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Numerical Solution of a Plane Jet Impingement on an Infinite Flat Surface

Abstract: In this paper numerical solution of the unsteady plane incompressible viscous jet impinging on to an infinite flat surface are presented for Re=450. In the present study, all calculations have been done by using Dufort Frankel scheme and over relaxation scheme. Result and graphs have been obtained by using MATLAB programming. The obtained results explain the flow of water after exhaling from nozzle and the streamlines and vorticity of flow of water after striking with flat infinite surface. The solutions obtained by proposed method indicate that this approach is easy to implement and computationally very attractive and the results of our investigation are in qualitative agreement with those available in the literature [1, 9]. This method is capable of greatly reducing the size of calculations while still maintaining high accuracy of the numerical solution.

Keywords: jet impingement; heat transfer; streamlines and vorticity; Dufort Frankel Scheme

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NOMENCLATURE

K = heat conductivity
ρ = density of fluid
μ= dynamic viscosity
$C_p\mu$ = viscous diffusion rate
ψ = stream function
ω = vorticity function
X_i = free jet thickness at a distance
ν = Kinematic viscosity $\nu = (\mu/\rho)$
R_e = Reynolds no.
P_r = Prandlt no.
L = characteristic linear dimension
T_i = ambient temperature
T_m = max temp values of the velocity on the axis
V_m = max values of the velocity on the axis

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1 Introduction

When the development of a free jet flow is interrupted by the presence of a surface, an impinging jet is created. Impinging jets are characterized by a rapid deceleration of the discharged fluid as it reaches the surface, which results in an exchange of momentum between the fluid and the impingement surface that leads to high rates of heat and mass transfer. The impingement of a jet on a solid surface is of interest in many practical problems, such as paint sprays, curtain coating in paper industries, shielded-arc welding and jet blast, particularly in connection with current schemes for vertical take-off aircrafts. Also with its effectiveness of localized heating or cooling, jet impingement has been widely applied in cooling of electronic package, cooling of turbine blade, drying of textile, and many other engineering areas. The regimes for which these base flow characteristics have to be determined are the cases of conventional subsonic and supersonic jet impingement on the flat surface. From a numerical modelling perspective, the study of impinging jets can be incorporated into the development of turbulence models as most are tested on flows, which are parallel to the wall and are, therefore not equipped to deal with flows on which the streamlines change orientation and become computational models have been made, they are being held back the lack of detailed experimental data, as a large quantity of the research on impinging jets is still directed toward understanding the heat transfer characteristics of impinging jets at the high Reynolds numbers, because they lead to the highest rates of heat transfer.

The flow field of an impinging jet is typically characterized by three distinct regions 1) the potential region 2) the impinging decaying or decelerating region 3) the wall jet region. The flow characteristics for each of these regions are clearly distinct from each other. There is a considerable literature available concerning the flow in the potential jet region and wall jet region while the impingement region is not yet completely investigated. Experimental and analytical data from literature has also been included in comparison. Some experimental results for the impingement region were obtained but they were not verified by theoretical investigations [1–4]. Numerical solution was obtained to the problem for inviscid fluid, impinging normally on to a plate by power series expansion [5].Some interesting numerical solutions were obtained for a steady, plane incompressible jet using finite-difference technique, leaving the heat transfer problem untouched [6]. The heat transfer of swirling and conventional CO2-air submerged jet impingement was experimentally studied with thermochromic crystal liquid technique [7].The heat transfer characteristics of a planar free water jet normally or obliquely impinging onto a flat substrate were investigated experimentally by Ibuki et al. [8]. Numerical study of normal impingement of a plane jet on a flat surface was proposed in [9–11]. Michael obtained results of jet impingement heat transfer at high Reynolds number and large temperature difference [10]. Recently, a systematic study of the effect of the Reynolds number on the fluid dynamics and turbulence statistics of pulsed jets impinging on a flat surface is presented in [14].

The key parameters determining the heat transfer characteristics of a single impinging jet are the Re, Pr and jet diameter. Nozzle geometry can also have a significant influence on heat transfer. Numerous studies have been conducted to investigate the influence of each of these parameters. For a constant jet diameter, heat transfer increases with increasing Reynolds number. For a constant Reynolds number, decreasing the jet diameter yields higher stagnation and average heat transfer coefficients.

