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Heat Transfer Analysis for Couple Stress Fluid over a Nonlinearly Stretching Sheet

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Abstract. An investigation has presented to analyze the heat transfer in a boundary layer flow of an incompressible couple stress fluid over a nonlinear stretching sheet. The heat transfer problem have been analyzed for the two heating processes, 1) PST (prescribed surface temperature case) and 2) PHF (prescribed heat flux case). The governing partial differential equations for flow and heat transfer are transformed to ordinary differential equations. The numerical solutions have been obtained by using fourth order Runge-Kutta method coupled with shooting method. The effects of dimensionless surface temperature, couple stress parameter β, Prandtl number P_r, Eckert number E_c, nonlinear stretching sheet parameter n and λ_1, λ_2 have been observed by graphical illsurations.

Keywords. stretching sheet; couple stress; heat transfer.

1 Introduction

The boundary layer flow over a stretching surface has become a significant area of study due to its applications in industrial and technological processes, mostly in extrusion of metals, polymers, plastic sheets, etc. The linear stretching of a sheet for a Newtonian fluid has first taken into account by Crane [1]. Ming et al. [2] consider heat transfer of a continuous stretching surface with suction or blowing, Dandapat et al. [3] found the exact analytical solution for the flow and heat transfer over a stretching sheet. Andersson [4] investigated magnetohydrodynamic flow of an electrically conducting power-law fluid over a stretching sheet. Bhatnagar et al. [5] examine the flow of an Oldroyd-B fluid due to the stretching of sheet. Shit [6] study the Hall effects on MHD free convective flow and mass transfer over a stretching sheet, Prasad [7] explain the shear-thinning phenomena for magneto hydrodynamic flow of an electrically conducting power law fluid over a vertical stretching sheet. Chen [8] described the effects of viscous dissipation on heat transfer in a non-Newtonian liquid film over an unsteady stretching sheet. Alinejad [9] investigate the flow and heat transfer characteristics of viscous flow over a nonlinearly stretching sheet with the presence of viscous dissipation, Khan et al. [10] presented an analysis of the axisymmetric flow of a non-Newtonian fluid over a radially stretching sheet, Mehmood et al. [11] investigates the theoretical study of steady stagnation point flow with heat transfer of a second grade nano fluid towards a stretching surface, Khan et al. [12] gives an analytical study for the effects of slip factors on unsteady stagnation point flow and heat transfer towards a stretching sheet. Asghar [13] reveals all possible similarity transformations for the flow of power-law fluid over a stretching surface by a Lie group analysis. The rotation of the fluid particles and its effect has studied under the theory of couple stresses which describes the rotation field in specification of the velocity field. The examples of couple stress fluids include blood, rheologically complex fluids, lubricant oil with long chain additives, polymeric suspensions, etc. Stokes [14] considered the effects of couple stresses in fluids. Soundalgekar [15] carried out an analysis for the dispersion of a solute in a channel flow of fluid with couple stresses, Ramanaiah [16] presented the theory of infinitely long slider bearings assuming the lubricant to be an incompressible fluid with couple stress. Chang et al. [17] have examined the bifurcation analysis of flexible rotor set up by couple-stress fluid film bearings with non-linear suspension systems. Manivasakan et al. [18] investigate the couple stress squeeze films in a curved circular geometry. Srinivasacharya et al. [19] analyzed the couple stress fluid in a porous channel with expanding or contracting walls. Srinivasacharya et al. [20] presented the Hall and Ion-slip effects on electrically conducting couple stress fluid flow between vertical parallel plates in the presence of a temperature dependent heat source. Very recently, Sajid et al. [21] and Khan et al. [22] respectively described, the two-dimensional flows of various kinds with couple stresses and the approximate solution of couple stress fluid with expanding or contracting porous channel.

In this paper, the study has presented to analyze the couple stress flow over a nonlinearly stretching sheet. A suitable similarity transformation has used to convert the governing partial differential equations to ordinary differential equations. The numerical result have been presented to examine its effect on the flow and heat transfer with its different parameter considering two cases for it, one with con-

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stant surface temperature and another with prescribed surface temperature. To, best of authors knowledge the influence of heat transfer on couple stress flow over a nonlinear stretching sheet has not been studied before.

