

Ashok Kumar*, Pravez Alam, and S. Kapoor

Numerical investigation of mixed convective flow in parallel channel filled with porous medium

Abstract: The present manuscript reports a numerical investigation of fully developed mixed convective flow in a parallel channel filled with porous medium. The motion in the channel is induced by external pressure gradient and buoyancy force. The non-Darcy Brinkman-Forchheimer extended model has been used. The Chebyshev spectral collocation method has been used to solve the coupled differential equations numerically. The main emphasis has been given on the influence of Forchheimer number (F), Darcy number (Da) and Rayleigh number (Ra) on velocity and temperature profiles. From this investigation we have found that the velocity as well as temperature profiles possess point of inflection beyond a threshold value of Ra . The back flow starts near the center of the channel on increasing the value of Ra beyond the threshold value of Ra . The point of inflection on the velocity as well as temperature profile dies out on decreasing of Da . The strength of the velocity decreases on the increasing of F' , it also changes the velocity profile drastically. The point of inflection on the basic flow dies out on increasing of F' .

Keywords: Mixed convection, Porous medium, Non-Darcy model, Chebyshev spectral collocation method

DOI 10.1515/nleng-2014-0016

Received May 1, 2014; accepted October 6, 2014.

1 Introduction

The concept of transport processes through porous media has substantially increased during recent years due to its many practical applications arises in engineering and sciences, such as in geothermal engineering, petroleum in-

dustries, thermal insulation, contaminant transport and marine science etc.

In expressing the equations for the flow in the porous medium, it should be noted that the Darcy model presents a linear relationship between the volume-averaged velocity of discharge and the pressure gradient. As the Darcy model is valid when the flow velocity is sufficiently small and small porosity. However, in numerous practical situations the porous medium is bounded by solid walls, has higher flow rates, and reveals non-uniform porosity distribution in the near wall region, making Darcy's law inapplicable. Extensions to this model known as Brinkman-extended or Forchheimer-extended models exist. In short, the Brinkman term is found to be needed for satisfying a no-slip boundary condition at solid walls, whereas the Forchheimer term accounts for the form drag. Brinkman-extended model is generally applicable for porous media with both low and high permeabilities also in analogy with the Navier-Stokes equations, the Darcy model has been extended by including the material derivative. Convection due to external pressure gradient and buoyancy forces (i.e. mixed convection) in a porous media is a fundamental problem in fluid dynamics and still a field of active and ongoing research. For example, convection in permeable sediment due to hydrothermal vent is a combination of forced and natural convection.

A recent comprehensive literature review on laminar wall bounded forced or mixed convection is amply documented in the book of Nield and Bejan [1]. Here, the literature pertinent to the laminar wall-bounded mixed convection in vertical systems has given. Some studies in wall bounded mixed convection through vertical channels ([2], [3], [4], [5], [6], [7], [6], [8], [9], [10]) pipe ([11]) and annulus ([12], [13]) filled with porous medium are reported.

In the vertical channel, when the walls are heated uniformly, Hadim [2] investigated the evolution of mixed convection in the entrance region for Darcy as well as non-Darcy cases. Hadim along with Chen [3] have studied the extension of the above study for asymmetric heating of the walls. It was found that distortions in the velocity profile lead to increased heat transfer rates when Da is decreased. The same problem with discrete heat sources at the walls, in place of asymmetric wall heating has been investigated

*Corresponding Author: Ashok Kumar: Department of Mathematics, H N B G University Srinagar (Garhwal), Uttarakhand, India, E-mail: ashdsdma@gmail.com

Pravez Alam: Department of Mathematics, H N B G University Srinagar (Garhwal), Uttarakhand, India

S. Kapoor: Department of Education in Science and Mathematics Regional Institute of Education, Bhubneshwar, (NCERT), India, Email: saurabh09.ncert@gmail.com, Tel: +919658236692

by Hadim and Chen [4]. They have found that the location of the flow separation from the cold wall does not change while reattachment moves further downstream. The Nusselt number increases with decreasing Da and the effect of Da is more pronounced over the first heat source and in non-Darcy regimes. Chen et al. [5] have indicated for non-Darcy mixed convection in a vertical channel filled with a porous medium, the buoyancy force can significantly affects on Nu for higher values of Ra and Da and lower values of Forchheimer number.