The objective of present study is to investigate the heat transfer characteristics of a planar water jet impinging normally of obliquely on an infinite flat surface. The impingement region has been divided into mesh with equal step size which has been solved using Matlab programming with the help of boundary conditions to obtain results. This paper is written to understand the hydrodynamics and heat transfer of the impingement process, particularly the complexities attributable to the asymmetric geometry of an oblique liquid plane jet. However, unsteady problem with heat transfer has been studied here in impingent region using Dufort Frankel scheme [14] and over relaxation techniques with Matlab programming.

Fig. 1 shows a schematic diagram of a planar impinging jet. The liquid jet issues from a slot nozzle into the air, and then impinges on the infinite flat surface. The heat transfer characteristics of the interaction between the jet and the flat surface depend on the liquid flow rate and the impingement angle.

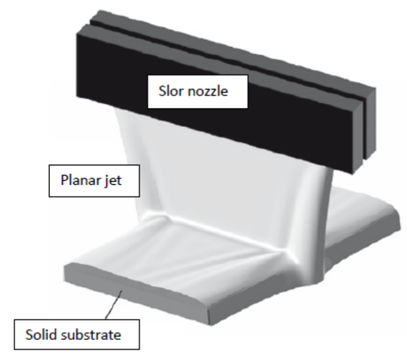


Fig. 1. Schematic diagram of a planar jet impinging on a solid surface.

2 Mathematical study

Consider the problem of two-dimensional viscous Incompressible impinging Jet upon a flat surface. The governing parameters are:

$$\omega = \frac{\partial u_2}{\partial X} - \frac{\partial u_1}{\partial Y} \ , \quad Re = \frac{vL}{v} \ ,$$
$$\mathrm{Pr} = \frac{C_p \mu}{k} \ , \quad \theta = \frac{T - T_w}{T_e - T_w}$$

$C_p\mu$ is viscous diffusion rate T_e and v are also assumed as constant. Stream function $\psi\psi$ is connected with the vorticity function ω *by* the Equation

$$\omega = -\left[\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2}\right] \tag{1}$$

2.1 Boundary conditions

Fig. 2 describes the jet impingement system. We define the boundaries as

$$Y_0 = 1 \ , \quad X_0 = X_i \tag{2}$$

Where X_i is the free jet thickness at a distance from the nozzle equal to $(Y - Y_0)$

From Schauer and Eustis's data (1963) on free jets, X_i is given by

$$X_i = 0.22(Y - Y_0) \tag{3}$$

Thus boundary values of impinging region are:

$$0 \le Y \le Y_0 = 1 \ , \ 0 \le X \le X_0 = 2.X_i = 0.44(Y - Y_0) \tag{4}$$

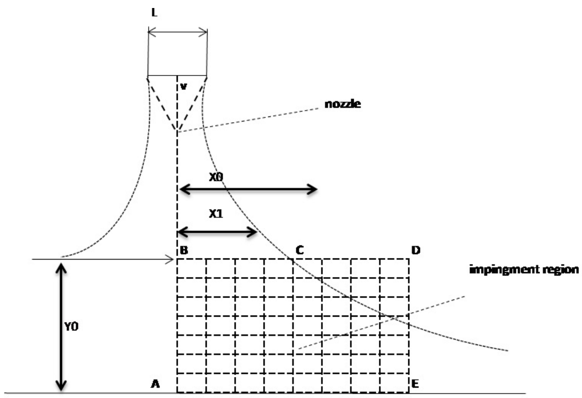


Fig. 2. Diagram of Impinged jet on Flat Surface.

The boundary conditions have been defined for the dynamical and heat transfer Problems separately for a hot jet impinging on a flat surface.