2 Flow Analysis

Consider the steady and incompressible flow of a couple stress fluid towards the stretching sheet in the plane $y = 0$, flowing with velocity $u_w\ (x)$. Forces have been applied along the x-axis to stretch the wall keeping the origin fixed. The governing equations for the boundary layer flow can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\ \frac{\partial u}{\partial x} + v\ \frac{\partial u}{\partial y} = \nu\ \frac{\partial^2 u}{\partial y^2}\ - \frac{\xi}{\rho}\ \frac{\partial^4 u}{\partial y^4} \tag{2}$$

Where u and v are the velocity components in the x and y-directions, respectively, ν is the kinematic viscosity and ξ is the coefficient of couple stress. The boundary conditions on the stretching sheet are as follows

$$u_w\ (x) = C\ x^n\ , \quad v = 0 \text{ at } y = 0 \tag{3}$$

$$u \to 0 \text{ as } y \to \infty \tag{4}$$

$$\frac{\partial^2 u}{\partial y^2} = 0 \ \text{ at } y = 0\ u \ \to\ 0\ ,\ \ \frac{\partial u}{\partial \mathrm{y}}\ \to\ 0 \ \text{ as } y \to \infty \tag{5}$$

Introducing the similarity transform as follows:

$$u = C\ x^n\ f'\ \ (\eta)\ ,$$

$$v = -\sqrt{\frac{C\ \ \nu\ \ (n+1)}{2}}\ x^{\frac{n-1}{2}}\ \left(\frac{n-1}{n+1}\ \ \eta\, f'\,(\eta) + f\ (\eta)\right) \tag{6}$$

$$\eta = \sqrt{\frac{C\ \ \ (n+1)}{2\nu}}\, x^{\frac{n-1}{2}}\ y \tag{7}$$

Which satisfies the equation of continuity and reduced Eq. (2) to the following form:

$$f^{\ (5)} - \beta^2 \left(f'''\ + f\ f''\ - \frac{2\,n}{n+1}\ f'^2\right) = 0 \tag{8}$$

subject to the boundary conditions

$$f\ \ (0) = 0\,, \quad f'\ \ (0) = 1, \quad f'''\,(0) = 0\,, \tag{9}$$

$$f'\ \ (\infty) = 0\,, \quad f''\,(\infty) = 0 \tag{10}$$

Where prime denotes differentiation with respect to the independent similarity variable η, n is the nonlinear stretching sheet parameter and $\beta = \frac{2\,x^{1-n}\ \nu^2}{C\ \xi\ (1+n)}$ is the velocity couple stress parameter.

The surface shear stress τ_w is given by:

$$\tau_w = \left(\mu\ \ \left(\frac{\partial u}{\partial y}\right) - \xi\ \left(\frac{\partial^3 u}{\partial y^3}\right)\right)\Bigg|_{y=0} \tag{11}$$

Using similarity transforms from Eqs. (5)-(6) in Eq. (11), we get

$$\tau_w = C\ \sqrt{\frac{C\ (n+1)}{2\upsilon}}\, x^{\frac{3n-1}{2}} \left(\mu\ \ f''\,(0) - \frac{C\ \ \ (1+n)\ \ x^{n-1}\xi}{2\mu\nu}\ \ f''''\ \ (0)\right) \tag{12}$$

The local skin friction coefficient is given by

$$Re_x^{\frac{1}{2}}\, C_f = f''\,(0) - \frac{1}{\beta^2}\, f''''\,(0) \tag{13}$$

where $Re_x = \sqrt{\frac{C\ \ x^{1+n}}{(1+n)\ \ \nu}}$ is the local Reynolds number.

3 Heat Transfer analysis

The energy equation for temperature T with dissipations can be written as:

$$u\frac{\partial T}{\partial x} + \upsilon\frac{\partial T}{\partial \upsilon} = \alpha\frac{\partial^2\ T}{\partial\ y^2} + \frac{\nu}{c_p}\ \left(\frac{\partial u}{\partial y}\right)^2 + \frac{\xi}{c_p}\ \left(\frac{\partial^2\ u}{\partial\ y^2}\right)^2 \tag{14}$$

Where c_p and α are the specific heat of the fluid and thermal conductivity.

3.1 Prescribed surface temperature (PST):

When the stretching sheet is provided with the constant temperature, the boundary conditions are as follows:

$$T = T_w\ \left(= T_\infty + A\,x^k\right) \text{ at } y = 0 : T \to T_\infty \text{ as } y \to \infty. \tag{15}$$

Where k is the surface temperature constant.

The non dimensional temperature $\theta\ \ (\eta)$ defined as

$$\theta\ \ (\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{16}$$

Here, T_w describes the wall temperature and T_∞ describes the temperature far away from the wall, respectively.