An extensive study of mixed convective flow and its stability in vertical channel filled with porous medium has been reported elsewhere ([6], [7], [8], [9], [10]). They have shown that the fully developed flow profiles contains point of separation as well as point of inflection and also discuss the stability in which they have given that one dimensional mixed convective flow in the vertical channel does not remain one dimensional always.

More recently, Kumar et al. ([11]) have reported an analytical as well as numerical study of non-Darcy fully developed mixed convection in a vertical pipe filled with porous medium. They have shown that the velocity profile contains a point of inflection in the center and point of separation at the near to the wall. A point of separation causes the back flow near the wall. Further, it is also found that in contrast to buoyancy opposed case, where increment in Ra increases the magnitude of temperature at the center as well as velocity till the distortion appears on the both profiles, in buoyancy assisted case both velocity as well as temperature decrease on increasing Ra at the center. The point of separation as well as inflection are died out on reducing the media permeability and increasing magnitude of the form drag coefficient. Parang and Keyhani [12] studied the fully developed buoyancy-added mixed convective flow in a vertical annulus employing Darcy-Brinkman model. They have concluded that the Brinkman term had a negligible effect for lower Darcy number (Da). According to Muralidhar [13], in the case of buoyancy-assisted mixed convection, the Nusselt number (Nu) increased with increasing Rayleigh number (Ra).

We focus of the present study on the numerical investigation of fully developed mixed convective flow in a parallel channel filled with porous medium by using the spectral collocation methods. The non-Darcy Brinkman-Forchheimer extended model has been used. The main emphasis has been given on the influence of Rayleigh number (Ra), Darcy number (Da) and Forchheimer number (F) on velocity and temperature profiles.

2 Mathematical formulation

We consider a fully developed mixed convection flow caused by an external pressure gradient and a buoyancy force in a parallel channel filled with porous medium. The wall temperature is linearly varying with y as $T_w = T_0 + CLy$, where C is a positive constant and T_0 is upstream reference wall temperature and $2L$ is width of the channel. The gravitational force is aligned in the negative x^* -direction. As shown schematically in figure In the formulation of porous media equations, the following assumptions are made.

- The convective fluid and porous media are everywhere in local thermal equilibrium.
- The thermo-physical properties of the fluid are assumed to be constant except for the density dependence of the buoyancy term in the momentum equation.
- The porous medium is saturated with a fluid that is in local thermodynamic equilibrium with the solid matrix. The medium is assumed to be isotropic in permeability.

Using non-dimensional quantities

$$X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{u}{U_0}, V = \frac{v}{U_0}, P = \frac{p}{\rho U_0^2}, t = \frac{t' U_0}{L}, \\ \Theta = \frac{T - T_w}{CLRePr},$$

the governing differential equations can be written as:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0; \quad (1)$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} + F(\sqrt{U^2 + V^2})U = -\frac{\partial P}{\partial X} \\ + \frac{1}{Re} [\nabla^2 U] - \frac{1}{ReDa} U; \quad (2)$$

$$\frac{\partial U}{\partial t} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} + F(\sqrt{U^2 + V^2})V = -\frac{\partial P}{\partial Y} + \frac{1}{Re} [\nabla^2 V] \\ - \frac{1}{ReDa} V + \frac{Ra}{Re} \Theta; \quad (3)$$

$$\frac{\partial \Theta}{\partial t} + U \frac{\partial \Theta}{\partial X} + V \frac{\partial \Theta}{\partial Y} = \frac{1}{RePr} \left(\frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} - U \right); \quad (4)$$

In this problem non-dimensional governing parameters are Rayleigh number (Ra), Darcy number (Da), Prandtl number (Pr), Reynolds number (Re) and Forchheimer number (F), which are defined by $\frac{g\beta_T CL^4}{\nu\alpha}$, $\frac{K}{L^2}$, ν/α , $\frac{U_0 CL}{\nu}$ and $\frac{Lc_F}{K^{1/2}}$ respectively.