2.2 Dynamical Problem

$$\text{On } AB: \psi = 0\,,\ \omega = 0\,,\ \frac{\partial \psi}{\partial Y} = 0 \text{ at } t \geq 0$$
$$\text{On } AE: \psi = 0\,,\ \omega = 0 \quad \psi = 0, \text{ at } t = 0$$
$$\psi = 0\,,\ \omega_0 = \nabla^2 \psi \text{ at } t > 0$$
$$\text{On } BC: \frac{\partial \psi}{\partial X} - \frac{\partial \omega}{\partial X} = 0 \text{ at } t \geq 0,$$
$$\text{On } CD: \frac{\partial \psi}{\partial Y} = 0 \ ,\ \omega = 0 \text{ for } t \geq 0$$
$$\text{On } BC: \psi = -\int V_\infty dX\ ,\ \omega = \frac{\partial V_\infty}{\partial x}$$
$$\psi = -\int V_\infty dX, \text{ for all } t \geq 0 \tag{5}$$

Let V_∞ be determined from the well-known known Schlichting's profile for a free jet

Thus

$$\frac{V_\infty}{V_{\infty m}} = \left[1 - \left(\frac{X}{X_i}\right)^{\frac{3}{2}}\right]^2 \tag{6}$$

Where $V_{\infty m}$ is the maximum velocity on the jet axis given by

$$V_{\infty m} = \frac{2.35}{(Y - Y_0)^{\frac{1}{2}}} \tag{7}$$

From equation (6) $V_\infty = V_{\infty m}\left[1 - \left(\frac{X}{X_i}\right)^{\frac{3}{2}}\right]^2$

From equation (5), (6) and (7),

$$\psi = -V_{\infty m} \int \left[1 - \left(\frac{X}{X_i}\right)^{\frac{3}{2}}\right]^2 dX$$

$$\psi = -V_{\infty m} \int \left[1 - \left(\frac{X}{X_i}\right)^3 - 2\left(\frac{X}{X_i}\right)^{\frac{3}{2}}\right] dX \quad ,$$

$$\psi = -V_{\infty m} \left[X + \frac{1}{4}\left(\frac{X}{X_i}\right)^4 \frac{1}{X_i} - 2 \times \frac{2}{5}\left(\frac{X}{X_i}\right)^{\frac{5}{2}} \frac{1}{X_i}\right]$$

$$\psi = -V_{\infty m} X_i \left[\frac{X}{X_i} + 0.25\left(\frac{X}{X_i}\right)^4 \frac{1}{X_i} - 0.8\left(\frac{X}{X_i}\right)^{\frac{5}{2}}\right] \tag{8}$$

$$\omega = \frac{\partial V_\infty}{\partial X} = \frac{\partial}{\partial X} V_{\infty m}\left[1 - \left(\frac{X}{X_i}\right)^{\frac{3}{2}}\right]^2$$

$$\omega = V_{\infty m} 2\left[1 - \left(\frac{X}{X_i}\right)^{\frac{3}{2}}\right]\left[-\frac{3}{2}\left(\frac{X}{X_i}\right)^{\frac{1}{2}} \frac{1}{X_i}\right]$$

$$\omega = \frac{-3V_{\infty m}}{X_i}\left[\left(\frac{X}{X_i}\right)^{0.5} - \left(\frac{X}{X_i}\right)^2\right]$$

$$\omega = \frac{3V_{\infty m}}{X_i}\left[\left(\frac{X}{X_i}\right)^2 - \left(\frac{X}{X_i}\right)^{0.5}\right] \tag{9}$$

2.3 Heat Transfer Problem

For a free two-dimensional jet we have [2]

$$\frac{T - T_i}{T_m - T_i} = \left(\frac{V}{V_m}\right)^{\frac{1}{2}} \tag{10}$$

Where T_i is the ambient temperature, T_m and V_m are the maximum values of the Temperature and velocity on the axis. T_m is a function of the distance from the exit section of the nozzle and the exit temperature. Hence equation (6)

$$\frac{T_\infty - T_i}{T_{\infty m} - T_i} = \left[1 - \left(\frac{X}{X_i}\right)^{\frac{3}{2}}\right] \tag{11}$$

Following [2], we have

$$T_{\infty m} - T_i = \frac{2.02(T_e - T_i)}{(Y - Y_0)^{\frac{1}{2}}} \tag{12}$$