By using Eqs. (6))-(7) and (16) in Eq. (14), obtained the equation

$$\theta'' + \Pr\left(f\,\theta' - \left(\frac{2k}{n+1}\right) f'\theta\right) = - P_r E_c \left(f''\right)^2 - E_c \lambda_1 \left(f'''\right)^2 \tag{17}$$

The boundary conditions are

$$\theta = 1 \text{ at } \eta = 0: \quad \theta \to 0 \text{ as } \eta \to \infty \tag{18}$$

Where E_c is the Eckert number, λ_1 is the energy couple stress parameter and P_r is the Prandtl number, which can be defined as follows:

$$E_c = \frac{C^2 x^{2n-k}}{A\ \ c_p},\quad \lambda_1 = \frac{C^3\,(1+n)\,x^{-1+3n-k}\xi}{2A\alpha\upsilon} \text{ and } P_r = \frac{\upsilon}{\alpha} \tag{19}$$

The local surface heat flux can be expressed as

$$q_w = -\lambda_T \left(\frac{\partial T}{\partial y}\right)_w = -\lambda_T\,\frac{A}{\sqrt{2}}\ x^{\frac{2k+n-1}{2}}\,\theta'(0)\,\sqrt{\frac{C\,(n+1)}{2\ \upsilon}} \tag{20}$$

where λ_T is the thermal conductivity.

3.2 Prescribed Heat Flux (PHF):

Here the boundary conditions are

$$-K_1 \frac{\partial T}{\partial y} = q_w = A\,(x)^k \text{ at } y = 0; T \to T_\infty \text{ as } y \to \infty \tag{21}$$

$$T - T_\infty = \frac{A x^k}{K_1} \frac{g\,(\eta)}{\sqrt{\frac{C(n+1)}{2\ \nu}}} \tag{22}$$

using Eqs. (6)-(7) and (22) in Eq. (14), the equation fro the prescribed heat flux (PHF) can be obtained as

$$g'' + P_r\left(f g' - \left(\frac{2k}{n+1}\right) f'\,g\right) = - P_r E'_c x^{2n-k}\left(f''\right)^2 - E'_c\lambda_2\left(f'''\right)^2 \tag{23}$$

with

$$g'\ \ (0) = 1, \quad g\ \ (\infty) = 0 \tag{24}$$

Where $E'_c = \frac{C^2 x^{2n-k} K_1}{A c_p}\sqrt{\frac{C(n+1)}{2\nu}}$, and $\lambda_2 = \frac{x^{-1+n}\xi}{2A\alpha}\frac{C(n+1)}{2\nu}$.

The local surface heat flux can be expressed as

$$q_w = -\lambda_T\left(\frac{\partial T}{\partial y}\right)_w = -\lambda_T\,\frac{A}{K_1} x^{\frac{2k+n-1}{2}} g'(0) \tag{25}$$

4 Numerical Results and Discussions

The results has been carried out for the presented problem by converting the BVP into IVP implementing the shooting technique and solved by using the fourth order Runge-Kutta method. The obtained results have analyzed for various values of pertaining parameters. Fig. 1 (a)-(b) displays the effect of couple stress parameter β on both velocity components u and v, when the non linear parameter $n = 0.5$. Fig. 1 (a) presents the decrease in the velocity component v as the couple stress parameter increases which predicts that the couple stress term does not supports the longitudinal velocity resisting the motion of the fluid. Fig. 1 (b) also show the same trend for the translational velocity u. It also depicts that the boundary layer thickness decreases with the couple stress term.

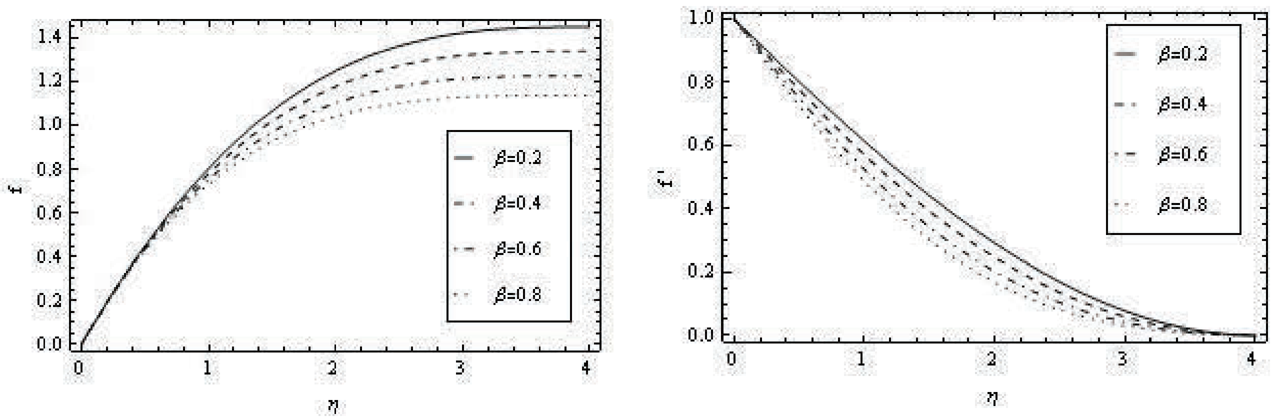
Fig. 2 (a)-(b) presents the behavior of nonlinear stretching parameter n when the couple stress parameter is kept constant at $\beta = 0.4$. It reveals that the longitudinal velocity decreases with the increase in n (Fig. 2 a), while the translational velocity shows the gradual decrease with the increase in $n$decreasing the thickness of the boundary wall (Fig. 2 b).