Where X and Y are the streamwise and normal coordinates respectively; U and V are the streamwise and normal velocity respectively; β_T and α are the volumetric thermal expansion coefficient and thermal diffusivity respectively; t is the time; P is the pressure; L is the length of channel; c_F is the form drag coefficient; K is the permeability parameter and U_0 is the characteristic fluid velocity.

Since the main objective of this paper is to understand the flow dynamics of the steady, unidirectional fully developed flow, therefore, the above governing differential equations are reduced into the following set of coupled differential equations.

$$\frac{d^2 U}{dY^2} - \frac{1}{Da} U - Ra\theta - Re \frac{dP}{dY} - F |U| U = 0 \quad (5)$$

$$\frac{d^2 \theta}{dY^2} = U. \quad (6)$$

The corresponding boundary conditions are given by:

$$\frac{dU}{dY} = \frac{d\theta}{dY} = 0 \quad \text{at} \quad Y = 0, \quad (7)$$

$$U = \theta = 0 \quad \text{at} \quad Y = 1, \quad (8)$$

with U and θ being the basic velocity and temperature, respectively. In order to solve the above equations numerically, Eqs. (5) and (6) are divided by the constant term ($Re \frac{dP}{dY}$). As a consequence of it, the above equations are modified by

$$\frac{d^2 U^*}{dY^2} - \frac{1}{Da} U^* - Ra\theta^* - F' |U^*| U^* = 1 \quad (9)$$

$$\frac{d^2 \theta^*}{dY^2} = U^*. \quad (10)$$

in which, $U^* = \frac{U}{(Re \frac{dP}{dY})}$, $\theta^* = \frac{\theta}{(Re \frac{dP}{dY})}$ and $F' = Re^2 F \frac{dP}{dY}$ are the modified basic velocity, temperature and modified Forchheimer number respectively. The corresponding boundary condition will remain same. Above coupled differential equation are solved using the Chebyshev spectral collocation method.

2.1 Chebyshev spectral collocation method

In spectral method is a higher order weighted residual method [14]. The key element of the method of weighted residual method are the trial functions (also basis or approximating functions) and the weight functions (also called the test functions). The trial functions are used as

the basis functions of the truncated series of expansion of the solution, which produces the residual, when it is substituted into the differential equations. The test functions are used to ensure that the differential equation is satisfied as closely as possible by the truncated series expansion. The error of the differential equation produced by the truncated expansion instead of the exact solution with respect to a suitable norm. The choice of the test function distinguishes between the most commonly used spectral schemes, the Galerkin, collocation and tau. In the Galerkin technique the test functions are the same as the basis functions, whereas, in the collocation approach the test functions are Dirac delta functions centered at collocation points. In the collocation approach the differential equation satisfies exactly at the collocation points. Spectral tau methods are close to Galerkin methods, but they differ in imposing the boundary conditions.

In the spectral methods the trial functions are global in nature. So it is usually applied to regular geometry problems with smooth solutions. this method we approximate the unknown function by Chebyshev polynomials. The Chebyshev polynomials distribute the error evenly, exhibit the exponential convergence rate with increasing number of terms and makes the cluster of collocation points at the boundaries of $[-1, 1]$. The collocation points are selected to be the extrema of N th order Chebyshev polynomials.