Thus the new variable ϑ at $Y = Y_0$ is denoted by ϑ_∞ with the help of equations (11) and (12) is given by

$$\vartheta_\infty = \frac{T_\infty - T_w}{T_e - T_w} = \frac{T_\infty - T_i}{T_e - T_w} + \frac{T_i - T_w}{T_e - T_w} \tag{13}$$

From (11), (12) and (13) [2, 9]

$$T_\infty - T_i = (T_{\infty m} - T_i)\left[1 - \left(\frac{X}{X_i}\right)^{\frac{3}{2}}\right]$$

$$T_\infty - T_i = \frac{2.02(T_e - T_a)}{(Y - Y_0)^{\frac{1}{2}}}\left[1 - \left(\frac{X}{X_i}\right)^{\frac{3}{2}}\right]$$

$$\vartheta_\infty = \frac{T_\infty - T_w}{T_e - T_w} = \frac{T_e - T_i}{T_e - T_w}\frac{2.02}{(Y - Y_0)^{\frac{1}{2}}}\left[1 - \left(\frac{X}{X_i}\right)^{\frac{3}{2}}\right] + \frac{T_i - T_w}{T_e - T_w}$$

for $0 \le X \le X_i$

and

$$\vartheta_\infty = \frac{T_i - T_w}{T_e - T_w}\vartheta_\infty = \frac{\mathrm{T_i - T_w}}{\mathrm{T_e - T_w}} \quad \text{for } X > X_i \tag{14}$$

Wall temperature has been determined from the consideration of the heat balance on the wall surface and it varies from surface to surface. Thus, for a hot jet and a cold plate; $T_i = T_w$.

Boundary conditions for heat transfer problem are:

$$\begin{aligned} &\text{On } BC: \vartheta_\infty = \frac{2.02(T_e - T_i)}{(Y - Y_0)^{\frac{1}{2}}}\left[1 - \left(\frac{X}{X_i}\right)^{\frac{3}{2}}\right] \quad 0 \le X \le X_i \\ &\text{On } CD: \vartheta_\infty = 0 \text{ for } t \ge 0 \\ &\text{On } AE: T = T_w, \ \vartheta = \vartheta_\infty = 0 \text{ for } t \ge 0 \\ &\text{On } AB: \text{ Assume } \frac{\partial \vartheta}{\partial X} = 0 \text{ for all } t \ge 0 \end{aligned} \tag{15}$$

In present study, Dufort Frankel numerical technique [14] has been used by dividing the region ABCDEA into meshes.

$$\psi_{i,j}(t) = \left[\frac{(\psi_{i-1,j} + \psi_{i+1,j})\Delta Y^2 + (\psi_{i,j-1} + \psi_{i,j+1})\Delta X^2 + \omega_{i,j}\Delta X^2 \Delta Y^2}{2(\Delta X^2 + \Delta Y^2)}\right]\psi_{\mathrm{i,j}}(\mathrm{t}) \tag{16}$$

vorticity transport equation is :

$$\left(\frac{\partial\omega}{\partial t} + u\frac{\partial\omega}{\partial X} + \upsilon\frac{\partial\omega}{\partial Y}\right) = \frac{1}{Re}\left(\frac{\partial^2\omega}{\partial X^2} + \frac{\partial^2\omega}{\partial Y^2}\right)$$

In above equation both time and space derivatives are replaced with their central difference approximation:

$$\frac{\partial\omega}{\partial t} = \frac{\omega_{i,j}^{n+1} - \omega_{i,j}^{n-1}}{2\Delta t}, \ \frac{\partial\omega}{\partial X} = \frac{\omega_{i,j}^{n+1} - \omega_{i,j}^{n-1}}{2\Delta X}, \ \frac{\partial\omega}{\partial Y} = \frac{\omega_{i,j}^{n+1} - \omega_{i,j}^{n-1}}{2\Delta Y}$$

$$u_1 = \frac{\partial\psi}{\partial Y} = \frac{\psi_{i,j}^{n+1} - \psi_{i,j}^{n-1}}{2\Delta Y}, \ u_2 = -\frac{\partial\psi}{\partial X} = \frac{-\psi_{i,j}^{n+1} + \psi_{i,j}^{n-1}}{2\Delta X}$$