The energy dissipation is analyzed with the prescribed surface temperature (PST) and prescribed heat flux (PHF) and the influence of various parameters on temperature θ has been observed from Fig. 3 - 8. In these figures, θ and g are the temperature variation that correspond to the PST and PHF cases. An increase in the value of β increases the temperature which is more in PHF case as compare to PST case, also the thermal boundary layer thickness increases by increasing β. (see Figure 3 a-b).

The general effect of Eckert numberEcon both PST and PHF case are increasing but its effect is more clear in PHF case (see Figure 4 a-b). The decreasing trend has been observed on the profiles of θ as the increase in the Prandtal number Pr has applied. It has also discovered that

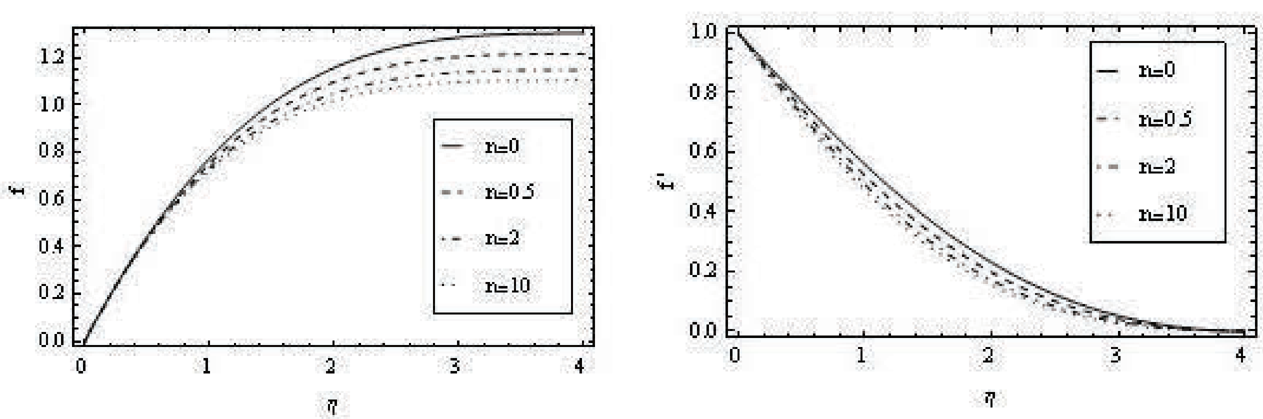
Table 1. Numerical values of $Re_x^{\frac{1}{2}} C_f$.

$n\backslash\beta$	0.3	0.5	0.7	0.9
0	1.169011	0.838708	0.749344	0.712857
0.25	1.432229	1.085753	0.981090	0.932119
0.5	1.603743	1.243044	1.126116	1.067790
0.75	1.724381	1.352201	1.225857	1.160551
2.5	2.077981	1.665982	1.509008	1.422009
5	2.221842	1.791319	1.620962	1.524776
10	2.312309	1.869639	1.690635	1.588582
25	2.374539	1.923260	1.738174	1.632085
50	2.396828	1.942373	1.755130	1.647586
100	2.408321	1.952239	1.763875	1.655544



