therefore, the ratio of rear to front stuffiness increases when the speed of the car decreases. Figures 6 and 9 also provide the same design graphs for $\xi_1 = 0.4$, 0.3, 0.2, 0.1, respectively.

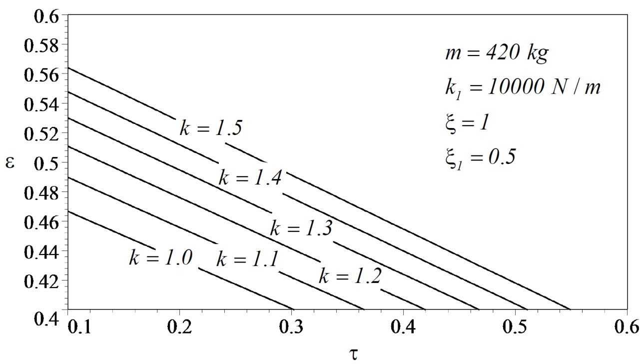

Figure 5. ε versus τ, for different k for $\xi_1 = 0.5$ to have near flat ride with ideal nonlinear damping.

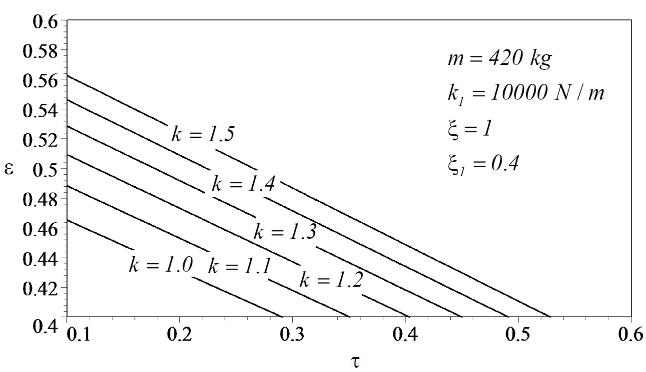

Figure 6. ε versus τ, for different k for $\xi_1 = 0.4$ to have near flat ride with ideal nonlinear damping.

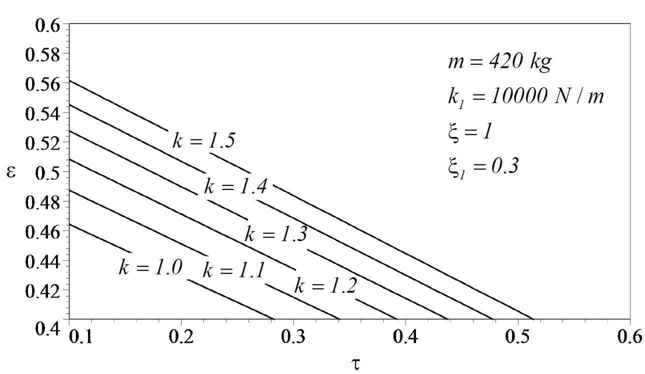

Figure 7. ε versus τ, for different k for $\xi_1 = 0.3$ to have near flat ride with ideal nonlinear damping.

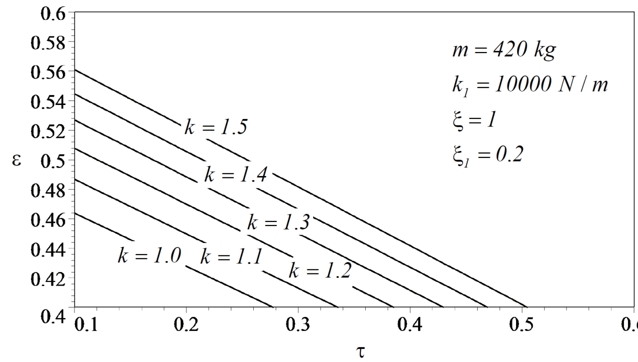

Figure 8. ε versus τ, for different k for $\xi_1 = 0.2$ to have near flat ride with ideal nonlinear damping.

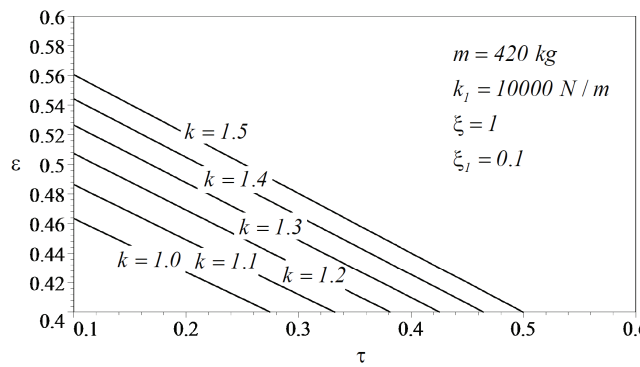

Figure 9. ε versus τ, for different k for $\xi_1 = 0.1$ to have near flat ride with ideal nonlinear damping.