From vorticity transport equation

$$\begin{aligned} &\frac{\omega_{i,j}^{n+1} - \omega_{i,j}^{n-1}}{2\Delta t} + \left(\frac{\psi_{i,j}^{n+1} - \psi_{i,j}^{n-1}}{2\Delta Y}\right)\left(\frac{\omega_{i,j}^{n+1} - \omega_{i,j}^{n-1}}{2\Delta X}\right) + \left(\frac{-\omega_{i,j}^{n+1} + \psi_{i,j}^{n-1}}{2\Delta X}\right)\left(\frac{\omega_{i,j}^{n+1} - \omega_{i,j}^{n-1}}{2\Delta Y}\right) = \\ &\frac{1}{Re}\left(\frac{\omega_{i-1,j}^n - 2\omega_{i,j}^n + \omega_{i+1,j}^n}{\Delta X^2} + \frac{\omega_{i,j-1}^n - 2\omega_{i,j}^n + \omega_{i,j+1}^n}{\Delta X^2}\right) = \\ &\frac{1}{Re}\left(\frac{\omega_{i-1,j}^n - (\omega_{i,j}^{n+1} + \omega_{i,j}^{n-1}) + \omega_{i+1,j}^n}{\Delta X^2} + \frac{\omega_{i,j-1}^n - (\omega_{i,j}^{n+1} + \omega_{i,j}^{n-1}) + \omega_{i,j+1}^n}{\Delta Y^2}\right) \end{aligned}$$

$$\omega_{i,j}^{n+1}\left[\frac{1}{Re} + \frac{1}{\Delta X^2} + \frac{1}{\Delta Y^2}\right] = \omega_{i,j}^{n-1}\left[\frac{1}{Re} + \frac{1}{\Delta X^2} + \frac{1}{\Delta Y^2}\right] + \frac{Re}{\Delta X \Delta Y}\left[\psi_{i,j}^n\omega_{i,j}^n - \psi_{i,j}^n\omega_{i,j}^n\right] + \frac{(\omega_{i+1,j}^n + \omega_{i-1,j}^n)}{\Delta X^2} + \frac{(\omega_{i,j-1}^n + \omega_{i\backslash,j+1}^n)}{\Delta Y^2}$$

Using finite difference scheme

$$\omega_{i,j}^{n+1}\left[\frac{1}{Re}+\frac{1}{\Delta X^2}+\frac{1}{\Delta Y^2}\right]=\omega_{i,j}^{n-1}\left[\frac{1}{Re}+\frac{1}{\Delta X^2}+\frac{1}{\Delta Y^2}\right]+\frac{Re}{4\Delta X\Delta Y}\left[(\psi_{i+1,j}^{n}-\psi_{i-1,j}^{n})(\omega_{i,j+1}^{n}-\omega_{i,j-1}^{n})\right.$$
$$\left.-(\psi_{i,j+1j}^{n}-\psi_{i,j-1}^{n})(\omega_{i+1,j}^{n}-\omega_{i-1,j}^{n}\right]+\frac{(\omega_{i+1,j}^{n}+\omega_{i-1,j}^{n})}{\Delta X^2}+\frac{(\omega_{i,j-1}^{n}+\omega_{i\backslash,j+1}^{n})}{\Delta Y^2} \tag{17}$$

The Energy equation is:

$$\left(\frac{\partial\vartheta}{\partial t}+u_1\frac{\partial\vartheta}{\partial X}+u_2\frac{\partial\vartheta}{\partial Y}\right)=\frac{1}{Re\,\mathrm{Pr}}\left(\frac{\partial^2\omega}{\partial X^2}+\frac{\partial^2\omega}{\partial Y^2}\right)$$

In above equation both time and space derivatives are replaced with their central difference approximation:

$$\left(\frac{\partial\vartheta}{\partial t}=\frac{\vartheta_{i,j}^{n+1}-\vartheta_{i,j}^{n-1}}{2\Delta t},\ \frac{\partial\vartheta}{\partial X}=\frac{\vartheta_{i,j}^{n+1}-\vartheta_{i,j}^{n-1}}{2\Delta X},\ \frac{\partial\vartheta}{\partial Y}=\frac{\vartheta_{i,j}^{n+1}-\vartheta_{i,j}^{n-1}}{2\Delta Y}\right)$$

$$u_1=\frac{\partial\psi}{\partial Y}=\frac{\psi_{i,j}^{n+1}-\psi_{i,j}^{n-1}}{2\Delta Y},\ u_2=-\frac{\partial\psi}{\partial X}=\frac{-\psi_{i,j}^{n+1}+\psi_{i,j}^{n-1}}{2\Delta X}$$