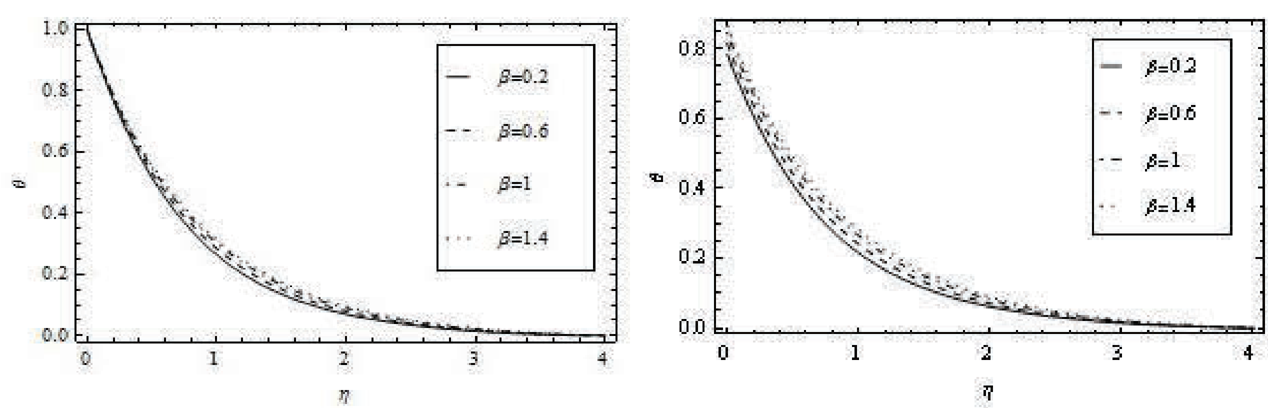
(a) (b)

Figure 1. The influence of couple stress parameter β when $n = 0.5$ on: (a) the stream function $f(\eta)$ and (b) the velocity $f'(\eta)$.



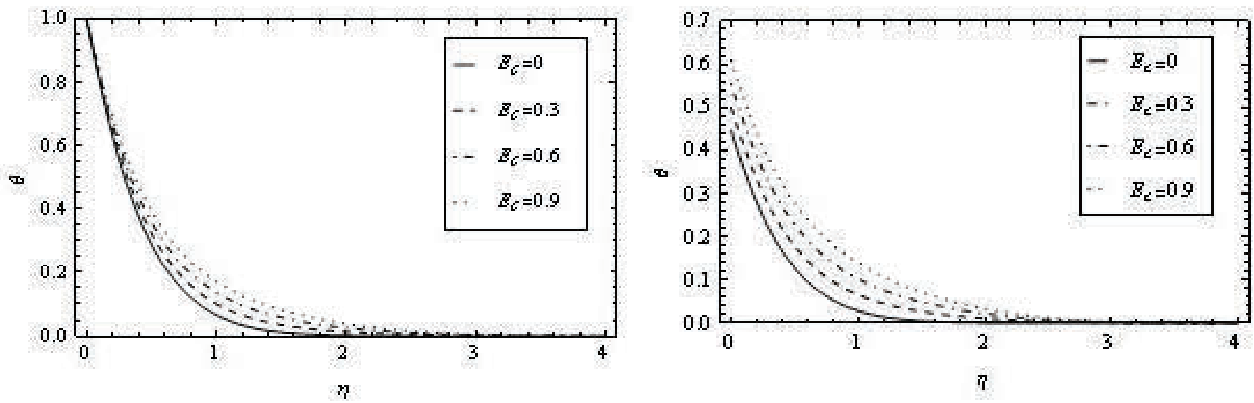
(a) (b)

Figure 2. The influence of non linear stretching sheet parameter n when $\beta = 0.4$ on: (a) the stream function $f(\eta)$ and (b) the velocity $f'(\eta)$.



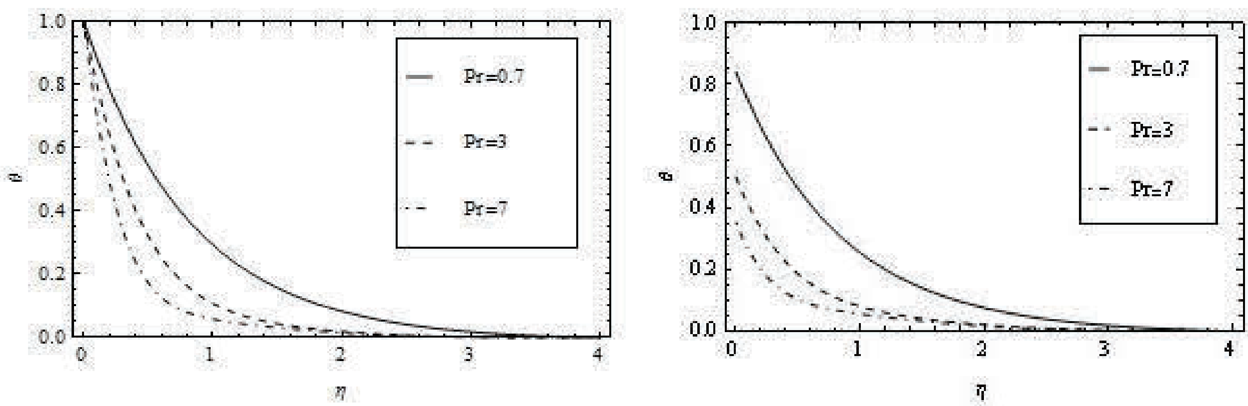
(a) (b)