The k th Chebyshev polynomial, $T_k(\xi)$, is define on interval $[-1, 1]$ by

$$T_k(\xi) = \cos k\theta \quad (11)$$

If $\xi = \cos(\theta)$, Then $T_k(\xi) = \cos(k\theta)$. The collocation points which are the extrema of the last retained Chebyshev polynomial $T_N(\xi)$ in the truncated series are defined by

$$\xi_j = \cos\left(\frac{\pi j}{N}\right), \quad j = 0, 1, \dots, N \quad (12)$$

An interpolant polynomial is constructed in terms of the flow variable at the collocation points by using a truncated Chebyshev series. For example, the unknown function $f(\xi)$ and its derivatives are approximated in terms of the values of that function at the collocation points and N^{th} order polynomial approximation of $f(\xi)$ is given by

$$f(\xi) = \sum_{j=0}^N g_j(\xi) f(\xi_j), \quad (13)$$

where the interpolant $g_j(\xi)$ is

$$g_j(\xi) = \frac{(-1)^{j+1} (1 - \xi^2) T'_N(\xi)}{N^2 c_j (\xi - \xi_j)}, \quad (14)$$

with, $c_0 = c_N = 2$ and $c_j = 1$ for $1 \leq j \leq N - 1$.

The interpolants also satisfy the cardinality condition

$$g_j(\xi_k) = \delta_{jk} \quad (15)$$

where δ_{jk} is Kroneker delta. The first and second derivatives are obtained explicitly by differentiating equation (6). The derivatives at ξ_j are written as:

$$\frac{df}{d\xi} \Big|_j = \sum_{k=0}^N A_{jk} f_k, \quad j = 0, 1, \dots, N \quad (16)$$

$$\frac{d^2f}{d\xi^2} \Big|_j = \sum_{k=0}^N B_{jk} f_k, \quad j = 0, 1, \dots, N \quad (17)$$

Where f_k is simply $f(\xi_j)$, A_{jk} and B_{jk} is defined as

$$A_{jk} = \begin{cases} \frac{c_j(-1)^{k+j}}{c_k(\xi_j - \xi_k)} & j \neq k, \\ \frac{2(1-\xi_j^2)}{2N^2+1} & 1 \leq j = k \leq N-1, \\ \frac{6}{-2N^2+1} & j = k = N. \end{cases}$$

and

$$B_{jk} = A_{jm} \cdot A_{mk}.$$

Similarly one can obtained the higher order derivatives of the unknown field variable. To approximate the field variables, U^* and Θ^* by Chebyshev polynomials, the range, $[0, 1]$, of the independent variable, Y , is mapped into $[-1, 1]$ by using the transformation $\xi = 1 - 2Y$. The governing equations (9)-(10) and the corresponding boundary conditions in terms of Chebyshev variable ξ are:

$$4. \frac{d^2 U^*}{d\xi^2} - \frac{1}{Da} U^* - Ra \Theta^* - F' \Big| U^* \Big| U^* = 1 \quad (18)$$

$$4. \frac{d^2 \Theta^*}{d\xi^2} = U^*. \quad (19)$$

with the following boundary conditions:

$$\frac{dU^*}{d\xi} = \frac{d\Theta^*}{d\xi} = 0 \quad \text{at} \quad \xi = 1,$$

$$U^* = \Theta^* = 0 \quad \text{at} \quad \xi = -1.$$

The discretized governing equations (18)-(19) in terms of Chebyshev variable ξ are:

$$4 \sum_{k=0}^N B_{jk} U_k^* - \frac{1}{Da} U_j^* - Ra \Theta_j^* - F' \Big| U_j^* \Big| U_j^* = 1 \quad (20)$$

$$4. \sum_{k=0}^N B_{jk} \Theta_k^* = U_j^*. \quad (21)$$

With the help of boundary conditions the above equations form the following system:

$$\mathbf{AX} = \mathbf{b} \quad (22)$$

Where $\mathbf{A}$, $\mathbf{X}$ and $\mathbf{b}$ are matrices of order $(2N+2) \times (2N+2)$, $(2N+2) \times 1$ and $(2N+2) \times 1$ respectively. The system of linear equations (22) was solved using the MATLAB command $\mathbf{A} \setminus \mathbf{b}$.