From energy equation:

$$\frac{\vartheta_{i,j}^{n+1}-\vartheta_{i,j}^{n-1}}{2\Delta t}+\left(\frac{\psi_{i,j}^{n+1}-\psi_{i,j}^{n-1}}{2\Delta Y}\right)\left(\frac{\vartheta_{i,j}^{n+1}-\vartheta_{i,j}^{n-1}}{2\Delta X}\right)+\left(\frac{-\psi_{i,j}^{n+1}+\psi_{i,j}^{n-1}}{2\Delta X}\right)\left(\frac{\vartheta_{i,j}^{n+1}-\vartheta_{i,j}^{n-1}}{2\Delta Y}\right)=$$
$$\frac{1}{Re}\left(\frac{\vartheta_{i-1,j}^{n}-2\vartheta_{i,j}^{n}+\vartheta_{i+1,j}^{n}}{\Delta X^2}+\frac{\vartheta_{i,j-1}^{n}-2\vartheta_{i,j}^{n}+\vartheta_{i,j+1}^{n}}{\Delta X^2}\right)=$$
$$\frac{1}{Re}\left(\frac{\vartheta_{i-1,j}^{n}-\left(\vartheta_{i,j}^{n+1}+\vartheta_{i,j}^{n-1}\right)+\vartheta_{i+1,j}^{n}}{\Delta X^2}+\frac{\vartheta_{i,j-1}^{n}-\left(\vartheta_{i,j}^{n+1}+\vartheta_{i,j}^{n-1}\right)+\vartheta_{i,j+1}^{n}}{\Delta Y^2}\right)$$

Therefore

$$\vartheta_{i,j}^{n+1}\left[\frac{1}{Re}+\frac{1}{\Delta X^2}+\frac{1}{\Delta Y^2}\right]=\vartheta_{i,j}^{n-1}\left[\frac{1}{Re}+\frac{1}{\Delta X^2}+\frac{1}{\Delta Y^2}\right]+\frac{Re}{\Delta X\Delta Y}\left[\psi_{i,j}^{n}\vartheta_{i,j}^{n}-\psi_{i,j}^{n}\vartheta_{i,j}^{n}\right]+\frac{\vartheta_{i+1,j}^{n}+\vartheta_{i-1,j}^{n}}{\Delta X^2}+\frac{\vartheta_{i,j-1}^{n}+\vartheta_{i,j+1}^{n}}{\Delta Y^2}$$

Using finite difference scheme

$$\vartheta_{i,j}^{n+1}\left[\frac{1}{Re}+\frac{1}{\Delta X^2}+\frac{1}{\Delta Y^2}\right]=\vartheta_{i,j}^{n-1}\left[\frac{1}{Re}+\frac{1}{\Delta X^2}+\frac{1}{\Delta Y^2}\right]+\frac{Re}{4\Delta X\Delta Y}\left[(\psi_{i+1,j}^{n}-\psi_{i-1,j}^{n})\left(\vartheta_{i,j+1}^{n}-\vartheta_{i,j-1}^{n}\right)-\left(\psi_{i,j+1}^{n}-\psi_{i,j-1}^{n}\right)\left(\vartheta_{i+1,j}^{n}-\vartheta_{i-1,j}^{n}\right)\right]$$
$$+\frac{\vartheta_{i+1,j}^{n}+\vartheta_{i-1,j}^{n}}{\Delta X^2}+\frac{\vartheta_{i,j-1}^{n}+\vartheta_{i,j+1}^{n}}{\Delta Y^2} \tag{18}$$

Here, the subscripts i and j correspond to the x and y coordinates, while the Superscript$n$indicates the time index and not a power. This discretization scheme has been discussed in Dufort frankel method [14].