Figure 3. The effect of couple stress parameter β on Temperature when $n = 0.25$, $E_c = 0.5$, $\lambda_1 = \lambda_2 = 0.5$, $P_r = 1$ and $k = 1$ (a) for PST and (b) PHF cases.



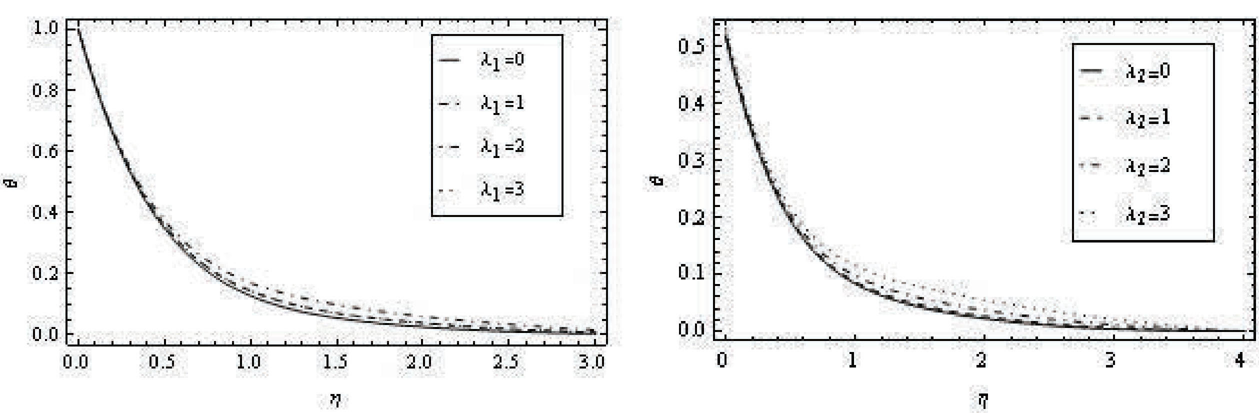
(a) (b)

Figure 4. The effect of Eckert number E_c on temperature when $n = 1.25, \beta = 0.6, \lambda_1 = \lambda_2 = 0.5, P_r = 3$ and $k = 1.5$ (a) for PST and (b) PHF cases.



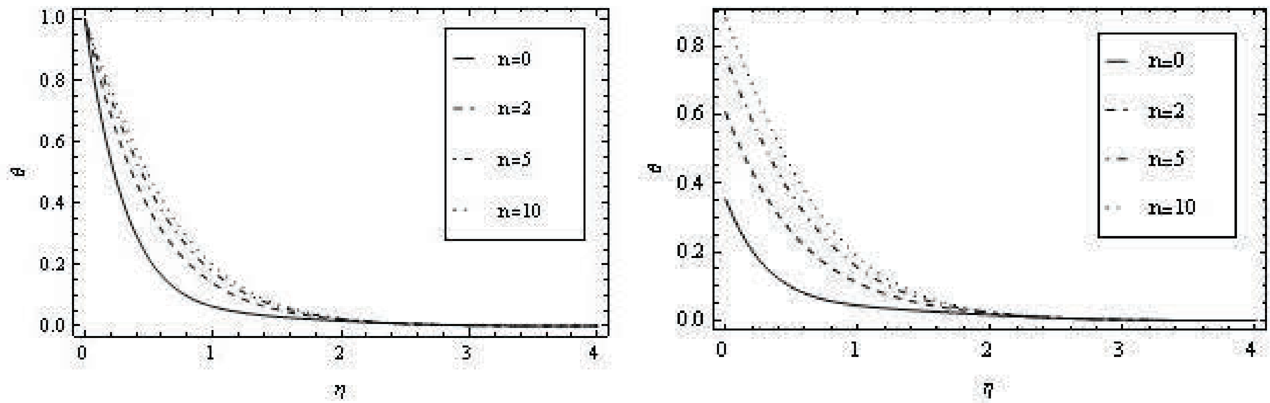
(a) (b)

Figure 5. The effect of Prandtl number P_r on temperature profiles when $n = 1, \beta = 0.5, \lambda_1 = \lambda_2 = 0.5, E_c = 0.5$ and $k = 1.5$ (a) for PST and (b) PHF cases.