Table 1. Dependence of the velocity and temperature at $Y = 0$, on the order of approximation polynomials at $Da = 10^{-2}$, $F' = 10^2$ and $Ra = 10^5$.

N(terms)	$U(Y=0)$	$\theta(Y=0)$
10	0.000345928846	0.000008718084
20	0.000741803488	0.000007253026
30	0.000741787346	0.000007252927
40	0.000741783963	0.000007252909
50	0.000741779332	0.000007252921
60	0.000741780250	0.000007252919
70	0.000741780406	0.000007252922
80	0.000741781143	0.000007252922

Table 2. Comparison between analytical and numerical velocity U at $Y = 0$ for different Da when $F' = 0$ and $Ra = 0$.

Da	Analytic Solution	Numerical Solution
10^8	0.499999997719	0.499999996961
0.1	0.091549297730	0.091549297730
0.01	0.009999092001	0.009999092001
0.001	0.001000000000	0.001000000000
0.0001	0.000100000000	0.000100000000
0.00001	0.000010000000	0.000010000000
0.000001	0.000001000000	0.000001000000

2.2 Validation of numerical solution

The validation of our numerical simulation has been shown in two ways. First, the independence of the solution was examined for variation of collocation points by changing order of polynomials. Second, results of our special case are compared with the published one. Table 1, illustrates the grid independence of the numerical solution, we have taken different order of the Chebyshev polynomials, ranging from 10 to 80, in spectral approximation to find the basic velocity at $Y = 0$ when $Da = 10^{-2}$, $Ra = 10^5$, and $F' = 10^2$. It can be seen that by changing the order of polynomial from 10 to 30 basic velocity as well as temperature change from 7 decimal onwards. As the number of the terms increases, the results remain consistent and the accuracy improved up to 9-digit. The comparison with the analytic solution was depicted in Table II as a special case when $F' = 0$, $Da = 10^{12}$ and $Ra = 0$. In this case the governing differential equation will be $\frac{d^2 U^*}{dY^2} - \frac{1}{Da} U^* = 1$ and its solution will be $U^*(Y) = Da \cdot \left[\frac{1 - \cosh(Y/\sqrt{Da})}{\cosh(1/\sqrt{Da})} \right]$. As can be seen from Table 2, that the solution generated by the spectral-collocation method is excellent agreement with the analytic solution.

3 Results and discussions

In this section a rigorous study is made to find the dependence of basic velocity and temperature on different controlling parameters. The heat source intensity in the channel filled with a porous medium is governed by Rayleigh number (Ra), whereas, media permeability and form drag are controlled by Darcy number (Da), and modified Forcheimer numbers (F') respectively. Here we have plotted the basic velocity and temperature in terms of U/U_A and Θ/Θ_A respectively, where U_A and Θ_A are average velocity and temperature respectively.

The influence of Rayleigh number (Ra) on basic velocity as well as temperature is plotted in Fig. 2 (a)-(b) at $Da = 10^{-2}$ and $F' = 1$. It can be seen from Fig. 2(a)-(b) that the basic velocity profile possess point of inflection for $Ra \geq 100$. For $Ra(< 100)$ flow is nearly flat in most of the domain of flow except near the wall of the channel. In case of isothermal i.e. $Ra \rightarrow 0$, the buoyancy force has no effect on the flow but in the case of non-isothermal fluid near the wall moves faster due to heating and the fluid motion near the center of the pipe is retarded in order to satisfy the global mass conservation imposed by some external pressure gradient. The strength of velocity (i.e. magnitude of the maximum velocity) of the fluid at the center point decreases on the increasing of Ra and it increases on the increasing of Ra at the vicinity of the wall. The back flow (i.e. negative velocity) starts near the center of the channel on increasing the value of Ra beyond its threshold value, which is consequence of enhancement of buoyancy force. The point of inflection on the velocity profile moves from center to wall of the channel on enhancing Rayleigh number (Ra). However, from the Fig. 2(b) the corresponding parabolic temperature profile becomes flat at the center zone of the channel when Ra is increased and it also shows that the maximum magnitude of the temperature decreases at the center of channel on the increasing of Ra .