3 Results

Since flow starts from rest, the initial values of ω, $\vartheta\omega$ and ψ have been set to zero at all mesh points except on BC. Where ω, $\vartheta\omega$ and ψ are computed using above discussed equations

$\psi=-V_{\infty m}X_i\left[\frac{X}{X_i}+0.25\left(\frac{X}{X_i}\right)^4-0.8\left(\frac{X}{X_i}\right)^{\frac{5}{2}}\right]$, $\omega=\frac{3V_{\infty m}}{X_i}\left(\frac{X}{X_i}\right)^2-\left(\frac{X}{X_i}\right)^{0.5}$

$V_{\infty m}=\frac{2.35}{(Y-Y_0)^{\frac{1}{2}}}$, Take $(Y-Y_0)=16$

$V_{\infty m}=\frac{2.35}{(16)^{\frac{1}{2}}}=0.5875$, $X_i=0.22(Y-Y_0)$, $X_i=0.22(16)=3$

Then

$\psi=-0.5875\times 3.52\left[\frac{X}{3.52}+0.25\left(\frac{X}{3.52}\right)^4-0.8\left(\frac{X}{3.52}\right)^{\frac{5}{2}}\right]$

$\psi = -0.5873X + 1.2863X^{2.5} - 0.002068X^{4}$
$\omega = \frac{3V_{\infty m}}{X_i}[\frac{X}{X_i}^{2} - \frac{X}{X_i}^{0.5}]$, $\omega = \frac{3\times 0.5875}{3.52}[(\frac{X}{3.52})^{2} - (\frac{X}{3.52})^{0.5}]$
$\omega = 0.04035X^{2} - 0.2665X^{0.5}$
From equation (15) θ = constant.

$$\text{On BC}\vartheta_{\infty} = \frac{2.02(T_e - T_i)}{(Y - Y_0)^{\frac{1}{2}}}\left[1 - \left(\frac{X}{X_i}\right)^{\frac{3}{2}}\right], \ \vartheta_{\infty} = \frac{2.02.}{(13)^{\frac{1}{2}}}\left[1 - \left(\frac{X}{3.52}\right)^{\frac{3}{2}}\right]$$

$$\vartheta_{\infty} = 0.505X - 0.332X^{1.5}$$

4 Calculations

Equation (16) has been solved by using over-relaxation method to obtain improved stream function values at all interior points.

$$\psi_{i,j}(t) = \left[\frac{(\psi_{i-1,j} + \psi_{i+1,j})\Delta Y^{2} + (\psi_{i,j-1} + \psi_{i,j+1})\Delta X^{2} + \Omega_{i,j}\Delta X^{2}\Delta Y^{2}}{2(\Delta X^{2} + \Delta Y^{2})}\right]$$

Over relaxation factor:

$$\psi_{i,j}^{k+1} = W\psi_{i,j}^{\star} + (1 - W)\psi_{i,j}^{k}$$

Take $\psi_{i,j}^{\star} = \psi_{i,j}(t)\psi_{i,j}^{\star}=\psi_{i,j}$ (t) in equation (16)

$$\psi_{i,j}^{k+1} = W\left[\frac{(\psi_{i-1,j} + \psi_{i+1,j})\Delta Y^{2} + (\psi_{i,j-1} + \psi_{i,j+1})\Delta X^{2} + \Omega_{i,j}\Delta X^{2}\Delta Y^{2}}{2(\Delta X^{2} + \Delta Y^{2})}\right] + \psi_{i,j}^{k} - W\psi_{i,j}^{k}$$

$$\psi_{i,j}^{k+1} = \psi_{i,j}^{k} + W\left[\frac{(\psi_{i-1,j} + \psi_{i+1,j})\Delta Y^{2} + (\psi_{i,j-1} + \psi_{i,j+1})\Delta X^{2} + \Omega_{i,j}\Delta X^{2}\Delta Y^{2}}{2(\Delta X^{2} + \Delta Y^{2})} - \psi_{i,j}^{k}\right] \tag{19}$$

Here W is the relaxation factor and superscripts (k) and (k+1) indicate the values at the (k) th and (k+1)th iteration respectively; superscripts * indicates the most recent corrected value. After some preliminary tests by using below equation, value of W has been fixed at 1.32.

$$W_{opt} = \frac{4}{2 + \sqrt{4 - C^{2}}} \tag{20}$$

Where $C = Cos\frac{\pi}{p} + Cos\frac{\pi}{q}$.