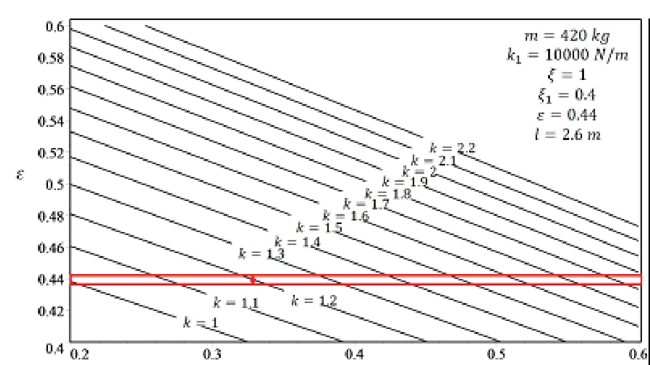

Figure 10. Design chart for a smart suspension with a non-linear damper.

There will be a possibility of using the τ vs. ϵ diagrams as a design chart, which has been illustrated using the values in Table 1, by figure 10.

The box in figure 10, is indicating the values that the spring rate should be having as the travelling speed of the vehicle changes to provide the passengers with a flat ride, in case of having a smart active suspension. The point on the figure is an example for a passive suspension vehicle. It is showing the required spring rate, for getting a flat ride in a car with a wheelbase of 2.6 meters, traveling at 28 km/h.

Table 1. Specification of a sample car.

Specification	Nominal value
m [kg]	420
a_1 [m]	1.4
a_2 [m]	1.47
l [m]	2.87
k_1 [N/m]	10000
k_2 [N/m]	13000
c_1 [Ns/m]	1000
c_2 [Ns/m]	1000
β	4.00238
γ	1.05
Ω_1	95.2947
Ω_2	123.8832
ξ_1	0.05
ξ_2	0.0384

7 Conclusion

Olley's flat ride tuning has been regarded as a rule for designing chassis. The fact that these rules were based on experimental results, motivated many researchers to study and validate these rules. In this study, has been tried to validate Olley's results analytically for the first time.

As a result of the dual behavior of the suspension which is required to get the optimal flat ride, more accurate results were looked for using a nonlinear suspension system for this analysis. The results prove that the forward speed of the vehicle affects the flat ride condition, which agrees with previous researchers' results. In a passive suspension syetem flat ride can be achieved at a certain speed only, so the suspension system of a car should be designed in a way which provides the flat ride at a certain forward speed.

A design chart based on the nonlinear analysis, for smart active suspension systems has been provided which enables a car with smart suspension system to provide flat ride at any forward speed of the vehicle. The design chart can be used for designing chassis with passive suspension for a specified speed as well. examples of both of the above mentioned conditions have been reviewed and discussed, by using some numerical values from a sample car. The research proves the effectiveness of Olley's flat ride for getting a more comfortable ride in cars, and considering the shortcomings of the principles suggests better ways of implementing them to the design of suspension systems for a better and more effective flat ride tuning.

References

[1] A. Best, 'Vehicle ride-stages in comprehension', *Physics in Technology*, vol. 15, no. 4, p. 205, 2002.

[2] D. Crolla, and R. King, 'Olley's" Flat Ride" revisited', 2000.

[3] J. Dixon, The shock absorber handbook, *Wiley*, 2008.

[4] W. F. Milliken, D. L. Milliken, and M. Olley, Chassis design, *Professional Engineering Publ.*, 2002.

[5] A. Odhams, and D. Cebon, 'An analysis of ride coupling in automobile suspensions', *Proceedings of the Institution of Mechanical Engineers*, Part D: Journal of Automobile Engineering, vol. 220, no. 8, pp. 1041-61, 2006.

[6] M. Olley, 'Independent wheel suspension–its whys and wherefores', *Society of Automotive Engineers Journal*, 1934.

[7] M. Olley, 'National influences on American passenger car design', *Proceedings of the Institute of Automobile Engineers*, vol. 32, no. 2, pp. 509-72, 1938.

[8] M. Olley, 'Road manners of the modern car', *Proceedings of the Institute of Automobile Engineers*, vol. 41, no. 1, pp. 147-82, 1946.

[9] H. S. Rowell, and J. J. Guest, Proc. Inst. Automobile Engineers, 18, PP 455, 1923.

[10] R. Sharp, 'Wheelbase filtering and automobile suspension tuning for minimizing motions in pitch', *Proceedings of the Institution of Mechanical Engineers*, Part D: Journal of Automobile Engineering, vol. 216, no. 12, pp. 933-46, 2002.

[11] R. Sharp, and C. Pilbeam, 'Achievability and value of passive suspension design for minimum pitch response', *Vehicle Ride and Handling*, vol. 39, pp. 243-59, 1993.

[12] Dai Liming, Reza N. Jazar, Eds., 'Nonlinear Approaches in Engineering Applications', *Springer, New York, Chapter 1: Smart Flat Ride Tuning* , Web Address: http://www.springer.com/materials/mechanics/book/978-1-4614-6876-9, 2013.