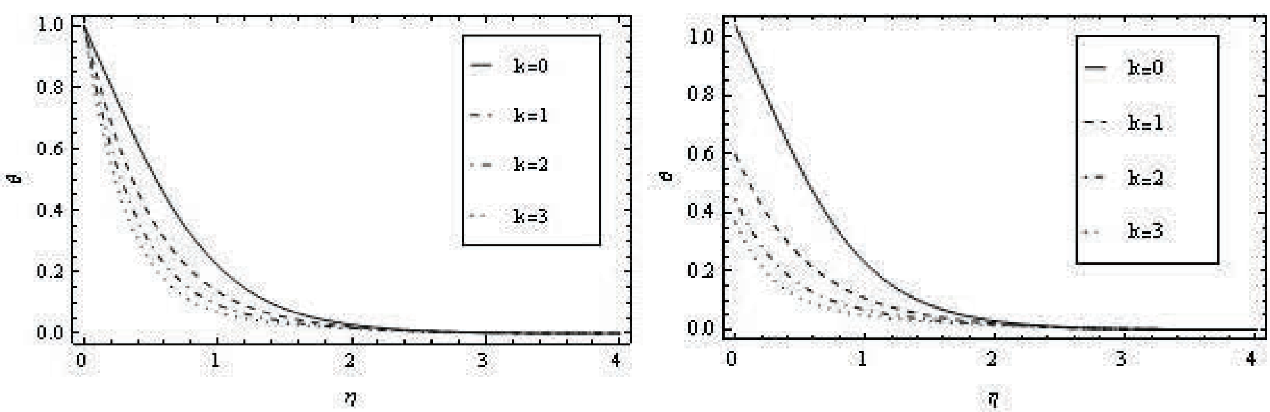
(a) (b)

Figure 6. The effect of couple stress parameter λ_1 and λ_2 on temperature profiles when $n = 1.25, \beta = 0.4, E_c = 0.5, P_r = 3$ and $k = 1.5$ (a) for PST and (b) PHF cases.



(a) (b)

Figure 7. The effect of non linear stretching sheet parameter n on temperature profiles when $E_c = .5, \beta = 0.6, \lambda_1 = \lambda_2 = 0.5$, $P_r = 3$ and $k = 1.5$ (a) for PST and (b) PHF cases.



(a) (b)

Figure 8. The effect of the surface temperature constant k on temperature profiles when $n = 1, \beta = 0.6, \lambda_1 = \lambda_2 = 0.5$, $P_r = 3$ and $E_c = 0.5$ (a) for PST and (b) PHF cases.

Table 2. Numerical values of $-\theta'(0)$.

$n\backslash\beta$	0.1	0.3	0.5	0.7	0.9
0.25	1.296025	1.279787	1.254498	1.227748	1.203783
0.5	1.200358	1.180879	1.151142	1.120283	1.093019
0.75	1.128323	1.106593	1.073854	1.040295	1.010909
2.5	0.889137	0.861276	0.820761	0.780583	0.74625
5	0.776562	0.746661	0.703750	0.661737	0.626186
10	0.699893	0.668971	0.624944	0.582176	0.546208
25	0.644183	0.612727	0.568168	0.525112	0.489054
50	0.623584	0.591977	0.547283	0.504178	0.468130
100	0.612854	0.581179	0.536428	0.493309	0.457278

the boundary layer thickeness has also decreased (see Figure 5 a-b).

The parameters λ_1 (for PST) and λ_2 (for PHF) have a very slight effect on the temperature variation, but when its value rises, the enlargement in the temperature of the fluid has been observed which increases the boundary layer thickness. (see Fig. 6 a-b)

When the non linear stretching has been applied to the fluid different increasing pattern has shown by the temperature profiles for both PST and PHF cases. But both shows that the non linear stretching parameter n increases the temperature of the fluid. (see Fig. 7 a-b)

The positive change in the surface temperature constant k effects the temperature of the fluid negatively, as the increase in $k$decrease the fluid temperature decreasing the boundary layer thickness. (see Fig. 8 a-b)

5 Conclusions

The flow and heat transfer of a couple stress fluid has been investigated. The governing problem has been transformed by employing similarity transformation and numerical solution has been obtained with the help Runge-Kutta method. Effects of various parameter has been observed on velocity and temperature profiles of both PST nd PHF cases. It has been concluded from the graphs that the rise in couple stress parameter increases the temperature of the fluid. Tables have also been made to evaluate the values of skin friction coefficient and temperature gradient.