Where p= mesh division along x-axis and q= mesh division along y-axis.

So p=10 ($\Delta x = 0.5$) and q=5 ($\Delta y = 0.2$).

1. The non-dimensional time has been advanced by Δt and at this new time t= Δt, the vorticity ω is computed at all interior points using equation (18).stream function ψ has been computed at different time steps using equation (17) and (20).
2. Steps (ii) to (iv) has been repeated. These time steps have been further repeated at latter times.
3. All above procedure has been done using programming language MATLAB. In which Re=450, $\Delta t = 0.02$, $\Delta x = 0.5$, $\Delta y = 0.2$

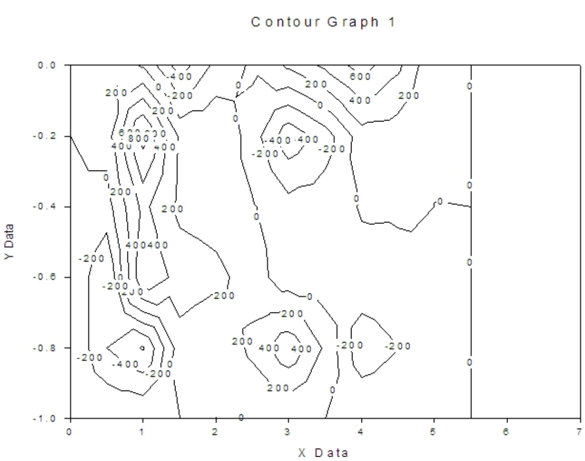
Fig. 3 illustrates the geometry and boundary conditions of impinged jet on flat surface. Graph computes the streamlines in the impingement region at t=6.0.these values have been calculated by doing programming in MATLAB.

Graphical results of impinging jet problem has been discussed by using Reynolds number Re=450, $\Delta t = 0.02$, $\Delta x = 0.5$, $\Delta y = 0.2$, $(Y - Y_0) = 16$.

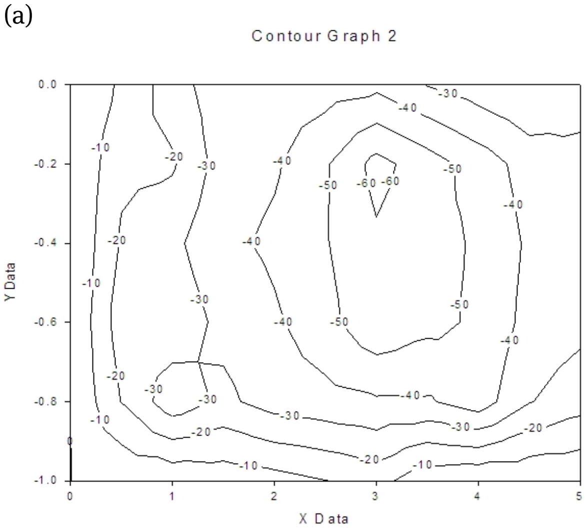
In Fig. 3, Reynolds number Re=450, $\Delta t = 0.02$, $\Delta x = 0.5$, $\Delta y = 0.2$ $(Y - Y_0) = 16$, L=1.0 (Diameter of nozzle).

5 Conclusion

Graphs has been obtained by doing Programming in MATLAB by using *Dufort Frankel scheme* [14] and over relaxation scheme. In this paper Re=450, $\Delta t = 0.02$, $\Delta x = 0.5$, $\Delta y = 0.2$, $(Y - Y_0) = 16$, L =1.0 which provides us approximate results but near to given problems required results. Results explains us the flow of water after exhaling from nozzle. It explains us the streamlines and vorticity of flow of water after striking with flat infinite surface. This method is capable of greatly reducing the size of calculations while still maintaining high accuracy of the numerical solution. The implementation of current approach in analogy to existed methods is more convenient and the accuracy is high.



(a)



(b)

Fig. 3. (a) Contour Graph 1; (b) Contour Graph 2.

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