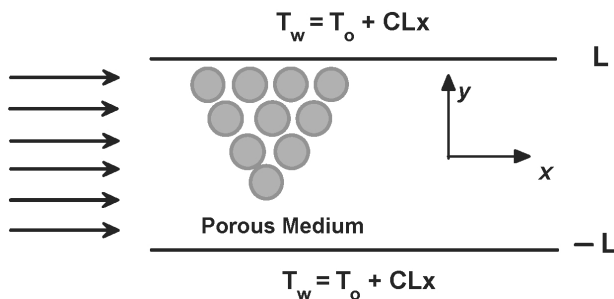


Fig. 1. Schematic of the dimensional physical problem considered

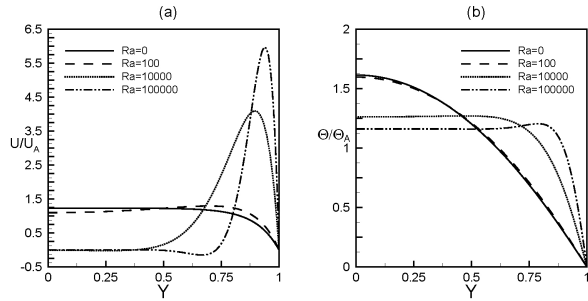


Fig. 2. The influence of Rayleigh number (Ra) on (a) velocity and (b) temperature, when $Da = 10^{-2}$ and $F' = 1$.

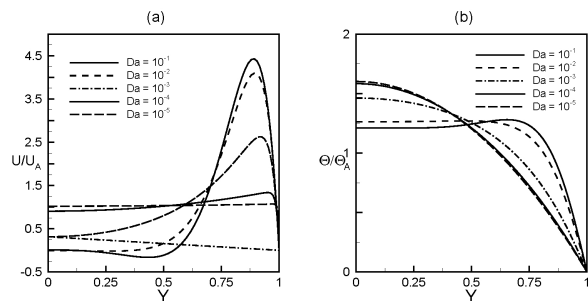


Fig. 3. The influence of Darcy number (Da) on (a) velocity and (b) temperature, when $Ra = 10^4$ and $F' = 1$.

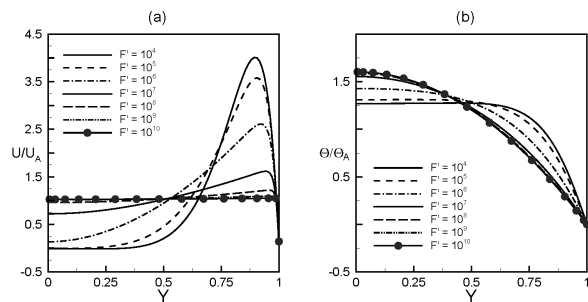


Fig. 4. The influence of modified Forcheimer number (F') on (a) velocity and (b) temperature, when $Ra = 10^4$ and $Da = 10^{-2}$.

The influence of media permeability via Darcy number (Da) on the velocity as well as on temperature is depicted in Fig. 3(a)-(b) when $Ra = 10^4$ and $F' = 1$. From Fig. 3(a)-(b) it can be observed that the strength of the velocity as well as temperature reduces on varying Da from 10^{-1} to 10^{-5} . The point of inflection on the velocity as well as temperature

profile dies out on decreasing of Da . Since the value of Da decreases, the penetrating capability of a porous medium diminishes and induces more drag force on fluid. As a consequence of it above result is expected. Furthermore, on decreasing of the media permeability, by varying Darcy number from 10^{-1} to 10^{-8} , it disappears the back flow near the center of the channel and also decreases the magnitude of the velocity significantly.

In order to understand the effect of inertia on the basic flow, both velocity as well as temperature as function of form drag i.e. F' , is plotted in Fig. 4(a)-(b) when $Ra = 10^4$ and $Da = 10^{-2}$. Since form drag in the medium reduces the flow, as a result, the strength of the velocity is expected to decrease on the increasing of F' , which is shown in Fig 4(a). It also changes the velocity profile drastically.