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References

[1] L.J. Crane, Flow past a stretching plate, ZAMP 1970, 21 (4), 645–7.
[2] M.-I. Char, C.-K. Chen, Temperature field in non-Newtonian flow over a stretching plate with variable heat flux, Int. J. Heat Mass Transfer 1988, 31 (5), 917-21.
[3] B.S. Dandapat, A.S. Gupta, Flow and heat transfer in a viscoelastic fluid over a stretching sheet, Int. J. Non-Linear Mech. 1989, 24 (3), 215-9.
[4] H.I. Andersson, K.H. Bech, B.S. Dandapat, Magnetohydrodynamic flow of a power-law fluid over a stretching sheet, Int. J. Non-Linear Mech. 1992, 27 (5), 929-36.
[5] R.K. Bhatnagar, G. Gupta, K.R. Rajagopal, Flow of an Oldroyd-B fluid due to a stretching sheet in the presence of a free stream velocity, Int. J. Non-Linear Mech. 1995, 30 (3), 391-405.
[6] G.C. Shit, Hall effects on MHD free convective flow and mass transfer over a stretching sheet, Int. J. Appl. Math. Mech. 2009, 5 (7), 22-38.
[7] K.V. Prasad, P.S. Datti, K. Vajravelu, Hydromagnetic flow and heat transfer of a non-Newtonian power law fluid over a vertical stretching sheet. Int. J. Heat Mass Transfer 2010, 53 (5-6), 879-88.
[8] C.-H. Chen, Effect of viscous dissipation on heat transfer in a non-Newtonian liquis film over an unsteady stretching sheet, J. Non-Newtonian Fluid Mech. 2006, 135 (2-3), 128-35.
[9] J. Alinejad, S. Samarbakhsh, Viscous flow over nonlinearly stretching sheet with effects of viscous dissipation, J. Appl. Math. 2012, Art. ID 587834.
[10] M. Khan, A. Shahzad, On axisymmetric flow of Sisko fluid over a radially stretching sheet, Int. J. Non-Linear Mech. 2012, 47 (8), 999-1007.
[11] R. Mehmood, S. Nadeem, N.S. Akbar, Non-orthogonal stagnation point flow of a nano non-Newtonian fluid towards a stretching surface with heat transfer, Int. J. Heat Mass Transfer 2013, 57 (2), 679-89.
[12] N.A. Khan, M. Jamil, A. Khan Nadeem, Effects of slip factors on unsteady stagnation point flow and heat transfer towards a stretching sheet: an analytical study, Heat Transfer Research 2012, 43 (7), 779-94.
[13] S. Asghar, M. Jalil, Flow of power-law fluid over a stretching surface: A Lie group analysis, Int. J. Non-Linear Mech. 2013, 48, 65-71.
[14] V.K. Stokes, Couple Stresses in fluid, Phy. Fluids 1966, 9, 1709–15.
[15] V.M. Soundalgekar, Effects of couple stresses in fluids on dispersion of a solute in a channel flow, Phy Fluids 1971, 14, Art. ID 19.
[16] G. Ramanaiah, P. Sarkar, Slider bearings lubricated by fluids with couple stress, Wear 1979, 52(1), pp. 27-36, 1979.
[17] C.-W. Chang-Jian, C.-K. Chen, Bifurcation analysis of flexible rotor supported by couple-stress fluid film bearings with non-linear suspension systems, Tribology Int. 2008, 41 (5), 367-86.
[18] V. Manivasakan, G. Sumathi, Theoretical investigation of couple stress squeeze films in a curved circular geometry, J. Tribology 2011, 133, Art. ID 041701.
[19] D. Srinivasacharya, N. Srinivasacharyulu, O. Odelu, Flow and heat transfer of couple stress fluid in a porous channel with expanding and contracting walls. Int Comm in Heat and Mass Transfer 2009, 36 (2), 180-5.
[20] D. Srinivasacharya, K. Kaladhar, Mixed convection flow of couple stress fluid between parallel vertical plates with Hall and Ion-slip effects, Commun. Nonlinear Sc. Numer. Simul. 2012, 17 (5), 2447-24.
[21] M. Sajid, N. Ali, Z. Abbas, T. Javed, M. Imran, Some two-dimensional flows with couple stresses, J. Eng. Phy. Thermophy 2012, 85 (3), 649-54.
[22] N.A. Khan, A. Mahmood, A. Ara, Approximate solution of couple stress fluid with expanding or contracting porous channel, Engg. Comput. 2013, 30 (3), 399-408.