4 Conclusions

An attempt has been made to investigate the dynamics of mixed convective flow in parallel channel by using the Chebyshev spectral collocation method. Its dependency on porous medium parameters, where flow is induced by external pressure gradient and buoyancy force. The wall temperature of the parallel channel varies linearly with streamwise coordinate. We have adopted the non Darcy Brinkmann-Forchheimer-Wooding extended model. The following conclusions can be drawn from this study.

- The velocity as well as temperature profiles possess point of inflection beyond a threshold value of Ra .
- The back flow starts near the center of the channel on increasing the value of Ra beyond the threshold value of Ra .
- The point of inflection on the velocity as well as temperature profile dies out on decreasing of Da .
- The strength of the velocity is decreases on the increasing of F' , it also changes the velocity profile drastically.
- The point of inflection on the basic flow dies out on increasing of F' .

Acknowledgement: One of the author, Ashok Kumar is grateful to University Grant Commission (UGC) India for providing financial support under BSR startup grant (No. F.30-57/2014(BSR)) during the preparation of this manuscript.

References

- [1] D.A. Nield, A. Bejan, Convection in Porous Media. Springer-Verlag, 2006.
- [2] A. Hadim, Numerical study of non-Darcy mixed convection in a vertical porous channel. *Journal of Thermophysics and Heat Transfer* 8 (1994) 371-373.
- [3] A. Hadim, G. Chen, Non-Darcy mixed convection in a vertical porous channel with asymmetric wall heating. *Journal of Thermophysics and Heat Transfer* 8 (1994) 805-808.
- [4] A. Hadim, G. Chen, Non-Darcy mixed convection in a vertical porous channel with discrete heat sources at the walls. *International Communications in Heat and Mass Transfer* 21 (1994) 377-387.
- [5] Y.C. Chen, J.N. Chung, C.S. Wu, Y.F. Lue, Non-Darcy flow stability of mixed convection in a vertical channel filled with a porous medium. *International Journal of Heat and Mass Transfer* 43 (2000), 2421-2429.
- [6] Y.C. Chen, Non-Darcy flow stability of mixed convection in a vertical channel filled with a porous medium, *International Journal of Heat and Mass Transfer* 47 (2004) 1257-1266.
- [7] P. Bera and A. Khalili, Stability of mixed convection in an anisotropic porous channel, *Physics of Fluids* 14 (2002) 1617-1630.
- [8] P. Bera and A. Khalili, Influence of Prandtl number on stability of mixed convective flow in a vertical channel filled with a porous medium, *Physics of Fluids* 18 (2006) 124103-1-10.
- [9] P. Bera and A. Khalili, Stability of Buoyancy Opposed Mixed Convection in a Vertical Channel and its Dependency on Permeability, *Advances in Water Resources* 30 (2007) 2296-2308.
- [10] J. Kumar, P. Bera and A. Khalili, Influence of inertia and drag terms on the stability of mixed convection in a vertical porous-medium channel. *International Journal of Heat and Mass Transfer* 53 (2010) 5261-5273.
- [11] A. Kumar, P. Bera and J. Kumar, Non-Darcy mixed convection in a vertical pipe filled with porous medium, *Int. J. Thermal Sciences*, 50 (2011) 725-735.
- [12] M. Parang, M. Keyhani, Boundary effects in laminar mixed convection flow through an annular porous medium. *Journal of Heat Transfer* 109 (1987) 1039-1041.
- [13] M. Murlidhari, Mixed convection flow in a saturated porous annulus. *International Journal of Heat and Mass Transfer* 32 (1989) 881-888.
- [14] C. Canuto, M.Y. Hussaine, A. Quarteroni, T.A. Zang, *Spectral Method in Fluid Dynamics*. Springer-Verlag, New York Berlin Heidelberg, 1988.